Computer algebra independent integration tests

Summer 2022 edition

3-Logarithms/58-3.1.5-u-a+b-log-c-x^n-^p

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 3:09am

Contents

1	**Introduction**	**3**
2	**detailed summary tables of results**	**19**
3	**Listing of integrals**	**85**
4	**Appendix**	**1275**

Chapter 1

Introduction

Local contents

1.1 Listing of CAS systems tested 4
1.2 Results 5
1.3 Time and leaf size Performance 9
1.4 list of integrals that has no closed form antiderivative 11
1.5 List of integrals solved by CAS but has no known antiderivative 12
1.6 list of integrals solved by CAS but failed verification 13
1.7 Timing 13
1.8 Verification 14
1.9 Important notes about some of the results 14
1.10 Design of the test system 17

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [249]. This is test number [58].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (249)	0.00 (0)
Mathematica	97.59 (243)	2.41 (6)
Maple	48.19 (120)	51.81 (129)
Fricas	36.14 (90)	63.86 (159)
Maxima	27.31 (68)	72.69 (181)
Giac	24.10 (60)	75.90 (189)
Mupad	18.47 (46)	81.53 (203)
Sympy	18.47 (46)	81.53 (203)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

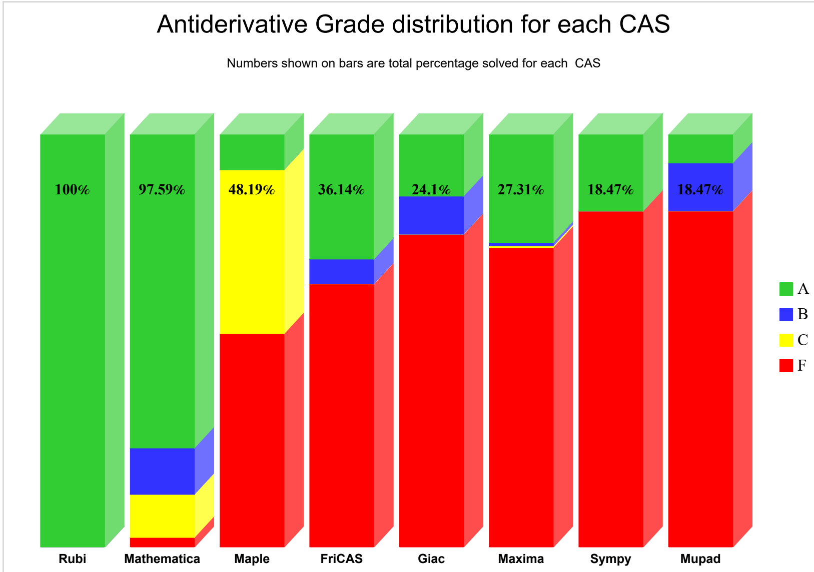
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

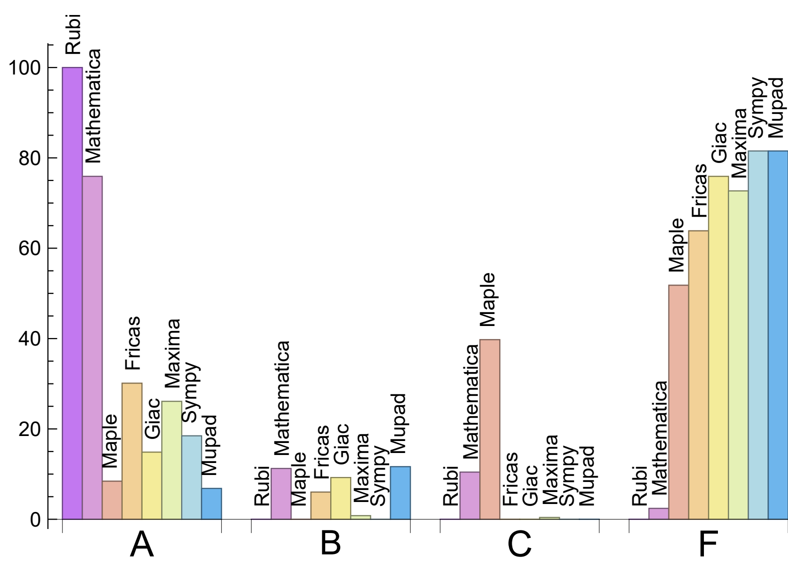
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.90	11.24	10.44	2.41
Fricas	30.12	6.02	0.00	63.86
Maxima	26.10	0.80	0.40	72.69
Sympy	18.47	0.00	0.00	81.53
Giac	14.86	9.24	0.00	75.90
Maple	8.43	0.00	39.76	51.81
Mupad	N/A	11.65	0.00	81.53

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	129	100.00 %	0.00 %	0.00 %
Fricas	159	100.00 %	0.00 %	0.00 %
Giac	189	95.77 %	0.00 %	4.23 %
Maxima	181	92.27 %	0.00 %	7.73 %
Sympy	203	26.60 %	58.13 %	15.27 %
Mupad	203	97.04 %	2.96 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

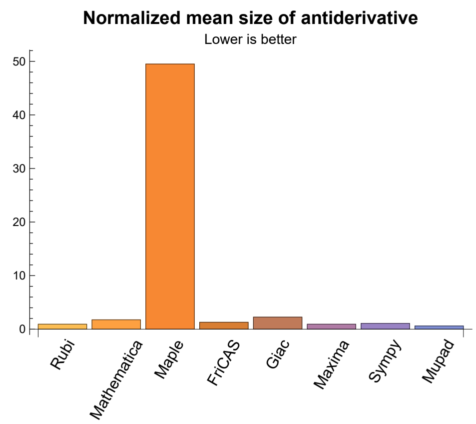
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

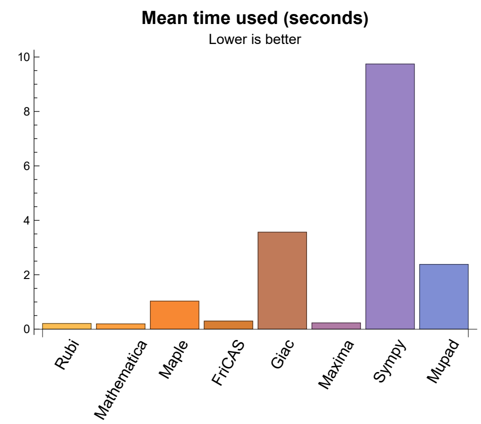
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	254.71	0.93	195.00	1.00
Mathematica	0.20	436.33	1.76	252.00	1.07
Maple	1.03	9987.33	49.50	2100.50	15.33
Maxima	0.23	107.51	0.93	69.50	1.08
Fricas	0.30	165.19	1.30	112.50	1.09
Sympy	9.74	99.76	1.08	47.00	1.05
Giac	3.56	296.87	2.26	91.00	1.46
Mupad	2.38	48.11	0.61	23.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
10
20
30
40
50
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {138, 144, 145, 146, 148, 149, 220}

Maple {220, 221, 222}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{150, 151, 152, 153, 154, 155, 197\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

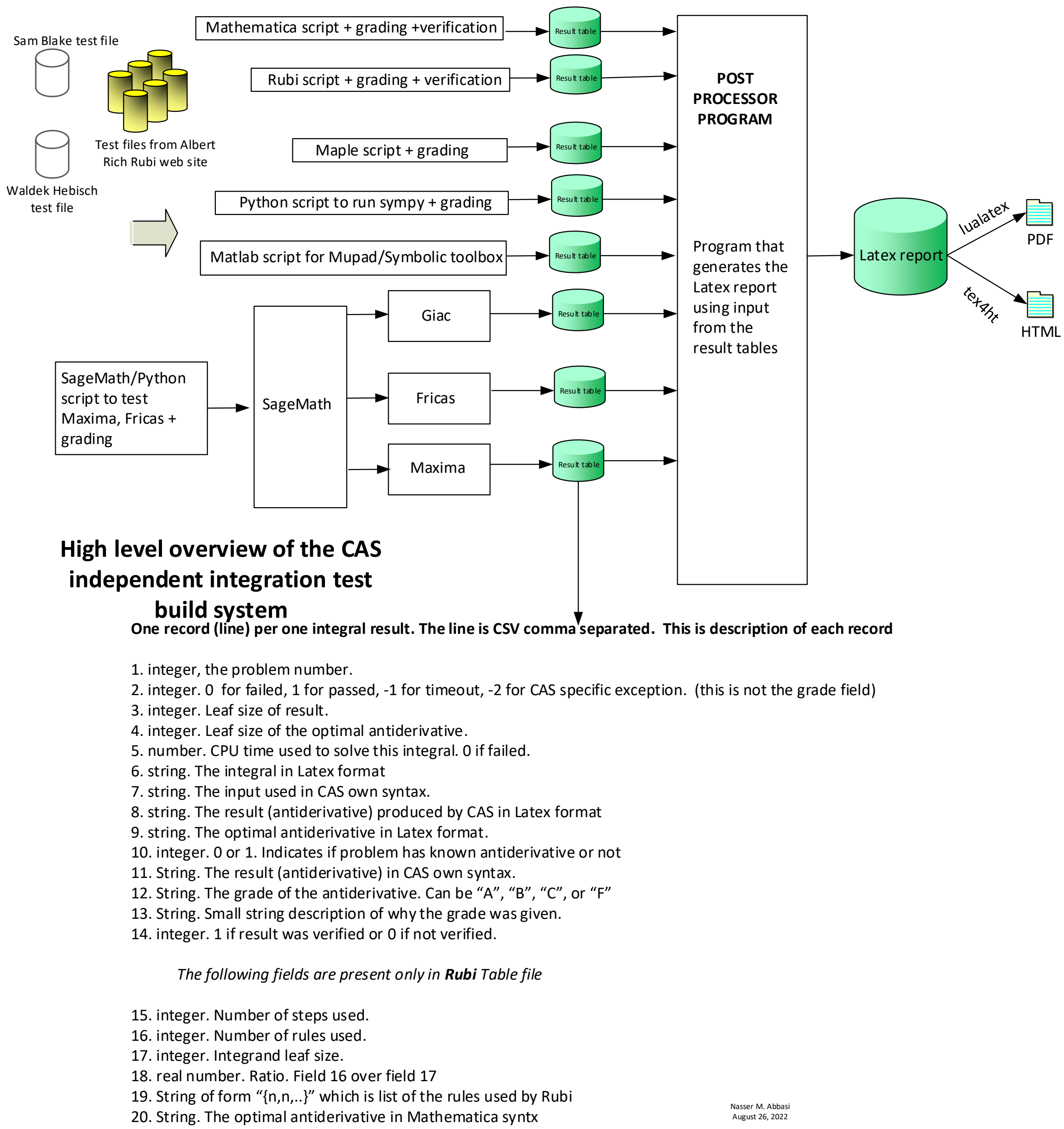
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS 20
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 76

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi 21
2.1.2 Mathematica 21
2.1.3 Maple 22
2.1.4 Maxima 22
2.1.5 FriCAS 22
2.1.6 Sympy 23
2.1.7 Giac 23
2.1.8 Mupad 24

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 28, 29, 30, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 95, 96, 97, 104, 105, 106, 107, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 217, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

B grade: { 22, 23, 62, 63, 64, 65, 66, 81, 82, 83, 84, 85, 86, 87, 88, 89, 111, 112, 113, 114, 128, 129, 130, 139, 140, 141, 147, 166 }

C grade: { 24, 25, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110 }

F grade: { 207, 214, 215, 216, 218, 219 }

2.1.3 Maple

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222, 225, 226, 233, 246 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 42, 43, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 182, 186, 187, 188, 189, 190, 191, 192, 193 }

F grade: { 10, 11, 12, 13, 17, 18, 19, 20, 32, 33, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 104, 105, 106, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 178, 179, 180, 181, 183, 184, 185, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 7, 8, 9, 68, 69, 70, 71, 72, 73, 75, 76, 77, 138, 142, 143, 144, 145, 146, 148, 149, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 173, 177, 182, 193, 198, 202, 203, 204, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 246 }

B grade: { 166, 237 }

C grade: { 192 }

F grade: { 1, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 170, 171, 172, 174, 175, 176, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.5 FriCAS

A grade: { 66, 67, 68, 69, 138, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 186, 187, 190, 191, 198, 202, 203, 204, 208, 209, 210, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 235, 236, 239, 240, 241, 245, 246 }

B grade: { 64, 65, 139, 140, 163, 164, 165, 166, 182, 230, 231, 232, 233, 237, 238 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 178, 179, 180, 183, 184, 185, 188, 189, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 211, 217, 242, 243, 244, 247, 248, 249 }

2.1.6 Sympy

A grade: { 5, 145, 146, 148, 156, 157, 158, 160, 161, 162, 163, 164, 165, 167, 168, 169, 177, 188, 189, 198, 202, 203, 204, 205, 208, 209, 210, 211, 217, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 246 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 149, 150, 151, 152, 153, 154, 155, 159, 166, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 206, 207, 212, 213, 214, 215, 216, 218, 219, 220, 221, 237, 238, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.1.7 Giac

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 158, 159, 160, 161, 162, 170, 171, 172, 173, 177, 198, 202, 203, 204, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 240, 246 }

B grade: { 156, 157, 163, 164, 165, 166, 167, 168, 169, 176, 182, 186, 187, 230, 231, 232, 233, 235, 236, 237, 238, 239, 241 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 174, 175, 178, 179, 180, 181, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 242, 243, 244, 245, 247, 248, 249 }

2.1.8 Mupad

A grade: { 68, 69, 138, 142, 143, 144, 145, 146, 148, 149, 198, 202, 203, 204, 220, 221, 222 }

B grade: { 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 246 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 147, 150, 151, 152, 153, 154, 155, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	173	555	0	0	0	0	-1
N.S.	1	1.00	1.00	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.180	0.243	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	188	1014	223	0	0	0	-1
N.S.	1	1.00	0.90	4.83	1.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.087	0.081	0.172	0.330	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	161	870	194	0	0	0	-1
N.S.	1	1.00	0.90	4.89	1.09	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.056	0.123	0.321	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	725	163	0	0	0	-1
N.S.	1	1.00	0.90	4.97	1.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.051	0.119	0.338	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	557	127	0	194	0	-1
N.S.	1	1.00	1.22	7.53	1.72	0.00	2.62	0.00	-0.01
time (sec)	N/A	0.062	0.028	0.093	0.334	0.000	176.720	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	143	0	0	0	0	-1
N.S.	1	1.00	1.21	5.11	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.011	0.115	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	69	481	130	0	0	0	-1
N.S.	1	1.00	0.64	4.50	1.21	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.050	0.115	0.320	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	215	647	178	0	0	0	-1
N.S.	1	1.00	1.32	3.97	1.09	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.056	0.127	0.341	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	206	796	208	0	0	0	-1
N.S.	1	1.00	1.06	4.08	1.07	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.062	0.134	0.342	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	594	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.115	0.014	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	506	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.088	0.018	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	416	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.076	0.021	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	294	0	0	0	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.059	0.012	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	835	0	0	0	0	-1
N.S.	1	1.00	0.96	15.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.043	0.162	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	183	3402	0	0	0	0	-1
N.S.	1	1.00	0.90	16.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.134	0.239	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	513	4445	0	0	0	0	-1
N.S.	1	1.00	1.79	15.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.115	0.278	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	710	710	1144	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	0.207	0.017	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	975	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	0.168	0.019	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	806	0	0	0	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.145	0.021	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	584	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.110	0.014	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	4058	0	0	0	0	-1
N.S.	1	1.00	0.95	50.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.067	0.250	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	770	14041	0	0	0	0	-1
N.S.	1	1.00	2.25	41.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.190	0.379	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	1047	17975	0	0	0	0	-1
N.S.	1	1.00	2.23	38.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	0.236	0.484	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	348	840	0	0	0	0	-1
N.S.	1	1.00	1.93	4.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.078	0.132	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	267	820	0	0	0	0	-1
N.S.	1	1.00	2.34	7.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.038	0.091	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	1026	0	0	0	0	-1
N.S.	1	1.00	1.28	26.31	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.011	0.127	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	241	619	0	0	0	0	-1
N.S.	1	1.00	1.71	4.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.069	0.092	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	364	891	0	0	0	0	-1
N.S.	1	1.00	1.51	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.069	0.152	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	254	643	0	0	0	0	-1
N.S.	1	1.00	1.40	3.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.069	0.136	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	221	547	0	0	0	0	-1
N.S.	1	1.00	1.31	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.069	0.142	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	285	734	0	0	0	0	-1
N.S.	1	1.00	1.35	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.134	0.147	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	654	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.208	0.015	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	519	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.161	0.014	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	484	6180	0	0	0	0	-1
N.S.	1	1.00	6.91	88.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.127	0.215	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	488	3493	0	0	0	0	-1
N.S.	1	1.00	1.90	13.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.227	0.250	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	703	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.694	0.380	0.012	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	544	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	0.221	0.012	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	414	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.187	0.015	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	585	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.349	0.014	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	1234	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	0.605	0.018	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1004	0	0	0	0	0	-1
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	0.399	0.019	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	754	23414	0	0	0	0	-1
N.S.	1	1.00	7.47	231.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.191	0.439	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	940	13973	0	0	0	0	-1
N.S.	1	1.00	2.21	32.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.238	0.424	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	938	938	1027	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.074	0.449	0.020	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	794	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.209	0.021	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	263	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.217	0.014	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	191	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.160	0.021	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.114	0.014	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.009	0.015	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	124	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.142	0.015	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	207	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	0.185	0.015	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	288	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.254	0.014	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	995	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	0.343	0.019	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	769	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.243	0.018	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	527	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.194	0.016	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.090	0.018	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	627	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.253	0.018	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	881	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.335	0.018	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	1432	0	0	0	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.368	0.023	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	986	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.313	0.018	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.122	0.022	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	1455	0	0	0	0	0	-1
N.S.	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.557	0.022	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	2009	0	0	0	0	0	-1
N.S.	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.788	0.673	0.031	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1700	38574	0	523	0	0	-1
N.S.	1	1.00	12.41	281.56	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.437	0.589	0.000	0.359	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	1035	11734	0	285	0	0	-1
N.S.	1	1.00	9.86	111.75	0.00	2.71	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.242	0.282	0.000	0.360	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	526	2578	0	131	0	0	-1
N.S.	1	1.00	7.21	35.32	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.146	0.187	0.000	0.355	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	308	0	42	0	0	-1
N.S.	1	1.00	1.30	7.70	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.013	0.151	0.000	0.376	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.045	0.012	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	1.911	0.008	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	290	2403	356	0	0	0	-1
N.S.	1	1.00	1.02	8.49	1.26	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.163	0.429	0.391	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	252	2222	311	0	0	0	-1
N.S.	1	1.00	1.04	9.14	1.28	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.108	0.386	0.392	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	208	2041	260	0	0	0	-1
N.S.	1	1.00	1.02	10.05	1.28	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.089	0.087	0.377	0.390	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	152	1762	193	0	0	0	-1
N.S.	1	1.00	1.30	15.06	1.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.051	0.266	0.376	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	147	1795	0	0	0	0	-1
N.S.	1	1.00	1.47	17.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.051	0.162	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	117	1892	198	0	0	0	-1
N.S.	1	1.00	0.71	11.54	1.21	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.082	0.296	0.389	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	2100	271	0	0	0	-1
N.S.	1	1.00	0.99	8.97	1.16	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	0.106	0.323	0.403	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	280	2282	320	0	0	0	-1
N.S.	1	1.00	1.02	8.33	1.17	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.122	0.361	0.398	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	788	12902	0	0	0	0	-1
N.S.	1	1.00	1.74	28.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.207	0.580	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	674	11828	0	0	0	0	-1
N.S.	1	1.00	1.81	31.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.162	0.545	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	507	10356	0	0	0	0	-1
N.S.	1	1.00	1.76	35.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.121	0.487	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	329	21792	0	0	0	0	-1
N.S.	1	1.00	2.51	166.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.091	0.551	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	600	10967	0	0	0	0	-1
N.S.	1	1.00	2.42	44.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.206	0.532	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	796	12159	0	0	0	0	-1
N.S.	1	1.00	2.31	35.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.235	0.533	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	909	13227	0	0	0	0	-1
N.S.	1	1.00	2.16	31.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.259	0.549	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	1431	44991	0	0	0	0	-1
N.S.	1	1.00	2.37	74.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	0.340	1.250	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1122	39644	0	0	0	0	-1
N.S.	1	1.00	2.37	83.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.258	1.099	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	602	60520	0	0	0	0	-1
N.S.	1	1.00	3.74	375.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.140	1.374	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1347	41923	0	0	0	0	-1
N.S.	1	1.00	3.28	102.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	0.413	1.216	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	1736	46399	0	0	0	0	-1
N.S.	1	1.00	3.13	83.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.593	0.470	1.270	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	324	2270	0	0	0	0	-1
N.S.	1	1.00	1.47	10.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.112	0.687	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	266	2068	0	0	0	0	-1
N.S.	1	1.00	1.80	13.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.060	0.291	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	297	2842	0	0	0	0	-1
N.S.	1	1.00	2.63	25.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.064	0.530	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	298	2101	0	0	0	0	-1
N.S.	1	1.00	1.53	10.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.091	0.305	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	363	2313	0	0	0	0	-1
N.S.	1	1.00	1.46	9.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.095	0.343	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	389	2321	0	0	0	0	-1
N.S.	1	1.00	1.55	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.099	0.190	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	332	2001	0	0	0	0	-1
N.S.	1	1.00	1.71	10.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.061	0.165	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	305	1972	0	0	0	0	-1
N.S.	1	1.00	1.70	11.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.066	0.172	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	362	2204	0	0	0	0	-1
N.S.	1	1.00	1.59	9.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.081	0.189	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	399	2385	0	0	0	0	-1
N.S.	1	1.00	1.49	8.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.125	0.195	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	814	12230	0	0	0	0	-1
N.S.	1	1.00	2.63	39.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.159	0.527	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	736	27214	0	0	0	0	-1
N.S.	1	1.00	5.01	185.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.134	1.251	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	946	12568	0	0	0	0	-1
N.S.	1	1.00	3.43	45.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.301	0.587	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	1111	13825	0	0	0	0	-1
N.S.	1	1.00	3.12	38.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.282	0.591	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1128	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	0.263	0.020	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	993	0	0	0	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.200	0.023	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	917	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.187	0.022	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	1083	0	0	0	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.589	0.254	0.024	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1911	47964	0	0	0	0	-1
N.S.	1	1.00	3.72	93.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	0.320	1.233	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	1348	77072	0	0	0	0	-1
N.S.	1	1.00	7.45	425.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.210	3.446	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	2248	49628	0	0	0	0	-1
N.S.	1	1.00	4.98	110.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.559	1.268	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	2544	0	0	0	0	0	-1
N.S.	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.232	0.567	0.024	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	977	977	2302	0	0	0	0	0	-1
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.998	0.442	0.028	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	879	879	2166	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	0.418	0.039	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1007	1007	2488	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.120	0.551	0.024	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	434	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.336	0.019	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	336	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.252	0.017	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	218	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.163	0.031	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	186	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.130	0.018	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	250	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.226	0.017	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	359	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.281	0.019	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	457	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.341	0.017	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	1319	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	0.549	0.022	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	960	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	0.289	0.020	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	718	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.233	0.019	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	263	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.147	0.019	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	821	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.312	0.019	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	1078	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.359	0.020	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	1968	0	0	0	0	0	-1
N.S.	1	1.00	2.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	0.469	0.021	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	1522	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	0.405	0.020	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	403	0	0	0	0	0	-1
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.240	0.022	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	976	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.769	0.740	0.040	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	914	914	1549	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.014	1.101	0.026	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	394	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.261	0.016	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	296	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.194	0.017	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	145	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.270	0.022	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	326	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.251	0.017	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	422	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.279	0.017	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	304	0	0	0	0	0	-1
N.S.	1	0.00	9.81	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.206	0.103	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	1395	0	0	763	0	0	-1
N.S.	1	1.00	7.54	0.00	0.00	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.376	0.034	0.000	0.378	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	741	0	0	405	0	0	-1
N.S.	1	1.00	4.94	0.00	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.215	0.030	0.000	0.368	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	277	0	0	173	0	0	-1
N.S.	1	1.00	2.43	0.00	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.118	0.028	0.000	0.361	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.117	0.024	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	1.786	0.024	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	292	0	0	0	0	0	-1
N.S.	1	0.00	10.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.134	0.028	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	292	0	0	0	0	0	-1
N.S.	1	0.00	10.81	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.125	0.029	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	165	0	0	0	0	0	-1
N.S.	1	0.00	6.35	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.003	0.119	0.025	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	277	0	0	173	0	0	-1
N.S.	1	1.00	2.43	0.00	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.125	0.029	0.000	0.370	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	282	0	0	0	0	0	-1
N.S.	1	0.00	9.72	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.118	0.028	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	292	0	0	0	0	0	-1
N.S.	1	0.00	10.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.116	0.028	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	410	0	0	364	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.252	0.101	0.000	0.375	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	352	0	0	301	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.232	0.099	0.000	0.370	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	268	0	0	202	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.147	0.100	0.000	0.362	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	162	0	0	247	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.223	0.101	0.000	0.376	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	302	0	0	334	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.227	0.104	0.000	0.382	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	358	0	0	395	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.258	0.108	0.000	0.373	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	1640	108	145	128	161	82
N.S.	1	1.00	0.85	19.52	1.29	1.73	1.52	1.92	0.98
time (sec)	N/A	0.052	0.037	0.167	0.305	0.366	2.401	4.656	4.038

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	1640	106	140	126	161	82
N.S.	1	1.00	0.81	19.52	1.26	1.67	1.50	1.92	0.98
time (sec)	N/A	0.037	0.032	0.180	0.290	0.372	1.078	4.013	4.003

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	1503	86	115	97	122	66
N.S.	1	1.00	0.75	19.52	1.12	1.49	1.26	1.58	0.86
time (sec)	N/A	0.024	0.020	0.141	0.279	0.364	0.435	4.537	3.779

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	1597	76	68	0	85	73
N.S.	1	1.00	1.26	28.02	1.33	1.19	0.00	1.49	1.28
time (sec)	N/A	0.047	0.033	0.235	0.292	0.379	0.000	4.201	3.868

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	1443	98	95	99	108	75
N.S.	1	1.00	0.79	20.04	1.36	1.32	1.38	1.50	1.04
time (sec)	N/A	0.048	0.035	0.146	0.294	0.348	0.475	5.149	3.809

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	64	1442	97	100	128	116	83
N.S.	1	1.00	0.77	17.37	1.17	1.20	1.54	1.40	1.00
time (sec)	N/A	0.052	0.038	0.143	0.297	0.365	1.231	3.860	3.941

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	1451	103	112	129	121	83
N.S.	1	1.00	0.83	17.48	1.24	1.35	1.55	1.46	1.00
time (sec)	N/A	0.051	0.043	0.155	0.298	0.380	2.664	6.282	3.935

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	157	9271	256	407	340	506	189
N.S.	1	1.00	0.76	44.79	1.24	1.97	1.64	2.44	0.91
time (sec)	N/A	0.138	0.071	0.385	0.307	0.361	5.382	5.414	3.989

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	154	9262	253	405	318	497	187
N.S.	1	1.00	0.75	44.96	1.23	1.97	1.54	2.41	0.91
time (sec)	N/A	0.116	0.068	0.369	0.311	0.353	2.446	5.341	4.146

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	8701	219	350	279	425	165
N.S.	1	1.00	0.96	59.19	1.49	2.38	1.90	2.89	1.12
time (sec)	N/A	0.058	0.049	0.354	0.288	0.353	1.098	7.320	3.879

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	129	9164	168	182	0	223	124
N.S.	1	1.00	2.26	160.77	2.95	3.19	0.00	3.91	2.18
time (sec)	N/A	0.065	0.068	0.700	0.290	0.350	0.000	4.620	3.988

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	138	8407	227	316	280	392	181
N.S.	1	1.00	0.76	46.45	1.25	1.75	1.55	2.17	1.00
time (sec)	N/A	0.129	0.076	0.470	0.287	0.364	1.212	6.799	4.013

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	151	8407	230	334	320	403	186
N.S.	1	1.00	0.74	41.21	1.13	1.64	1.57	1.98	0.91
time (sec)	N/A	0.142	0.084	0.429	0.308	0.347	1.252	5.268	4.082

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	155	8407	236	337	342	403	190
N.S.	1	1.00	0.76	41.01	1.15	1.64	1.67	1.97	0.93
time (sec)	N/A	0.144	0.085	0.451	0.300	0.364	2.715	6.980	4.197

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	93	2350	0	95	0	206	-1
N.S.	1	1.00	0.66	16.67	0.00	0.67	0.00	1.46	-0.01
time (sec)	N/A	0.125	0.106	0.405	0.000	0.352	0.000	3.841	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	93	2350	0	95	0	206	-1
N.S.	1	1.00	0.66	16.67	0.00	0.67	0.00	1.46	-0.01
time (sec)	N/A	0.104	0.095	0.398	0.000	0.356	0.000	4.756	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	86	2329	0	87	0	179	-1
N.S.	1	1.00	0.66	17.92	0.00	0.67	0.00	1.38	-0.01
time (sec)	N/A	0.081	0.086	0.410	0.000	0.353	0.000	3.664	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	58	1744	120	56	0	85	-1
N.S.	1	1.00	0.82	24.56	1.69	0.79	0.00	1.20	-0.01
time (sec)	N/A	0.069	0.034	0.148	0.295	0.368	0.000	5.491	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	87	2296	0	83	0	0	-1
N.S.	1	1.00	0.65	17.26	0.00	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.078	0.406	0.000	0.410	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	2341	0	92	0	0	-1
N.S.	1	1.00	0.67	16.60	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.080	0.406	0.000	0.369	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	135	87	370	0	151	0	661	-1
N.S.	1	1.52	0.98	4.16	0.00	1.70	0.00	7.43	-0.01
time (sec)	N/A	0.095	0.092	0.264	0.000	0.364	0.000	4.716	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	131	32	16	32	17	-1
N.S.	1	1.00	0.97	4.52	1.10	0.55	1.10	0.59	-0.03
time (sec)	N/A	0.035	0.017	0.048	0.272	0.365	5.211	3.925	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	179	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.665	0.105	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	156	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.274	0.025	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	156	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.261	0.024	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	146	0	0	134	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.233	0.026	0.000	0.093	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	854	97	229	0	246	-1
N.S.	1	1.00	1.00	12.03	1.37	3.23	0.00	3.46	-0.01
time (sec)	N/A	0.103	0.080	0.431	0.288	0.358	0.000	5.008	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	141	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.242	0.026	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	154	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.261	0.026	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	154	0	0	0	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.263	0.026	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	248	6894	0	231	0	5311	-1
N.S.	1	1.00	1.01	28.02	0.00	0.94	0.00	21.59	-0.00
time (sec)	N/A	0.164	0.124	2.980	0.000	0.569	0.000	6.325	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	248	5619	0	320	0	2136	-1
N.S.	1	1.00	1.01	22.93	0.00	1.31	0.00	8.72	-0.00
time (sec)	N/A	0.158	0.128	2.758	0.000	0.634	0.000	7.257	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	165	78943	0	0	221	0	-1
N.S.	1	1.00	0.91	433.75	0.00	0.00	1.21	0.00	-0.01
time (sec)	N/A	0.113	0.092	13.609	0.000	0.000	54.387	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	152337	0	0	231	0	-1
N.S.	1	1.00	0.98	837.02	0.00	0.00	1.27	0.00	-0.01
time (sec)	N/A	0.106	0.094	14.166	0.000	0.000	52.739	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	240	4077	0	429	0	0	-1
N.S.	1	1.00	0.98	16.71	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.111	3.937	0.000	0.547	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	145	4732	0	390	0	0	-1
N.S.	1	1.00	0.46	15.17	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.165	3.382	0.000	0.485	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	167	90894	364	0	0	0	-1
N.S.	1	1.00	0.93	504.97	2.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.110	13.385	0.315	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	178	1911	328	0	0	0	-1
N.S.	1	1.00	0.99	10.62	1.82	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.101	29.138	0.371	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	456	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.599	3.983	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	564	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.467	0.578	2.937	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	516	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.513	1.642	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	619	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.981	2.475	2.072	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	0.037	0.019	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	99	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.036	0.018	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.014	0.016	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	51	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.007	0.014	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	0.032	0.018	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.034	0.018	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.035	0.017	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	15	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.05
time (sec)	N/A	0.016	0.003	0.008	0.000	0.000	1.135	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.004	0.013	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.074	0.073	0.027	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	196	0	0	230	250	0	-1
N.S.	1	1.00	0.90	0.00	0.00	1.06	1.15	0.00	-0.00
time (sec)	N/A	0.129	0.391	0.036	0.000	0.360	76.595	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	168	0	0	196	206	0	-1
N.S.	1	1.00	0.91	0.00	0.00	1.06	1.11	0.00	-0.01
time (sec)	N/A	0.098	0.243	0.036	0.000	0.373	24.940	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	113	0	0	146	136	0	-1
N.S.	1	1.00	1.07	0.00	0.00	1.38	1.28	0.00	-0.01
time (sec)	N/A	0.079	0.058	0.033	0.000	0.355	6.964	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	-0.04
time (sec)	N/A	0.020	0.007	0.035	0.000	0.000	9.093	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	115	0	0	144	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.121	0.052	0.000	0.388	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	163	0	0	182	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.155	0.067	0.000	0.385	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	0	0	0	274	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.104	0.022	0.000	0.380	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	241	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.087	0.019	0.000	0.357	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	184	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.068	0.017	0.000	0.367	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	0	0	0	26	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	1.00	0.00	-0.04
time (sec)	N/A	0.020	0.006	0.019	0.000	0.000	5.105	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	167	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.108	0.033	0.000	0.409	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	217	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.091	0.049	0.000	0.365	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	266	844	0	0	0	0	-1
N.S.	1	0.00	8.87	28.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.144	0.399	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	0	0	867	0	0	0	0	-1
N.S.	1	0.00	0.00	4.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.077	0.212	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	0	0	1065	0	0	0	0	-1
N.S.	1	0.00	0.00	4.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.038	0.860	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	32	22	32	23
N.S.	1	1.00	1.00	0.00	0.85	1.19	0.81	1.19	0.85
time (sec)	N/A	0.021	0.004	0.018	0.276	0.339	0.407	3.532	3.838

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	32	22	32	23
N.S.	1	1.00	1.00	0.00	0.85	1.19	0.81	1.19	0.85
time (sec)	N/A	0.012	0.002	0.014	0.282	0.347	0.217	4.566	3.799

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	21	15	21	17
N.S.	1	1.00	1.00	1.06	1.00	1.17	0.83	1.17	0.94
time (sec)	N/A	0.005	0.002	0.023	0.286	0.355	0.116	5.823	0.027

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	37	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.86	1.68	0.91	0.91
time (sec)	N/A	0.020	0.003	0.030	0.284	0.351	1.004	3.755	3.801

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	23	20	17	25	19
N.S.	1	1.00	1.00	0.00	1.00	0.87	0.74	1.09	0.83
time (sec)	N/A	0.020	0.002	0.013	0.281	0.338	0.222	6.393	3.881

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	24	24	28	23
N.S.	1	1.00	1.00	0.00	0.85	0.89	0.89	1.04	0.85
time (sec)	N/A	0.020	0.002	0.014	0.284	0.333	0.485	5.651	3.842

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	23	24	24	28	23
N.S.	1	1.00	1.00	0.00	0.85	0.89	0.89	1.04	0.85
time (sec)	N/A	0.020	0.002	0.014	0.285	0.364	0.893	3.015	3.868

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	46	113	49	115	46
N.S.	1	1.00	1.00	0.00	0.88	2.17	0.94	2.21	0.88
time (sec)	N/A	0.050	0.004	0.014	0.294	0.345	0.799	5.444	3.798

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	0	46	113	46	112	46
N.S.	1	1.00	0.83	0.00	0.88	2.17	0.88	2.15	0.88
time (sec)	N/A	0.029	0.006	0.015	0.273	0.337	0.417	7.380	3.848

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	0	39	90	39	92	39
N.S.	1	1.00	0.95	0.00	1.00	2.31	1.00	2.36	1.00
time (sec)	N/A	0.015	0.004	0.013	0.283	0.353	0.218	4.761	3.844

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	54	41	59	20
N.S.	1	1.00	1.00	0.95	0.91	2.45	1.86	2.68	0.91
time (sec)	N/A	0.032	0.002	0.030	0.271	0.423	0.778	5.350	3.749

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	0	46	81	41	90	40
N.S.	1	1.00	0.87	0.00	1.00	1.76	0.89	1.96	0.87
time (sec)	N/A	0.047	0.005	0.014	0.272	0.412	0.223	2.838	3.851

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	0	46	87	48	94	46
N.S.	1	1.00	0.83	0.00	0.88	1.67	0.92	1.81	0.88
time (sec)	N/A	0.047	0.005	0.015	0.276	0.337	0.489	5.073	3.785

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	46	88	51	95	46
N.S.	1	1.00	1.00	0.00	0.88	1.69	0.98	1.83	0.88
time (sec)	N/A	0.047	0.003	0.014	0.279	0.335	0.894	4.673	3.881

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	0	285	1355	0	1811	-1
N.S.	1	1.00	0.67	0.00	2.11	10.04	0.00	13.41	-0.01
time (sec)	N/A	0.153	0.043	0.030	0.290	0.354	0.000	4.388	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	152	388	0	561	-1
N.S.	1	1.00	0.97	0.00	1.63	4.17	0.00	6.03	-0.01
time (sec)	N/A	0.085	0.032	0.030	0.279	0.391	0.000	3.118	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	37	0	63	69	110	111	-1
N.S.	1	1.00	0.73	0.00	1.24	1.35	2.16	2.18	-0.02
time (sec)	N/A	0.032	0.012	0.029	0.282	0.373	4.273	4.958	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	0	0	104	0	140	-1
N.S.	1	1.00	0.99	0.00	0.00	1.21	0.00	1.63	-0.01
time (sec)	N/A	0.127	0.105	0.029	0.000	0.368	0.000	3.626	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	112	0	0	198	0	1540	-1
N.S.	1	1.00	0.88	0.00	0.00	1.56	0.00	12.13	-0.01
time (sec)	N/A	0.167	0.190	0.028	0.000	0.375	0.000	4.486	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	133	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.167	0.126	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.129	0.030	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.120	0.031	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	73	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.108	0.028	0.000	0.094	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	33	49	80	36	33
N.S.	1	1.00	1.00	1.03	1.00	1.48	2.42	1.09	1.00
time (sec)	N/A	0.059	0.008	0.045	0.286	0.379	1.460	5.620	4.072

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.116	0.029	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.120	0.028	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.065	0.055	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [57]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	23	0.130
2	A	6	4	1.00	20	0.200
3	A	6	4	1.00	20	0.200
4	A	6	4	1.00	18	0.222
5	A	7	7	1.00	17	0.412
6	A	2	2	1.00	20	0.100
7	A	8	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	7	5	1.00	20	0.250
10	A	15	9	1.00	22	0.409
11	A	14	9	1.00	22	0.409
12	A	13	9	1.00	20	0.450
13	A	14	12	1.00	19	0.632
14	A	3	3	1.00	22	0.136
15	A	10	10	1.00	22	0.454
16	A	14	9	1.00	22	0.409
17	A	29	12	1.00	22	0.546
18	A	26	12	1.00	22	0.546
19	A	23	12	1.00	20	0.600
20	A	24	16	1.00	19	0.842
21	A	4	3	1.00	22	0.136
22	A	14	11	1.00	22	0.500
23	A	22	10	1.00	22	0.454
24	A	7	5	1.00	26	0.192
25	A	8	9	1.00	24	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	26	0.077
27	A	9	8	1.00	26	0.308
28	A	9	7	1.00	26	0.269
29	A	8	7	1.00	23	0.304
30	A	7	6	1.00	26	0.231
31	A	8	7	1.00	26	0.269
32	A	13	9	1.00	28	0.321
33	A	15	16	1.00	26	0.615
34	A	3	3	1.00	28	0.107
35	A	11	11	1.00	28	0.393
36	A	30	17	1.00	28	0.607
37	A	26	16	1.00	25	0.640
38	A	16	12	1.00	28	0.429
39	A	22	14	1.00	28	0.500
40	A	22	11	1.00	28	0.393
41	A	24	21	1.00	26	0.808
42	A	4	3	1.00	28	0.107
43	A	15	12	1.00	28	0.429
44	A	42	17	1.00	25	0.680
45	A	26	13	1.00	28	0.464
46	A	7	5	1.00	28	0.179
47	A	7	5	1.00	26	0.192
48	A	7	5	1.00	25	0.200
49	A	2	2	1.00	28	0.071
50	A	8	6	1.00	28	0.214
51	A	8	6	1.00	28	0.214
52	A	8	6	1.00	28	0.214
53	A	18	10	1.00	30	0.333
54	A	16	10	1.00	28	0.357
55	A	14	9	1.00	27	0.333
56	A	3	3	1.00	30	0.100
57	A	17	14	1.00	30	0.467
58	A	19	14	1.00	30	0.467
59	A	30	13	1.00	28	0.464
60	A	24	12	1.00	27	0.444

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	30	0.100
62	A	28	16	1.00	30	0.533
63	A	34	16	1.00	30	0.533
64	A	5	3	1.00	28	0.107
65	A	4	3	1.00	28	0.107
66	A	3	3	1.00	28	0.107
67	A	2	2	1.00	26	0.077
68	A	0	0	0.00	0	0.000
69	A	0	0	0.00	0	0.000
70	A	7	5	1.00	24	0.208
71	A	7	5	1.00	24	0.208
72	A	7	5	1.00	22	0.227
73	A	8	8	1.00	21	0.381
74	A	4	4	1.00	24	0.167
75	A	9	8	1.00	24	0.333
76	A	8	6	1.00	24	0.250
77	A	8	6	1.00	24	0.250
78	A	24	12	1.00	26	0.462
79	A	21	12	1.00	24	0.500
80	A	18	11	1.00	23	0.478
81	A	5	5	1.00	26	0.192
82	A	10	10	1.00	26	0.385
83	A	14	9	1.00	26	0.346
84	A	19	9	1.00	26	0.346
85	A	34	13	1.00	24	0.542
86	A	28	12	1.00	23	0.522
87	A	6	5	1.00	26	0.192
88	A	14	11	1.00	26	0.423
89	A	22	10	1.00	26	0.385
90	A	9	6	1.00	26	0.231
91	A	9	10	1.00	24	0.417
92	A	4	4	1.00	26	0.154
93	A	11	9	1.00	26	0.346
94	A	10	7	1.00	26	0.269
95	A	9	6	1.00	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	6	1.00	23	0.261
97	A	7	5	1.00	26	0.192
98	A	8	6	1.00	26	0.231
99	A	9	6	1.00	26	0.231
100	A	17	11	1.00	26	0.423
101	A	5	5	1.00	28	0.179
102	A	11	11	1.00	28	0.393
103	A	15	10	1.00	28	0.357
104	A	30	17	1.00	28	0.607
105	A	26	16	1.00	25	0.640
106	A	16	12	1.00	28	0.429
107	A	22	14	1.00	28	0.500
108	A	26	12	1.00	26	0.462
109	A	6	5	1.00	28	0.179
110	A	15	12	1.00	28	0.429
111	A	49	18	1.00	28	0.643
112	A	42	17	1.00	25	0.680
113	A	26	13	1.00	28	0.464
114	A	36	15	1.00	28	0.536
115	A	9	6	1.00	28	0.214
116	A	9	6	1.00	26	0.231
117	A	9	7	1.00	25	0.280
118	A	4	4	1.00	28	0.143
119	A	10	7	1.00	28	0.250
120	A	10	7	1.00	28	0.250
121	A	10	7	1.00	28	0.250
122	A	22	13	1.00	28	0.464
123	A	20	13	1.00	26	0.500
124	A	18	13	1.00	25	0.520
125	A	5	5	1.00	28	0.179
126	A	21	17	1.00	28	0.607
127	A	23	17	1.00	28	0.607
128	A	36	16	1.00	26	0.615
129	A	30	16	1.00	25	0.640
130	A	6	5	1.00	28	0.179

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	34	19	1.00	28	0.679
132	A	40	19	1.00	28	0.679
133	A	9	6	1.00	30	0.200
134	A	9	6	1.00	30	0.200
135	A	11	9	1.00	30	0.300
136	A	10	7	1.00	30	0.233
137	A	10	7	1.00	30	0.233
138	A	0	0	0.00	0	0.000
139	A	6	5	1.00	28	0.179
140	A	5	5	1.00	28	0.179
141	A	4	4	1.00	26	0.154
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	4	4	1.00	26	0.154
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	18	12	1.00	32	0.375
151	A	16	12	1.00	32	0.375
152	A	14	11	1.00	30	0.367
153	A	15	12	1.00	32	0.375
154	A	16	12	1.00	32	0.375
155	A	18	12	1.00	32	0.375
156	A	3	3	1.00	24	0.125
157	A	3	3	1.00	22	0.136
158	A	3	2	1.00	21	0.095
159	A	4	5	1.00	24	0.208
160	A	2	2	1.00	24	0.083
161	A	3	3	1.00	24	0.125
162	A	3	3	1.00	24	0.125
163	A	7	5	1.00	26	0.192
164	A	7	5	1.00	24	0.208
165	A	6	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	4	1.00	26	0.154
167	A	6	4	1.00	26	0.154
168	A	7	5	1.00	26	0.192
169	A	7	5	1.00	26	0.192
170	A	6	6	1.00	26	0.231
171	A	6	6	1.00	24	0.250
172	A	6	6	1.00	23	0.261
173	A	5	6	1.00	26	0.231
174	A	6	6	1.00	26	0.231
175	A	6	6	1.00	26	0.231
176	A	7	4	1.52	23	0.174
177	A	2	4	1.00	18	0.222
178	A	8	7	1.00	28	0.250
179	A	7	7	1.00	26	0.269
180	A	7	7	1.00	24	0.292
181	A	7	7	1.00	23	0.304
182	A	4	4	1.00	26	0.154
183	A	7	7	1.00	26	0.269
184	A	7	7	1.00	26	0.269
185	A	7	7	1.00	26	0.269
186	A	17	11	1.00	18	0.611
187	A	17	11	1.00	18	0.611
188	A	9	10	1.00	18	0.556
189	A	9	10	1.00	18	0.556
190	A	17	11	1.00	18	0.611
191	A	12	11	1.00	18	0.611
192	A	9	10	1.00	18	0.556
193	A	9	10	1.00	18	0.556
194	A	21	14	1.00	20	0.700
195	A	21	14	1.00	20	0.700
196	A	21	14	1.00	20	0.700
197	A	22	15	1.00	20	0.750
198	A	0	0	0.00	0	0.000
199	A	4	2	1.00	23	0.087
200	A	3	2	1.00	23	0.087

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	2	2	1.00	21	0.095
202	A	0	0	0.00	0	0.000
203	A	0	0	0.00	0	0.000
204	A	0	0	0.00	0	0.000
205	A	2	2	1.00	11	0.182
206	A	3	2	1.00	13	0.154
207	A	2	1	1.00	57	0.018
208	A	10	5	1.00	19	0.263
209	A	10	5	1.00	17	0.294
210	A	10	8	1.00	16	0.500
211	A	2	2	1.00	19	0.105
212	A	13	8	1.00	19	0.421
213	A	11	6	1.00	19	0.316
214	A	15	6	1.00	19	0.316
215	A	15	6	1.00	17	0.353
216	A	14	9	1.00	16	0.562
217	A	2	2	1.00	19	0.105
218	A	19	9	1.00	19	0.474
219	A	16	7	1.00	19	0.368
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	2	2	1.00	14	0.143
224	A	2	2	1.00	12	0.167
225	A	2	2	1.00	10	0.200
226	A	2	2	1.00	14	0.143
227	A	2	2	1.00	14	0.143
228	A	2	2	1.00	14	0.143
229	A	2	2	1.00	14	0.143
230	A	3	3	1.00	16	0.188
231	A	3	3	1.00	14	0.214
232	A	3	3	1.00	12	0.250
233	A	3	3	1.00	16	0.188
234	A	3	3	1.00	16	0.188
235	A	3	3	1.00	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	16	0.188
237	A	4	3	1.00	22	0.136
238	A	3	3	1.00	22	0.136
239	A	2	2	1.00	20	0.100
240	A	3	3	1.00	22	0.136
241	A	4	4	1.00	22	0.182
242	A	3	3	1.00	22	0.136
243	A	3	3	1.00	20	0.150
244	A	3	3	1.00	18	0.167
245	A	3	3	1.00	16	0.188
246	A	3	3	1.00	20	0.150
247	A	3	3	1.00	20	0.150
248	A	3	3	1.00	20	0.150
249	A	6	6	1.00	24	0.250

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$ . . . 86
3.2 $\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$ . . . 90
3.3 $\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$ . . . 94
3.4 $\int x(a+b\log(cx^n))\log(1+ex)\,dx$ . . . 98
3.5 $\int (a+b\log(cx^n))\log(1+ex)\,dx$ . . . 102
3.6 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx$ . . . 106
3.7 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx$ . . . 109
3.8 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx$ . . . 113
3.9 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx$ . . . 117
3.10 $\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx$ . . . 121
3.11 $\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx$ . . . 126
3.12 $\int x(a+b\log(cx^n))^2\log(1+ex)\,dx$ . . . 131
3.13 $\int (a+b\log(cx^n))^2\log(1+ex)\,dx$ . . . 136
3.14 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx$ . . . 141
3.15 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx$ . . . 145
3.16 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx$ . . . 151
3.17 $\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx$ . . . 157
3.18 $\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx$ . . . 163
3.19 $\int x(a+b\log(cx^n))^3\log(1+ex)\,dx$ . . . 169
3.20 $\int (a+b\log(cx^n))^3\log(1+ex)\,dx$ . . . 175
3.21 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx$ . . . 181
3.22 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx$ . . . 186
3.23 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx$ . . . 191
3.24 $\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ . . . 196
3.25 $\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ . . . 200

3.26 $\int \frac{(a+b\log(cx^n))\log(d(\frac{1}{d}+fx^2))}{x}\,dx$ 205
3.27 $\int \frac{(a+b\log(cx^n))\log(d(\frac{1}{d}+fx^2))}{x^3}\,dx$ 209
3.28 $\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 213
3.29 $\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 218
3.30 $\int \frac{(a+b\log(cx^n))\log(d(\frac{1}{d}+fx^2))}{x^2}\,dx$ 223
3.31 $\int \frac{(a+b\log(cx^n))\log(d(\frac{1}{d}+fx^2))}{x^4}\,dx$ 227
3.32 $\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 232
3.33 $\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 237
3.34 $\int \frac{(a+b\log(cx^n))^2\log(d(\frac{1}{d}+fx^2))}{x}\,dx$ 243
3.35 $\int \frac{(a+b\log(cx^n))^2\log(d(\frac{1}{d}+fx^2))}{x^3}\,dx$ 247
3.36 $\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 253
3.37 $\int (a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 260
3.38 $\int \frac{(a+b\log(cx^n))^2\log(d(\frac{1}{d}+fx^2))}{x^2}\,dx$ 266
3.39 $\int \frac{(a+b\log(cx^n))^2\log(d(\frac{1}{d}+fx^2))}{x^4}\,dx$ 272
3.40 $\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 278
3.41 $\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 285
3.42 $\int \frac{(a+b\log(cx^n))^3\log(d(\frac{1}{d}+fx^2))}{x}\,dx$ 293
3.43 $\int \frac{(a+b\log(cx^n))^3\log(d(\frac{1}{d}+fx^2))}{x^3}\,dx$ 297
3.44 $\int (a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$ 303
3.45 $\int \frac{(a+b\log(cx^n))^3\log(d(\frac{1}{d}+fx^2))}{x^2}\,dx$ 310
3.46 $\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$ 316
3.47 $\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$ 320
3.48 $\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx$ 324
3.49 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx$ 328
3.50 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx$ 331
3.51 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx$ 335
3.52 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$ 339
3.53 $\int x^2\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 343
3.54 $\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 349
3.55 $\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 355
3.56 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$ 360
3.57 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$ 363
3.58 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx$ 369
3.59 $\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$ 375
3.60 $\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$ 382

3.61 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x}\,dx$ 388
3.62 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$ 391
3.63 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$ 398
3.64 $\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$ 406
3.65 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$ 411
3.66 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$ 415
3.67 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$ 420
3.68 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))}\,dx$ 423
3.69 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))^2}\,dx$ 426
3.70 $\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$ 429
3.71 $\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$ 434
3.72 $\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$ 439
3.73 $\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx$ 444
3.74 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx$ 449
3.75 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$ 453
3.76 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx$ 458
3.77 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx$ 463
3.78 $\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$ 468
3.79 $\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$ 474
3.80 $\int (a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$ 480
3.81 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx$ 485
3.82 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$ 489
3.83 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx$ 494
3.84 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx$ 499
3.85 $\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx$ 504
3.86 $\int (a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx$ 511
3.87 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx$ 518
3.88 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx$ 522
3.89 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx$ 528
3.90 $\int x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 534
3.91 $\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 540
3.92 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$ 546
3.93 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$ 551
3.94 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$ 557
3.95 $\int x^2(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 563
3.96 $\int (a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 569
3.97 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$ 574

3.98 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$ 579
3.99 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^6}\,dx$ 585
3.100 $\int x(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 591
3.101 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$ 596
3.102 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$ 600
3.103 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$ 605
3.104 $\int x^2(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 610
3.105 $\int (a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 617
3.106 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$ 624
3.107 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$ 630
3.108 $\int x(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 636
3.109 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$ 643
3.110 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$ 648
3.111 $\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 654
3.112 $\int (a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx$ 663
3.113 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$ 671
3.114 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$ 678
3.115 $\int x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$ 686
3.116 $\int x\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$ 691
3.117 $\int \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$ 696
3.118 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx$ 701
3.119 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx$ 705
3.120 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx$ 710
3.121 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^4}\,dx$ 715
3.122 $\int x^2\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 720
3.123 $\int x\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 726
3.124 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx$ 732
3.125 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$ 738
3.126 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$ 742
3.127 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx$ 749
3.128 $\int x\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$ 756
3.129 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx$ 764

3.130 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x}\,dx$ 772
3.131 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$ 776
3.132 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$ 783
3.133 $\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$ 792
3.134 $\int \sqrt{x}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$ 797
3.135 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx$ 802
3.136 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx$ 807
3.137 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx$ 812
3.138 $\int (gx)^q(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 817
3.139 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx$ 820
3.140 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$ 825
3.141 $\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$ 830
3.142 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx$ 834
3.143 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))^2}\,dx$ 836
3.144 $\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 839
3.145 $\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 842
3.146 $\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 845
3.147 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx$ 848
3.148 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$ 852
3.149 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$ 855
3.150 $\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 858
3.151 $\int (gx)^{-1+2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 864
3.152 $\int (gx)^{-1+m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 869
3.153 $\int (gx)^{-1-m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 874
3.154 $\int (gx)^{-1-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 879
3.155 $\int (gx)^{-1-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$ 885
3.156 $\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx$ 891
3.157 $\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx$ 895
3.158 $\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx$ 899
3.159 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$ 903

3.160 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx$ . . . 907
3.161 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx$ . . . 911
3.162 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx$ . . . 915
3.163 $\int x^2(a+b\log(cx^n))^2\,(d+e\log(fx^r))\,dx$ . . . 919
3.164 $\int x(a+b\log(cx^n))^2\,(d+e\log(fx^r))\,dx$ . . . 924
3.165 $\int (a+b\log(cx^n))^2\,(d+e\log(fx^r))\,dx$ . . . 928
3.166 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx$ . . . 932
3.167 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$ . . . 936
3.168 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$ . . . 940
3.169 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$ . . . 944
3.170 $\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$ . . . 948
3.171 $\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$ . . . 953
3.172 $\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx$ . . . 958
3.173 $\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx$ . . . 963
3.174 $\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx$ . . . 968
3.175 $\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx$ . . . 973
3.176 $\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx$ . . . 978
3.177 $\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx$ . . . 982
3.178 $\int (gx)^m\,(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx$ . . . 985
3.179 $\int x^2(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx$ . . . 990
3.180 $\int x(a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx$ . . . 994
3.181 $\int (a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx$ . . . 998
3.182 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx$ . . . 1003
3.183 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^2}\,dx$ . . . 1007
3.184 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx$ . . . 1012
3.185 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx$ . . . 1017
3.186 $\int (d+ex^2)\sin^{-1}(ax)\log(cx^n)\,dx$ . . . 1022
3.187 $\int (d+ex^2)\cos^{-1}(ax)\log(cx^n)\,dx$ . . . 1029
3.188 $\int (d+ex^2)\tan^{-1}(ax)\log(cx^n)\,dx$ . . . 1036
3.189 $\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx$ . . . 1041
3.190 $\int (d+ex^2)\sinh^{-1}(ax)\log(cx^n)\,dx$ . . . 1046
3.191 $\int (d+ex^2)\cosh^{-1}(ax)\log(cx^n)\,dx$ . . . 1053
3.192 $\int (d+ex^2)\tanh^{-1}(ax)\log(cx^n)\,dx$ . . . 1060
3.193 $\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx$ . . . 1065
3.194 $\int (d+ex^2)\sin^{-1}(ax)^2\log(cx^n)\,dx$ . . . 1071
3.195 $\int (d+ex^2)\cos^{-1}(ax)^2\log(cx^n)\,dx$ . . . 1077
3.196 $\int (d+ex^2)\sinh^{-1}(ax)^2\log(cx^n)\,dx$ . . . 1083
3.197 $\int (d+ex^2)\cosh^{-1}(ax)^2\log(cx^n)\,dx$ . . . 1089
3.198 $\int \frac{(a+b\log(cx^n))^p\mathrm{Li}_k(ex^q)}{x}\,dx$ . . . 1096

3.199 $\int \frac{(a+b\log(cx^n))^3\text{Li}_k(ex^q)}{x}\,dx$ 1099
3.200 $\int \frac{(a+b\log(cx^n))^2\text{Li}_k(ex^q)}{x}\,dx$ 1102
3.201 $\int \frac{(a+b\log(cx^n))\text{Li}_k(ex^q)}{x}\,dx$ 1105
3.202 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))}\,dx$ 1108
3.203 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^2}\,dx$ 1111
3.204 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^3}\,dx$ 1114
3.205 $\int \frac{\log(x)\text{Li}_n(ax)}{x}\,dx$ 1117
3.206 $\int \frac{\log^2(x)\text{Li}_n(ax)}{x}\,dx$ 1120
3.207 $\int \left(\frac{q\text{Li}_{-1+k}(ex^q)}{bnx(a+b\log(cx^n))} - \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^2}\right)\,dx$ 1123
3.208 $\int x^2(a+b\log(cx^n))\,\text{Li}_2(ex)\,dx$ 1126
3.209 $\int x(a+b\log(cx^n))\,\text{Li}_2(ex)\,dx$ 1130
3.210 $\int (a+b\log(cx^n))\,\text{Li}_2(ex)\,dx$ 1134
3.211 $\int \frac{(a+b\log(cx^n))\text{Li}_2(ex)}{x}\,dx$ 1139
3.212 $\int \frac{(a+b\log(cx^n))\text{Li}_2(ex)}{x^2}\,dx$ 1142
3.213 $\int \frac{(a+b\log(cx^n))\text{Li}_2(ex)}{x^3}\,dx$ 1146
3.214 $\int x^2(a+b\log(cx^n))\,\text{Li}_3(ex)\,dx$ 1150
3.215 $\int x(a+b\log(cx^n))\,\text{Li}_3(ex)\,dx$ 1155
3.216 $\int (a+b\log(cx^n))\,\text{Li}_3(ex)\,dx$ 1160
3.217 $\int \frac{(a+b\log(cx^n))\text{Li}_3(ex)}{x}\,dx$ 1165
3.218 $\int \frac{(a+b\log(cx^n))\text{Li}_3(ex)}{x^2}\,dx$ 1168
3.219 $\int \frac{(a+b\log(cx^n))\text{Li}_3(ex)}{x^3}\,dx$ 1173
3.220 $\int -(dx)^m(a+b\log(cx^n))\log(1-ex^q)\,dx$ 1178
3.221 $\int (dx)^m(a+b\log(cx^n))\,\text{Li}_2(ex^q)\,dx$ 1181
3.222 $\int (dx)^m(a+b\log(cx^n))\,\text{Li}_3(ex^q)\,dx$ 1184
3.223 $\int x^2\log(c(bx^n)^p)\,dx$ 1188
3.224 $\int x\log(c(bx^n)^p)\,dx$ 1191
3.225 $\int \log(c(bx^n)^p)\,dx$ 1194
3.226 $\int \frac{\log(c(bx^n)^p)}{x}\,dx$ 1197
3.227 $\int \frac{\log(c(bx^n)^p)}{x^2}\,dx$ 1200
3.228 $\int \frac{\log(c(bx^n)^p)}{x^3}\,dx$ 1203
3.229 $\int \frac{\log(c(bx^n)^p)}{x^4}\,dx$ 1206
3.230 $\int x^2\log^2(c(bx^n)^p)\,dx$ 1209
3.231 $\int x\log^2(c(bx^n)^p)\,dx$ 1212
3.232 $\int \log^2(c(bx^n)^p)\,dx$ 1215
3.233 $\int \frac{\log^2(c(bx^n)^p)}{x}\,dx$ 1218
3.234 $\int \frac{\log^2(c(bx^n)^p)}{x^2}\,dx$ 1221
3.235 $\int \frac{\log^2(c(bx^n)^p)}{x^3}\,dx$ 1224
3.236 $\int \frac{\log^2(c(bx^n)^p)}{x^4}\,dx$ 1227

3.237 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^3 dx$ 1230
3.238 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2 dx$ 1235
3.239 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right) dx$ 1239
3.240 $\int \frac{(ex)^q}{a+b\log(c(dx^m)^n)} dx$ 1242
3.241 $\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2} dx$ 1245
3.242 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^p dx$ 1249
3.243 $\int x^2\left(a+b\log\left(c(dx^m)^n\right)\right)^p dx$ 1252
3.244 $\int x\left(a+b\log\left(c(dx^m)^n\right)\right)^p dx$ 1255
3.245 $\int \left(a+b\log\left(c(dx^m)^n\right)\right)^p dx$ 1258
3.246 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x} dx$ 1261
3.247 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2} dx$ 1265
3.248 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3} dx$ 1268
3.249 $\int \frac{a+b\log(c(dx^m)^n)}{e+fx^2} dx$ 1271

3.1 $\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=173

$$\frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}-\frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}+\frac{bn\mathrm{Li}_2\left(-\frac{2fx}{e-\sqrt{e^2-}\right.}{\sqrt{e^2-4df}}$$

```
[Out] (a+b*ln(c*x^n))*ln(1+2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-(a+b*
ln(c*x^n))*ln(1+2*f*x/(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+b*n*polylo
g(2,-2*f*x/(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)-b*n*polylog(2,-2*f*x/
(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2404, 2354, 2438}

$$\frac{bn\text{PolyLog}\left(2,-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}-\frac{bn\text{PolyLog}\left(2,-\frac{2fx}{\sqrt{e^2-4df}+e}\right)}{\sqrt{e^2-4df}}+\frac{\log\left(\frac{2fx}{e-\sqrt{e^2-4df}}+1\right)(a+b\log(cx^n))}{\sqrt{e^2-4df}}-\frac{\log\left(\frac{2fx}{\sqrt{e^2-4df}+e}+1\right)(a+b\log(cx^n))}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
```

```
[Out] ((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4*
d*f] - ((a + b*Log[c*x^n])*Log[1 + (2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt[e
^2 - 4*d*f] + (b*n*PolyLog[2, (-2*f*x)/(e - Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 -
 4*d*f] - (b*n*PolyLog[2, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f])])/Sqrt[e^2 - 4*d
*f]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{d+ex+fx^2}\,dx = \int \left(\frac{2f(a+b\log(cx^n))}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)} - \frac{2f(a+b\log(cx^n))}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right.$$
$$= \frac{(2f)\int \frac{a+b\log(cx^n)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{a+b\log(cx^n)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}$$
$$= \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right.}{\sqrt{e^2-4df}}$$
$$= \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}} - \frac{(a+b\log(cx^n))\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right.}{\sqrt{e^2-4df}}$$

Mathematica [A]
time = 0.18, size = 173, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)(a-bn\log(x)+b\log(cx^n))}{\sqrt{-e^2+4df}} + \frac{bn\left(\log(x)\left(\log\left(1+\frac{2fx}{e-\sqrt{e^2-4df}}\right)-\log\left(1+\frac{2fx}{e+\sqrt{e^2-4df}}\right)\right)+\text{Li}_2\left(\frac{2fx}{-e+\sqrt{e^2-4df}}\right)-\text{Li}_2\left(-\frac{2fx}{e+\sqrt{e^2-4df}}\right)\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x + f*x^2),x]
```

```
[Out] (2*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*(a - b*n*Log[x] + b*Log[c*x^n]))/
Sqrt[-e^2 + 4*d*f] + (b*n*(Log[x]*(Log[1 + (2*f*x)/(e - Sqrt[e^2 - 4*d*f])]
 - Log[1 + (2*f*x)/(e + Sqrt[e^2 - 4*d*f])]) + PolyLog[2, (2*f*x)/(-e + Sqr
t[e^2 - 4*d*f])] - PolyLog[2, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2
- 4*d*f]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 555, normalized size = 3.21

method	result
risch	$-\frac{2b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)n\ln(x)}{\sqrt{4df-e^2}} + \frac{2b\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)\ln(x^n)}{\sqrt{4df-e^2}} + \frac{bn\ln(x)\ln\left(\frac{-2fx+\sqrt{-4df+e^2}-e}{-e+\sqrt{-4df+e^2}}\right)}{\sqrt{-4df+e^2}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*n*ln(x)+2*b/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*ln(x^n)+b*n/(-4*d*f+e^2)^
(1/2)*ln(x)*ln((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-4*d*f+e^2)^(1/2)))-b*n/(
-4*d*f+e^2)^(1/2)*ln(x)*ln((2*f*x+(-4*d*f+e^2)^(1/2)+e)/(e+(-4*d*f+e^2)^(1/
2)))+b*n/(-4*d*f+e^2)^(1/2)*dilog((-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(-e+(-4*d*f
+e^2)^(1/2)))-b*n/(-4*d*f+e^2)^(1/2)*dilog((2*f*x+(-4*d*f+e^2)^(1/2)+e)/(e+
(-4*d*f+e^2)^(1/2)))-I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)
)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I/(4*d*f-e^2)^(1/2)*arctan((2*f*
x+e)/(4*d*f-e^2)^(1/2))*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I/(4*d*f-e^2)^(1/2)*
arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*Pi*csgn(I*c*x^n)^3+2/(4
*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*ln(c)+2*a/(4*d*f-e^2)
^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)/(f*x^2 + x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log{\left(cx^{n} \right)}}{d + ex + fx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/(f*x^2 + x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*x + f*x^2), x)
```

3.2 $\int x^3(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal. Leaf size=210

$$-\frac{5bnx}{16e^3}+\frac{3bnx^2}{32e^2}-\frac{7bnx^3}{144e}+\frac{1}{32}bnx^4+\frac{x(a+b\log(cx^n))}{4e^3}-\frac{x^2(a+b\log(cx^n))}{8e^2}+\frac{x^3(a+b\log(cx^n))}{12e}-\frac{1}{16}x^4(a+b$$

```
[Out] -5/16*b*n*x/e^3+3/32*b*n*x^2/e^2-7/144*b*n*x^3/e+1/32*b*n*x^4+1/4*x*(a+b*ln
(c*x^n))/e^3-1/8*x^2*(a+b*ln(c*x^n))/e^2+1/12*x^3*(a+b*ln(c*x^n))/e-1/16*x^
4*(a+b*ln(c*x^n))+1/16*b*n*ln(e*x+1)/e^4-1/16*b*n*x^4*ln(e*x+1)-1/4*(a+b*ln
(c*x^n))*ln(e*x+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*b*n*polylog(2,
-e*x)/e^4
```

Rubi [A]
time = 0.09, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2442, 45, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-ex)}{4e^4}-\frac{\log(ex+1)(a+b\log(cx^n))}{4e^4}+\frac{x(a+b\log(cx^n))}{4e^3}-\frac{x^2(a+b\log(cx^n))}{8e^2}+\frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))+\frac{x^3(a+b\log(cx^n))}{12e}-\frac{1}{16}x^4(a+b\log(cx^n))+\frac{bn\log(ex+1)}{16e^4}-\frac{5bnx}{16e^3}+\frac{3bnx^2}{32e^2}-\frac{1}{16}bnx^4\log(ex+1)-\frac{7bnx^3}{144e}+\frac{1}{32}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-5*b*n*x)/(16*e^3) + (3*b*n*x^2)/(32*e^2) - (7*b*n*x^3)/(144*e) + (b*n*x^4
)/32 + (x*(a + b*Log[c*x^n]))/(4*e^3) - (x^2*(a + b*Log[c*x^n]))/(8*e^2) +
(x^3*(a + b*Log[c*x^n]))/(12*e) - (x^4*(a + b*Log[c*x^n]))/16 + (b*n*Log[1
+ e*x])/(16*e^4) - (b*n*x^4*Log[1 + e*x])/16 - ((a + b*Log[c*x^n])*Log[1 +
e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n])*Log[1 + e*x])/4 - (b*n*PolyLog[2, -
(e*x)])/(4*e^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^3(a+b\log(cx^n))\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))}{4e^3} - \frac{x^2(a+b\log(cx^n))}{8e^2} + \frac{x^3(a+b\log(cx^n))}{12e} - \frac{1}{16}x^4 \cdots$$
$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x(a+b\log(cx^n))}{4e^3} - \frac{x^2(a+b\log(\cdots}{8e^2}$$
$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x(a+b\log(cx^n))}{4e^3} - \frac{x^2(a+b\log(\cdots}{8e^2}$$
$$= -\frac{bnx}{4e^3} + \frac{bnx^2}{16e^2} - \frac{bnx^3}{36e} + \frac{1}{64}bnx^4 + \frac{x(a+b\log(cx^n))}{4e^3} - \frac{x^2(a+b\log(\cdots}{8e^2}$$
$$= -\frac{5bnx}{16e^3} + \frac{3bnx^2}{32e^2} - \frac{7bnx^3}{144e} + \frac{1}{32}bnx^4 + \frac{x(a+b\log(cx^n))}{4e^3} - \frac{x^2(a+b\,l \cdots}{8e \cdots}$$

Mathematica [A]
time = 0.08, size = 188, normalized size = 0.90

$$\frac{72aex-90benx-36ae^2x^2+27be^2nx^2+24ae^3x^3-14be^3nx^3-18ae^4x^4+9be^4nx^4-72a\log(1+ex)+18bn\log(1+ex)+72ae^4x^4\log(1+ex)-18be^4nx^4\log(1+ex)+6b\log(cx^n)\left(ex(12-6ex+4e^2x^2-3e^3x^3)+12(-1+e^4x^4)\log(1+ex)\right)-72bn\text{Li}_2(-ex)}{288e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (72*a*e*x - 90*b*e*n*x - 36*a*e^2*x^2 + 27*b*e^2*n*x^2 + 24*a*e^3*x^3 - 14*
b*e^3*n*x^3 - 18*a*e^4*x^4 + 9*b*e^4*n*x^4 - 72*a*Log[1 + e*x] + 18*b*n*Log
[1 + e*x] + 72*a*e^4*x^4*Log[1 + e*x] - 18*b*e^4*n*x^4*Log[1 + e*x] + 6*b*L
og[c*x^n]*(e*x*(12 - 6*e*x + 4*e^2*x^2 - 3*e^3*x^3) + 12*(-1 + e^4*x^4)*Log
[1 + e*x]) - 72*b*n*PolyLog[2, -(e*x)])/(288*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 1014, normalized size = 4.83

method	result	size
risch	Expression too large to display	1014

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 25/48*a/e^4-1/8*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)*x^4+1/24*I/e*x^3*Pi*b*csgn
(I*c)*csgn(I*c*x^n)^2+1/12*a/e*x^3+1/4*a*ln(e*x+1)*x^4-1/4*a/e^4*ln(e*x+1)+
1/32*I*Pi*b*x^4*csgn(I*c*x^n)^3-25/96*I/e^4*Pi*b*csgn(I*c*x^n)^3+1/4*a/e^3*
x-1/16*ln(c)*b*x^4+1/12*b*ln(c)/e*x^3-1/8*a/e^2*x^2-1/8*b*ln(c)/e^2*x^2+1/3
2*b*n*x^4-1/16*a*x^4+(1/4*x^4*b*ln(e*x+1)-1/48*b*(3*e^4*x^4-4*e^3*x^3+6*e^2
*x^2-12*e*x+12*ln(e*x+1))/e^4)*ln(x^n)-1/4*b*n/e^4*dilog(e*x+1)+1/4*b*ln(c)
*ln(e*x+1)*x^4-1/4/e^4*ln(e*x+1)*b*ln(c)+1/8*I/e^4*ln(e*x+1)*Pi*b*csgn(I*c)
*csgn(I*x^n)*csgn(I*c*x^n)+1/4*b*ln(c)/e^3*x+1/16*I/e^2*x^2*Pi*b*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)-1/24*I/e*x^3*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)-1/8*I/e^3*x*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*Pi*b*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x+1)*x^4+25/48/e^4*b*ln(c)-35/72*b*n/e^4
-7/144*b*n*x^3/e+3/32*b*n*x^2/e^2+1/8*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(e
*x+1)*x^4+1/8*I/e^4*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^3-1/32*I*Pi*b*x^4*csgn(I*x
^n)*csgn(I*c*x^n)^2+25/96*I/e^4*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/e^4*
ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*b*n*ln(e*x+1)/e^4-1/16*b*n*
x^4*ln(e*x+1)+1/24*I/e*x^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/e^4*ln(e*
x+1)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*I/e^3*x*Pi*b*csgn(I*c)*csgn(I*c*x^n
)^2-25/96*I/e^4*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-5/16*b*n*x/e^3+1/8
*I/e^3*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/16*I/e^2*x^2*Pi*b*csgn(I*c)*csg
n(I*c*x^n)^2-1/24*I/e*x^3*Pi*b*csgn(I*c*x^n)^3-1/32*I*Pi*b*x^4*csgn(I*c)*cs
gn(I*c*x^n)^2+1/16*I/e^2*x^2*Pi*b*csgn(I*c*x^n)^3+25/96*I/e^4*Pi*b*csgn(I*c
)*csgn(I*c*x^n)^2-1/8*I/e^3*x*Pi*b*csgn(I*c*x^n)^3+1/32*I*Pi*b*x^4*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)-1/16*I/e^2*x^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2
+1/8*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x+1)*x^4
```

Maxima [A]
time = 0.33, size = 223, normalized size = 1.06

$$-\frac{1}{4}\left(\log\left(xe+1\right)\log\left(x\right)+\mathrm{Li}_2(-xe)\right)bne^{(-4)}+\frac{1}{16}\left(b(n-4\log\left(c\right))-4a\right)e^{(-4)}\log\left(xe+1\right)+\frac{1}{288}\left(9\left(b(n-2\log\left(c\right))-2a\right)x^4e^4-2\left(b(7n-12\log\left(c\right))-12a\right)x^3e^3+9\left(b(3n-4\log\left(c\right))-4a\right)x^2e^2-18\left(b(5n-4\log\left(c\right))-4a\right)xe-18\left(\left(b(n-4\log\left(c\right))-4a\right)x^4e^4-4bn\log\left(x\right)\right)\log\left(xe+1\right)-6\left(3bx^4e^4-4bx^3e^3+6bx^2e^2-12bxe-12\left(bx^4e^4-b\right)\log\left(xe+1\right)\right)\log\left(x^n\right)\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^(-4) + 1/16*(b*(n - 4*log(c)
) - 4*a)*e^(-4)*log(x*e + 1) + 1/288*(9*(b*(n - 2*log(c)) - 2*a)*x^4*e^4 -
2*(b*(7*n - 12*log(c)) - 12*a)*x^3*e^3 + 9*(b*(3*n - 4*log(c)) - 4*a)*x^2*e
^2 - 18*(b*(5*n - 4*log(c)) - 4*a)*x*e - 18*((b*(n - 4*log(c)) - 4*a)*x^4*e
```

```
^4 - 4*b*n*log(x))*log(x*e + 1) - 6*(3*b*x^4*e^4 - 4*b*x^3*e^3 + 6*b*x^2*e^
2 - 12*b*x*e - 12*(b*x^4*e^4 - b)*log(x*e + 1))*log(x^n))*e^(-4)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
[Out] integral(b*x^3*log(c*x^n)*log(x*e + 1) + a*x^3*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(e*x+1),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln(e\,x+1)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n)),x)
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

3.3 $\int x^2(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal. Leaf size=178

$$\frac{4bnx}{9e^2}-\frac{5bnx^2}{36e}+\frac{2}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))-\frac{bn\log(1+ex)}{9e^3}-\frac{1}{9}bn$$

```
[Out] 4/9*b*n*x/e^2-5/36*b*n*x^2/e+2/27*b*n*x^3-1/3*x*(a+b*ln(c*x^n))/e^2+1/6*x^2
*(a+b*ln(c*x^n))/e-1/9*x^3*(a+b*ln(c*x^n))-1/9*b*n*ln(e*x+1)/e^3-1/9*b*n*x^
3*ln(e*x+1)+1/3*(a+b*ln(c*x^n))*ln(e*x+1)/e^3+1/3*x^3*(a+b*ln(c*x^n))*ln(e*
x+1)+1/3*b*n*polylog(2,-e*x)/e^3
```

Rubi [A]
time = 0.07, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2442, 45, 2423, 2438}

$$\frac{bn\text{PolyLog}(2,-ex)}{3e^3}+\frac{\log(ex+1)(a+b\log(cx^n))}{3e^3}-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{1}{3}x^3\log(ex+1)(a+b\log(cx^n))+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))-\frac{bn\log(ex+1)}{9e^3}+\frac{4bnx}{9e^2}-\frac{1}{9}bnx^3\log(ex+1)-\frac{5bnx^2}{36e}+\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (4*b*n*x)/(9*e^2) - (5*b*n*x^2)/(36*e) + (2*b*n*x^3)/27 - (x*(a + b*Log[c*x
^n]))/(3*e^2) + (x^2*(a + b*Log[c*x^n]))/(6*e) - (x^3*(a + b*Log[c*x^n]))/9
 - (b*n*Log[1 + e*x])/(9*e^3) - (b*n*x^3*Log[1 + e*x])/9 + ((a + b*Log[c*x^
n])*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/3 + (b*n*
PolyLog[2, -(e*x)])/(3*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))\log(1+ex)\,dx = -\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))+\frac{(a}{}$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}$$

$$=\frac{bnx}{3e^2}-\frac{bnx^2}{12e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-\frac{1}{9}$$

$$=\frac{4bnx}{9e^2}-\frac{5bnx^2}{36e}+\frac{2}{27}bnx^3-\frac{x(a+b\log(cx^n))}{3e^2}+\frac{x^2(a+b\log(cx^n))}{6e}-$$

Mathematica [A]
time = 0.06, size = 161, normalized size = 0.90

$$\frac{-36aex+48benx+18ae^2x^2-15be^2nx^2-12ae^3x^3+8be^3nx^3+36a\log(1+ex)-12bn\log(1+ex)+36ae^3x^3\log(1+ex)-12be^3nx^3\log(1+ex)+6b\log(cx^n)\left(ex(-6+3ex-2e^2x^2)+6(1+e^3x^3)\log(1+ex)\right)+36bn\text{Li}_2(-ex)}{108e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-36*a*e*x + 48*b*e*n*x + 18*a*e^2*x^2 - 15*b*e^2*n*x^2 - 12*a*e^3*x^3 + 8*
b*e^3*n*x^3 + 36*a*Log[1 + e*x] - 12*b*n*Log[1 + e*x] + 36*a*e^3*x^3*Log[1
+ e*x] - 12*b*e^3*n*x^3*Log[1 + e*x] + 6*b*Log[c*x^n]*(e*x*(-6 + 3*e*x - 2*
e^2*x^2) + 6*(1 + e^3*x^3)*Log[1 + e*x]) + 36*b*n*PolyLog[2, -(e*x)])/(108*
e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 870, normalized size = 4.89

method	result	size
risch	Expression too large to display	870

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(e*x+1)*a-11/18*a/e^3+1/3*b*ln(c)*ln(e*x+1)*x^3+1/3/e^3*ln(e*x+1)
*b*ln(c)+1/3*a/e^3*ln(e*x+1)+1/6*a/e*x^2-1/9*ln(c)*b*x^3-1/9*x^3*a+(1/3*x^3
*b*ln(e*x+1)+1/18*b*(-2*e^3*x^3+3*e^2*x^2-6*e*x+6*ln(e*x+1))/e^3)*ln(x^n)-1
/3*b*ln(c)*x/e^2+2/27*b*n*x^3-1/18*I*Pi*b*x^3*csgn(I*c)*csgn(I*c*x^n)^2+1/6
*I/e^2*x*Pi*b*csgn(I*c*x^n)^3+71/108/e^3*b*n+1/6*b*ln(c)/e*x^2+11/36*I/e^3*
Pi*b*csgn(I*c*x^n)^3+1/18*I*Pi*b*x^3*csgn(I*c*x^n)^3-1/12*I/e*x^2*Pi*b*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-11/18/e^3*b*ln(c)-1/6*I*Pi*b*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)*ln(e*x+1)*x^3-1/6*I/e^3*ln(e*x+1)*Pi*b*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+1/6*I/e^2*x*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+
1/3/e^3*b*n*dilog(e*x+1)-5/36*b*n*x^2/e+4/9*b*n*x/e^2-1/6*I/e^2*x*Pi*b*csgn
(I*c)*csgn(I*c*x^n)^2-1/6*I/e^2*x*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/9*b*n*
ln(e*x+1)/e^3-1/9*b*n*x^3*ln(e*x+1)-1/3*a*x/e^2+1/12*I/e*x^2*Pi*b*csgn(I*x^
n)*csgn(I*c*x^n)^2+11/36*I/e^3*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/1
2*I/e*x^2*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/6*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+
1)*x^3-11/36*I/e^3*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-11/36*I/e^3*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)^2-1/6*I/e^3*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^3-1/18*I*Pi*b*x^
3*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*I/e*x^2*Pi*b*csgn(I*c*x^n)^3+1/6*I*Pi*b*
csgn(I*c)*csgn(I*c*x^n)^2*ln(e*x+1)*x^3+1/18*I*Pi*b*x^3*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)+1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x+1)*x^3+1/6*I
/e^3*ln(e*x+1)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I/e^3*ln(e*x+1)*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [A]
time = 0.32, size = 194, normalized size = 1.09

$$\frac{1}{3}\left(\log\left(xe+1\right)\log\left(x\right)+\mathrm{Li}_2(-xe)\right)bne^{(-3)}-\frac{1}{9}\left(b(n-3\log\left(c\right))-3a\right)e^{(-3)}\log\left(xe+1\right)+\frac{1}{108}\left(4\left(b(2n-3\log\left(c\right))-3a\right)x^3e^3-3\left(b(5n-6\log\left(c\right))-6a\right)x^2e^2+12\left(b(4n-3\log\left(c\right))-3a\right)xe-12\left(\left(b(n-3\log\left(c\right))-3a\right)x^3e^3+3bn\log\left(x\right)\right)\log\left(xe+1\right)-6\left(2bx^3e^3-3bx^2e^2+6bxe-6\left(bx^3e^3+b\right)\log\left(xe+1\right)\right)\log\left(x^n\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^(-3) - 1/9*(b*(n - 3*log(c))
- 3*a)*e^(-3)*log(x*e + 1) + 1/108*(4*(b*(2*n - 3*log(c)) - 3*a)*x^3*e^3 -
3*(b*(5*n - 6*log(c)) - 6*a)*x^2*e^2 + 12*(b*(4*n - 3*log(c)) - 3*a)*x*e -
12*((b*(n - 3*log(c)) - 3*a)*x^3*e^3 + 3*b*n*log(x))*log(x*e + 1) - 6*(2*b*
x^3*e^3 - 3*b*x^2*e^2 + 6*b*x*e - 6*(b*x^3*e^3 + b)*log(x*e + 1))*log(x^n))
*e^(-3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*log(c*x^n)*log(x*e + 1) + a*x^2*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

3.4 $\int x(a+b\log(cx^n))\log(1+ex)\,dx$

Optimal. Leaf size=146

$$-\frac{3bnx}{4e}+\frac{1}{4}bnx^2+\frac{x(a+b\log(cx^n))}{2e}-\frac{1}{4}x^2(a+b\log(cx^n))+\frac{bn\log(1+ex)}{4e^2}-\frac{1}{4}bnx^2\log(1+ex)-\frac{(a+b\log(cx$$

```
[Out] -3/4*b*n*x/e+1/4*b*n*x^2+1/2*x*(a+b*ln(c*x^n))/e-1/4*x^2*(a+b*ln(c*x^n))+1/
4*b*n*ln(e*x+1)/e^2-1/4*b*n*x^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/e^2
+1/2*x^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*b*n*polylog(2,-e*x)/e^2
```

Rubi [A]
time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2442, 45, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-ex)}{2e^2}-\frac{\log(ex+1)(a+b\log(cx^n))}{2e^2}+\frac{x(a+b\log(cx^n))}{2e}+\frac{1}{2}x^2\log(ex+1)(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n))+\frac{bn\log(ex+1)}{4e^2}-\frac{1}{4}bnx^2\log(ex+1)-\frac{3bnx}{4e}+\frac{1}{4}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-3*b*n*x)/(4*e) + (b*n*x^2)/4 + (x*(a + b*Log[c*x^n]))/(2*e) - (x^2*(a + b
*Log[c*x^n]))/4 + (b*n*Log[1 + e*x])/(4*e^2) - (b*n*x^2*Log[1 + e*x])/4 - (
(a + b*Log[c*x^n])*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 +
e*x])/2 - (b*n*PolyLog[2, -(e*x)])/(2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x(a+b\log(cx^n))\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))}{2e} - \frac{1}{4}x^2(a+b\log(cx^n)) - \frac{(a+b\log(cx^n))\log(1+ex)}{2e^2}$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x(a+b\log(cx^n))}{2e} - \frac{1}{4}x^2(a+b\log(cx^n)) - \frac{(a+b\log}{}$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x(a+b\log(cx^n))}{2e} - \frac{1}{4}x^2(a+b\log(cx^n)) - \frac{1}{4}bnx^2\log$$

$$= -\frac{bnx}{2e} + \frac{1}{8}bnx^2 + \frac{x(a+b\log(cx^n))}{2e} - \frac{1}{4}x^2(a+b\log(cx^n)) - \frac{1}{4}bnx^2\log$$

$$= -\frac{3bnx}{4e} + \frac{1}{4}bnx^2 + \frac{x(a+b\log(cx^n))}{2e} - \frac{1}{4}x^2(a+b\log(cx^n)) + \frac{bn\log(}{4e}$$

Mathematica [A]
time = 0.05, size = 131, normalized size = 0.90

$$\frac{2aex - 3benx - ae^2x^2 + be^2nx^2 - 2a\log(1+ex) + bn\log(1+ex) + 2ae^2x^2\log(1+ex) - be^2nx^2\log(1+ex) + b\log(cx^n)(ex(2-ex) + 2(-1+e^2x^2)\log(1+ex)) - 2bn\text{Li}_2(-ex)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (2*a*e*x - 3*b*e*n*x - a*e^2*x^2 + b*e^2*n*x^2 - 2*a*Log[1 + e*x] + b*n*Log
[1 + e*x] + 2*a*e^2*x^2*Log[1 + e*x] - b*e^2*n*x^2*Log[1 + e*x] + b*Log[c*x
^n]*(e*x*(2 - e*x) + 2*(-1 + e^2*x^2)*Log[1 + e*x]) - 2*b*n*PolyLog[2, -(e*
x)])/(4*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 725, normalized size = 4.97

method	result
risch	$\frac{x^2\ln(ex+1)a}{2} + \frac{i\pi b\,x^2\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)}{8} - \frac{i\ln(ex+1)\pi b\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2}{4e^2} + \frac{3a}{4e^2} - \frac{\ln(ex+1)b\ln(c)}{2e^2} + \frac{b\ln(c)\ln(}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(e*x+1)*a+3/4*a/e^2-1/2/e^2*ln(e*x+1)*b*ln(c)+1/2*b*ln(c)*ln(e*x+
1)*x^2-1/2*a/e^2*ln(e*x+1)-1/4*ln(c)*b*x^2+(1/2*b*x^2*ln(e*x+1)-1/4*b*(e^2*
x^2-2*e*x+2*ln(e*x+1))/e^2)*ln(x^n)-1/4*x^2*a-1/2/e^2*b*n*dilog(e*x+1)-1/4*
I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x+1)*x^2+1/4*I/e^2*ln(e*x+1
)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*b*n*x^2-3/8*I/e^2*Pi*b*csgn(
I*c*x^n)^3+1/8*I*Pi*b*x^2*csgn(I*c*x^n)^3+3/8*I/e^2*Pi*b*csgn(I*c)*csgn(I*c
*x^n)^2+3/8*I/e^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*x/e*Pi*b*csgn(I*c)
*csgn(I*x^n)*csgn(I*c*x^n)-1/e^2*b*n+1/2*b*ln(c)/e*x+1/2*a*x/e-3/4*b*n*x/e+
3/4/e^2*b*ln(c)-1/4*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)*x^2+1/8*I*Pi*b*x^2*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*b*n*ln(e*x+1)/e^2-1/4*b*n*x^2*ln(e*x+1
)-1/4*I/e^2*ln(e*x+1)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I/e^2*ln(e*x+1)*Pi
*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*
x+1)*x^2+1/4*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(e*x+1)*x^2-1/4*I*x/e*Pi*b*
csgn(I*c*x^n)^3+1/4*I/e^2*ln(e*x+1)*Pi*b*csgn(I*c*x^n)^3-1/8*I*Pi*b*x^2*csg
n(I*c)*csgn(I*c*x^n)^2-1/8*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*x/e
*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*x/e*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-
3/8*I/e^2*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
```

Maxima [**A**]
time = 0.34, size = 163, normalized size = 1.12

$$-\frac{1}{2}\left(\log\left(xe+1\right)\log\left(x\right)+\mathrm{Li}_2(-xe)\right)bne^{(-2)}+\frac{1}{4}\left(b(n-2\log\left(c\right))-2a\right)e^{(-2)}\log\left(xe+1\right)+\frac{1}{4}\left(\left(b(n-\log\left(c\right))-a\right)x^2e^2-\left(b(3n-2\log\left(c\right))-2a\right)xe-\left(\left(b(n-2\log\left(c\right))-2a\right)x^2e^2-2bn\log\left(x\right)\right)\log\left(xe+1\right)-\left(bx^2e^2-2bxe-2\left(bx^2e^2-b\right)\log\left(xe+1\right)\right)\log\left(x^n\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^(-2) + 1/4*(b*(n - 2*log(c))
 - 2*a)*e^(-2)*log(x*e + 1) + 1/4*((b*(n - log(c)) - a)*x^2*e^2 - (b*(3*n -
 2*log(c)) - 2*a)*x*e - ((b*(n - 2*log(c)) - 2*a)*x^2*e^2 - 2*b*n*log(x))*l
og(x*e + 1) - (b*x^2*e^2 - 2*b*x*e - 2*(b*x^2*e^2 - b)*log(x*e + 1))*log(x^
n))*e^(-2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*x*log(c*x^n)*log(x*e + 1) + a*x*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log(x*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n)), x)
```

3.5 $\int (a+b\log(cx^n))\log(1+ex)\,dx$

Optimal. Leaf size=74

$$2bnx-x(a+b\log(cx^n))-\frac{bn(1+ex)\log(1+ex)}{e}+\frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e}+\frac{bn\mathrm{Li}_2(-ex)}{e}$$

```
[Out] 2*b*n*x-x*(a+b*ln(c*x^n))-b*n*(e*x+1)*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))*l
n(e*x+1)/e+b*n*polylog(2,-e*x)/e
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2436, 2332, 2417, 2458, 45, 2393, 2352}

$$\frac{bn\text{PolyLog}(2,-ex)}{e}+\frac{(ex+1)\log(ex+1)(a+b\log(cx^n))}{e}-x(a+b\log(cx^n))-\frac{bn(ex+1)\log(ex+1)}{e}+2bnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] 2*b*n*x - x*(a + b*Log[c*x^n]) - (b*n*(1 + e*x)*Log[1 + e*x])/e + ((1 + e*x
)*(a + b*Log[c*x^n])*Log[1 + e*x])/e + (b*n*PolyLog[2, -(e*x)])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
```

```
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\int (a+b\log(cx^n))\log(1+ex)\,dx = -x(a+b\log(cx^n)) + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} - (bn)\int \Big(\cdots$$

$$= bnx - x(a+b\log(cx^n)) + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} - \frac{(bn)\cdots}{\cdots}$$

$$= bnx - x(a+b\log(cx^n)) + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} - \frac{(bn)\cdots}{\cdots}$$

$$= bnx - x(a+b\log(cx^n)) + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} - \frac{(bn)\cdots}{\cdots}$$

$$= bnx - x(a+b\log(cx^n)) + \frac{(1+ex)(a+b\log(cx^n))\log(1+ex)}{e} - \frac{(bn)\cdots}{\cdots}$$

$$= 2bnx - x(a+b\log(cx^n)) - \frac{bn(1+ex)\log(1+ex)}{e} + \frac{(1+ex)(a+b\log\cdots}{\cdots}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 1.22

$$\frac{-aex+2benx+a\log(1+ex)-bn\log(1+ex)+aex\log(1+ex)-benx\log(1+ex)+b\log\left(cx^n\right)\left(-ex+(1+ex)\log(1+ex)\right)+bn\text{Li}_2(-ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[1 + e*x],x]
```

```
[Out] (-(a*e*x) + 2*b*e*n*x + a*Log[1 + e*x] - b*n*Log[1 + e*x] + a*e*x*Log[1 + e
*x] - b*e*n*x*Log[1 + e*x] + b*Log[c*x^n]*(-(e*x) + (1 + e*x)*Log[1 + e*x])
 + b*n*PolyLog[2, -(e*x)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 557, normalized size = 7.53

method	result
risch	$x\ln\left(ex+1\right)a-\frac{bn\ln(ex+1)}{e}+\frac{a\ln(ex+1)}{e}-\ln\left(c\right)bx-\frac{i\pi b\,\text{csgn}(ix^n)\text{csgn}(ic\,x^n)^2}{2e}-\frac{i\pi b\,\text{csgn}(ic)\text{csgn}(ic\,x^n)^2 x}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(e*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(e*x+1)*a-1/e*b*n*ln(e*x+1)+a/e*ln(e*x+1)-ln(c)*b*x+1/2*I/e*ln(e*x+1)*P
i*b*csgn(I*c)*csgn(I*c*x^n)^2-1/e*b*ln(c)-1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*c*x
^n)^3+2*b*n*x-1/2*I/e*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I/e*ln(e*x+1)*Pi*b
*csgn(I*c*x^n)^3-a/e-1/2*I/e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*cs
gn(I*c)*csgn(I*c*x^n)^2*x+2/e*b*n-1/2*I/e*ln(e*x+1)*Pi*b*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)-1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)-n*b*x*ln(e*x+1)-a*x-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*x+ln(e*x+1)*ln
(c)*x*b+1/e*ln(e*x+1)*b*ln(c)+(b*x*ln(e*x+1)+b*(-e*x+ln(e*x+1))/e)*ln(x^n)+
1/e*b*n*dilog(e*x+1)+1/2*I/e*Pi*b*csgn(I*c*x^n)^3+1/2*I*Pi*b*csgn(I*c*x^n)^
3*x+1/2*I*ln(e*x+1)*Pi*x*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I/e*Pi*b*csgn(I*c)
*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I/e*ln(e*x+1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)
^2+1/2*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x+1/2*I*ln(e*x+1)*Pi*x*b*
csgn(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [A]
time = 0.33, size = 127, normalized size = 1.72

$$(\log(xe+1)\log(x)+\text{Li}_2(-xe))bne^{(-1)}-(b(n-\log(c))-a)e^{(-1)}\log(xe+1)+((b(2n-\log(c))-a)xe-((b(n-\log(c))-a)xe+bn\log(x))\log(xe+1)-(bxe-(bxe+b)\log(xe+1))\log(x^n))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="maxima")
```

```
[Out] (log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^(-1) - (b*(n - log(c)) - a)*e^(-1
)*log(x*e + 1) + ((b*(2*n - log(c)) - a)*x*e - ((b*(n - log(c)) - a)*x*e +
```

```
b*n*log(x))*log(x*e + 1) - (b*x*e - (b*x*e + b)*log(x*e + 1))*log(x^n))*e^(
-1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b*log(c*x^n)*log(x*e + 1) + a*log(x*e + 1), x)
```

Sympy [A]
time = 176.72, size = 194, normalized size = 2.62

$$a\left(\begin{cases} 0 & \text{for}\: e = 0 \\ x \log{\left(e x + 1 \right)} - x + \frac{\log{\left(e x + 1 \right)}}{e} - \frac{1}{e} & \text{otherwise} \end{cases}\right) - b e^{2} n \left(\begin{cases} \frac{x}{e^{2}} - \frac{\log{\left(e x + 1 \right)}}{e^{3}} & \text{for}\: e = 0 \\ \frac{\log{\left(e x + 1 \right)}^{2}}{2 e^{3}} & \text{otherwise} \end{cases}\right) - b n x \log{\left(e x + 1 \right)} + 2 b n x - b n \left(\begin{cases} 0 & \text{for}\: e = 0 \\ \frac{\log{\left(e x + 1 \right)}^{2}}{2 e} & \text{otherwise} \end{cases}\right) + b n \left(\begin{cases} x & \text{for}\: e = 0 \\ \frac{\log{\left(e x + 1 \right)}}{e} & \text{otherwise} \end{cases}\right) \log{\left(e x + 1 \right)} - b n \left(\begin{cases} x & \text{for}\: e = 0 \\ \frac{\log{\left(e x + 1 \right)}}{e} & \text{otherwise} \end{cases}\right) - b n \left(\begin{cases} x & \text{for}\: e = 0 \\ - \frac{\operatorname{Li}_{2}\left(e x e^{i \pi}\right)}{e} & \text{otherwise} \end{cases}\right) + b x \log{\left(c x^{n} \right)} \log{\left(e x + 1 \right)} - b x \log{\left(c x^{n} \right)} + b \left(\begin{cases} x & \text{for}\: e = 0 \\ \frac{\log{\left(e x + 1 \right)}}{e} & \text{otherwise} \end{cases}\right) \log{\left(c x^{n} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1),x)
```

```
[Out] a*Piecewise((0, Eq(e, 0)), (x*log(e*x + 1) - x + log(e*x + 1)/e - 1/e, True
)) - b*e**2*n*Piecewise((x/e**2 - log(e*x + 1)/e**3, Eq(e, 0)), (log(e*x +
1)**2/(2*e**3), True)) - b*n*x*log(e*x + 1) + 2*b*n*x - b*n*Piecewise((0, E
q(e, 0)), (log(e*x + 1)**2/(2*e), True)) + b*n*Piecewise((x, Eq(e, 0)), (lo
g(e*x + 1)/e, True))*log(e*x + 1) - b*n*Piecewise((x, Eq(e, 0)), (log(e*x +
 1)/e, True)) - b*n*Piecewise((x, Eq(e, 0)), (-polylog(2, e*x*exp_polar(I*p
i))/e, True)) + b*x*log(c*x**n)*log(e*x + 1) - b*x*log(c*x**n) + b*Piecewis
e((x, Eq(e, 0)), (log(e*x + 1)/e, True))*log(c*x**n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(e*x + 1)*(a + b*log(c*x^n)), x)
```

3.6 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx$

Optimal. Leaf size=28

$$-((a+b\log(cx^n))\operatorname{Li}_2(-ex))+bn\operatorname{Li}_3(-ex)$$

```
[Out] -(a+b*ln(c*x^n))*polylog(2,-e*x)+b*n*polylog(3,-e*x)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2421, 6724}

$$bn\text{PolyLog}(3,-ex)-\text{PolyLog}(2,-ex)\,(a+b\log(cx^n))$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])*PolyLog[2, -(e*x)]) + b*n*PolyLog[3, -(e*x)]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(1+ex)}{x}\,dx &= -(a+b\log(cx^n))\operatorname{Li}_2(-ex)+(bn)\int \frac{\operatorname{Li}_2(-ex)}{x}\,dx\\ &= -(a+b\log(cx^n))\operatorname{Li}_2(-ex)+bn\operatorname{Li}_3(-ex)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.21

$$-a\operatorname{Li}_2(-ex)-b\log(cx^n)\operatorname{Li}_2(-ex)+bn\operatorname{Li}_3(-ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x,x]
```

```
[Out] -(a*PolyLog[2, -(e*x)]) - b*Log[c*x^n]*PolyLog[2, -(e*x)] + b*n*PolyLog[3,
-(e*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 143, normalized size = 5.11

method	result
risch	$-\ln(x)\operatorname{polylog}(2,-ex)\,bn+\ln(x)\operatorname{dilog}(ex+1)\,bn-\ln(x^n)\operatorname{dilog}(ex+1)\,b+bn\operatorname{polylog}(3,-e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)*polylog(2,-e*x)*b*n+ln(x)*dilog(e*x+1)*b*n-ln(x^n)*dilog(e*x+1)*b+b*
n*polylog(3,-e*x)-(-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*b
*Pi*csgn(I*c*x^n)^3+b*ln(c)+a)*dilog(e*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(x*e + 1) + a*log(x*e + 1))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x, x)
```

3.7 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx$

Optimal. Leaf size=107

$$ben\log(x)-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-ben\log(1+ex)-\frac{bn\log(1+ex)}{x}-e(a+b\log\left(cx^n\right))\log(1-$$

```
[Out] b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-b*e*n*ln(e*x+1)-b*n*l
n(e*x+1)/x-e*(a+b*ln(c*x^n))*ln(e*x+1)-(a+b*ln(c*x^n))*ln(e*x+1)/x-b*e*n*po
lylog(2,-e*x)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2442, 36, 29, 31, 2423, 2338, 2438}

$$-ben\text{PolyLog}(2,-ex)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(ex+1)\left(a+b\log\left(cx^n\right)\right)-\frac{\log(ex+1)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{1}{2}ben\log^2(x)+ben\log(x)-ben\log(ex+1)-\frac{bn\log(ex+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]
```

```
[Out] b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - b*e*n*Log
[1 + e*x] - (b*n*Log[1 + e*x])/x - e*(a + b*Log[c*x^n])*Log[1 + e*x] - ((a
+ b*Log[c*x^n])*Log[1 + e*x])/x - b*e*n*PolyLog[2, -(e*x)]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^2}\,dx = e\log(x)\,(a+b\log(cx^n)) - e(a+b\log(cx^n))\log(1+ex) - \frac{(a+b\log(}{}$$

$$= e\log(x)\,(a+b\log(cx^n)) - e(a+b\log(cx^n))\log(1+ex) - \frac{(a+b\log(}{}$$

$$= -\frac{1}{2}ben\log^2(x) + e\log(x)\,(a+b\log(cx^n)) - \frac{bn\log(1+ex)}{x} - e(a+b$$

$$= -\frac{1}{2}ben\log^2(x) + e\log(x)\,(a+b\log(cx^n)) - \frac{bn\log(1+ex)}{x} - e(a+b$$

$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)\,(a+b\log(cx^n)) - ben\log(1+ex$$

Mathematica [A]
time = 0.05, size = 69, normalized size = 0.64

$$-\frac{1}{2}ben\log^2(x) + e\log(x)\,(a+bn+b\log(cx^n)) - \frac{(1+ex)\,(a+bn+b\log(cx^n))\log(1+ex)}{x} - ben\mathrm{Li}_2(-ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^2,x]
```

```
[Out] -1/2*(b*e*n*Log[x]^2) + e*Log[x]*(a + b*n + b*Log[c*x^n]) - ((1 + e*x)*(a +
 b*n + b*Log[c*x^n])*Log[1 + e*x])/x - b*e*n*PolyLog[2, -(e*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 481, normalized size = 4.50

method	result
risch	$\left(-\frac{b\ln(ex+1)}{x}+be\ln(x)-be\ln(ex+1)\right)\ln(x^n)-\frac{ben\ln(x)^2}{2}+nbe\ln(ex)-ben\ln(ex+1)-\frac{bn\ln(e}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-b/x*ln(e*x+1)+b*e*ln(x)-b*e*ln(e*x+1))*ln(x^n)-1/2*b*e*n*ln(x)^2+n*b*e*ln
(e*x)-b*e*n*ln(e*x+1)-b*n*ln(e*x+1)/x-e*b*n*dilog(e*x+1)-1/2*I*Pi*b*csgn(I*
c)*csgn(I*c*x^n)^2*ln(e*x+1)/x-1/2*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(
e*x+1)+1/2*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)/x+1/2*I*e*Pi*b*csgn(I*c)*csgn(I
*c*x^n)^2*ln(e*x)+1/2*I*e*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x+1
)+1/2*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x)+1/2*I*Pi*b*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)*ln(e*x+1)/x-1/2*I*e*Pi*b*csgn(I*c*x^n)^3*ln(e*x)+1/2
*I*e*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)-1/2*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*
ln(e*x+1)/x-1/2*I*e*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(e*x+1)-1/2*I*e*Pi*b*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x)+e*b*ln(c)*ln(e*x)-e*b*ln(c)*ln(e
*x+1)-b*ln(c)*ln(e*x+1)/x+a*e*ln(e*x)-a*e*ln(e*x+1)-ln(e*x+1)/x*a
```

Maxima [A]
time = 0.32, size = 130, normalized size = 1.21

$$-(\log(xe+1)\log(x)+\mathrm{Li}_2(-xe))bne-(b(n+\log(c))+a)e\log(xe+1)+(b(n+\log(c))+a)e\log(x)-\frac{bnxe\log(x)^2-2\,(bnxe\log(x)-b(n+\log(c))-a)\log(xe+1)-2\,(bxe\log(x)-(bxe+b)\log(xe+1))\log(x^n)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e - (b*(n + log(c)) + a)*e*log(x*e
 + 1) + (b*(n + log(c)) + a)*e*log(x) - 1/2*(b*n*x*e*log(x)^2 - 2*(b*n*x*e*
log(x) - b*(n + log(c)) - a)*log(x*e + 1) - 2*(b*x*e*log(x) - (b*x*e + b)*l
og(x*e + 1))*log(x^n))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(x*e + 1) + a*log(x*e + 1))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log(cx^n))\log(ex + 1)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**2,x)
[Out] Integral((a + b*log(c*x**n))*log(e*x + 1)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(e\,x + 1)\,\left(a + b\,\ln(c\,x^n)\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2,x)
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^2, x)
```

3.8 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{3ben}{4x}-\frac{1}{4}be^2n\log(x)+\frac{1}{4}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)\left(a+b\log(cx^n)\right)+\frac{1}{4}be^2n\log(1+ex)-\frac{bn}{}$$

```
[Out] -3/4*b*e*n/x-1/4*b*e^2*n*ln(x)+1/4*b*e^2*n*ln(x)^2-1/2*e*(a+b*ln(c*x^n))/x-
1/2*e^2*ln(x)*(a+b*ln(c*x^n))+1/4*b*e^2*n*ln(e*x+1)-1/4*b*n*ln(e*x+1)/x^2+1
/2*e^2*(a+b*ln(c*x^n))*ln(e*x+1)-1/2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2+1/2*b*e^
2*n*polylog(2,-e*x)
```

Rubi [A]

time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 46, 2423, 2338, 2438}

$$\frac{1}{2}be^2n\text{PolyLog}(2,-ex)-\frac{1}{2}e^2\log(x)\left(a+b\log(cx^n)\right)+\frac{1}{2}e^2\log(ex+1)\left(a+b\log(cx^n)\right)-\frac{e(a+b\log(cx^n))}{2x}-\frac{\log(ex+1)\left(a+b\log(cx^n)\right)}{2x^2}+\frac{1}{4}be^2n\log^2(x)-\frac{1}{4}be^2n\log(x)+\frac{1}{4}be^2n\log(ex+1)-\frac{bn\log(ex+1)}{4x^2}-\frac{3ben}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

```
[Out] (-3*b*e*n)/(4*x) - (b*e^2*n*Log[x])/4 + (b*e^2*n*Log[x]^2)/4 - (e*(a + b*Lo
g[c*x^n]))/(2*x) - (e^2*Log[x]*(a + b*Log[c*x^n]))/2 + (b*e^2*n*Log[1 + e*x
])/4 - (b*n*Log[1 + e*x])/(4*x^2) + (e^2*(a + b*Log[c*x^n])*Log[1 + e*x])/2
 - ((a + b*Log[c*x^n])*Log[1 + e*x])/(2*x^2) + (b*e^2*n*PolyLog[2, -(e*x)])
/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^3}\,dx &= -\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)(a+b\log(cx^n))+\frac{1}{2}e^2(a+b\log(cx^n \\
&= -\frac{ben}{2x}-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)(a+b\log(cx^n))+\frac{1}{2}e^2(a+b\l \\
&= -\frac{ben}{2x}+\frac{1}{4}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)(a+b\log(cx \\
&= -\frac{ben}{2x}+\frac{1}{4}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log(x)(a+b\log(cx \\
&= -\frac{3ben}{4x}-\frac{1}{4}be^2n\log(x)+\frac{1}{4}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{2x}-\frac{1}{2}e^2\log
\end{aligned}
$$

Mathematica [A]

time = 0.06, size = 215, normalized size = 1.32

$$-\frac{1}{4}be^2\log(x)(n+2(-n\log(x)+\log(cx^n)))+\frac{b(-en-2e(-n\log(x)+\log(cx^n)))}{4x}-\frac{a\log(1+ex)}{2x^2}+\frac{1}{4}be^2(n+2(-n\log(x)+\log(cx^n)))\log(1+ex)-\frac{b(n+2n\log(x)+2(-n\log(x)+\log(cx^n)))\log(1+ex)}{4x^2}+\frac{1}{2}ae\left(-\frac{1}{x}-e\log(x)+e\log(1+ex)\right)+\frac{1}{2}ben\left(-\frac{1}{x}-\frac{\log(x)}{x}-\frac{1}{2}e\log^2(x)+e^2\left(\frac{\log(x)\log(1+ex)}{e}+\frac{\mathrm{Li}_2(-ex)}{e}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/4*(b*e^2*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n]))) + (b*(-(e*n) - 2*e*(
-(n*Log[x]) + Log[c*x^n])))/(4*x) - (a*Log[1 + e*x])/(2*x^2) + (b*e^2*(n +
2*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/4 - (b*(n + 2*n*Log[x] + 2*(-(n
*Log[x]) + Log[c*x^n]))*Log[1 + e*x])/(4*x^2) + (a*e*(-x^(-1) - e*Log[x] +
e*Log[1 + e*x]))/2 + (b*e*n*(-x^(-1) - Log[x]/x - (e*Log[x]^2)/2 + e^2*((Lo
g[x]*Log[1 + e*x])/e + PolyLog[2, -(e*x)]/e)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 647, normalized size = 3.97

method	result
risch	$-\frac{\ln(ex+1)a}{2x^2}-\frac{e^2b\ln(c)\ln(ex)}{2}-\frac{b\ln(c)\ln(ex+1)}{2x^2}-\frac{eb\ln(c)}{2x}+\frac{b\ln(c)e^2\ln(ex+1)}{2}-\frac{a\,e^2\ln(ex)}{2}+\frac{a\,e^2\ln(ex+1)}{2}-\frac{ae}{2x}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(e*x+1)/x^2*a-1/4*I*e^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x)-1/4
*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(e*x+1)/x^2+1/4*I*Pi*b*csgn(I*c*x^n)^3*
ln(e*x+1)/x^2-1/4*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2/x+1/4*I*Pi*b*csgn(I*
x^n)*csgn(I*c*x^n)^2*e^2*ln(e*x+1)-1/4*I*e^2*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2
*ln(e*x)-1/4*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e^2*ln(e*x+1)+1/4*I
*e^2*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x)+1/4*I*Pi*b*csgn(I*c)*
csgn(I*c*x^n)^2*e^2*ln(e*x+1)+1/4*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)*ln(e*x+1)/x^2+1/4*I*e*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)/x-1/4*I*Pi
*b*csgn(I*c*x^n)^3*e^2*ln(e*x+1)+1/4*I*e^2*Pi*b*csgn(I*c*x^n)^3*ln(e*x)-1/4
*I*e*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2/x-1/2*e^2*b*ln(c)*ln(e*x)-1/2*b*ln(c)*l
n(e*x+1)/x^2-1/2*e*b*ln(c)/x+1/2*b*ln(c)*e^2*ln(e*x+1)-1/2*a*e^2*ln(e*x)+1/
2*a*e^2*ln(e*x+1)-1/2*a*e/x-1/4*n*e^2*b*ln(e*x)+(-1/2*b/x^2*ln(e*x+1)-1/2*b
*e*(e*x*ln(x)-e*ln(e*x+1)*x+1)/x)*ln(x^n)+1/2*e^2*b*n*dilog(e*x+1)-3/4*b*e*
n/x-1/4*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(e*x+1)/x^2-1/4*b*n*ln(e*x+1)/
x^2+1/4*b*e^2*n*ln(x)^2+1/4*b*e^2*n*ln(e*x+1)+1/4*I*e*Pi*b*csgn(I*c*x^n)^3/
x
```

Maxima [A]
time = 0.34, size = 178, normalized size = 1.09

$$\frac{1}{2}\left(\log\left(xe+1\right)\log\left(x\right)+\mathrm{Li}_2(-xe)\right)bne^2+\frac{1}{4}\left(b(n+2\,\log\left(c\right))+2\,a\right)e^2\log\left(xe+1\right)+\frac{bnx^2e^2\log\left(x\right)^2-\left(b(n+2\,\log\left(c\right))+2\,a\right)x^2e^2\log\left(x\right)-\left(b(3\,n+2\,\log\left(c\right))+2\,a\right)xe-\left(2\,bnx^2e^2\log\left(x\right)+b(n+2\,\log\left(c\right))+2\,a\right)\log\left(xe+1\right)-2\left(bx^2e^2\log\left(x\right)+bxe-\left(bx^2e^2-b\right)\log\left(xe+1\right)\right)\log\left(x^n\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^2 + 1/4*(b*(n + 2*log(c)) + 2
*a)*e^2*log(x*e + 1) + 1/4*(b*n*x^2*e^2*log(x)^2 - (b*(n + 2*log(c)) + 2*a)
*x^2*e^2*log(x) - (b*(3*n + 2*log(c)) + 2*a)*x*e - (2*b*n*x^2*e^2*log(x) +
b*(n + 2*log(c)) + 2*a)*log(x*e + 1) - 2*(b*x^2*e^2*log(x) + b*x*e - (b*x^2
*e^2 - b)*log(x*e + 1))*log(x^n))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*log(x*e + 1) + a*log(x*e + 1))/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,x+1\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^3, x)
```

3.9 $\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx$

Optimal. Leaf size=195

$$-\frac{5ben}{36x^2}+\frac{4be^2n}{9x}+\frac{1}{9}be^3n\log(x)-\frac{1}{6}be^3n\log^2(x)-\frac{e(a+b\log(cx^n))}{6x^2}+\frac{e^2(a+b\log(cx^n))}{3x}+\frac{1}{3}e^3\log(x)(a+b\log($$

```
[Out] -5/36*b*e*n/x^2+4/9*b*e^2*n/x+1/9*b*e^3*n*ln(x)-1/6*b*e^3*n*ln(x)^2-1/6*e*(
a+b*ln(c*x^n))/x^2+1/3*e^2*(a+b*ln(c*x^n))/x+1/3*e^3*ln(x)*(a+b*ln(c*x^n))-
1/9*b*e^3*n*ln(e*x+1)-1/9*b*n*ln(e*x+1)/x^3-1/3*e^3*(a+b*ln(c*x^n))*ln(e*x+
1)-1/3*(a+b*ln(c*x^n))*ln(e*x+1)/x^3-1/3*b*e^3*n*polylog(2,-e*x)
```

Rubi [A]
time = 0.08, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 46, 2423, 2338, 2438}

$$-\frac{1}{3}be^3n\text{PolyLog}(2,-ex)+\frac{1}{3}e^3\log(x)(a+b\log(cx^n))-\frac{1}{3}e^3\log(ex+1)(a+b\log(cx^n))+\frac{e^2(a+b\log(cx^n))}{3x}-\frac{\log(ex+1)(a+b\log(cx^n))}{3x^3}-\frac{e(a+b\log(cx^n))}{6x^2}-\frac{1}{6}be^3n\log^2(x)+\frac{1}{9}be^3n\log(x)-\frac{1}{9}be^3n\log(ex+1)+\frac{4be^2n}{9x}-\frac{bn\log(ex+1)}{9x^3}-\frac{5ben}{36x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

```
[Out] (-5*b*e*n)/(36*x^2) + (4*b*e^2*n)/(9*x) + (b*e^3*n*Log[x])/9 - (b*e^3*n*Log
[x]^2)/6 - (e*(a + b*Log[c*x^n]))/(6*x^2) + (e^2*(a + b*Log[c*x^n]))/(3*x)
+ (e^3*Log[x]*(a + b*Log[c*x^n]))/3 - (b*e^3*n*Log[1 + e*x])/9 - (b*n*Log[1
 + e*x])/(9*x^3) - (e^3*(a + b*Log[c*x^n])*Log[1 + e*x])/3 - ((a + b*Log[c*
x^n])*Log[1 + e*x])/(3*x^3) - (b*e^3*n*PolyLog[2, -(e*x)])/3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log(1+ex)}{x^4}\,dx = -\frac{e(a+b\log(cx^n))}{6x^2} + \frac{e^2(a+b\log(cx^n))}{3x} + \frac{1}{3}e^3\log(x)\,(a+b\log(cx^n)$$

$$= -\frac{ben}{12x^2} + \frac{be^2n}{3x} - \frac{e(a+b\log(cx^n))}{6x^2} + \frac{e^2(a+b\log(cx^n))}{3x} + \frac{1}{3}e^3\log(x)$$

$$= -\frac{ben}{12x^2} + \frac{be^2n}{3x} - \frac{1}{6}be^3n\log^2(x) - \frac{e(a+b\log(cx^n))}{6x^2} + \frac{e^2(a+b\log(cx}{3x}$$

$$= -\frac{ben}{12x^2} + \frac{be^2n}{3x} - \frac{1}{6}be^3n\log^2(x) - \frac{e(a+b\log(cx^n))}{6x^2} + \frac{e^2(a+b\log(cx}{3x}$$

$$= -\frac{5ben}{36x^2} + \frac{4be^2n}{9x} + \frac{1}{9}be^3n\log(x) - \frac{1}{6}be^3n\log^2(x) - \frac{e(a+b\log(cx^n))}{6x^2}$$

Mathematica [A]
time = 0.06, size = 206, normalized size = 1.06

$$-\frac{6aex+5benx-12ae^2x^2-16be^2nx^2+6be^3nx^3\log^2(x)+6bex\log(cx^n)-12be^2x^2\log(cx^n)-4e^3x^3\log(x)\,(3a+bn+3b\log(cx^n))+12a\log(1+ex)+4bn\log(1+ex)+12ae^3x^3\log(1+ex)+4be^3nx^3\log(1+ex)+12b\log(cx^n)\log(1+ex)+12be^3x^3\log(cx^n)\log(1+ex)+12be^3nx^3\mathrm{Li}_2(-ex)}{36x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[1 + e*x])/x^4,x]
```

```
[Out] -1/36*(6*a*e*x + 5*b*e*n*x - 12*a*e^2*x^2 - 16*b*e^2*n*x^2 + 6*b*e^3*n*x^3*
Log[x]^2 + 6*b*e*x*Log[c*x^n] - 12*b*e^2*x^2*Log[c*x^n] - 4*e^3*x^3*Log[x]*
(3*a + b*n + 3*b*Log[c*x^n]) + 12*a*Log[1 + e*x] + 4*b*n*Log[1 + e*x] + 12*
a*e^3*x^3*Log[1 + e*x] + 4*b*e^3*n*x^3*Log[1 + e*x] + 12*b*Log[c*x^n]*Log[1
 + e*x] + 12*b*e^3*x^3*Log[c*x^n]*Log[1 + e*x] + 12*b*e^3*n*x^3*PolyLog[2,
-(e*x)])/x^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 796, normalized size = 4.08

method	result
risch	$-\frac{\ln(ex+1)a}{3x^3}+\left(-\frac{b\ln(ex+1)}{3x^3}-\frac{be(2e^2\ln(ex+1)x^2-2e^2\ln(x)x^2-2ex+1)}{6x^2}\right)\ln\left(x^n\right)+\frac{i\pi b\operatorname{csgn}(ic\,x^n)^3\ln(ex+1)}{6x^3}-\frac{i\pi b\operatorname{csgn}(}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(e*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(e*x+1)/x^3*a+(-1/3*b/x^3*ln(e*x+1)-1/6*b*e*(2*e^2*ln(e*x+1)*x^2-2*e
^2*ln(x)*x^2-2*e*x+1)/x^2)*ln(x^n)+1/6*I*e^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)
^2*ln(e*x)-1/6*I*e^3*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x)+1/6*I
*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(e*x+1)/x^3+1/12*I*e*Pi*b*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)/x^2+1/6*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)*e^3*ln(e*x+1)-1/3*b*ln(c)*ln(e*x+1)/x^3+1/3*e^3*b*ln(c)*ln(e*x)+1/9*
n*b*e^3*ln(e*x)-1/6*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e^2/x-1/6*e*
a/x^2+1/3*e^3*a*ln(e*x)-1/3*e^3*a*ln(e*x+1)+1/3*e^2*a/x-1/6*I*Pi*b*csgn(I*x
^n)*csgn(I*c*x^n)^2*ln(e*x+1)/x^3+1/6*I*e^3*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*
ln(e*x)-1/6*I*e^3*Pi*b*csgn(I*c*x^n)^3*ln(e*x)-1/3*e^3*b*n*dilog(e*x+1)-1/1
2*I*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2/x^2-1/12*I*e*Pi*b*csgn(I*c)*csgn(I*c
*x^n)^2/x^2+1/3*b*ln(c)*e^2/x-1/6*e*b*ln(c)/x^2-1/3*b*ln(c)*e^3*ln(e*x+1)+4
/9*b*e^2*n/x-1/6*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*e^3*ln(e*x+1)+1/6*I*Pi*b*
csgn(I*x^n)*csgn(I*c*x^n)^2*e^2/x-5/36*b*e*n/x^2-1/9*b*e^3*n*ln(e*x+1)-1/9*
b*n*ln(e*x+1)/x^3+1/6*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*e^2/x-1/6*b*e^3*n*ln
(x)^2-1/6*I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*e^3*ln(e*x+1)-1/6*I*Pi*b*csgn(
I*c)*csgn(I*c*x^n)^2*ln(e*x+1)/x^3+1/6*I*Pi*b*csgn(I*c*x^n)^3*ln(e*x+1)/x^3
-1/6*I*Pi*b*csgn(I*c*x^n)^3*e^2/x+1/6*I*Pi*b*csgn(I*c*x^n)^3*e^3*ln(e*x+1)+
1/12*I*e*Pi*b*csgn(I*c*x^n)^3/x^2
```

Maxima [A]
time = 0.34, size = 208, normalized size = 1.07

$$-\frac{1}{3}\left(\log\left(xe+1\right)\log\left(x\right)+\mathrm{Li}_2(-xe)\right)bne^3-\frac{1}{9}\left(b(n+3\log\left(c\right))+3a\right)e^3\log\left(xe+1\right)-\frac{6bnx^3e^3\log\left(x\right)^2-4\left(b(n+3\log\left(c\right))+3a\right)x^3e^3\log\left(x\right)-4\left(b(4n+3\log\left(c\right))+3a\right)x^2e^2+\left(b(5n+6\log\left(c\right))+6a\right)xe-4\left(3bnx^3e^3\log\left(x\right)-b(n+3\log\left(c\right))-3a\right)\log\left(xe+1\right)-6\left(2bx^3e^3\log\left(x\right)+2bx^2e^2-bxe-2\left(bx^3e^3+b\right)\log\left(xe+1\right)\right)\log\left(x^n\right)}{36x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(x*e + 1)*log(x) + dilog(-x*e))*b*n*e^3 - 1/9*(b*(n + 3*log(c)) +
3*a)*e^3*log(x*e + 1) - 1/36*(6*b*n*x^3*e^3*log(x)^2 - 4*(b*(n + 3*log(c))
+ 3*a)*x^3*e^3*log(x) - 4*(b*(4*n + 3*log(c)) + 3*a)*x^2*e^2 + (b*(5*n + 6*
log(c)) + 6*a)*x*e - 4*(3*b*n*x^3*e^3*log(x) - b*(n + 3*log(c)) - 3*a)*log(
x*e + 1) - 6*(2*b*x^3*e^3*log(x) + 2*b*x^2*e^2 - b*x*e - 2*(b*x^3*e^3 + b)*
log(x*e + 1))*log(x^n))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n)*log(x*e + 1) + a*log(x*e + 1))/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(e*x+1)/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(e*x+1)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log(x*e + 1)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (e\,x+1)\,\left(a+b\,\ln (c\,x^n)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(e*x + 1)*(a + b*log(c*x^n)))/x^4, x)
```

3.10 $\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal. Leaf size=456

$$-\frac{abnx}{2e^3}+\frac{21b^2n^2x}{32e^3}-\frac{7b^2n^2x^2}{64e^2}+\frac{37b^2n^2x^3}{864e}-\frac{3}{128}b^2n^2x^4-\frac{b^2nx\log(cx^n)}{2e^3}-\frac{bnx(a+b\log(cx^n))}{8e^3}+\frac{3bnx^2(a+b\log(}{16e^2}$$

```
[Out] -1/2*a*b*n*x/e^3+21/32*b^2*n^2*x/e^3-7/64*b^2*n^2*x^2/e^2+37/864*b^2*n^2*x^
3/e-3/128*b^2*n^2*x^4-1/2*b^2*n*x*ln(c*x^n)/e^3-1/8*b*n*x*(a+b*ln(c*x^n))/e
^3+3/16*b*n*x^2*(a+b*ln(c*x^n))/e^2-7/72*b*n*x^3*(a+b*ln(c*x^n))/e+1/16*b*n
*x^4*(a+b*ln(c*x^n))+1/4*x*(a+b*ln(c*x^n))^2/e^3-1/8*x^2*(a+b*ln(c*x^n))^2/
e^2+1/12*x^3*(a+b*ln(c*x^n))^2/e-1/16*x^4*(a+b*ln(c*x^n))^2-1/32*b^2*n^2*ln
(e*x+1)/e^4+1/32*b^2*n^2*x^4*ln(e*x+1)+1/8*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^
4-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln(e*x+1)-1/4*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^4
+1/4*x^4*(a+b*ln(c*x^n))^2*ln(e*x+1)+1/8*b^2*n^2*polylog(2,-e*x)/e^4-1/2*b*
n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^4+1/2*b^2*n^2*polylog(3,-e*x)/e^4
```

Rubi [A]
time = 0.24, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2442, 45, 2424, 2332, 2341, 2421, 6724, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{2e^4}+\frac{b^2n^2\text{PolyLog}(2,-ex)}{8e^4}+\frac{b^2n^2\text{PolyLog}(3,-ex)}{2e^4}-\frac{\log(ex+1)(a+b\log(cx^n))^2}{4e^4}+\frac{bn\log(ex+1)(a+b\log(cx^n))}{8e^4}+\frac{x(a+b\log(cx^n))^2}{4e^3}-\frac{bnx(a+b\log(cx^n))}{8e^3}-\frac{x^2(a+b\log(cx^n))^2}{8e^2}+\frac{3bnx^2(a+b\log(cx^n))}{16e^2}+\frac{1}{4}x^4\log(ex+1)(a+b\log(cx^n))^2-\frac{1}{8}bnx^4\log(ex+1)(a+b\log(cx^n))+\frac{x^3(a+b\log(cx^n))^2}{12e}-\frac{7bnx^3(a+b\log(cx^n))}{72e}-\frac{1}{16}x^4(a+b\log(cx^n))^2+\frac{1}{16}bnx^4(a+b\log(cx^n))-\frac{abnx}{2e^3}-\frac{b^2nx\log(cx^n)}{2e^3}-\frac{b^2n^2\log(ex+1)}{32e^4}+\frac{21b^2n^2x}{32e^3}-\frac{7b^2n^2x^2}{64e^2}+\frac{1}{32}b^2n^2x^4\log(ex+1)+\frac{37b^2n^2x^3}{864e}-\frac{3}{128}b^2n^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] -1/2*(a*b*n*x)/e^3 + (21*b^2*n^2*x)/(32*e^3) - (7*b^2*n^2*x^2)/(64*e^2) + (
37*b^2*n^2*x^3)/(864*e) - (3*b^2*n^2*x^4)/128 - (b^2*n*x*Log[c*x^n])/(2*e^3
) - (b*n*x*(a + b*Log[c*x^n]))/(8*e^3) + (3*b*n*x^2*(a + b*Log[c*x^n]))/(16
*e^2) - (7*b*n*x^3*(a + b*Log[c*x^n]))/(72*e) + (b*n*x^4*(a + b*Log[c*x^n])
)/16 + (x*(a + b*Log[c*x^n])^2)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^2)/(8*e^2
) + (x^3*(a + b*Log[c*x^n])^2)/(12*e) - (x^4*(a + b*Log[c*x^n])^2)/16 - (b^
2*n^2*Log[1 + e*x])/(32*e^4) + (b^2*n^2*x^4*Log[1 + e*x])/32 + (b*n*(a + b*
Log[c*x^n])*Log[1 + e*x])/(8*e^4) - (b*n*x^4*(a + b*Log[c*x^n])*Log[1 + e*x
])/8 - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(4*e^4) + (x^4*(a + b*Log[c*x^n]
)^2*Log[1 + e*x])/4 + (b^2*n^2*PolyLog[2, -(e*x)])/(8*e^4) - (b*n*(a + b*Lo
g[c*x^n])*PolyLog[2, -(e*x)])/(2*e^4) + (b^2*n^2*PolyLog[3, -(e*x)])/(2*e^4
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3(a+b\log(cx^n))^2\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))^2}{4e^3} - \frac{x^2(a+b\log(cx^n))^2}{8e^2} + \frac{x^3(a+b\log(cx^n))^2}{12e} - \cdots$$

$$= \frac{x(a+b\log(cx^n))^2}{4e^3} - \frac{x^2(a+b\log(cx^n))^2}{8e^2} + \frac{x^3(a+b\log(cx^n))^2}{12e} - \cdots$$

$$= -\frac{abnx}{2e^3} - \frac{b^2n^2x^2}{16e^2} + \frac{b^2n^2x^3}{54e} - \frac{1}{128}b^2n^2x^4 - \frac{bnx(a+b\log(cx^n))}{8e^3} + \cdots$$

$$= -\frac{abnx}{2e^3} + \frac{5b^2n^2x}{8e^3} - \frac{3b^2n^2x^2}{32e^2} + \frac{7b^2n^2x^3}{216e} - \frac{1}{64}b^2n^2x^4 - \frac{b^2nx\log(cx^n \cdots}{2e^3}$$

$$= -\frac{abnx}{2e^3} + \frac{5b^2n^2x}{8e^3} - \frac{3b^2n^2x^2}{32e^2} + \frac{7b^2n^2x^3}{216e} - \frac{1}{64}b^2n^2x^4 - \frac{b^2nx\log(cx^n \cdots}{2e^3}$$

$$= -\frac{abnx}{2e^3} + \frac{5b^2n^2x}{8e^3} - \frac{3b^2n^2x^2}{32e^2} + \frac{7b^2n^2x^3}{216e} - \frac{1}{64}b^2n^2x^4 - \frac{b^2nx\log(cx^n \cdots}{2e^3}$$

$$= -\frac{abnx}{2e^3} + \frac{21b^2n^2x}{32e^3} - \frac{7b^2n^2x^2}{64e^2} + \frac{37b^2n^2x^3}{864e} - \frac{3}{128}b^2n^2x^4 - \frac{b^2nx\log(\cdots}{2e^3}$$

Mathematica [A]
time = 0.12, size = 594, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (864*a^2*e*x - 2160*a*b*e*n*x + 2268*b^2*e*n^2*x - 432*a^2*e^2*x^2 + 648*a*
b*e^2*n*x^2 - 378*b^2*e^2*n^2*x^2 + 288*a^2*e^3*x^3 - 336*a*b*e^3*n*x^3 + 1
48*b^2*e^3*n^2*x^3 - 216*a^2*e^4*x^4 + 216*a*b*e^4*n*x^4 - 81*b^2*e^4*n^2*x
^4 + 1728*a*b*e*x*Log[c*x^n] - 2160*b^2*e*n*x*Log[c*x^n] - 864*a*b*e^2*x^2*
Log[c*x^n] + 648*b^2*e^2*n*x^2*Log[c*x^n] + 576*a*b*e^3*x^3*Log[c*x^n] - 33
6*b^2*e^3*n*x^3*Log[c*x^n] - 432*a*b*e^4*x^4*Log[c*x^n] + 216*b^2*e^4*n*x^4
```

```
*Log[c*x^n] + 864*b^2*e*x*Log[c*x^n]^2 - 432*b^2*e^2*x^2*Log[c*x^n]^2 + 288
*b^2*e^3*x^3*Log[c*x^n]^2 - 216*b^2*e^4*x^4*Log[c*x^n]^2 - 864*a^2*Log[1 +
e*x] + 432*a*b*n*Log[1 + e*x] - 108*b^2*n^2*Log[1 + e*x] + 864*a^2*e^4*x^4*
Log[1 + e*x] - 432*a*b*e^4*n*x^4*Log[1 + e*x] + 108*b^2*e^4*n^2*x^4*Log[1 +
 e*x] - 1728*a*b*Log[c*x^n]*Log[1 + e*x] + 432*b^2*n*Log[c*x^n]*Log[1 + e*x
] + 1728*a*b*e^4*x^4*Log[c*x^n]*Log[1 + e*x] - 432*b^2*e^4*n*x^4*Log[c*x^n]
*Log[1 + e*x] - 864*b^2*Log[c*x^n]^2*Log[1 + e*x] + 864*b^2*e^4*x^4*Log[c*x
^n]^2*Log[1 + e*x] + 432*b*n*(-4*a + b*n - 4*b*Log[c*x^n])*PolyLog[2, -(e*x
)] + 1728*b^2*n^2*PolyLog[3, -(e*x)])/(3456*e^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
[Out] int(x^3*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
[Out] -1/48*(3*b^2*x^4*e^4 - 4*b^2*x^3*e^3 + 6*b^2*x^2*e^2 - 12*b^2*x*e - 12*(b^2
*x^4*e^4 - b^2)*log(x*e + 1))*e^(-4)*log(x^n)^2 + 1/24*e^(-4)*integrate((24
*(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x^4*e^4*log(x*e + 1) + (3*b^2*n*x^4*e^
4 - 4*b^2*n*x^3*e^3 + 6*b^2*n*x^2*e^2 - 12*b^2*n*x*e - 12*((b^2*(n - 4*log(
c)) - 4*a*b)*x^4*e^4 - b^2*n)*log(x*e + 1))*log(x^n))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
[Out] integral(b^2*x^3*log(c*x^n)^2*log(x*e + 1) + 2*a*b*x^3*log(c*x^n)*log(x*e +
 1) + a^2*x^3*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^3*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln(e\,x+1)\,\left(a+b\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

3.11 $\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal. Leaf size=396

$$\frac{2abnx}{3e^2}-\frac{26b^2n^2x}{27e^2}+\frac{19b^2n^2x^2}{108e}-\frac{2}{27}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bnx(a+b\log(cx^n))}{9e^2}-\frac{5bnx^2(a+b\log(cx^n))}{18e}+$$

```
[Out] 2/3*a*b*n*x/e^2-26/27*b^2*n^2*x/e^2+19/108*b^2*n^2*x^2/e-2/27*b^2*n^2*x^3+2
/3*b^2*n*x*ln(c*x^n)/e^2+2/9*b*n*x*(a+b*ln(c*x^n))/e^2-5/18*b*n*x^2*(a+b*ln
(c*x^n))/e+4/27*b*n*x^3*(a+b*ln(c*x^n))-1/3*x*(a+b*ln(c*x^n))^2/e^2+1/6*x^2
*(a+b*ln(c*x^n))^2/e-1/9*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*n^2*ln(e*x+1)/e^3+2
/27*b^2*n^2*x^3*ln(e*x+1)-2/9*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^3-2/9*b*n*x^3
*(a+b*ln(c*x^n))*ln(e*x+1)+1/3*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^3+1/3*x^3*(a+b
*ln(c*x^n))^2*ln(e*x+1)-2/9*b^2*n^2*polylog(2,-e*x)/e^3+2/3*b*n*(a+b*ln(c*x
^n))*polylog(2,-e*x)/e^3-2/3*b^2*n^2*polylog(3,-e*x)/e^3
```

Rubi [A]

time = 0.20, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2442, 45, 2424, 2332, 2341, 2421, 6724, 2423, 2438}

$$\frac{2bn\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{3e^3}-\frac{2b^2n^2\text{PolyLog}(2,-ex)}{9e^3}-\frac{2b^2n^2\text{PolyLog}(3,-ex)}{3e^3}+\frac{\log(ex+1)(a+b\log(cx^n))^2}{3e^3}-\frac{2bn\log(ex+1)(a+b\log(cx^n))}{9e^3}-\frac{x(a+b\log(cx^n))^2}{3e^2}+\frac{2bnx(a+b\log(cx^n))}{9e^2}+\frac{1}{3}x^3\log(ex+1)(a+b\log(cx^n))^2-\frac{2}{9}bnx^3\log(ex+1)(a+b\log(cx^n))+\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{5bnx^2(a+b\log(cx^n))}{18e}-\frac{1}{9}x^3(a+b\log(cx^n))^2+\frac{4}{27}bnx^3(a+b\log(cx^n))+\frac{2abnx}{3e^2}+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2b^2n^2\log(ex+1)}{27e^3}-\frac{26b^2n^2x}{27e^2}+\frac{2}{27}b^2n^2x^3\log(ex+1)+\frac{19b^2n^2x^2}{108e}-\frac{2}{27}b^2n^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (2*a*b*n*x)/(3*e^2) - (26*b^2*n^2*x)/(27*e^2) + (19*b^2*n^2*x^2)/(108*e) -
(2*b^2*n^2*x^3)/27 + (2*b^2*n*x*Log[c*x^n])/(3*e^2) + (2*b*n*x*(a + b*Log[c
*x^n]))/(9*e^2) - (5*b*n*x^2*(a + b*Log[c*x^n]))/(18*e) + (4*b*n*x^3*(a + b
*Log[c*x^n]))/27 - (x*(a + b*Log[c*x^n])^2)/(3*e^2) + (x^2*(a + b*Log[c*x^n
])^2)/(6*e) - (x^3*(a + b*Log[c*x^n])^2)/9 + (2*b^2*n^2*Log[1 + e*x])/(27*e
^3) + (2*b^2*n^2*x^3*Log[1 + e*x])/27 - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e
*x])/(9*e^3) - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/9 + ((a + b*Log[
c*x^n])^2*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n])^2*Log[1 + e*x])/3
 - (2*b^2*n^2*PolyLog[2, -(e*x)])/(9*e^3) + (2*b*n*(a + b*Log[c*x^n])*PolyL
og[2, -(e*x)])/(3*e^3) - (2*b^2*n^2*PolyLog[3, -(e*x)])/(3*e^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^2\log(1+ex)\,dx = -\frac{x(a+b\log(cx^n))^2}{3e^2}+\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^2$$

$$= -\frac{x(a+b\log(cx^n))^2}{3e^2}+\frac{x^2(a+b\log(cx^n))^2}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^2$$

$$= \frac{2abnx}{3e^2}+\frac{b^2n^2x^2}{12e}-\frac{2}{81}b^2n^2x^3+\frac{2bnx(a+b\log(cx^n))}{9e^2}-\frac{5bnx^2(a+}{1}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bnx}{}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bnx}{}$$

$$= \frac{2abnx}{3e^2}-\frac{8b^2n^2x}{9e^2}+\frac{5b^2n^2x^2}{36e}-\frac{4}{81}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bnx}{}$$

$$= \frac{2abnx}{3e^2}-\frac{26b^2n^2x}{27e^2}+\frac{19b^2n^2x^2}{108e}-\frac{2}{27}b^2n^2x^3+\frac{2b^2nx\log(cx^n)}{3e^2}+\frac{2bn}{}$$

Mathematica [A]
time = 0.09, size = 506, normalized size = 1.28

$$\frac{-36a^2ex+96abenx-104b^2en^2x+18a^2e^2x^2-30abe^2nx^2+19b^2e^2n^2x^2-12a^2e^3x^3+16abe^3nx^3-8b^2e^3n^2x^3-72abex\log(cx^n)+96b^2enx\log(cx^n)+36abe^2x^2\log(cx^n)-30b^2e^2nx^2\log(cx^n)-24abe^3x^3\log(cx^n)+16b^2e^3nx^3\log(cx^n)-36b^2ex\log^2(cx^n)+18b^2e^2x^2\log^2(cx^n)-12b^2e^3x^3\log^2(cx^n)+36a^2\log(1+ex)-24abn\log(1+ex)+8b^2n^2\log(1+ex)+36a^2e^3x^3\log(1+ex)-24abe^3nx^3\log(1+ex)+8b^2e^3n^2x^3\log(1+ex)+72ab\log(cx^n)\log(1+ex)-24b^2n\log(cx^n)\log(1+ex)+72abe^3x^3\log(cx^n)\log(1+ex)-24b^2e^3nx^3\log(cx^n)\log(1+ex)+36b^2\log^2(cx^n)\log(1+ex)+36b^2e^3x^3\log^2(cx^n)\log(1+ex)+24bn(3a-bn+3b\log(cx^n))\,\mathrm{Li}_2(-ex)-72b^2n^2\,\mathrm{Li}_3(-ex)}{108e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (-36*a^2*e*x + 96*a*b*e*n*x - 104*b^2*e*n^2*x + 18*a^2*e^2*x^2 - 30*a*b*e^2
*n*x^2 + 19*b^2*e^2*n^2*x^2 - 12*a^2*e^3*x^3 + 16*a*b*e^3*n*x^3 - 8*b^2*e^3
*n^2*x^3 - 72*a*b*e*x*Log[c*x^n] + 96*b^2*e*n*x*Log[c*x^n] + 36*a*b*e^2*x^2
*Log[c*x^n] - 30*b^2*e^2*n*x^2*Log[c*x^n] - 24*a*b*e^3*x^3*Log[c*x^n] + 16*
b^2*e^3*n*x^3*Log[c*x^n] - 36*b^2*e*x*Log[c*x^n]^2 + 18*b^2*e^2*x^2*Log[c*x
^n]^2 - 12*b^2*e^3*x^3*Log[c*x^n]^2 + 36*a^2*Log[1 + e*x] - 24*a*b*n*Log[1
+ e*x] + 8*b^2*n^2*Log[1 + e*x] + 36*a^2*e^3*x^3*Log[1 + e*x] - 24*a*b*e^3*
```

```
n*x^3*Log[1 + e*x] + 8*b^2*e^3*n^2*x^3*Log[1 + e*x] + 72*a*b*Log[c*x^n]*Log
[1 + e*x] - 24*b^2*n*Log[c*x^n]*Log[1 + e*x] + 72*a*b*e^3*x^3*Log[c*x^n]*Lo
g[1 + e*x] - 24*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 36*b^2*Log[c*x^n]^2
*Log[1 + e*x] + 36*b^2*e^3*x^3*Log[c*x^n]^2*Log[1 + e*x] + 24*b*n*(3*a - b*
n + 3*b*Log[c*x^n])*PolyLog[2, -(e*x)] - 72*b^2*n^2*PolyLog[3, -(e*x)])/(10
8*e^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\ln (c\,x^{n}))^{2}\ln (ex+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
[Out] int(x^2*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
[Out] -1/18*(2*b^2*x^3*e^3 - 3*b^2*x^2*e^2 + 6*b^2*x*e - 6*(b^2*x^3*e^3 + b^2)*lo
g(x*e + 1))*e^(-3)*log(x^n)^2 + 1/9*e^(-3)*integrate((9*(b^2*log(c)^2 + 2*a
*b*log(c) + a^2)*x^3*e^3*log(x*e + 1) + (2*b^2*n*x^3*e^3 - 3*b^2*n*x^2*e^2
+ 6*b^2*n*x*e - 6*((b^2*(n - 3*log(c)) - 3*a*b)*x^3*e^3 + b^2*n)*log(x*e +
1))*log(x^n))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
[Out] integral(b^2*x^2*log(c*x^n)^2*log(x*e + 1) + 2*a*b*x^2*log(c*x^n)*log(x*e +
 1) + a^2*x^2*log(x*e + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log(x*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

3.12 $\int x(a+b\log(cx^n))^2\log(1+ex)\,dx$

Optimal. Leaf size=327

$$-\frac{abnx}{e}+\frac{7b^2n^2x}{4e}-\frac{3}{8}b^2n^2x^2-\frac{b^2nx\log(cx^n)}{e}-\frac{bnx(a+b\log(cx^n))}{2e}+\frac{1}{2}bnx^2(a+b\log(cx^n))+\frac{x(a+b\log(cx^n))}{2e}$$

```
[Out] -a*b*n*x/e+7/4*b^2*n^2*x/e-3/8*b^2*n^2*x^2-b^2*n*x*ln(c*x^n)/e-1/2*b*n*x*(a
+b*ln(c*x^n))/e+1/2*b*n*x^2*(a+b*ln(c*x^n))+1/2*x*(a+b*ln(c*x^n))^2/e-1/4*x
^2*(a+b*ln(c*x^n))^2-1/4*b^2*n^2*ln(e*x+1)/e^2+1/4*b^2*n^2*x^2*ln(e*x+1)+1/
2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/e^2-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(e*x+1)-1
/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2+1/2*x^2*(a+b*ln(c*x^n))^2*ln(e*x+1)+1/2*
b^2*n^2*polylog(2,-e*x)/e^2-b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^2+b^2*n^2
*polylog(3,-e*x)/e^2
```

Rubi [A]
time = 0.16, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2442, 45, 2424, 2332, 2341, 2421, 6724, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{e^2}+\frac{b^2n^2\text{PolyLog}(2,-ex)}{2e^2}+\frac{b^2n^2\text{PolyLog}(3,-ex)}{e^2}+\frac{bn\log(ex+1)(a+b\log(cx^n))}{2e^2}-\frac{\log(ex+1)(a+b\log(cx^n))^2}{2e^2}-\frac{bnx(a+b\log(cx^n))}{2e}+\frac{x(a+b\log(cx^n))^2}{2e}-\frac{1}{2}bnx^2\log(ex+1)(a+b\log(cx^n))+\frac{1}{2}x^2\log(ex+1)(a+b\log(cx^n))^2+\frac{1}{2}bnx^2(a+b\log(cx^n))-\frac{1}{4}x^2(a+b\log(cx^n))^2-\frac{abnx}{e}-\frac{b^2nx\log(cx^n)}{e}-\frac{b^2n^2\log(ex+1)}{4e^2}+\frac{1}{4}b^2n^2x^2\log(ex+1)+\frac{7b^2n^2x}{4e}-\frac{3}{8}b^2n^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] -((a*b*n*x)/e) + (7*b^2*n^2*x)/(4*e) - (3*b^2*n^2*x^2)/8 - (b^2*n*x*Log[c*x
^n])/e - (b*n*x*(a + b*Log[c*x^n]))/(2*e) + (b*n*x^2*(a + b*Log[c*x^n]))/2
+ (x*(a + b*Log[c*x^n])^2)/(2*e) - (x^2*(a + b*Log[c*x^n])^2)/4 - (b^2*n^2*
Log[1 + e*x])/(4*e^2) + (b^2*n^2*x^2*Log[1 + e*x])/4 + (b*n*(a + b*Log[c*x^
n])*Log[1 + e*x])/(2*e^2) - (b*n*x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/2 - (
(a + b*Log[c*x^n])^2*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^2*Log[
1 + e*x])/2 + (b^2*n^2*PolyLog[2, -(e*x)])/(2*e^2) - (b*n*(a + b*Log[c*x^n]
)*PolyLog[2, -(e*x)])/e^2 + (b^2*n^2*PolyLog[3, -(e*x)])/e^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^2\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))^2}{2e} - \frac{1}{4}x^2(a+b\log(cx^n))^2 - \frac{(a+b\log(cx^n))^2\log(1 \cdots}{2e^2}$$

$$= \frac{x(a+b\log(cx^n))^2}{2e} - \frac{1}{4}x^2(a+b\log(cx^n))^2 - \frac{(a+b\log(cx^n))^2\log(1 \cdots}{2e^2}$$

$$= -\frac{abnx}{e} - \frac{1}{8}b^2n^2x^2 - \frac{bnx(a+b\log(cx^n))}{2e} + \frac{1}{2}bnx^2(a+b\log(cx^n)) + \cdots$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log(cx^n)}{e} - \frac{bnx(a+b\log(cx^n))}{2e}$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log(cx^n)}{e} - \frac{bnx(a+b\log(cx^n))}{2e}$$

$$= -\frac{abnx}{e} + \frac{3b^2n^2x}{2e} - \frac{1}{4}b^2n^2x^2 - \frac{b^2nx\log(cx^n)}{e} - \frac{bnx(a+b\log(cx^n))}{2e}$$

$$= -\frac{abnx}{e} + \frac{7b^2n^2x}{4e} - \frac{3}{8}b^2n^2x^2 - \frac{b^2nx\log(cx^n)}{e} - \frac{bnx(a+b\log(cx^n))}{2e}$$

Mathematica [A]
time = 0.08, size = 416, normalized size = 1.27

$$\frac{4a^2ex-12abenx+14b^2en^2x-2a^2e^2x^2+4abe^2nx^2-3b^2e^2n^2x^2+8abex\log(cx^n)-12b^2enx\log(cx^n)-4abe^2x^2\log(cx^n)+4b^2e^2nx^2\log(cx^n)+4b^2ex\log^2(cx^n)-2b^2e^2x^2\log^2(cx^n)-4a^2\log(1+ex)+4abn\log(1+ex)-2b^2n^2\log(1+ex)+4a^2e^2x^2\log(1+ex)-4abe^2nx^2\log(1+ex)+2b^2e^2n^2x^2\log(1+ex)-8ab\log(cx^n)\log(1+ex)+4b^2n\log(cx^n)\log(1+ex)+8abe^2x^2\log(cx^n)\log(1+ex)-4b^2e^2nx^2\log(cx^n)\log(1+ex)-4b^2\log^2(cx^n)\log(1+ex)+4b^2e^2x^2\log^2(cx^n)\log(1+ex)+4bn(-2a+bn-2b\log(cx^n))\,\mathrm{Li}_2(-ex)+8b^2n^2\mathrm{Li}_3(-ex)}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (4*a^2*e*x - 12*a*b*e*n*x + 14*b^2*e*n^2*x - 2*a^2*e^2*x^2 + 4*a*b*e^2*n*x^
2 - 3*b^2*e^2*n^2*x^2 + 8*a*b*e*x*Log[c*x^n] - 12*b^2*e*n*x*Log[c*x^n] - 4*
a*b*e^2*x^2*Log[c*x^n] + 4*b^2*e^2*n*x^2*Log[c*x^n] + 4*b^2*e*x*Log[c*x^n]^
2 - 2*b^2*e^2*x^2*Log[c*x^n]^2 - 4*a^2*Log[1 + e*x] + 4*a*b*n*Log[1 + e*x]
- 2*b^2*n^2*Log[1 + e*x] + 4*a^2*e^2*x^2*Log[1 + e*x] - 4*a*b*e^2*n*x^2*Log
[1 + e*x] + 2*b^2*e^2*n^2*x^2*Log[1 + e*x] - 8*a*b*Log[c*x^n]*Log[1 + e*x]
+ 4*b^2*n*Log[c*x^n]*Log[1 + e*x] + 8*a*b*e^2*x^2*Log[c*x^n]*Log[1 + e*x] -
 4*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 4*b^2*Log[c*x^n]^2*Log[1 + e*x]
+ 4*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] + 4*b*n*(-2*a + b*n - 2*b*Log[c*x
^n])*PolyLog[2, -(e*x)] + 8*b^2*n^2*PolyLog[3, -(e*x)])/(8*e^2)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a + b\ln{(c\,x^n)})^2 \ln{(ex + 1)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*x^2*e^2 - 2*b^2*x*e - 2*(b^2*x^2*e^2 - b^2)*log(x*e + 1))*e^(-2)*
log(x^n)^2 + 1/2*e^(-2)*integrate((2*(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x^
2*e^2*log(x*e + 1) + (b^2*n*x^2*e^2 - 2*b^2*n*x*e - 2*((b^2*(n - 2*log(c))
- 2*a*b)*x^2*e^2 - b^2*n)*log(x*e + 1))*log(x^n))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*x^n)^2*log(x*e + 1) + 2*a*b*x*log(c*x^n)*log(x*e + 1)
+ a^2*x*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log(x*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

3.13 $\int (a+b\log(cx^n))^2 \log(1+ex)\,dx$

Optimal. Leaf size=193

$$2abnx-6b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\log(cx^n))^2+\frac{2b^2n^2(1+ex)\log(1+ex)}{e}-\frac{2b$$

```
[Out] 2*a*b*n*x-6*b^2*n^2*x+2*b^2*n*x*ln(c*x^n)+2*b*n*x*(a+b*ln(c*x^n))-x*(a+b*ln
(c*x^n))^2+2*b^2*n^2*(e*x+1)*ln(e*x+1)/e-2*b*n*(e*x+1)*(a+b*ln(c*x^n))*ln(e
*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^2*ln(e*x+1)/e-2*b^2*n^2*polylog(2,-e*x)/e+2
*b*n*(a+b*ln(c*x^n))*polylog(2,-e*x)/e-2*b^2*n^2*polylog(3,-e*x)/e
```

Rubi [A]
time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2436, 2332, 2417, 2388, 2338, 6874, 2458, 45, 2393, 2352, 2421, 6724}

$$\frac{2bn\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{e}-\frac{2b^2n^2\text{PolyLog}(2,-ex)}{e}-\frac{2b^2n^2\text{PolyLog}(3,-ex)}{e}-\frac{2bn(ex+1)\log(ex+1)(a+b\log(cx^n))}{e}+\frac{(ex+1)\log(ex+1)(a+b\log(cx^n))^2}{e}+2bnx(a+b\log(cx^n))-x(a+b\log(cx^n))^2+2abnx+2b^2nx\log(cx^n)+\frac{2b^2n^2(ex+1)\log(ex+1)}{e}-6b^2n^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] 2*a*b*n*x - 6*b^2*n^2*x + 2*b^2*n*x*Log[c*x^n] + 2*b*n*x*(a + b*Log[c*x^n])
 - x*(a + b*Log[c*x^n])^2 + (2*b^2*n^2*(1 + e*x)*Log[1 + e*x])/e - (2*b*n*(
1 + e*x)*(a + b*Log[c*x^n])*Log[1 + e*x])/e + ((1 + e*x)*(a + b*Log[c*x^n])
^2*Log[1 + e*x])/e - (2*b^2*n^2*PolyLog[2, -(e*x)])/e + (2*b*n*(a + b*Log[c
*x^n])*PolyLog[2, -(e*x)])/e - (2*b^2*n^2*PolyLog[3, -(e*x)])/e
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+b\log(cx^n))^2\log(1+ex)\,dx = -x(a+b\log(cx^n))^2+\frac{(1+ex)\,(a+b\log(cx^n))^2\log(1+ex)}{e}-(2bn)$$

$$=2abnx-x(a+b\log(cx^n))^2+\frac{(1+ex)\,(a+b\log(cx^n))^2\log(1+ex)}{e}$$

$$=2abnx-2b^2n^2x+2b^2nx\log(cx^n)-x(a+b\log(cx^n))^2+\frac{(1+ex)\,(a}{}$$

$$=2abnx-2b^2n^2x+2b^2nx\log(cx^n)-x(a+b\log(cx^n))^2+\frac{(1+ex)\,(a}{}$$

$$=2abnx-2b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

$$=2abnx-4b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

$$=2abnx-4b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

$$=2abnx-4b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

$$=2abnx-4b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

$$=2abnx-6b^2n^2x+2b^2nx\log(cx^n)+2bnx(a+b\log(cx^n))-x(a+b\,\mathrm{l}$$

Mathematica [A]

time = 0.06, size = 294, normalized size = 1.52

$$\frac{-a^2ex + 4abenx - 6b^2en^2x - 2abex\log(cx^n) + 4b^2enx\log(cx^n) - b^2ex\log^2(cx^n) + a^2\log(1+ex) - 2abn\log(1+ex) + 2b^2n^2\log(1+ex) + a^2ex\log(1+ex) - 2abenx\log(1+ex) + 2b^2en^2x\log(1+ex) + 2ab\log(cx^n)\log(1+ex) - 2b^2n\log(cx^n)\log(1+ex) + 2abex\log(cx^n)\log(1+ex) - 2b^2enx\log(cx^n)\log(1+ex) + b^2\log^2(cx^n)\log(1+ex) + b^2ex\log^2(cx^n)\log(1+ex) + 2bn(a - bn + b\log(cx^n))\operatorname{Li}_2(-ex) - 2b^2n^2\operatorname{Li}_3(-ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[1 + e*x],x]
```

```
[Out] (-(a^2*e*x) + 4*a*b*e*n*x - 6*b^2*e*n^2*x - 2*a*b*e*x*Log[c*x^n] + 4*b^2*e*
n*x*Log[c*x^n] - b^2*e*x*Log[c*x^n]^2 + a^2*Log[1 + e*x] - 2*a*b*n*Log[1 +
e*x] + 2*b^2*n^2*Log[1 + e*x] + a^2*e*x*Log[1 + e*x] - 2*a*b*e*n*x*Log[1 +
e*x] + 2*b^2*e*n^2*x*Log[1 + e*x] + 2*a*b*Log[c*x^n]*Log[1 + e*x] - 2*b^2*n
*Log[c*x^n]*Log[1 + e*x] + 2*a*b*e*x*Log[c*x^n]*Log[1 + e*x] - 2*b^2*e*n*x*
Log[c*x^n]*Log[1 + e*x] + b^2*Log[c*x^n]^2*Log[1 + e*x] + b^2*e*x*Log[c*x^n
]^2*Log[1 + e*x] + 2*b*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, -(e*x)] - 2*b^
2*n^2*PolyLog[3, -(e*x)])/e
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (a + b\ln(c\,x^n))^2 \ln(ex+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -(b^2*x*e - (b^2*x*e + b^2)*log(x*e + 1))*e^(-1)*log(x^n)^2 + e^(-1)*integr
ate(((b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x*e*log(x*e + 1) + 2*(b^2*n*x*e -
(b^2*n + (b^2*(n - log(c)) - a*b)*x*e)*log(x*e + 1))*log(x^n))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*x^n)^2*log(x*e + 1) + 2*a*b*log(c*x^n)*log(x*e + 1) + a^
2*log(x*e + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\log{(cx^n)})^2 \log{(ex + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1),x)
[Out] Integral((a + b*log(c*x**n))**2*log(e*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log(x*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n))^2,x)
[Out] int(log(e*x + 1)*(a + b*log(c*x^n))^2, x)
```

3.14 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx$

Optimal. Leaf size=55

$$-(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)+2bn(a+b\log(cx^n))\,\mathrm{Li}_3(-ex)-2b^2n^2\mathrm{Li}_4(-ex)$$

```
[Out] -(a+b*ln(c*x^n))^2*polylog(2,-e*x)+2*b*n*(a+b*ln(c*x^n))*polylog(3,-e*x)-2*
b^2*n^2*polylog(4,-e*x)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2421, 2430, 6724}

$$2bn\mathrm{PolyLog}(3,-ex)\,(a+b\log(cx^n))-\mathrm{PolyLog}(2,-ex)\,(a+b\log(cx^n))^2-2b^2n^2\mathrm{PolyLog}(4,-ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*(a + b*Log[c*x^n])*PolyL
og[3, -(e*x)] - 2*b^2*n^2*PolyLog[4, -(e*x)]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)+(2bn)\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_2(-ex)}{x}\,dx$$

$$= -(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)+2bn(a+b\log(cx^n))\,\mathrm{Li}_3(-ex)-\left(2b^2n\right.$$

$$= -(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)+2bn(a+b\log(cx^n))\,\mathrm{Li}_3(-ex)-2b^2n^2$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.96

$$-(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)+2bn((a+b\log(cx^n))\,\mathrm{Li}_3(-ex)-bn\mathrm{Li}_4(-ex))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)]) + 2*b*n*((a + b*Log[c*x^n])*Poly
Log[3, -(e*x)] - b*n*PolyLog[4, -(e*x)])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 835, normalized size = 15.18

method	result	size
risch	Expression too large to display	835

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(x^n)*Pi*dilog(e*x+1)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I*ln(x)*Pi*polyl
og(2,-e*x)*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*polylog(3,-e*x)*b^2*n*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*ln(x)*Pi*polylog(2,-e*x)*b^2*n*csgn(I*c)
*csgn(I*c*x^n)^2+I*ln(x^n)*Pi*dilog(e*x+1)*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)-I*ln(x^n)*Pi*dilog(e*x+1)*b^2*csgn(I*c)*csgn(I*c*x^n)^2+I*ln(x)*Pi*
dilog(e*x+1)*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2+2*polylog(3,-e*x)*a*b*n+2*ln(c
)*polylog(3,-e*x)*b^2*n-ln(x^n)^2*dilog(e*x+1)*b^2+2*ln(x)*ln(x^n)*dilog(e*
x+1)*b^2*n+I*ln(x)*Pi*polylog(2,-e*x)*b^2*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+I*ln(x)*Pi*dilog(e*x+1)*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-2*ln(x^n)*di
log(e*x+1)*a*b-2*ln(c)*ln(x^n)*dilog(e*x+1)*b^2-I*ln(x)*Pi*dilog(e*x+1)*b^2
*n*csgn(I*c*x^n)^3+I*Pi*polylog(3,-e*x)*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2+I*P
i*polylog(3,-e*x)*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+I*ln(x)*Pi*polylog(2,-e
*x)*b^2*n*csgn(I*c*x^n)^3-2*ln(c)*ln(x)*polylog(2,-e*x)*b^2*n+2*ln(c)*ln(x)
*dilog(e*x+1)*b^2*n+ln(x)^2*polylog(2,-e*x)*b^2*n^2-ln(x)^2*dilog(e*x+1)*b^
2*n^2+2*ln(x^n)*polylog(3,-e*x)*b^2*n-I*ln(x)*Pi*dilog(e*x+1)*b^2*n*csgn(I*
```

```
c)*csgn(I*x^n)*csgn(I*c*x^n)-2*ln(x)*ln(x^n)*polylog(2,-e*x)*b^2*n+I*ln(x^n
)*Pi*dilog(e*x+1)*b^2*csgn(I*c*x^n)^3-I*Pi*polylog(3,-e*x)*b^2*n*csgn(I*c*x
^n)^3-1/4*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csg
n(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*
ln(c)+2*a)^2*dilog(e*x+1)-2*ln(x)*polylog(2,-e*x)*a*b*n+2*ln(x)*dilog(e*x+1
)*a*b*n-2*b^2*n^2*polylog(4,-e*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(x*e + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(x*e + 1) + 2*a*b*log(c*x^n)*log(x*e + 1) + a
^2*log(x*e + 1))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(x*e + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x, x)
```

3.15 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx$

Optimal. Leaf size=203

$$2b^2en^2\log(x)-2ben\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))-e\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2-2b^2en^2\log(1+ex)-\frac{2b}{}$$

```
[Out] 2*b^2*e*n^2*ln(x)-2*b*e*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-e*ln(1+1/e/x)*(a+b*ln
(c*x^n))^2-2*b^2*e*n^2*ln(e*x+1)-2*b^2*n^2*ln(e*x+1)/x-2*b*n*(a+b*ln(c*x^n)
)*ln(e*x+1)/x-(a+b*ln(c*x^n))^2*ln(e*x+1)/x+2*b^2*e*n^2*polylog(2,-1/e/x)+2
*b*e*n*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+2*b^2*e*n^2*polylog(3,-1/e/x)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$,
Rules used = {2342, 2341, 2425, 36, 29, 31, 2379, 2438, 2421, 6724}

$$2ben\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))+2b^2en^2\text{PolyLog}\left(2,-\frac{1}{ex}\right)+2b^2en^2\text{PolyLog}\left(3,-\frac{1}{ex}\right)-2ben\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))-\frac{2bn\log(ex+1)(a+b\log(cx^n))}{x}-e\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^2-\frac{\log(ex+1)(a+b\log(cx^n))^2}{x}+2b^2en^2\log(x)-2b^2en^2\log(ex+1)-\frac{2b^2n^2\log(ex+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

```
[Out] 2*b^2*e*n^2*Log[x] - 2*b*e*n*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n]) - e*Log[1
+ 1/(e*x)]*(a + b*Log[c*x^n])^2 - 2*b^2*e*n^2*Log[1 + e*x] - (2*b^2*n^2*Log
[1 + e*x])/x - (2*b*n*(a + b*Log[c*x^n])*Log[1 + e*x])/x - ((a + b*Log[c*x^
n])^2*Log[1 + e*x])/x + 2*b^2*e*n^2*PolyLog[2, -(1/(e*x))] + 2*b*e*n*(a + b
*Log[c*x^n])*PolyLog[2, -(1/(e*x))] + 2*b^2*e*n^2*PolyLog[3, -(1/(e*x))]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^2}\,dx = -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn(a+b\log(cx^n))\log(1+ex)}{x} - \frac{(a+b\log(cx\ldots}{}$$

$$= -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn(a+b\log(cx^n))\log(1+ex)}{x} - \frac{(a+b\log(cx\ldots}{}$$

$$= -\frac{2b^2n^2\log(1+ex)}{x} - \frac{2bn(a+b\log(cx^n))\log(1+ex)}{x} - \frac{(a+b\log(cx\ldots}{}$$

$$= 2b^2en^2\log(x) + e(a+b\log(cx^n))^2 - 2b^2en^2\log(1+ex) - \frac{2b^2n^2\log(1\ldots}{x}$$

$$= 2b^2en^2\log(x) + e(a+b\log(cx^n))^2 + \frac{e(a+b\log(cx^n))^3}{3bn} - 2b^2en^2\log(1\ldots$$

$$= 2b^2en^2\log(x) + e(a+b\log(cx^n))^2 + \frac{e(a+b\log(cx^n))^3}{3bn} - 2b^2en^2\log(1\ldots$$

Mathematica [**A**]
time = 0.13, size = 183, normalized size = 0.90

$$\frac{1}{3}b^2en^2\log^3(x) - ben\log^2(x)(a+bn+b\log(cx^n)) + e\log(x)\left(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n)\right) - \frac{(1+ex)\left(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n)\right)\log(1+ex)}{x} - 2ben(a+bn+b\log(cx^n))\,\mathrm{Li}_2(-ex) + 2b^2en^2\mathrm{Li}_3(-ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^2,x]
```

```
[Out] (b^2*e*n^2*Log[x]^3)/3 - b*e*n*Log[x]^2*(a + b*n + b*Log[c*x^n]) + e*Log[x]
*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)
- ((1 + e*x)*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Lo
g[c*x^n]^2)*Log[1 + e*x])/x - 2*b*e*n*(a + b*n + b*Log[c*x^n])*PolyLog[2, -
(e*x)] + 2*b^2*e*n^2*PolyLog[3, -(e*x)]
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 3402, normalized size = 16.76

method	result	size
risch	Expression too large to display	3402

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(e*x+1)/x*a^2-I*e*ln(e*x+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*x^n)^2+I*ln(
e*x+1)*ln(x^n)*e*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I/x*ln(e*x+1)*l
n(x^n)*b^2*Pi*csgn(I*c*x^n)^3-1/2*e*ln(e*x+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x
^n)*csgn(I*c*x^n)^3-2*e*ln(e*x+1)*ln(c)*a*b-2*ln(e*x+1)/x*ln(c)*a*b-2*n/x*l
```

n(e*x+1)*b^2*ln(c)+2*n*ln(x)*e*b^2*ln(c)-2*n*e*ln(e*x+1)*b^2*ln(c)+n*e*ln(x
)^2*b^2*ln(c)-2*n*e*polylog(2,-e*x)*b^2*ln(c)+2*e*ln(e*x)*ln(c)*a*b+b^2*ln(
e*x)*ln(x)^2*e*n^2-I*n*e*polylog(2,-e*x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2
-I*e*ln(e*x+1)*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-ln(e*x+1)/x*ln(c)^2*b^2+e*l
n(e*x)*ln(c)^2*b^2-e*ln(e*x+1)*ln(c)^2*b^2+1/2*I*n*e*ln(x)^2*b^2*Pi*csgn(I*
c)*csgn(I*c*x^n)^2-2*b*ln(e*x)*ln(x)*e*n*a+I*ln(e*x+1)/x*Pi*ln(c)*b^2*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+2*b*ln(e*x)*ln(x^n)*e*a+1/4*ln(e*x+1)/x*Pi^2
*b^2*csgn(I*c*x^n)^6-1/4*e*ln(e*x)*Pi^2*b^2*csgn(I*c*x^n)^6+1/4*e*ln(e*x+1)
*Pi^2*b^2*csgn(I*c*x^n)^6-2*b^2*n/x*ln(e*x+1)*ln(x^n)+I*ln(e*x+1)/x*Pi*a*b*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*ln(e*x)*ln(x^n)*e*b^2*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)-1/2*I*n*e*ln(x)^2*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)-I*ln(e*x)*ln(x)*e*n*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*ln(e*x
+1)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+e*a^2*ln(e*x)-e*a^2*ln(e*x+1)+2*ln(e
*x)*ln(x^n)*e*b^2*ln(c)-2*ln(e*x+1)*ln(x^n)*e*b^2*ln(c)-2/x*ln(e*x+1)*ln(x^
n)*b^2*ln(c)+2*b^2*ln(x)*ln(x^n)*e*n-2*b^2*ln(e*x+1)*ln(x^n)*e*n+b^2*ln(x)^
2*ln(x^n)*e*n-2*b^2*ln(x^n)*polylog(2,-e*x)*e*n-I*ln(e*x+1)*ln(x^n)*e*b^2*P
i*csgn(I*c)*csgn(I*c*x^n)^2+I*n*e*ln(e*x+1)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*cs
gn(I*c*x^n)+1/2*I*n*e*ln(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-2*b*n/x*ln
(e*x+1)*a+2*b*n*ln(x)*e*a-2*b*n*e*ln(e*x+1)*a+b*n*e*ln(x)^2*a-2*b*n*e*polyl
og(2,-e*x)*a-b^2*n^2*e*ln(x)^2-2*b^2*n^2*e*polylog(2,-e*x)-2/3*b^2*n^2*e*ln
(x)^3+2*b^2*n^2*e*polylog(3,-e*x)-I*n*e*ln(e*x+1)*b^2*Pi*csgn(I*c)*csgn(I*c
*x^n)^2-I*n*e*polylog(2,-e*x)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-I*ln(e*x+1)/
x*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*ln(e*x+1)*Pi*ln(c)*b^2*csgn(
I*x^n)*csgn(I*c*x^n)^2+I*e*ln(e*x)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*x^n)^2-2
*b*ln(e*x+1)*ln(x^n)*e*a-2*b/x*ln(e*x+1)*ln(x^n)*a+I*ln(e*x)*ln(x)*e*n*b^2*
Pi*csgn(I*c*x^n)^3+I*ln(e*x)*ln(x^n)*e*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-b
^2*ln(e*x+1)*ln(x^n)^2*e-I/x*ln(e*x+1)*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x^
n)^2-I*ln(e*x+1)/x*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I/x*ln(e*x+1)*ln(x^n)
*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*n/x*ln(e*x+1)*b^2*Pi*csgn(I*c)*csgn(I
*c*x^n)^2-1/2*ln(e*x+1)/x*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+I*e*ln(e*x+1)*
Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*n*ln(x)*e*b^2*Pi*csgn(I*c)*csg
n(I*c*x^n)^2+I*n*ln(x)*e*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-b^2*ln(e*x+1)/x
*ln(x^n)^2+b^2*ln(e*x)*ln(x^n)^2*e-I*n*e*ln(e*x+1)*b^2*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2-I*ln(e*x+1)/x*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-I*ln(e*x+1)*ln(x^
n)*e*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+I*e*ln(e*x)*Pi*a*b*csgn(I*c)*csgn(I
*c*x^n)^2+I*e*ln(e*x)*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I*ln(e*x+1)/
x*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*x^n)^2-I*n/x*ln(e*x+1)*b^2*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2+I*e*ln(e*x)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*ln(e*x)*l
n(x^n)*e*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*e*ln(e*x+1)*Pi*a*b*csgn(I*c*x^n
)^3+I*ln(e*x+1)*ln(x^n)*e*b^2*Pi*csgn(I*c*x^n)^3-I*e*ln(e*x)*Pi*a*b*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)-I*e*ln(e*x)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)-I*ln(e*x)*ln(x)*e*n*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+2*b^2*e
*n^2*ln(x)-2*b^2*e*n^2*ln(e*x+1)-2*b^2*n^2*ln(e*x+1)/x-e*ln(e*x)*Pi^2*b^2*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-1/2*e*ln(e*x+1)*Pi^2*b^2*csgn(I*c)*csg
n(I*c*x^n)^5+1/4*e*ln(e*x+1)*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-1/2*e*l

```
n(e*x+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+I*ln(e*x)*ln(x)*e*n*b^2*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*n/x*ln(e*x+1)*b^2*Pi*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)+I*n/x*ln(e*x+1)*b^2*Pi*csgn(I*c*x^n)^3+I*e*ln(e*x+1)*Pi*l
n(c)*b^2*csgn(I*c*x^n)^3+I/x*ln(e*x+1)*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)-1/2*e*ln(e*x+1)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^
n)^3-1/2*ln(e*x+1)/x*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-1/4*e
*ln(e*x)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-I*n*ln(x)*e*b^2
*Pi*csgn(I*c*x^n)^3+1/4*ln(e*x+1)/x*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn
(I*c*x^n)^2+1/2*e*ln(e*x)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+
I*n*e*polylog(2,-e*x)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+e*ln(e*x+1
)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-2*ln(e*x)*ln(x)*e*n*b^2*ln
(c)+I*n*e*ln(e*x+1)*b^2*Pi*csgn(I*c*x^n)^3+I*n*e*polylog(2,-e*x)*b^2*Pi*csg
n(I*c*x^n)^3+1/2*e*ln(e*x)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3
+1/4*e*ln(e*x+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-1/2*ln(
e*x+1)/x*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+1/4*ln(e*x+1)/x*P
i^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+1/4*e*ln(e*x+1)*Pi^2*b^2*csgn(I*c)^2*
csgn(I*c*x^n)^4-I*n*ln(x)*e*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*e*
ln(e*x+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*x^n)*cs...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*x*e*log(x) - (b^2*x*e + b^2)*log(x*e + 1))*log(x^n)^2/x + integrate(((
b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x*e + 1) - 2*(b^2*n*x*e*log(x) - (b^
2*n*x*e + b^2*(n + log(c)) + a*b)*log(x*e + 1))*log(x^n))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(x*e + 1) + 2*a*b*log(c*x^n)*log(x*e + 1) + a
^2*log(x*e + 1))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log{(cx^n)})^2 \log{(ex + 1)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*log(e*x + 1)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(x*e + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^2, x)
```

3.16 $\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=287

$$-\frac{7b^2en^2}{4x}-\frac{1}{4}b^2e^2n^2\log(x)-\frac{3ben(a+b\log(cx^n))}{2x}+\frac{1}{2}be^2n\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))^2}{2x}$$

```
[Out] -7/4*b^2*e*n^2/x-1/4*b^2*e^2*n^2*ln(x)-3/2*b*e*n*(a+b*ln(c*x^n))/x+1/2*b*e^
2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))-1/2*e*(a+b*ln(c*x^n))^2/x+1/2*e^2*ln(1+1/e/
x)*(a+b*ln(c*x^n))^2+1/4*b^2*e^2*n^2*ln(e*x+1)-1/4*b^2*n^2*ln(e*x+1)/x^2-1/
2*b*n*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(e*x+1)/x^2-1/2
*b^2*e^2*n^2*polylog(2,-1/e/x)-b*e^2*n*(a+b*ln(c*x^n))*polylog(2,-1/e/x)-b^
2*e^2*n^2*polylog(3,-1/e/x)
```

Rubi [A]
time = 0.31, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2342, 2341, 2425, 46, 2380, 2379, 2438, 2421, 6724}

$$-be^2n\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))-\frac{1}{2}b^2e^2n^2\text{PolyLog}\left(2,-\frac{1}{ex}\right)-b^2e^2n^2\text{PolyLog}\left(3,-\frac{1}{ex}\right)+\frac{1}{2}e^2\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^2+\frac{1}{2}be^2n\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))^2}{2x}-\frac{3ben(a+b\log(cx^n))}{2x}-\frac{\log(ex+1)(a+b\log(cx^n))^2}{2x^2}-\frac{bn\log(ex+1)(a+b\log(cx^n))}{2x^2}-\frac{1}{4}b^2e^2n^2\log(x)+\frac{1}{4}b^2e^2n^2\log(ex+1)-\frac{b^2n^2\log(ex+1)}{4x^2}-\frac{7b^2en^2}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

```
[Out] (-7*b^2*e*n^2)/(4*x) - (b^2*e^2*n^2*Log[x])/4 - (3*b*e*n*(a + b*Log[c*x^n])
)/(2*x) + (b*e^2*n*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n]))/2 - (e*(a + b*Log[c
*x^n])^2)/(2*x) + (e^2*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^2)/2 + (b^2*e^2*
n^2*Log[1 + e*x])/4 - (b^2*n^2*Log[1 + e*x])/(4*x^2) - (b*n*(a + b*Log[c*x^
n])*Log[1 + e*x])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[1 + e*x])/(2*x^2) - (
b^2*e^2*n^2*PolyLog[2, -(1/(e*x))])/2 - b*e^2*n*(a + b*Log[c*x^n])*PolyLog[
2, -(1/(e*x))] - b^2*e^2*n^2*PolyLog[3, -(1/(e*x))]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(1+ex)}{x^3}\,dx = -\frac{b^2n^2\log(1+ex)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(1+ex)}{2x^2} - \frac{(a+b\log(cx^n)}{2x}$$

$$= -\frac{b^2n^2\log(1+ex)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(1+ex)}{2x^2} - \frac{(a+b\log(cx^n)}{2x}$$

$$= -\frac{b^2n^2\log(1+ex)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(1+ex)}{2x^2} - \frac{(a+b\log(cx^n)}{2x}$$

$$= -\frac{b^2en^2}{4x} - \frac{1}{4}b^2e^2n^2\log(x) + \frac{1}{4}b^2e^2n^2\log(1+ex) - \frac{b^2n^2\log(1+ex)}{4x^2} -$$

$$= -\frac{3b^2en^2}{4x} - \frac{1}{4}b^2e^2n^2\log(x) - \frac{ben(a+b\log(cx^n))}{2x} - \frac{1}{4}e^2(a+b\log(cx^n$$

$$= -\frac{7b^2en^2}{4x} - \frac{1}{4}b^2e^2n^2\log(x) - \frac{3ben(a+b\log(cx^n))}{2x} - \frac{1}{4}e^2(a+b\log(cx$$

$$= -\frac{7b^2en^2}{4x} - \frac{1}{4}b^2e^2n^2\log(x) - \frac{3ben(a+b\log(cx^n))}{2x} - \frac{1}{4}e^2(a+b\log(cx$$

Mathematica [A]
time = 0.12, size = 513, normalized size = 1.79

$$-\frac{6a^2ex+18abenx+21b^2en^2x+6a^2e^2x^2\log(x)+6abe^2nx^2\log(x)+3b^2e^2n^2x^2\log(x)-6abe^2nx^2\log^2(x)-3b^2e^2n^2x^2\log^2(x)+2b^2e^2n^2x^2\log^3(x)+12abex\log(cx^n)+18b^2enx\log(cx^n)+12abe^2x^2\log(x)\log(cx^n)+6b^2e^2nx^2\log(x)\log(cx^n)-6b^2e^2nx^2\log^2(x)\log(cx^n)+6b^2ex\log^2(cx^n)+6b^2e^2x^2\log(x)\log^2(cx^n)+6a^2\log(1+ex)+6abn\log(1+ex)+3b^2n^2\log(1+ex)-6a^2e^2x^2\log(1+ex)-6abe^2nx^2\log(1+ex)-3b^2e^2n^2x^2\log(1+ex)+12ab\log(cx^n)\log(1+ex)+6b^2n\log(cx^n)\log(1+ex)-12abe^2x^2\log(cx^n)\log(1+ex)-6b^2e^2nx^2\log(cx^n)\log(1+ex)+6b^2\log^2(cx^n)\log(1+ex)-6b^2e^2x^2\log^2(cx^n)\log(1+ex)-6be^2nx^2(2a+bn+2b\log(cx^n))\mathrm{Li}_2(-ex)+12b^2e^2n^2x^2\mathrm{Li}_3(-ex)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/12*(6*a^2*e*x + 18*a*b*e*n*x + 21*b^2*e*n^2*x + 6*a^2*e^2*x^2*Log[x] + 6
*a*b*e^2*n*x^2*Log[x] + 3*b^2*e^2*n^2*x^2*Log[x] - 6*a*b*e^2*n*x^2*Log[x]^2
 - 3*b^2*e^2*n^2*x^2*Log[x]^2 + 2*b^2*e^2*n^2*x^2*Log[x]^3 + 12*a*b*e*x*Log
[c*x^n] + 18*b^2*e*n*x*Log[c*x^n] + 12*a*b*e^2*x^2*Log[x]*Log[c*x^n] + 6*b^
2*e^2*n*x^2*Log[x]*Log[c*x^n] - 6*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] + 6*b^2
*e*x*Log[c*x^n]^2 + 6*b^2*e^2*x^2*Log[x]*Log[c*x^n]^2 + 6*a^2*Log[1 + e*x]
+ 6*a*b*n*Log[1 + e*x] + 3*b^2*n^2*Log[1 + e*x] - 6*a^2*e^2*x^2*Log[1 + e*x
] - 6*a*b*e^2*n*x^2*Log[1 + e*x] - 3*b^2*e^2*n^2*x^2*Log[1 + e*x] + 12*a*b*
Log[c*x^n]*Log[1 + e*x] + 6*b^2*n*Log[c*x^n]*Log[1 + e*x] - 12*a*b*e^2*x^2*
Log[c*x^n]*Log[1 + e*x] - 6*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] + 6*b^2*L
og[c*x^n]^2*Log[1 + e*x] - 6*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b*e^
2*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -(e*x)] + 12*b^2*e^2*n^2*x^
2*PolyLog[3, -(e*x)])/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 4445, normalized size = 15.49

method	result	size
risch	Expression too large to display	4445

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(e*x+1)/x^2*a^2-1/2*I*e^2*ln(e*x+1)*Pi*a*b*csgn(I*c*x^n)^3-1/2*I*ln(
x)*ln(e*x)*e^2*n*b^2*Pi*csgn(I*c*x^n)^3+1/8*ln(e*x+1)/x^2*Pi^2*b^2*csgn(I*x
^n)^2*csgn(I*c*x^n)^4-1/4*ln(e*x+1)/x^2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^
5-1/2*I*e/x*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*e/x*ln(c)*Pi*b^2*c
sgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*e^2*ln(e*x+1)*ln(c)*Pi*b^2*csgn(I*c)*csgn(
I*c*x^n)^2-1/2*I*e^2*ln(e*x)*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*e/x*Pi*
a*b*csgn(I*c)*csgn(I*c*x^n)^2-b*e/x*ln(x^n)*a-3/4*I*n*e/x*b^2*Pi*csgn(I*x^n
)*csgn(I*c*x^n)^2-b/x^2*ln(e*x+1)*ln(x^n)*a-1/2*I*ln(e*x)*ln(x^n)*e^2*b^2*P
i*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*e^2*ln(e*x)*ln(c)*Pi*b^2*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/4*I*n*e^2*ln(x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/x^
2*ln(e*x+1)*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*e/x*ln(c)^2*b^2-1/
2*e^2*ln(e*x)*ln(c)^2*b^2+1/2*b^2*ln(e*x+1)*ln(x^n)*e^2*n-3/2*b^2*n*e/x*ln(
x^n)-1/2*b^2*n/x^2*ln(e*x+1)*ln(x^n)-1/2*b^2*ln(x)^2*ln(x^n)*e^2*n-1/2*b^2*
ln(x)*ln(x^n)*e^2*n-1/2*e*a^2/x-1/2*e^2*a^2*ln(e*x)+1/2*e^2*a^2*ln(e*x+1)-1
/4*I*n*e^2*ln(e*x+1)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*e^2*l
n(e*x)*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*b*n/x^2*ln(e*x+
1)*a-1/2*b*n*e^2*ln(x)^2*a-1/2*b*n*e^2*ln(x)*a+b*n*e^2*polylog(2,-e*x)*a+1/
2*b*n*e^2*ln(e*x+1)*a-3/2*b*n*e/x*a-1/2*n/x^2*ln(e*x+1)*b^2*ln(c)-1/2*n*e^2
*ln(x)^2*b^2*ln(c)-1/2*n*e^2*ln(x)*b^2*ln(c)+n*e^2*polylog(2,-e*x)*b^2*ln(c
)-1/2*b^2*ln(x)^2*ln(e*x)*e^2*n^2+1/2*I*e^2*ln(e*x+1)*Pi*a*b*csgn(I*x^n)*cs
gn(I*c*x^n)^2-1/2*I*e^2*ln(e*x)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*ln(e
*x+1)/x^2*Pi^2*b^2*csgn(I*c*x^n)^6+1/8*e^2*ln(e*x)*Pi^2*b^2*csgn(I*c*x^n)^6
-1/8*e^2*ln(e*x+1)*Pi^2*b^2*csgn(I*c*x^n)^6+1/8*e/x*Pi^2*b^2*csgn(I*c*x^n)^
6+1/2*n*e^2*ln(e*x+1)*b^2*ln(c)-e/x*ln(c)*a*b-e^2*ln(e*x)*ln(c)*a*b+e^2*ln(
e*x+1)*ln(c)*a*b-ln(e*x+1)/x^2*ln(c)*a*b-3/2*n*e/x*b^2*ln(c)+1/2*e^2*ln(e*x
+1)*ln(c)^2*b^2-1/2*ln(e*x+1)/x^2*ln(c)^2*b^2+1/2*I*e^2*ln(e*x+1)*ln(c)*Pi*
b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*e^2*ln(e*x+1)*Pi*a*b*csgn(I*c)*csgn(I
*c*x^n)^2-1/2*I*n*e^2*polylog(2,-e*x)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/2*I*n*e^2*polylog(2,-e*x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*
e/x*ln(x^n)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*b^2*n^2*e^2*ln(x)^2+1/2*
b^2*n^2*e^2*polylog(2,-e*x)+1/3*b^2*n^2*ln(x)^3*e^2-b^2*n^2*e^2*polylog(3,-
e*x)-1/2*I*e/x*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*n/x^2*ln(e*x+
1)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3/4*I*n*e/x*b^2*Pi*csgn(I*c)*csgn(I*c
*x^n)^2-1/4*I*n*e^2*ln(x)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*n*e^2*ln(x
)^2*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*ln(e*x+1)*ln(x^n)*e^2*b^2*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/2*I*e^2*ln(e*x)*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*
x^n)^2+1/2*I*ln(e*x+1)*ln(x^n)*e^2*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*l
```

```
n(e*x+1)/x^2*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*n/x^2*ln(e*x+1)*b^2*Pi*
csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(e*x+1)/x^2*Pi*a*b*csgn(I*x^n)*csgn(I*c*x
^n)^2-1/2*I*ln(e*x+1)/x^2*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln
(e*x+1)/x^2*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*n*e^2*ln(e*x+1)*b^
2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*n*e^2*ln(e*x+1)*b^2*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-1/2*I*e/x*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*n*e^2*polyl
og(2,-e*x)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+b^2*polylog(2,-e*x)*ln(x^n)*e^2
*n-e/x*ln(x^n)*b^2*ln(c)-ln(e*x)*ln(x^n)*e^2*b^2*ln(c)+ln(e*x+1)*ln(x^n)*e^
2*b^2*ln(c)-1/x^2*ln(e*x+1)*ln(x^n)*b^2*ln(c)-1/4*e/x*Pi^2*b^2*csgn(I*c)*cs
gn(I*x^n)^2*csgn(I*c*x^n)^3-1/4*ln(e*x+1)/x^2*Pi^2*b^2*csgn(I*c)^2*csgn(I*x
^n)*csgn(I*c*x^n)^3+1/2*I*ln(e*x+1)/x^2*Pi*a*b*csgn(I*c*x^n)^3-7/4*b^2*e*n^
2/x+1/2*I*e^2*ln(e*x)*Pi*a*b*csgn(I*c*x^n)^3-b*ln(e*x)*ln(x^n)*e^2*a+b*ln(e
*x+1)*ln(x^n)*e^2*a-1/2*b^2*e/x*ln(x^n)^2-1/2*b^2*ln(e*x)*ln(x^n)^2*e^2+1/2
*b^2*ln(e*x+1)*ln(x^n)^2*e^2-1/2*b^2*ln(e*x+1)/x^2*ln(x^n)^2-1/4*I*n*e^2*ln
(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln(e*x)*ln(x^n)*e^2*b^2*Pi*c
sgn(I*c)*csgn(I*c*x^n)^2-1/2*I/x^2*ln(e*x+1)*ln(x^n)*b^2*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2+3/4*I*n*e/x*b^2*Pi*csgn(I*c*x^n)^3-1/2*I*ln(e*x+1)*ln(x^n)*e^2
*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*e/x*ln(x^n)*b^2*Pi*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*e^2*ln(e*x+1)*Pi*a*b*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)-1/4*I*n*e^2*ln(e*x+1)*b^2*Pi*csgn(I*c*x^n)^3+1/2*ln(e*x+1
)/x^2*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+1/2*I*ln(e*x+1)/x^2*ln
(c)*Pi*b^2*csgn(I*c*x^n)^3+1/2*I/x^2*ln(e*x+1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n)
^3+1/2*I*ln(e*x)*ln(x^n)*e^2*b^2*Pi*csgn(I*c*x^n)^3+1/4*I*n*e^2*ln(x)*b^2*P
i*csgn(I*c*x^n)^3-1/4*e/x*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-
1/4*e^2*ln(e*x)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+1/2*e^2*ln
(e*x)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+1/2*I*e/x*Pi*a*b*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*e^2*ln(e*x)*Pi^2*b^2*csgn(I*c)^2*csgn(I*
x^n)*csgn(I*c*x^n)^3+1/8*ln(e*x+1)/x^2*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*c
sgn(I*c*x^n)^2-1/2*e^2*ln(e*x+1)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)^4+1/8*e/x*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*x^2*e^2*log(x) + b^2*x*e - (b^2*x^2*e^2 - b^2)*log(x*e + 1))*log(
x^n)^2/x^2 - integrate(-((b^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x*e + 1) +
 (b^2*n*x^2*e^2*log(x) + b^2*n*x*e - (b^2*n*x^2*e^2 - b^2*(n + 2*log(c)) -
2*a*b)*log(x*e + 1))*log(x^n))/x^3, x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2*log(x*e + 1) + 2*a*b*log(c*x^n)*log(x*e + 1) + a
^2*log(x*e + 1))/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log(x*e + 1)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^2)/x^3, x)
```

3.17 $\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal. Leaf size=710

$$\frac{15ab^2n^2x}{8e^3}-\frac{255b^3n^3x}{128e^3}+\frac{45b^3n^3x^2}{256e^2}-\frac{175b^3n^3x^3}{3456e}+\frac{3}{128}b^3n^3x^4+\frac{15b^3n^2x\log(cx^n)}{8e^3}+\frac{3b^2n^2x(a+b\log(cx^n))}{32e^3}-\frac{21b^2}{}$$

```
[Out] -255/128*b^3*n^3*x/e^3+45/256*b^3*n^3*x^2/e^2-175/3456*b^3*n^3*x^3/e-3/32*b
^3*n^3*polylog(2,-e*x)/e^4-3/8*b^3*n^3*polylog(3,-e*x)/e^4-3/2*b^3*n^3*poly
log(4,-e*x)/e^4-1/8*x^2*(a+b*ln(c*x^n))^3/e^2+1/12*x^3*(a+b*ln(c*x^n))^3/e-
1/4*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^4+1/4*x^4*(a+b*ln(c*x^n))^3*ln(e*x+1)+3/3
2*b*n*x^4*(a+b*ln(c*x^n))^2+3/128*b^3*n^3*ln(e*x+1)/e^4-3/128*b^3*n^3*x^4*l
n(e*x+1)-9/128*b^2*n^2*x^4*(a+b*ln(c*x^n))+3/128*b^3*n^3*x^4+15/8*b^3*n^2*x
*ln(c*x^n)/e^3+3/32*b^2*n^2*x*(a+b*ln(c*x^n))/e^3-21/64*b^2*n^2*x^2*(a+b*ln
(c*x^n))/e^2+37/288*b^2*n^2*x^3*(a+b*ln(c*x^n))/e-15/16*b*n*x*(a+b*ln(c*x^n
))^2/e^3+9/32*b*n*x^2*(a+b*ln(c*x^n))^2/e^2-7/48*b*n*x^3*(a+b*ln(c*x^n))^2/
e-3/32*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^4+3/32*b^2*n^2*x^4*(a+b*ln(c*x^n
))*ln(e*x+1)+3/16*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^4-3/16*b*n*x^4*(a+b*ln(
c*x^n))^2*ln(e*x+1)+3/8*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^4-3/4*b*n
*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^4+3/2*b^2*n^2*(a+b*ln(c*x^n))*polylog(
3,-e*x)/e^4+15/8*a*b^2*n^2*x/e^3-1/16*x^4*(a+b*ln(c*x^n))^3+1/4*x*(a+b*ln(c
*x^n))^3/e^3
```

Rubi [A]
time = 0.50, antiderivative size = 710, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2442, 45, 2424, 2333, 2332, 2342, 2341, 2421, 2430, 6724, 2423, 2438}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (15*a*b^2*n^2*x)/(8*e^3) - (255*b^3*n^3*x)/(128*e^3) + (45*b^3*n^3*x^2)/(25
6*e^2) - (175*b^3*n^3*x^3)/(3456*e) + (3*b^3*n^3*x^4)/128 + (15*b^3*n^2*x*L
og[c*x^n])/(8*e^3) + (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(32*e^3) - (21*b^2*n^
2*x^2*(a + b*Log[c*x^n]))/(64*e^2) + (37*b^2*n^2*x^3*(a + b*Log[c*x^n]))/(2
88*e) - (9*b^2*n^2*x^4*(a + b*Log[c*x^n]))/128 - (15*b*n*x*(a + b*Log[c*x^n
])^2)/(16*e^3) + (9*b*n*x^2*(a + b*Log[c*x^n])^2)/(32*e^2) - (7*b*n*x^3*(a
+ b*Log[c*x^n])^2)/(48*e) + (3*b*n*x^4*(a + b*Log[c*x^n])^2)/32 + (x*(a + b
*Log[c*x^n])^3)/(4*e^3) - (x^2*(a + b*Log[c*x^n])^3)/(8*e^2) + (x^3*(a + b*
Log[c*x^n])^3)/(12*e) - (x^4*(a + b*Log[c*x^n])^3)/16 + (3*b^3*n^3*Log[1 +
e*x])/(128*e^4) - (3*b^3*n^3*x^4*Log[1 + e*x])/128 - (3*b^2*n^2*(a + b*Log[
c*x^n])*Log[1 + e*x])/(32*e^4) + (3*b^2*n^2*x^4*(a + b*Log[c*x^n])*Log[1 +
e*x])/32 + (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/(16*e^4) - (3*b*n*x^4*
```

```
(a + b*Log[c*x^n])^2*Log[1 + e*x])/16 - ((a + b*Log[c*x^n])^3*Log[1 + e*x])
/(4*e^4) + (x^4*(a + b*Log[c*x^n])^3*Log[1 + e*x])/4 - (3*b^3*n^3*PolyLog[2
, -(e*x)])/(32*e^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(8*
e^4) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/(4*e^4) - (3*b^3*n^3
*PolyLog[3, -(e*x)])/(8*e^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e
*x)])/(2*e^4) - (3*b^3*n^3*PolyLog[4, -(e*x)])/(2*e^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3(a+b\log(cx^n))^3\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))^3}{4e^3} - \frac{x^2(a+b\log(cx^n))^3}{8e^2} + \frac{x^3(a+b\log(cx^n))^3}{12e} -$$

$$= \frac{x(a+b\log(cx^n))^3}{4e^3} - \frac{x^2(a+b\log(cx^n))^3}{8e^2} + \frac{x^3(a+b\log(cx^n))^3}{12e} -$$

$$= -\frac{15bnx(a+b\log(cx^n))^2}{16e^3} + \frac{9bnx^2(a+b\log(cx^n))^2}{32e^2} - \frac{7bnx^3(a+b}{48}$$

$$= \frac{3ab^2n^2x}{2e^3} + \frac{3b^3n^3x^2}{32e^2} - \frac{b^3n^3x^3}{54e} + \frac{3}{512}b^3n^3x^4 - \frac{3b^2n^2x^2(a+b\log(cx}{16e^2}$$

$$= \frac{15ab^2n^2x}{8e^3} - \frac{3b^3n^3x}{2e^3} + \frac{9b^3n^3x^2}{64e^2} - \frac{7b^3n^3x^3}{216e} + \frac{3}{256}b^3n^3x^4 + \frac{3b^3n^2x\,\mathrm{l}}{2e}$$

$$= \frac{15ab^2n^2x}{8e^3} - \frac{63b^3n^3x}{32e^3} + \frac{21b^3n^3x^2}{128e^2} - \frac{37b^3n^3x^3}{864e} + \frac{9}{512}b^3n^3x^4 + \frac{15b^3}{}$$

$$= \frac{15ab^2n^2x}{8e^3} - \frac{63b^3n^3x}{32e^3} + \frac{21b^3n^3x^2}{128e^2} - \frac{37b^3n^3x^3}{864e} + \frac{9}{512}b^3n^3x^4 + \frac{15b^3}{}$$

$$= \frac{15ab^2n^2x}{8e^3} - \frac{63b^3n^3x}{32e^3} + \frac{21b^3n^3x^2}{128e^2} - \frac{37b^3n^3x^3}{864e} + \frac{9}{512}b^3n^3x^4 + \frac{15b^3}{}$$

$$= \frac{15ab^2n^2x}{8e^3} - \frac{255b^3n^3x}{128e^3} + \frac{45b^3n^3x^2}{256e^2} - \frac{175b^3n^3x^3}{3456e} + \frac{3}{128}b^3n^3x^4 + \frac{15}{}$$

Mathematica [A]
time = 0.21, size = 1144, normalized size = 1.61

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (1728*a^3*e*x - 6480*a^2*b*e*n*x + 13608*a*b^2*e*n^2*x - 13770*b^3*e*n^3*x
- 864*a^3*e^2*x^2 + 1944*a^2*b*e^2*n*x^2 - 2268*a*b^2*e^2*n^2*x^2 + 1215*b^
3*e^2*n^3*x^2 + 576*a^3*e^3*x^3 - 1008*a^2*b*e^3*n*x^3 + 888*a*b^2*e^3*n^2*
x^3 - 350*b^3*e^3*n^3*x^3 - 432*a^3*e^4*x^4 + 648*a^2*b*e^4*n*x^4 - 486*a*b
^2*e^4*n^2*x^4 + 162*b^3*e^4*n^3*x^4 + 5184*a^2*b*e*x*Log[c*x^n] - 12960*a*
b^2*e*n*x*Log[c*x^n] + 13608*b^3*e*n^2*x*Log[c*x^n] - 2592*a^2*b*e^2*x^2*Lo
g[c*x^n] + 3888*a*b^2*e^2*n*x^2*Log[c*x^n] - 2268*b^3*e^2*n^2*x^2*Log[c*x^n
] + 1728*a^2*b*e^3*x^3*Log[c*x^n] - 2016*a*b^2*e^3*n*x^3*Log[c*x^n] + 888*b
^3*e^3*n^2*x^3*Log[c*x^n] - 1296*a^2*b*e^4*x^4*Log[c*x^n] + 1296*a*b^2*e^4*
n*x^4*Log[c*x^n] - 486*b^3*e^4*n^2*x^4*Log[c*x^n] + 5184*a*b^2*e*x*Log[c*x^
n]^2 - 6480*b^3*e*n*x*Log[c*x^n]^2 - 2592*a*b^2*e^2*x^2*Log[c*x^n]^2 + 1944
```

```
*b^3*e^2*n*x^2*Log[c*x^n]^2 + 1728*a*b^2*e^3*x^3*Log[c*x^n]^2 - 1008*b^3*e^
3*n*x^3*Log[c*x^n]^2 - 1296*a*b^2*e^4*x^4*Log[c*x^n]^2 + 648*b^3*e^4*n*x^4*
Log[c*x^n]^2 + 1728*b^3*e*x*Log[c*x^n]^3 - 864*b^3*e^2*x^2*Log[c*x^n]^3 + 5
76*b^3*e^3*x^3*Log[c*x^n]^3 - 432*b^3*e^4*x^4*Log[c*x^n]^3 - 1728*a^3*Log[1
 + e*x] + 1296*a^2*b*n*Log[1 + e*x] - 648*a*b^2*n^2*Log[1 + e*x] + 162*b^3*
n^3*Log[1 + e*x] + 1728*a^3*e^4*x^4*Log[1 + e*x] - 1296*a^2*b*e^4*n*x^4*Log
[1 + e*x] + 648*a*b^2*e^4*n^2*x^4*Log[1 + e*x] - 162*b^3*e^4*n^3*x^4*Log[1
+ e*x] - 5184*a^2*b*Log[c*x^n]*Log[1 + e*x] + 2592*a*b^2*n*Log[c*x^n]*Log[1
 + e*x] - 648*b^3*n^2*Log[c*x^n]*Log[1 + e*x] + 5184*a^2*b*e^4*x^4*Log[c*x^
n]*Log[1 + e*x] - 2592*a*b^2*e^4*n*x^4*Log[c*x^n]*Log[1 + e*x] + 648*b^3*e^
4*n^2*x^4*Log[c*x^n]*Log[1 + e*x] - 5184*a*b^2*Log[c*x^n]^2*Log[1 + e*x] +
1296*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 5184*a*b^2*e^4*x^4*Log[c*x^n]^2*Log[
1 + e*x] - 1296*b^3*e^4*n*x^4*Log[c*x^n]^2*Log[1 + e*x] - 1728*b^3*Log[c*x^
n]^3*Log[1 + e*x] + 1728*b^3*e^4*x^4*Log[c*x^n]^3*Log[1 + e*x] - 648*b*n*(8
*a^2 - 4*a*b*n + b^2*n^2 - 4*b*(-4*a + b*n)*Log[c*x^n] + 8*b^2*Log[c*x^n]^2
)*PolyLog[2, -(e*x)] + 2592*b^2*n^2*(4*a - b*n + 4*b*Log[c*x^n])*PolyLog[3,
 -(e*x)] - 10368*b^3*n^3*PolyLog[4, -(e*x)])/(6912*e^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
[Out] int(x^3*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
[Out] -1/48*(3*b^3*x^4*e^4 - 4*b^3*x^3*e^3 + 6*b^3*x^2*e^2 - 12*b^3*x*e - 12*(b^3
*x^4*e^4 - b^3)*log(x*e + 1))*e^(-4)*log(x^n)^3 + 1/16*e^(-4)*integrate((48
*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^4*e^4*log(x*e + 1)*log(x^n) + 16
*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^4*e^4*log(x*e +
 1) + (3*b^3*n*x^4*e^4 - 4*b^3*n*x^3*e^3 + 6*b^3*n*x^2*e^2 - 12*b^3*n*x*e -
 12*((b^3*(n - 4*log(c)) - 4*a*b^2)*x^4*e^4 - b^3*n)*log(x*e + 1))*log(x^n)
^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*x^3*log(c*x^n)^2*log(x
*e + 1) + 3*a^2*b*x^3*log(c*x^n)*log(x*e + 1) + a^3*x^3*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x^3*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^3*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

3.18 $\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal. Leaf size=615

$$-\frac{8ab^2n^2x}{3e^2}+\frac{80b^3n^3x}{27e^2}-\frac{65b^3n^3x^2}{216e}+\frac{8}{81}b^3n^3x^3-\frac{8b^3n^2x\log(cx^n)}{3e^2}-\frac{2b^2n^2x(a+b\log(cx^n))}{9e^2}+\frac{19b^2n^2x^2(a+b\log(}{36e}$$

```
[Out] 80/27*b^3*n^3*x/e^2-65/216*b^3*n^3*x^2/e+2/9*b^3*n^3*polylog(2,-e*x)/e^3+2/
3*b^3*n^3*polylog(3,-e*x)/e^3+2*b^3*n^3*polylog(4,-e*x)/e^3-1/3*x*(a+b*ln(c
*x^n))^3/e^2+1/6*x^2*(a+b*ln(c*x^n))^3/e+1/3*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^
3-2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))+2/9*b*n*x^3*(a+b*ln(c*x^n))^2-2/27*b^3*n^
3*ln(e*x+1)/e^3-2/27*b^3*n^3*x^3*ln(e*x+1)+8/81*b^3*n^3*x^3+b*n*(a+b*ln(c*x
^n))^2*polylog(2,-e*x)/e^3-2/3*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e^3-
2*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^3-8/3*b^3*n^2*x*ln(c*x^n)/e^2-2
/9*b^2*n^2*x*(a+b*ln(c*x^n))/e^2+19/36*b^2*n^2*x^2*(a+b*ln(c*x^n))/e+4/3*b*
n*x*(a+b*ln(c*x^n))^2/e^2-5/12*b*n*x^2*(a+b*ln(c*x^n))^2/e+2/9*b^2*n^2*(a+b
*ln(c*x^n))*ln(e*x+1)/e^3+2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))*ln(e*x+1)-1/3*b*n
*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^3-1/3*b*n*x^3*(a+b*ln(c*x^n))^2*ln(e*x+1)-8/
3*a*b^2*n^2*x/e^2-1/9*x^3*(a+b*ln(c*x^n))^3+1/3*x^3*(a+b*ln(c*x^n))^3*ln(e*
x+1)
```

Rubi [A]
time = 0.44, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2442, 45, 2424, 2333, 2332, 2342, 2341, 2421, 2430, 6724, 2423, 2438}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-8*a*b^2*n^2*x)/(3*e^2) + (80*b^3*n^3*x)/(27*e^2) - (65*b^3*n^3*x^2)/(216*
e) + (8*b^3*n^3*x^3)/81 - (8*b^3*n^2*x*Log[c*x^n])/(3*e^2) - (2*b^2*n^2*x*(
a + b*Log[c*x^n]))/(9*e^2) + (19*b^2*n^2*x^2*(a + b*Log[c*x^n]))/(36*e) - (
2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 + (4*b*n*x*(a + b*Log[c*x^n])^2)/(3*e^2
) - (5*b*n*x^2*(a + b*Log[c*x^n])^2)/(12*e) + (2*b*n*x^3*(a + b*Log[c*x^n])
^2)/9 - (x*(a + b*Log[c*x^n])^3)/(3*e^2) + (x^2*(a + b*Log[c*x^n])^3)/(6*e)
 - (x^3*(a + b*Log[c*x^n])^3)/9 - (2*b^3*n^3*Log[1 + e*x])/(27*e^3) - (2*b^
3*n^3*x^3*Log[1 + e*x])/27 + (2*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(9
*e^3) + (2*b^2*n^2*x^3*(a + b*Log[c*x^n])*Log[1 + e*x])/9 - (b*n*(a + b*Log
[c*x^n])^2*Log[1 + e*x])/(3*e^3) - (b*n*x^3*(a + b*Log[c*x^n])^2*Log[1 + e*
x])/3 + ((a + b*Log[c*x^n])^3*Log[1 + e*x])/(3*e^3) + (x^3*(a + b*Log[c*x^n
])^3*Log[1 + e*x])/3 + (2*b^3*n^3*PolyLog[2, -(e*x)])/(9*e^3) - (2*b^2*n^2*
(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/(3*e^3) + (b*n*(a + b*Log[c*x^n])^2*
PolyLog[2, -(e*x)])/e^3 + (2*b^3*n^3*PolyLog[3, -(e*x)])/(3*e^3) - (2*b^2*n
```

```
^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)])/e^3 + (2*b^3*n^3*PolyLog[4, -(e*x
)])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
```

```
u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^3\log(1+ex)\,dx = -\frac{x(a+b\log(cx^n))^3}{3e^2}+\frac{x^2(a+b\log(cx^n))^3}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^3$$

$$= -\frac{x(a+b\log(cx^n))^3}{3e^2}+\frac{x^2(a+b\log(cx^n))^3}{6e}-\frac{1}{9}x^3(a+b\log(cx^n))^3$$

$$= \frac{4bnx(a+b\log(cx^n))^2}{3e^2}-\frac{5bnx^2(a+b\log(cx^n))^2}{12e}+\frac{2}{9}bnx^3(a+b\log$$

$$= -\frac{2ab^2n^2x}{e^2}-\frac{b^3n^3x^2}{8e}+\frac{2}{81}b^3n^3x^3+\frac{b^2n^2x^2(a+b\log(cx^n))}{4e}-\frac{2}{27}b^2n$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{2b^3n^3x}{e^2}-\frac{5b^3n^3x^2}{24e}+\frac{4}{81}b^3n^3x^3-\frac{2b^3n^2x\log(cx^n)}{e^2}-$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log(cx^n)}{3e^2}$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log(cx^n)}{3e^2}$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{26b^3n^3x}{9e^2}-\frac{19b^3n^3x^2}{72e}+\frac{2}{27}b^3n^3x^3-\frac{8b^3n^2x\log(cx^n)}{3e^2}$$

$$= -\frac{8ab^2n^2x}{3e^2}+\frac{80b^3n^3x}{27e^2}-\frac{65b^3n^3x^2}{216e}+\frac{8}{81}b^3n^3x^3-\frac{8b^3n^2x\log(cx^n)}{3e^2}$$

Mathematica [A]
time = 0.17, size = 975, normalized size = 1.59

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-216*a^3*e*x + 864*a^2*b*e*n*x - 1872*a*b^2*e*n^2*x + 1920*b^3*e*n^3*x + 1
08*a^3*e^2*x^2 - 270*a^2*b*e^2*n*x^2 + 342*a*b^2*e^2*n^2*x^2 - 195*b^3*e^2*
n^3*x^2 - 72*a^3*e^3*x^3 + 144*a^2*b*e^3*n*x^3 - 144*a*b^2*e^3*n^2*x^3 + 64
*b^3*e^3*n^3*x^3 - 648*a^2*b*e*x*Log[c*x^n] + 1728*a*b^2*e*n*x*Log[c*x^n] -
 1872*b^3*e*n^2*x*Log[c*x^n] + 324*a^2*b*e^2*x^2*Log[c*x^n] - 540*a*b^2*e^2
*n*x^2*Log[c*x^n] + 342*b^3*e^2*n^2*x^2*Log[c*x^n] - 216*a^2*b*e^3*x^3*Log[
c*x^n] + 288*a*b^2*e^3*n*x^3*Log[c*x^n] - 144*b^3*e^3*n^2*x^3*Log[c*x^n] -
648*a*b^2*e*x*Log[c*x^n]^2 + 864*b^3*e*n*x*Log[c*x^n]^2 + 324*a*b^2*e^2*x^2
*Log[c*x^n]^2 - 270*b^3*e^2*n*x^2*Log[c*x^n]^2 - 216*a*b^2*e^3*x^3*Log[c*x^
n]^2 + 144*b^3*e^3*n*x^3*Log[c*x^n]^2 - 216*b^3*e*x*Log[c*x^n]^3 + 108*b^3*
e^2*x^2*Log[c*x^n]^3 - 72*b^3*e^3*x^3*Log[c*x^n]^3 + 216*a^3*Log[1 + e*x] -
```

```
 216*a^2*b*n*Log[1 + e*x] + 144*a*b^2*n^2*Log[1 + e*x] - 48*b^3*n^3*Log[1 +
 e*x] + 216*a^3*e^3*x^3*Log[1 + e*x] - 216*a^2*b*e^3*n*x^3*Log[1 + e*x] + 1
44*a*b^2*e^3*n^2*x^3*Log[1 + e*x] - 48*b^3*e^3*n^3*x^3*Log[1 + e*x] + 648*a
^2*b*Log[c*x^n]*Log[1 + e*x] - 432*a*b^2*n*Log[c*x^n]*Log[1 + e*x] + 144*b^
3*n^2*Log[c*x^n]*Log[1 + e*x] + 648*a^2*b*e^3*x^3*Log[c*x^n]*Log[1 + e*x] -
 432*a*b^2*e^3*n*x^3*Log[c*x^n]*Log[1 + e*x] + 144*b^3*e^3*n^2*x^3*Log[c*x^
n]*Log[1 + e*x] + 648*a*b^2*Log[c*x^n]^2*Log[1 + e*x] - 216*b^3*n*Log[c*x^n
]^2*Log[1 + e*x] + 648*a*b^2*e^3*x^3*Log[c*x^n]^2*Log[1 + e*x] - 216*b^3*e^
3*n*x^3*Log[c*x^n]^2*Log[1 + e*x] + 216*b^3*Log[c*x^n]^3*Log[1 + e*x] + 216
*b^3*e^3*x^3*Log[c*x^n]^3*Log[1 + e*x] + 72*b*n*(9*a^2 - 6*a*b*n + 2*b^2*n^
2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] +
432*b^2*n^2*(-3*a + b*n - 3*b*Log[c*x^n])*PolyLog[3, -(e*x)] + 1296*b^3*n^3
*PolyLog[4, -(e*x)])/(648*e^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2} \left(a +b \ln \left(c \,x^{n}\right)\right)^{3} \ln \left(e x +1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/18*(2*b^3*x^3*e^3 - 3*b^3*x^2*e^2 + 6*b^3*x*e - 6*(b^3*x^3*e^3 + b^3)*lo
g(x*e + 1))*e^(-3)*log(x^n)^3 + 1/6*e^(-3)*integrate((18*(b^3*log(c)^2 + 2*
a*b^2*log(c) + a^2*b)*x^3*e^3*log(x*e + 1)*log(x^n) + 6*(b^3*log(c)^3 + 3*a
*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^3*e^3*log(x*e + 1) + (2*b^3*n*x^3*e
^3 - 3*b^3*n*x^2*e^2 + 6*b^3*n*x*e - 6*((b^3*(n - 3*log(c)) - 3*a*b^2)*x^3*
e^3 + b^3*n)*log(x*e + 1))*log(x^n)^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*x^2*log(c*x^n)^2*log(x
*e + 1) + 3*a^2*b*x^2*log(c*x^n)*log(x*e + 1) + a^3*x^2*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x^2*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x^2*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

3.19 $\int x(a+b\log(cx^n))^3\log(1+ex)\,dx$

Optimal. Leaf size=530

$$\frac{9ab^2n^2x}{2e}-\frac{45b^3n^3x}{8e}+\frac{3}{4}b^3n^3x^2+\frac{9b^3n^2x\log(cx^n)}{2e}+\frac{3b^2n^2x(a+b\log(cx^n))}{4e}-\frac{9}{8}b^2n^2x^2(a+b\log(cx^n))-\frac{9bnx(a}{}$$

```
[Out] 9/2*a*b^2*n^2*x/e-45/8*b^3*n^3*x/e+3/4*b^3*n^3*x^2+9/2*b^3*n^2*x*ln(c*x^n)/
e+3/4*b^2*n^2*x*(a+b*ln(c*x^n))/e-9/8*b^2*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*n*x
*(a+b*ln(c*x^n))^2/e+3/4*b*n*x^2*(a+b*ln(c*x^n))^2+1/2*x*(a+b*ln(c*x^n))^3/
e-1/4*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*n^3*ln(e*x+1)/e^2-3/8*b^3*n^3*x^2*ln(e*
x+1)-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/e^2+3/4*b^2*n^2*x^2*(a+b*ln(c*x^
n))*ln(e*x+1)+3/4*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/e^2-3/4*b*n*x^2*(a+b*ln(c
*x^n))^2*ln(e*x+1)-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/e^2+1/2*x^2*(a+b*ln(c*x^
n))^3*ln(e*x+1)-3/4*b^3*n^3*polylog(2,-e*x)/e^2+3/2*b^2*n^2*(a+b*ln(c*x^n))
*polylog(2,-e*x)/e^2-3/2*b*n*(a+b*ln(c*x^n))^2*polylog(2,-e*x)/e^2-3/2*b^3*
n^3*polylog(3,-e*x)/e^2+3*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-e*x)/e^2-3*b^3
*n^3*polylog(4,-e*x)/e^2
```

Rubi [A]
time = 0.34, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2442, 45, 2424, 2333, 2332, 2342, 2341, 2421, 2430, 6724, 2423, 2438}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (9*a*b^2*n^2*x)/(2*e) - (45*b^3*n^3*x)/(8*e) + (3*b^3*n^3*x^2)/4 + (9*b^3*n
^2*x*Log[c*x^n])/(2*e) + (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(4*e) - (9*b^2*n^
2*x^2*(a + b*Log[c*x^n]))/8 - (9*b*n*x*(a + b*Log[c*x^n])^2)/(4*e) + (3*b*n
*x^2*(a + b*Log[c*x^n])^2)/4 + (x*(a + b*Log[c*x^n])^3)/(2*e) - (x^2*(a + b
*Log[c*x^n])^3)/4 + (3*b^3*n^3*Log[1 + e*x])/(8*e^2) - (3*b^3*n^3*x^2*Log[1
 + e*x])/8 - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(4*e^2) + (3*b^2*n
^2*x^2*(a + b*Log[c*x^n])*Log[1 + e*x])/4 + (3*b*n*(a + b*Log[c*x^n])^2*Log
[1 + e*x])/(4*e^2) - (3*b*n*x^2*(a + b*Log[c*x^n])^2*Log[1 + e*x])/4 - ((a
+ b*Log[c*x^n])^3*Log[1 + e*x])/(2*e^2) + (x^2*(a + b*Log[c*x^n])^3*Log[1 +
 e*x])/2 - (3*b^3*n^3*PolyLog[2, -(e*x)])/(4*e^2) + (3*b^2*n^2*(a + b*Log[c
*x^n])*PolyLog[2, -(e*x)])/(2*e^2) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2,
 -(e*x)])/(2*e^2) - (3*b^3*n^3*PolyLog[3, -(e*x)])/(2*e^2) + (3*b^2*n^2*(a
+ b*Log[c*x^n])*PolyLog[3, -(e*x)])/e^2 - (3*b^3*n^3*PolyLog[4, -(e*x)])/e^
2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^3\log(1+ex)\,dx = \frac{x(a+b\log(cx^n))^3}{2e} - \frac{1}{4}x^2(a+b\log(cx^n))^3 - \frac{(a+b\log(cx^n))^3\log(1}{2e^2}$$

$$= \frac{x(a+b\log(cx^n))^3}{2e} - \frac{1}{4}x^2(a+b\log(cx^n))^3 - \frac{(a+b\log(cx^n))^3\log(1}{2e^2}$$

$$= -\frac{9bnx(a+b\log(cx^n))^2}{4e} + \frac{3}{4}bnx^2(a+b\log(cx^n))^2 + \frac{x(a+b\log(cx}{2e}$$

$$= \frac{3ab^2n^2x}{e} + \frac{3}{16}b^3n^3x^2 - \frac{3}{8}b^2n^2x^2(a+b\log(cx^n)) - \frac{9bnx(a+b\log(c}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{3b^3n^3x}{e} + \frac{3}{8}b^3n^3x^2 + \frac{3b^3n^2x\log(cx^n)}{e} + \frac{3b^2n^2x(a+b\log}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log(cx^n)}{2e} + \frac{3b^2n^2x(a+b}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log(cx^n)}{2e} + \frac{3b^2n^2x(a+b}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{21b^3n^3x}{4e} + \frac{9}{16}b^3n^3x^2 + \frac{9b^3n^2x\log(cx^n)}{2e} + \frac{3b^2n^2x(a+b}{4e}$$

$$= \frac{9ab^2n^2x}{2e} - \frac{45b^3n^3x}{8e} + \frac{3}{4}b^3n^3x^2 + \frac{9b^3n^2x\log(cx^n)}{2e} + \frac{3b^2n^2x(a+b\,\mathrm{lo}}{4e}$$

Mathematica [A]
time = 0.14, size = 806, normalized size = 1.52

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (4*a^3*e*x - 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x - 45*b^3*e*n^3*x - 2*a^3*e^2
*x^2 + 6*a^2*b*e^2*n*x^2 - 9*a*b^2*e^2*n^2*x^2 + 6*b^3*e^2*n^3*x^2 + 12*a^2
*b*e*x*Log[c*x^n] - 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n] -
 6*a^2*b*e^2*x^2*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[c*x^n] - 9*b^3*e^2*n^2
*x^2*Log[c*x^n] + 12*a*b^2*e*x*Log[c*x^n]^2 - 18*b^3*e*n*x*Log[c*x^n]^2 - 6
*a*b^2*e^2*x^2*Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[c*x^n]^2 + 4*b^3*e*x*Log[
c*x^n]^3 - 2*b^3*e^2*x^2*Log[c*x^n]^3 - 4*a^3*Log[1 + e*x] + 6*a^2*b*n*Log[
1 + e*x] - 6*a*b^2*n^2*Log[1 + e*x] + 3*b^3*n^3*Log[1 + e*x] + 4*a^3*e^2*x^
2*Log[1 + e*x] - 6*a^2*b*e^2*n*x^2*Log[1 + e*x] + 6*a*b^2*e^2*n^2*x^2*Log[1
 + e*x] - 3*b^3*e^2*n^3*x^2*Log[1 + e*x] - 12*a^2*b*Log[c*x^n]*Log[1 + e*x]
 + 12*a*b^2*n*Log[c*x^n]*Log[1 + e*x] - 6*b^3*n^2*Log[c*x^n]*Log[1 + e*x] +
```

```
 12*a^2*b*e^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a*b^2*e^2*n*x^2*Log[c*x^n]*L
og[1 + e*x] + 6*b^3*e^2*n^2*x^2*Log[c*x^n]*Log[1 + e*x] - 12*a*b^2*Log[c*x^
n]^2*Log[1 + e*x] + 6*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 12*a*b^2*e^2*x^2*Lo
g[c*x^n]^2*Log[1 + e*x] - 6*b^3*e^2*n*x^2*Log[c*x^n]^2*Log[1 + e*x] - 4*b^3
*Log[c*x^n]^3*Log[1 + e*x] + 4*b^3*e^2*x^2*Log[c*x^n]^3*Log[1 + e*x] - 6*b*
n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^
n]^2)*PolyLog[2, -(e*x)] + 12*b^2*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[
3, -(e*x)] - 24*b^3*n^3*PolyLog[4, -(e*x)])/(8*e^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^3\ln(ex+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*x^2*e^2 - 2*b^3*x*e - 2*(b^3*x^2*e^2 - b^3)*log(x*e + 1))*e^(-2)*
log(x^n)^3 + 1/4*e^(-2)*integrate((12*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*
b)*x^2*e^2*log(x*e + 1)*log(x^n) + 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a
^2*b*log(c) + a^3)*x^2*e^2*log(x*e + 1) + 3*(b^3*n*x^2*e^2 - 2*b^3*n*x*e +
2*(b^3*n - (b^3*(n - 2*log(c)) - 2*a*b^2)*x^2*e^2)*log(x*e + 1))*log(x^n)^2
)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*x*log(c*x^n)^2*log(x*e +
 1) + 3*a^2*b*x*log(c*x^n)*log(x*e + 1) + a^3*x*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

3.20 $\int (a+b\log(cx^n))^3 \log(1+ex)\,dx$

Optimal. Leaf size=327

$$-12ab^2n^2x+24b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))+6bnx(a+b\log(cx^n))^2-x(a+b\log(cx^n))^3$$

```
[Out] -12*a*b^2*n^2*x+24*b^3*n^3*x-12*b^3*n^2*x*ln(c*x^n)-6*b^2*n^2*x*(a+b*ln(c*x
^n))+6*b*n*x*(a+b*ln(c*x^n))^2-x*(a+b*ln(c*x^n))^3-6*b^3*n^3*(e*x+1)*ln(e*x
+1)/e+6*b^2*n^2*(e*x+1)*(a+b*ln(c*x^n))*ln(e*x+1)/e-3*b*n*(e*x+1)*(a+b*ln(c
*x^n))^2*ln(e*x+1)/e+(e*x+1)*(a+b*ln(c*x^n))^3*ln(e*x+1)/e+6*b^3*n^3*polylo
g(2,-e*x)/e-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-e*x)/e+3*b*n*(a+b*ln(c*x^n
))^2*polylog(2,-e*x)/e+6*b^3*n^3*polylog(3,-e*x)/e-6*b^2*n^2*(a+b*ln(c*x^n)
)*polylog(3,-e*x)/e+6*b^3*n^3*polylog(4,-e*x)/e
```

Rubi [A]
time = 0.51, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {2436, 2332, 2417, 2333, 2388, 2339, 30, 6874, 2338, 2458, 45, 2393, 2352, 2421, 6724, 2430}

$$-\frac{6b^2n^2\text{PolyLog}(2,-ex)(a+b\log(cx^n))}{e}-\frac{6b^2n^2\text{PolyLog}(3,-ex)(a+b\log(cx^n))}{e}+\frac{3bn\text{PolyLog}(2,-ex)(a+b\log(cx^n))^2}{e}+\frac{6b^3n^3\text{PolyLog}(2,-ex)}{e}+\frac{6b^3n^3\text{PolyLog}(3,-ex)}{e}+\frac{6b^3n^3\text{PolyLog}(4,-ex)}{e}+\frac{6b^2n^2(ex+1)\log(ex+1)(a+b\log(cx^n))}{e}-6b^2n^2x(a+b\log(cx^n))-12ab^2n^2x-\frac{3bn(ex+1)\log(ex+1)(a+b\log(cx^n))^2}{e}+\frac{(ex+1)\log(ex+1)(a+b\log(cx^n))^3}{e}+6bnx(a+b\log(cx^n))^2-x(a+b\log(cx^n))^3-12b^3n^2x\log(cx^n)-\frac{6b^3n^3(ex+1)\log(ex+1)}{e}+24b^3n^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] -12*a*b^2*n^2*x + 24*b^3*n^3*x - 12*b^3*n^2*x*Log[c*x^n] - 6*b^2*n^2*x*(a +
 b*Log[c*x^n]) + 6*b*n*x*(a + b*Log[c*x^n])^2 - x*(a + b*Log[c*x^n])^3 - (6
*b^3*n^3*(1 + e*x)*Log[1 + e*x])/e + (6*b^2*n^2*(1 + e*x)*(a + b*Log[c*x^n]
)*Log[1 + e*x])/e - (3*b*n*(1 + e*x)*(a + b*Log[c*x^n])^2*Log[1 + e*x])/e +
 ((1 + e*x)*(a + b*Log[c*x^n])^3*Log[1 + e*x])/e + (6*b^3*n^3*PolyLog[2, -(
e*x)])/e - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e*x)])/e + (3*b*n*(a
+ b*Log[c*x^n])^2*PolyLog[2, -(e*x)])/e + (6*b^3*n^3*PolyLog[3, -(e*x)])/e
- (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e*x)])/e + (6*b^3*n^3*PolyLog[
4, -(e*x)])/e
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+b\log(cx^n))^3\log(1+ex)\,dx = -x(a+b\log(cx^n))^3+\frac{(1+ex)(a+b\log(cx^n))^3\log(1+ex)}{e}-(3bn)$$

$$= -x(a+b\log(cx^n))^3+\frac{(1+ex)(a+b\log(cx^n))^3\log(1+ex)}{e}+(3bn)$$

$$= 3bnx(a+b\log(cx^n))^2-x(a+b\log(cx^n))^3+\frac{(1+ex)(a+b\log(cx^n}{e}$$

$$= -6ab^2n^2x+3bnx(a+b\log(cx^n))^2-x(a+b\log(cx^n))^3+\frac{(1+ex)(}{}$$

$$= -6ab^2n^2x+6b^3n^3x-6b^3n^2x\log(cx^n)+6bnx(a+b\log(cx^n))^2-x($$

$$= -12ab^2n^2x+6b^3n^3x-6b^3n^2x\log(cx^n)+6bnx(a+b\log(cx^n))^2-x($$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log(cx^n)+6bnx(a+b\log(cx^n))^2-$$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log(cx^n)+6bnx(a+b\log(cx^n))^2-$$

$$= -12ab^2n^2x+12b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

$$= -12ab^2n^2x+18b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

$$= -12ab^2n^2x+24b^3n^3x-12b^3n^2x\log(cx^n)-6b^2n^2x(a+b\log(cx^n))-$$

Mathematica [A]
time = 0.11, size = 584, normalized size = 1.79

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[1 + e*x],x]
```

```
[Out] (-(a^3*e*x) + 6*a^2*b*e*n*x - 18*a*b^2*e*n^2*x + 24*b^3*e*n^3*x - 3*a^2*b*e
*x*Log[c*x^n] + 12*a*b^2*e*n*x*Log[c*x^n] - 18*b^3*e*n^2*x*Log[c*x^n] - 3*a
*b^2*e*x*Log[c*x^n]^2 + 6*b^3*e*n*x*Log[c*x^n]^2 - b^3*e*x*Log[c*x^n]^3 + a
^3*Log[1 + e*x] - 3*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*Log[1 + e*x] - 6*b^3
*n^3*Log[1 + e*x] + a^3*e*x*Log[1 + e*x] - 3*a^2*b*e*n*x*Log[1 + e*x] + 6*a
*b^2*e*n^2*x*Log[1 + e*x] - 6*b^3*e*n^3*x*Log[1 + e*x] + 3*a^2*b*Log[c*x^n]
*Log[1 + e*x] - 6*a*b^2*n*Log[c*x^n]*Log[1 + e*x] + 6*b^3*n^2*Log[c*x^n]*Lo
g[1 + e*x] + 3*a^2*b*e*x*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*x*Log[c*x^n]
*Log[1 + e*x] + 6*b^3*e*n^2*x*Log[c*x^n]*Log[1 + e*x] + 3*a*b^2*Log[c*x^n]^
2*Log[1 + e*x] - 3*b^3*n*Log[c*x^n]^2*Log[1 + e*x] + 3*a*b^2*e*x*Log[c*x^n]
^2*Log[1 + e*x] - 3*b^3*e*n*x*Log[c*x^n]^2*Log[1 + e*x] + b^3*Log[c*x^n]^3*
Log[1 + e*x] + b^3*e*x*Log[c*x^n]^3*Log[1 + e*x] + 3*b*n*(a^2 - 2*a*b*n + 2
*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)]
- 6*b^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(e*x)] + 6*b^3*n^3*PolyLog
[4, -(e*x)])/e
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(ex+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(e*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="maxima")
```

```
[Out] -(b^3*x*e - (b^3*x*e + b^3)*log(x*e + 1))*e^(-1)*log(x^n)^3 + e^(-1)*integr
ate((3*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e*log(x*e + 1)*log(x^n) +
(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x*e*log(x*e + 1) +
 3*(b^3*n*x*e - (b^3*n + (b^3*(n - log(c)) - a*b^2)*x*e)*log(x*e + 1))*log(
x^n)^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*log(c*x^n)^2*log(x*e + 1)
+ 3*a^2*b*log(c*x^n)*log(x*e + 1) + a^3*log(x*e + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(x*e + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln (e\,x+1)\,\left(a+b\,\ln (c\,x^n)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x + 1)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(e*x + 1)*(a + b*log(c*x^n))^3, x)
```

3.21 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx$

Optimal. Leaf size=81

$$-(a+b\log(cx^n))^3\operatorname{Li}_2(-ex)+3bn(a+b\log(cx^n))^2\operatorname{Li}_3(-ex)-6b^2n^2(a+b\log(cx^n))\operatorname{Li}_4(-ex)+6b^3n^3\operatorname{Li}_5(-ex)$$

```
[Out] -(a+b*ln(c*x^n))^3*polylog(2,-e*x)+3*b*n*(a+b*ln(c*x^n))^2*polylog(3,-e*x)-
6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-e*x)+6*b^3*n^3*polylog(5,-e*x)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2421, 2430, 6724}

$$-6b^2n^2\text{PolyLog}(4,-ex)(a+b\log(cx^n))+3bn\text{PolyLog}(3,-ex)(a+b\log(cx^n))^2-\text{PolyLog}(2,-ex)(a+b\log(cx^n))^3+6b^3n^3\text{PolyLog}(5,-ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*(a + b*Log[c*x^n])^2*Pol
yLog[3, -(e*x)] - 6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4, -(e*x)] + 6*b^3*n
^3*PolyLog[5, -(e*x)]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x}\,dx = -(a+b\log(cx^n))^3\,\mathrm{Li}_2(-ex) + (3bn)\int \frac{(a+b\log(cx^n))^2\,\mathrm{Li}_2(-ex)}{x}\,d$$

$$= -(a+b\log(cx^n))^3\,\mathrm{Li}_2(-ex) + 3bn(a+b\log(cx^n))^2\,\mathrm{Li}_3(-ex) - (6b^2$$

$$= -(a+b\log(cx^n))^3\,\mathrm{Li}_2(-ex) + 3bn(a+b\log(cx^n))^2\,\mathrm{Li}_3(-ex) - 6b^2n$$

$$= -(a+b\log(cx^n))^3\,\mathrm{Li}_2(-ex) + 3bn(a+b\log(cx^n))^2\,\mathrm{Li}_3(-ex) - 6b^2n$$

Mathematica [**A**]
time = 0.07, size = 77, normalized size = 0.95

$$-(a+b\log(cx^n))^3\,\mathrm{Li}_2(-ex) + 3bn\big((a+b\log(cx^n))^2\,\mathrm{Li}_3(-ex) + 2bn(-((a+b\log(cx^n))\,\mathrm{Li}_4(-ex)) + bn\mathrm{Li}_5(-ex))\big)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x,x]
```

```
[Out] -((a + b*Log[c*x^n])^3*PolyLog[2, -(e*x)]) + 3*b*n*((a + b*Log[c*x^n])^2*Po
lyLog[3, -(e*x)] + 2*b*n*(-((a + b*Log[c*x^n])*PolyLog[4, -(e*x)]) + b*n*Po
lyLog[5, -(e*x)]))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 4058, normalized size = 50.10

method	result	size
risch	Expression too large to display	4058

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(e*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)^3*dilog(e*x+1)-3/2*I*Pi*ln(x^n)^2*dilog(e*x+1)*b^3*csgn(I*c)*csgn(I*c
*x^n)^2+6*ln(c)*ln(x)*ln(x^n)*dilog(e*x+1)*b^3*n+3/2*ln(x)*Pi^2*dilog(e*x+1
)*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5-3/4*Pi^2*polylog(3,-e*x)*b^3*n*csgn(I*c
)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+3/2*Pi^2*polylog(3,-e*x)*b^3*n*csgn(I*c)*
csgn(I*x^n)^2*csgn(I*c*x^n)^3-3*Pi^2*polylog(3,-e*x)*b^3*n*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)^4+3*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)^4+3/4*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*c*x^n)^6-3/4*Pi^2
*polylog(3,-e*x)*b^3*n*csgn(I*c*x^n)^6+3/2*ln(x)*Pi^2*dilog(e*x+1)*b^3*n*cs
gn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-3/2*ln(x)*Pi^2*polylog(2,-e*x)*b^3*n*
csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+3*I*ln(c)*Pi*polylog(3,-e*x)*b^3*n*
```

csgn(I*c)*csgn(I*c*x^n)^2+3/4*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*c)^2*csg
n(I*c*x^n)^4-3/2*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*c)*csgn(I*c*x^n)^5-6*
ln(c)*ln(x^n)*dilog(e*x+1)*a*b^2+6*ln(c)*polylog(3,-e*x)*ln(x^n)*b^3*n-3*ln
(x^n)*dilog(e*x+1)*a^2*b-3*ln(c)^2*ln(x^n)*dilog(e*x+1)*b^3-3*ln(c)*ln(x^n)
^2*dilog(e*x+1)*b^3+3*polylog(3,-e*x)*a^2*b*n+3*ln(c)^2*polylog(3,-e*x)*b^3
*n-6*ln(c)*polylog(4,-e*x)*b^3*n^2+3*I*ln(x)*Pi*ln(x^n)*dilog(e*x+1)*b^3*n*
csgn(I*x^n)*csgn(I*c*x^n)^2+3*I*Pi*polylog(3,-e*x)*ln(x^n)*b^3*n*csgn(I*c)*
csgn(I*c*x^n)^2+3*I*Pi*polylog(4,-e*x)*b^3*n^2*csgn(I*c*x^n)^3-3/4*Pi^2*pol
ylog(3,-e*x)*b^3*n*csgn(I*c)^2*csgn(I*c*x^n)^4+3*I*ln(c)*Pi*ln(x^n)*dilog(e
*x+1)*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-3*I*ln(x)*Pi*polylog(2,-e*x)*
a*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2+3/4*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*
c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-3*I*ln(x)*Pi*dilog(e*x+1)*a*b^2*n*csgn(I
*c*x^n)^3-6*ln(x)*polylog(2,-e*x)*ln(x^n)*a*b^2*n+6*ln(x)*ln(x^n)*dilog(e*x
+1)*a*b^2*n-6*ln(c)*ln(x)*polylog(2,-e*x)*ln(x^n)*b^3*n+3/2*ln(x)*Pi^2*dilo
g(e*x+1)*b^3*n*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-3/2*ln(x)*Pi^2*polyl
og(2,-e*x)*b^3*n*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-3*I*ln(x)*Pi*polyl
og(2,-e*x)*ln(x^n)*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^2-3*ln(x)*Pi^2*dilog(e*x
+1)*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-3/4*ln(x)*Pi^2*dilog(e*x+1)
*b^3*n*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+3/4*ln(x)*Pi^2*polylog(2,-
e*x)*b^3*n*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-3/2*I*ln(x)^2*Pi*dilog
(e*x+1)*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2-3*ln(x)*polylog(2,-e*x)*ln(x^n)
^2*b^3*n+3*ln(x)*ln(x^n)^2*dilog(e*x+1)*b^3*n+3*ln(x)^2*polylog(2,-e*x)*ln(
x^n)*b^3*n^2-3*ln(x)^2*ln(x^n)*dilog(e*x+1)*b^3*n^2-3*ln(x)*polylog(2,-e*x)
*a^2*b*n+3*ln(x)*dilog(e*x+1)*a^2*b*n-3*ln(c)^2*ln(x)*polylog(2,-e*x)*b^3*n
+3*ln(c)^2*ln(x)*dilog(e*x+1)*b^3*n+6*ln(c)*polylog(3,-e*x)*a*b^2*n+3*ln(c)
*ln(x)^2*polylog(2,-e*x)*b^3*n^2-3*ln(c)*ln(x)^2*dilog(e*x+1)*b^3*n^2+3*I*P
i*polylog(3,-e*x)*a*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2+3*I*ln(x)*Pi*polylog(2,
-e*x)*ln(x^n)*b^3*n*csgn(I*c*x^n)^3+6*polylog(3,-e*x)*ln(x^n)*a*b^2*n+3/2*P
i^2*polylog(3,-e*x)*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5-3*I*ln(c)*Pi*ln(x^n)*
dilog(e*x+1)*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+3*I*ln(x)*Pi*polylog(2,-e*x)*a
*b^2*n*csgn(I*c*x^n)^3+3*I*Pi*polylog(4,-e*x)*b^3*n^2*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)-3*I*Pi*polylog(4,-e*x)*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+3
*I*Pi*polylog(3,-e*x)*ln(x^n)*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^2+3*ln(x)*Pi^
2*polylog(2,-e*x)*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+3*I*Pi*polylo
g(3,-e*x)*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+3*I*ln(c)*ln(x)*Pi*polylog(2,
-e*x)*b^3*n*csgn(I*c*x^n)^3+3/2*I*Pi*ln(x^n)^2*dilog(e*x+1)*b^3*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)-3*I*ln(c)*ln(x)*Pi*dilog(e*x+1)*b^3*n*csgn(I*c*x^n
)^3-3/2*I*ln(x)^2*Pi*dilog(e*x+1)*b^3*n^2*csgn(I*c)*csgn(I*c*x^n)^2-3*I*Pi*
ln(x^n)*dilog(e*x+1)*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-3/2*Pi^2*ln(x^n)*dil
og(e*x+1)*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+3/4*Pi^2*ln(x^n)*dilo
g(e*x+1)*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4-6*ln(c)*ln(x)*polylog(2,-e*x)*a*
b^2*n+6*ln(c)*ln(x)*dilog(e*x+1)*a*b^2*n-ln(x^n)^3*dilog(e*x+1)*b^3+3*I*ln(
x)*Pi*ln(x^n)*dilog(e*x+1)*b^3*n*csgn(I*c)*csgn(I*c*x^n)^2-3*ln(x^n)^2*dilo
g(e*x+1)*a*b^2+3*polylog(3,-e*x)*ln(x^n)^2*b^3*n-6*polylog(4,-e*x)*ln(x^n)*
b^3*n^2-3*I*ln(c)*Pi*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*c)*csgn(I*c*x^n)^2-6*p

```
olylog(4,-e*x)*a*b^2*n^2-3/2*Pi^2*ln(x^n)*dilog(e*x+1)*b^3*csgn(I*x^n)*csgn
(I*c*x^n)^5+3/4*ln(x)*Pi^2*polylog(2,-e*x)*b^3*n*csgn(I*c*x^n)^6-3/4*ln(x)*
Pi^2*dilog(e*x+1)*b^3*n*csgn(I*c*x^n)^6+3/2*Pi^2*polylog(3,-e*x)*b^3*n*csgn
(I*c)*csgn(I*c*x^n)^5-3/4*Pi^2*polylog(3,-e*x)*b^3*n*csgn(I*x^n)^2*csgn(I*c
*x^n)^4+3/2*I*Pi*ln(x^n)^2*dilog(e*x+1)*b^3*csgn(I*c*x^n)^3-3*I*Pi*ln(x^n)*
dilog(e*x+1)*a*b^2*csgn(I*c)*csgn(I*c*x^n)^2+3*I*ln(c)*Pi*polylog(3,-e*x)*b
^3*n*csgn(I*x^n)*csgn(I*c*x^n)^2-3/2*ln(x)*Pi^2*polylog(2,-e*x)*b^3*n*csgn(
I*c)*csgn(I*c*x^n)^5+3/4*ln(x)*Pi^2*polylog(2,-e*x)*b^3*n*csgn(I*x^n)^2*csg
n(I*c*x^n)^4-3/2*ln(x)*Pi^2*polylog(2,-e*x)*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)
^5+3*ln(x)^2*polylog(2,-e*x)*a*b^2*n^2-3*ln(x)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^3*log(x*e + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*log(c*x^n)^2*log(x*e + 1)
 + 3*a^2*b*log(c*x^n)*log(x*e + 1) + a^3*log(x*e + 1))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log(x*e + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(e\,x+1)\,\left(a+b\,\ln(c\,x^n)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x,x)
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x, x)
```

3.22 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx$

Optimal. Leaf size=342

$$6b^3en^3\log(x)-6b^2en^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))-3ben\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))^2-e\log\left(1+\frac{1}{ex}\right.$$

[Out] 6*b^3*e*n^3*ln(x)-6*b^2*e*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-3*b*e*n*ln(1+1/e/x)*(a+b*ln(c*x^n))^2-e*ln(1+1/e/x)*(a+b*ln(c*x^n))^3-6*b^3*e*n^3*ln(e*x+1)-6*b^3*n^3*ln(e*x+1)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(e*x+1)/x-(a+b*ln(c*x^n))^3*ln(e*x+1)/x+6*b^3*e*n^3*polylog(2,-1/e/x)+6*b^2*e*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)+3*b*e*n*(a+b*ln(c*x^n))^2*polylog(2,-1/e/x)+6*b^3*e*n^3*polylog(3,-1/e/x)+6*b^2*e*n^2*(a+b*ln(c*x^n))*polylog(3,-1/e/x)+6*b^3*e*n^3*polylog(4,-1/e/x)

Rubi [A]
time = 0.30, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2342, 2341, 2425, 36, 29, 31, 2379, 2438, 2421, 6724, 2430}

$$6b^2en^2\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))+6b^2en^2\text{PolyLog}\left(3,-\frac{1}{ex}\right)(a+b\log(cx^n))+3ben\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))^2+6b^3en^3\text{PolyLog}\left(2,-\frac{1}{ex}\right)+6b^3en^3\text{PolyLog}\left(3,-\frac{1}{ex}\right)+6b^3en^3\text{PolyLog}\left(4,-\frac{1}{ex}\right)-6b^2en^2\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))-\frac{6b^2n^2\log(ex+1)(a+b\log(cx^n))}{x}-3ben\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^2-\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{x}-e\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^3-\frac{\log(ex+1)(a+b\log(cx^n))^3}{x}+6b^3en^3\log(x)-6b^3en^3\log(ex+1)-\frac{6b^3n^3\log(ex+1)}{x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]

[Out] 6*b^3*e*n^3*Log[x] - 6*b^2*e*n^2*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n]) - 3*b*e*n*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^2 - e*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n])^3 - 6*b^3*e*n^3*Log[1 + e*x] - (6*b^3*n^3*Log[1 + e*x])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/x - ((a + b*Log[c*x^n])^3*Log[1 + e*x])/x + 6*b^3*e*n^3*PolyLog[2, -(1/(e*x))] + 6*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(1/(e*x))] + 3*b*e*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(1/(e*x))] + 6*b^3*e*n^3*PolyLog[3, -(1/(e*x))] + 6*b^2*e*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(1/(e*x))] + 6*b^3*e*n^3*PolyLog[4, -(1/(e*x))]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^2}\,dx = -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+ex)}{x} - \frac{3bn(a+b$$

$$= -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+ex)}{x} - \frac{3bn(a+b$$

$$= -\frac{6b^3n^3\log(1+ex)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+ex)}{x} - \frac{3bn(a+b$$

$$= 6b^3en^3\log(x) + 3ben(a+b\log(cx^n))^2 - 6b^3en^3\log(1+ex) - \frac{6b^3n^3\,\mathrm{l}$$

$$= 6b^3en^3\log(x) + 3ben(a+b\log(cx^n))^2 + e(a+b\log(cx^n))^3 + \frac{e(a+b$$

$$= 6b^3en^3\log(x) + 3ben(a+b\log(cx^n))^2 + e(a+b\log(cx^n))^3 + \frac{e(a+b$$

$$= 6b^3en^3\log(x) + 3ben(a+b\log(cx^n))^2 + e(a+b\log(cx^n))^3 + \frac{e(a+b$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 770 vs. 2(342) = 684.
time = 0.19, size = 770, normalized size = 2.25

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^2,x]
```

```
[Out] a^3*e*Log[x] + 3*a^2*b*e*n*Log[x] + 6*a*b^2*e*n^2*Log[x] + 6*b^3*e*n^3*Log[
x] - (3*a^2*b*e*n*Log[x]^2)/2 - 3*a*b^2*e*n^2*Log[x]^2 - 3*b^3*e*n^3*Log[x]
^2 + a*b^2*e*n^2*Log[x]^3 + b^3*e*n^3*Log[x]^3 - (b^3*e*n^3*Log[x]^4)/4 + 3
```

```
*a^2*b*e*Log[x]*Log[c*x^n] + 6*a*b^2*e*n*Log[x]*Log[c*x^n] + 6*b^3*e*n^2*Lo
g[x]*Log[c*x^n] - 3*a*b^2*e*n*Log[x]^2*Log[c*x^n] - 3*b^3*e*n^2*Log[x]^2*Lo
g[c*x^n] + b^3*e*n^2*Log[x]^3*Log[c*x^n] + 3*a*b^2*e*Log[x]*Log[c*x^n]^2 +
3*b^3*e*n*Log[x]*Log[c*x^n]^2 - (3*b^3*e*n*Log[x]^2*Log[c*x^n]^2)/2 + b^3*e
*Log[x]*Log[c*x^n]^3 - a^3*e*Log[1 + e*x] - 3*a^2*b*e*n*Log[1 + e*x] - 6*a*
b^2*e*n^2*Log[1 + e*x] - 6*b^3*e*n^3*Log[1 + e*x] - (a^3*Log[1 + e*x])/x -
(3*a^2*b*n*Log[1 + e*x])/x - (6*a*b^2*n^2*Log[1 + e*x])/x - (6*b^3*n^3*Log[
1 + e*x])/x - 3*a^2*b*e*Log[c*x^n]*Log[1 + e*x] - 6*a*b^2*e*n*Log[c*x^n]*Lo
g[1 + e*x] - 6*b^3*e*n^2*Log[c*x^n]*Log[1 + e*x] - (3*a^2*b*Log[c*x^n]*Log[
1 + e*x])/x - (6*a*b^2*n*Log[c*x^n]*Log[1 + e*x])/x - (6*b^3*n^2*Log[c*x^n]
*Log[1 + e*x])/x - 3*a*b^2*e*Log[c*x^n]^2*Log[1 + e*x] - 3*b^3*e*n*Log[c*x^
n]^2*Log[1 + e*x] - (3*a*b^2*Log[c*x^n]^2*Log[1 + e*x])/x - (3*b^3*n*Log[c*
x^n]^2*Log[1 + e*x])/x - b^3*e*Log[c*x^n]^3*Log[1 + e*x] - (b^3*Log[c*x^n]^
3*Log[1 + e*x])/x - 3*b*e*n*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[
c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 6*b^2*e*n^2*(a + b*n + b*Lo
g[c*x^n])*PolyLog[3, -(e*x)] - 6*b^3*e*n^3*PolyLog[4, -(e*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 14041, normalized size = 41.06

method	result	size
risch	Expression too large to display	14041

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(e*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] (b^3*x*e*log(x) - (b^3*x*e + b^3)*log(x*e + 1))*log(x^n)^3/x + integrate((3
*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x*e + 1)*log(x^n) - 3*(b^3*n*x
*e*log(x) - (b^3*n*x*e + b^3*(n + log(c)) + a*b^2)*log(x*e + 1))*log(x^n)^2
 + (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(x*e + 1))/x
^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*log(c*x^n)^2*log(x*e + 1)
 + 3*a^2*b*log(c*x^n)*log(x*e + 1) + a^3*log(x*e + 1))/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(x*e + 1)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^2, x)
```

3.23 $\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx$

Optimal. Leaf size=470

$$-\frac{45b^3en^3}{8x}-\frac{3}{8}b^3e^2n^3\log(x)-\frac{21b^2en^2(a+b\log(cx^n))}{4x}+\frac{3}{4}b^2e^2n^2\log\left(1+\frac{1}{ex}\right)(a+b\log(cx^n))-\frac{9ben(a+b\log}{4x}$$

```
[Out] -45/8*b^3*e*n^3/x-3/8*b^3*e^2*n^3*ln(x)-21/4*b^2*e*n^2*(a+b*ln(c*x^n))/x+3/
4*b^2*e^2*n^2*ln(1+1/e/x)*(a+b*ln(c*x^n))-9/4*b*e*n*(a+b*ln(c*x^n))^2/x+3/4
*b*e^2*n*ln(1+1/e/x)*(a+b*ln(c*x^n))^2-1/2*e*(a+b*ln(c*x^n))^3/x+1/2*e^2*ln
(1+1/e/x)*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*n^3*ln(e*x+1)-3/8*b^3*n^3*ln(e*x+1)
/x^2-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(e*x+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln
(e*x+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(e*x+1)/x^2-3/4*b^3*e^2*n^3*polylog(2,-
1/e/x)-3/2*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(2,-1/e/x)-3/2*b*e^2*n*(a+b*l
n(c*x^n))^2*polylog(2,-1/e/x)-3/2*b^3*e^2*n^3*polylog(3,-1/e/x)-3*b^2*e^2*n
^2*(a+b*ln(c*x^n))*polylog(3,-1/e/x)-3*b^3*e^2*n^3*polylog(4,-1/e/x)
```

Rubi [A]
time = 0.53, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2342, 2341, 2425, 46, 2380, 2379, 2438, 2421, 6724, 2430}

$$-\frac{3}{2}b^2e^2n^2\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))-3b^2e^2n^2\text{PolyLog}\left(3,-\frac{1}{ex}\right)(a+b\log(cx^n))-\frac{3}{2}be^2n\text{PolyLog}\left(2,-\frac{1}{ex}\right)(a+b\log(cx^n))^2-\frac{3}{4}b^3e^2n^3\text{PolyLog}\left(2,-\frac{1}{ex}\right)-\frac{3}{2}b^3e^2n^3\text{PolyLog}\left(3,-\frac{1}{ex}\right)-3b^3e^2n^3\text{PolyLog}\left(4,-\frac{1}{ex}\right)+\frac{3}{4}b^2e^2n^2\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))-\frac{21b^2en^2(a+b\log(cx^n))}{4x}-\frac{3b^2n^2\log(ex+1)(a+b\log(cx^n))}{4x^2}+\frac{3}{4}be^2n\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^2+\frac{1}{2}e^2\log\left(\frac{1}{ex}+1\right)(a+b\log(cx^n))^3-\frac{9ben(a+b\log(cx^n))^2}{4x}-\frac{e(a+b\log(cx^n))^3}{2x}-\frac{3bn\log(ex+1)(a+b\log(cx^n))^2}{4x^2}-\frac{\log(ex+1)(a+b\log(cx^n))^3}{2x^2}-\frac{3}{8}b^3e^2n^3\log(x)+\frac{3}{8}b^3e^2n^3\log(ex+1)-\frac{3b^3n^3\log(ex+1)}{8x^2}-\frac{45b^3en^3}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

```
[Out] (-45*b^3*e*n^3)/(8*x) - (3*b^3*e^2*n^3*Log[x])/8 - (21*b^2*e*n^2*(a + b*Log
[c*x^n]))/(4*x) + (3*b^2*e^2*n^2*Log[1 + 1/(e*x)]*(a + b*Log[c*x^n]))/4 - (
9*b*e*n*(a + b*Log[c*x^n])^2)/(4*x) + (3*b*e^2*n*Log[1 + 1/(e*x)]*(a + b*Lo
g[c*x^n])^2)/4 - (e*(a + b*Log[c*x^n])^3)/(2*x) + (e^2*Log[1 + 1/(e*x)]*(a
+ b*Log[c*x^n])^3)/2 + (3*b^3*e^2*n^3*Log[1 + e*x])/8 - (3*b^3*n^3*Log[1 +
e*x])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[1 + e*x])/(4*x^2) - (3*b*
n*(a + b*Log[c*x^n])^2*Log[1 + e*x])/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[1
+ e*x])/(2*x^2) - (3*b^3*e^2*n^3*PolyLog[2, -(1/(e*x))])/4 - (3*b^2*e^2*n^2
*(a + b*Log[c*x^n])*PolyLog[2, -(1/(e*x))])/2 - (3*b*e^2*n*(a + b*Log[c*x^n
])^2*PolyLog[2, -(1/(e*x))])/2 - (3*b^3*e^2*n^3*PolyLog[3, -(1/(e*x))])/2 -
 3*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(1/(e*x))] - 3*b^3*e^2*n^3*Po
lyLog[4, -(1/(e*x))]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(1+ex)}{x^3}\,dx = -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{4x^2} - \frac{3bn(a+b\,\mathrm{l}$$

$$= -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{4x^2} - \frac{3bn(a+b\,\mathrm{l}$$

$$= -\frac{3b^3n^3\log(1+ex)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(1+ex)}{4x^2} - \frac{3bn(a+b\,\mathrm{l}$$

$$= -\frac{3b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) + \frac{3}{8}b^3e^2n^3\log(1+ex) - \frac{3b^3n^3\log(1+ex)}{8x^2}$$

$$= -\frac{9b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{3b^2en^2(a+b\log(cx^n))}{4x} - \frac{3}{8}be^2n(a+b\,\mathrm{log}$$

$$= -\frac{21b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{9b^2en^2(a+b\log(cx^n))}{4x} - \frac{3}{8}be^2n(a+b\,\mathrm{lo}$$

$$= -\frac{45b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{21b^2en^2(a+b\log(cx^n))}{4x} - \frac{3}{8}be^2n(a+b$$

$$= -\frac{45b^3en^3}{8x} - \frac{3}{8}b^3e^2n^3\log(x) - \frac{21b^2en^2(a+b\log(cx^n))}{4x} - \frac{3}{8}be^2n(a+b$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1047 vs. $2(470) = 940$.
time = 0.24, size = 1047, normalized size = 2.23

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[1 + e*x])/x^3,x]
```

```
[Out] -1/8*(4*a^3*e*x + 18*a^2*b*e*n*x + 42*a*b^2*e*n^2*x + 45*b^3*e*n^3*x + 4*a^
3*e^2*x^2*Log[x] + 6*a^2*b*e^2*n*x^2*Log[x] + 6*a*b^2*e^2*n^2*x^2*Log[x] +
3*b^3*e^2*n^3*x^2*Log[x] - 6*a^2*b*e^2*n*x^2*Log[x]^2 - 6*a*b^2*e^2*n^2*x^2
*Log[x]^2 - 3*b^3*e^2*n^3*x^2*Log[x]^2 + 4*a*b^2*e^2*n^2*x^2*Log[x]^3 + 2*b
^3*e^2*n^3*x^2*Log[x]^3 - b^3*e^2*n^3*x^2*Log[x]^4 + 12*a^2*b*e*x*Log[c*x^n
] + 36*a*b^2*e*n*x*Log[c*x^n] + 42*b^3*e*n^2*x*Log[c*x^n] + 12*a^2*b*e^2*x^
2*Log[x]*Log[c*x^n] + 12*a*b^2*e^2*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*e^2*n^2*
x^2*Log[x]*Log[c*x^n] - 12*a*b^2*e^2*n*x^2*Log[x]^2*Log[c*x^n] - 6*b^3*e^2*
n^2*x^2*Log[x]^2*Log[c*x^n] + 4*b^3*e^2*n^2*x^2*Log[x]^3*Log[c*x^n] + 12*a*
b^2*e*x*Log[c*x^n]^2 + 18*b^3*e*n*x*Log[c*x^n]^2 + 12*a*b^2*e^2*x^2*Log[x]*
Log[c*x^n]^2 + 6*b^3*e^2*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*e^2*n*x^2*Log[x]
^2*Log[c*x^n]^2 + 4*b^3*e*x*Log[c*x^n]^3 + 4*b^3*e^2*x^2*Log[x]*Log[c*x^n]^
3 + 4*a^3*Log[1 + e*x] + 6*a^2*b*n*Log[1 + e*x] + 6*a*b^2*n^2*Log[1 + e*x]
+ 3*b^3*n^3*Log[1 + e*x] - 4*a^3*e^2*x^2*Log[1 + e*x] - 6*a^2*b*e^2*n*x^2*L
og[1 + e*x] - 6*a*b^2*e^2*n^2*x^2*Log[1 + e*x] - 3*b^3*e^2*n^3*x^2*Log[1 +
e*x] + 12*a^2*b*Log[c*x^n]*Log[1 + e*x] + 12*a*b^2*n*Log[c*x^n]*Log[1 + e*x
] + 6*b^3*n^2*Log[c*x^n]*Log[1 + e*x] - 12*a^2*b*e^2*x^2*Log[c*x^n]*Log[1 +
 e*x] - 12*a*b^2*e^2*n*x^2*Log[c*x^n]*Log[1 + e*x] - 6*b^3*e^2*n^2*x^2*Log[
c*x^n]*Log[1 + e*x] + 12*a*b^2*Log[c*x^n]^2*Log[1 + e*x] + 6*b^3*n*Log[c*x^
n]^2*Log[1 + e*x] - 12*a*b^2*e^2*x^2*Log[c*x^n]^2*Log[1 + e*x] - 6*b^3*e^2*
n*x^2*Log[c*x^n]^2*Log[1 + e*x] + 4*b^3*Log[c*x^n]^3*Log[1 + e*x] - 4*b^3*e
^2*x^2*Log[c*x^n]^3*Log[1 + e*x] - 6*b*e^2*n*x^2*(2*a^2 + 2*a*b*n + b^2*n^2
 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -(e*x)] + 12
*b^2*e^2*n^2*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[3, -(e*x)] - 24*b^3*e
^2*n^3*x^2*PolyLog[4, -(e*x)])/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 17975, normalized size = 38.24

method	result	size
risch	Expression too large to display	17975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(e*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*x^2*e^2*log(x) + b^3*x*e - (b^3*x^2*e^2 - b^3)*log(x*e + 1))*log(
x^n)^3/x^2 - 1/2*integrate(-(6*(b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(
x*e + 1)*log(x^n) + 3*(b^3*n*x^2*e^2*log(x) + b^3*n*x*e - (b^3*n*x^2*e^2 -
b^3*(n + 2*log(c)) - 2*a*b^2)*log(x*e + 1))*log(x^n)^2 + 2*(b^3*log(c)^3 +
3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*log(x*e + 1))/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3*log(x*e + 1) + 3*a*b^2*log(c*x^n)^2*log(x*e + 1)
 + 3*a^2*b*log(c*x^n)*log(x*e + 1) + a^3*log(x*e + 1))/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(e*x+1)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(e*x+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log(x*e + 1)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,x+1\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(e*x + 1)*(a + b*log(c*x^n))^3)/x^3, x)
```

3.24 $\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=180

$$-\frac{3bnx^2}{16df}+\frac{1}{16}bnx^4+\frac{x^2(a+b\log(cx^n))}{4df}-\frac{1}{8}x^4(a+b\log(cx^n))+\frac{bn\log(1+dfx^2)}{16d^2f^2}-\frac{1}{16}bnx^4\log(1+dfx^2)-\frac{(a}{}$$

```
[Out] -3/16*b*n*x^2/d/f+1/16*b*n*x^4+1/4*x^2*(a+b*ln(c*x^n))/d/f-1/8*x^4*(a+b*ln(
c*x^n))+1/16*b*n*ln(d*f*x^2+1)/d^2/f^2-1/16*b*n*x^4*ln(d*f*x^2+1)-1/4*(a+b*
ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))*ln(d*f*x^2+1)-1/8*
b*n*polylog(2,-d*f*x^2)/d^2/f^2
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2504, 2442, 45, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-dfx^2)}{8d^2f^2}-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{4d^2f^2}+\frac{x^2(a+b\log(cx^n))}{4df}+\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))-\frac{1}{8}x^4(a+b\log(cx^n))+\frac{bn\log(dfx^2+1)}{16d^2f^2}-\frac{3bnx^2}{16df}-\frac{1}{16}bnx^4\log(dfx^2+1)+\frac{1}{16}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-3*b*n*x^2)/(16*d*f) + (b*n*x^4)/16 + (x^2*(a + b*Log[c*x^n]))/(4*d*f) - (
x^4*(a + b*Log[c*x^n]))/8 + (b*n*Log[1 + d*f*x^2])/(16*d^2*f^2) - (b*n*x^4*
Log[1 + d*f*x^2])/16 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(4*d^2*f^2) +
(x^4*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/4 - (b*n*PolyLog[2, -(d*f*x^2)])/
(8*d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx^n)) - \frac{(a+b\log(cx^n)}{4d^2}$$

$$= -\frac{bnx^2}{8df} + \frac{1}{32}bnx^4 + \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx^n$$

$$= -\frac{bnx^2}{8df} + \frac{1}{32}bnx^4 + \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx^n$$

$$= -\frac{bnx^2}{8df} + \frac{1}{32}bnx^4 + \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx^n$$

$$= -\frac{bnx^2}{8df} + \frac{1}{32}bnx^4 + \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx^n$$

$$= -\frac{3bnx^2}{16df} + \frac{1}{16}bnx^4 + \frac{x^2(a+b\log(cx^n))}{4df} - \frac{1}{8}x^4(a+b\log(cx$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 348, normalized size = 1.93

$$\frac{ax^2}{4df} - \frac{ax^4}{8} + \frac{1}{32}bx^4(n-4(-n\log(x)+\log(cx^n))) + \frac{bx^2(-n+4(-n\log(x)+\log(cx^n)))}{16df} - \frac{a\log(1+dfx^2)}{4d^2f^2} + \frac{1}{4}ax^4\log(1+dfx^2) + \frac{b(n-4(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{16d^2f^2} + \frac{1}{16}bx^4(-n+4n\log(x)+4(-n\log(x)+\log(cx^n)))\log(1+dfx^2) - \frac{1}{2}bdfn\left(-\frac{-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)}{d^2f^2} + \frac{-\frac{x^4}{16}+\frac{1}{4}x^4\log(x)}{df} + \frac{\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)}{2d^3f^3} + \frac{\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)}{2d^3f^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (a*x^2)/(4*d*f) - (a*x^4)/8 + (b*x^4*(n - 4*(-(n*Log[x]) + Log[c*x^n])))/32
 + (b*x^2*(-n + 4*(-(n*Log[x]) + Log[c*x^n])))/(16*d*f) - (a*Log[1 + d*f*x^
```

```
2])/(4*d^2*f^2) + (a*x^4*Log[1 + d*f*x^2])/4 + (b*(n - 4*(-(n*Log[x]) + Log
[c*x^n]))*Log[1 + d*f*x^2])/(16*d^2*f^2) + (b*x^4*(-n + 4*n*Log[x] + 4*(-(n
*Log[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/16 - (b*d*f*n*(-((-1/4*x^2 + (x^2
*Log[x])/2)/(d^2*f^2)) + (-1/16*x^4 + (x^4*Log[x])/4)/(d*f) + (Log[x]*Log[1
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2*d^3*f^3) +
 (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(2
*d^3*f^3)))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 840, normalized size = 4.67

method	result
risch	$-\frac{i\pi b\,x^4\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2}{16}-\frac{i\pi b\,x^4\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2}{16}-\frac{i\pi b\mathrm{csgn}(ic\,x^n)^3x^4\ln(df\,x^2+1)}{8}+\frac{ix^2\pi b\,\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2}{8df}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(c)*b*x^4-1/4/d^2/f^2*b*n*ln(x)*ln(1-x*(-d*f)^(1/2))-1/4/d^2/f^2*b*n
*ln(x)*ln(1+x*(-d*f)^(1/2))+1/4*a*x^4*ln(d*f*x^2+1)+1/4*a/d/f*x^2-1/4*a/d^2
/f^2*ln(d*f*x^2+1)-1/4/d^2/f^2*b*n*dilog(1+x*(-d*f)^(1/2))-1/4/d^2/f^2*b*n*
dilog(1-x*(-d*f)^(1/2))-1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*c)*csgn(I*c
*x^n)^2+1/16*b*n*x^4-1/8*a*x^4-1/4/d^2/f^2*ln(d*f*x^2+1)*b*ln(c)+1/4/d/f*x^
2*b*ln(c)-1/16*I*Pi*b*x^4*csgn(I*c)*csgn(I*c*x^n)^2-1/16*I*Pi*b*x^4*csgn(I*
x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*b*csgn(I*c*x^n)^3*x^4*ln(d*f*x^2+1)-1/8*I*Pi*
b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x^4*ln(d*f*x^2+1)+(1/4*x^4*b*ln(d*(1/
d+f*x^2))-1/8*b*(d^2*f^2*x^4-2*d*f*x^2+2*ln(d*(1/d+f*x^2))+1)/d^2/f^2)*ln(x
^n)+1/8*b*n/d^2/f^2*ln(x)+1/16*I*Pi*b*x^4*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)+1/4*b*ln(c)*x^4*ln(d*f*x^2+1)-1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*x^
n)*csgn(I*c*x^n)^2+1/8*I/d/f*x^2*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*I/d/f*x
^2*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I/d/f*x^2*Pi*b*csgn(I*c*x^n)^3+1/16
*I*Pi*b*x^4*csgn(I*c*x^n)^3-3/16*b*n*x^2/d/f-1/8*I/d/f*x^2*Pi*b*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)-1/16*b*n*x^4*ln(d*f*x^2+1)+1/8*I*Pi*b*csgn(I*x^n)*csgn(I
*c*x^n)^2*x^4*ln(d*f*x^2+1)+1/4*n*b/d^2/f^2*ln(x)*ln(d*f*x^2+1)+1/16*b*n*ln
(d*f*x^2+1)/d^2/f^2+1/8*I/d^2/f^2*ln(d*f*x^2+1)*Pi*b*csgn(I*c*x^n)^3+1/8*I*
Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*x^4*ln(d*f*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x^4*log(x^n) - (b*(n - 4*log(c)) - 4*a)*x^4)*log(d*f*x^2 + 1) - i
ntegrate(1/8*(4*b*d*f*x^5*log(x^n) + (4*a*d*f - (d*f*n - 4*d*f*log(c))*b)*x
^5)/(d*f*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a*x^3*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

3.25 $\int x(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=114

$$\frac{1}{2}bnx^2-\frac{1}{2}x^2(a+b\log(cx^n))-\frac{bn(1+dfx^2)\log(1+dfx^2)}{4df}+\frac{(1+dfx^2)(a+b\log(cx^n))\log(1+dfx^2)}{2df}+\frac{bn\mathrm{L}}{}$$

```
[Out] 1/2*b*n*x^2-1/2*x^2*(a+b*ln(c*x^n))-1/4*b*n*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+1
/2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f+1/4*b*n*polylog(2,-d*f*x^2
)/d/f
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2504, 2436, 2332, 2423, 2525, 2458, 45, 2393, 2352}

$$\frac{bn\text{PolyLog}(2,-dfx^2)}{4df}+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{2df}-\frac{1}{2}x^2(a+b\log(cx^n))-\frac{bn(dfx^2+1)\log(dfx^2+1)}{4df}+\frac{1}{2}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (b*n*x^2)/2 - (x^2*(a + b*Log[c*x^n]))/2 - (b*n*(1 + d*f*x^2)*Log[1 + d*f*x
^2])/(4*d*f) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(2*d*f)
+ (b*n*PolyLog[2, -(d*f*x^2)])/(4*d*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
```

```
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x(a + b\log(cx^n))\log\left(d\left(\frac{1}{d} + fx^2\right)\right)\,dx = -\frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))\log(1 +}{2df}$$

$$= \frac{1}{4}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))}{2df}$$

$$= \frac{1}{4}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))}{2df}$$

$$= \frac{1}{4}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))}{2df}$$

$$= \frac{1}{4}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))}{2df}$$

$$= \frac{1}{4}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) + \frac{(1 + dfx^2)(a + b\log(cx^n))}{2df}$$

$$= \frac{1}{2}bnx^2 - \frac{1}{2}x^2(a + b\log(cx^n)) - \frac{bn(1 + dfx^2)\log(1 + dfx^2)}{4df}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 267, normalized size = 2.34

$$\frac{1}{4}bx^2(n - 2(-n\log(x) + \log(cx^n))) + \frac{b(-n + 2(-n\log(x) + \log(cx^n)))\log(1 + dfx^2)}{4df} + \frac{1}{4}bx^2(-n + 2n\log(x) + 2(-n\log(x) + \log(cx^n)))\log(1 + dfx^2) + \frac{1}{2}a\left(-x^2 + \frac{(1 + dfx^2)\log(1 + dfx^2)}{df}\right) - bdfn\left(\frac{-\frac{x^2}{4} + \frac{1}{2}x^2\log(x)}{df} - \frac{\log(x)\log\left(1 + i\sqrt{d}\sqrt{f}x\right) + \text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)}{2d^2f^2} - \frac{\log(x)\log\left(1 - i\sqrt{d}\sqrt{f}x\right) + \text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)}{2d^2f^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (b*x^2*(n - 2*(-(n*Log[x]) + Log[c*x^n])))/4 + (b*(-n + 2*(-(n*Log[x]) + Lo
g[c*x^n]))*Log[1 + d*f*x^2])/(4*d*f) + (b*x^2*(-n + 2*n*Log[x] + 2*(-(n*Log
[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 + (a*(-x^2 + ((1 + d*f*x^2)*Log[1 +
 d*f*x^2])/(d*f)))/2 - b*d*f*n*((-1/4*x^2 + (x^2*Log[x])/2)/(d*f) - (Log[x]
*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(2*d^2*
f^2) - (Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*
x])/(2*d^2*f^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 820, normalized size = 7.19

method	result

risch	$-\frac{nb\,x^2\ln(df\,x^2+1)}{4}+\frac{\ln(c)\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)x^2b}{2}-\frac{b\ln(c)}{2df}-\frac{a}{2df}-\frac{nb\ln(x)\ln(df\,x^2+1)}{2df}+\frac{bn\,\mathrm{dilog}\left(1+x\sqrt{-df}\right)}{2fd}+\frac{\ln(c)\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*n*b*x^2*ln(d*f*x^2+1)+1/2*ln(c)*ln(d*(1/d+f*x^2))*x^2*b-1/2/d/f*b*ln(c
)-1/2*a/d/f+1/4*I*Pi*b*x^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I/d/f*ln
(d*(1/d+f*x^2))*Pi*b*csgn(I*c*x^n)^3-1/2*n*b/d/f*ln(x)*ln(d*f*x^2+1)+1/2/d/
f*ln(c)*ln(d*(1/d+f*x^2))*b-1/2*ln(c)*b*x^2+1/4*I*Pi*b*x^2*csgn(I*c*x^n)^3-
1/4*I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*x^2*a+1/2*b*n*x^2+1/2/d/f*ln
(d*(1/d+f*x^2))*a+1/2*ln(d*(1/d+f*x^2))*x^2*a+1/2/f*b*n/d*dilog(1+x*(-d*f)^
(1/2))+1/2/f*b*n/d*dilog(1-x*(-d*f)^(1/2))-1/4*b*n/d/f*ln(d*f*x^2+1)+1/4*I/
d/f*ln(d*(1/d+f*x^2))*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I/d/f*ln(d*(1/d+f*
x^2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I/d/f*Pi*b*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)-1/4*I*ln(d*(1/d+f*x^2))*Pi*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)+(1/2*b*x^2*ln(d*(1/d+f*x^2))+1/2*b*(-d*f*x^2+ln(d*(1/d+f*x^2)))/d/
f)*ln(x^n)+1/4*I*ln(d*(1/d+f*x^2))*Pi*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4
*I/d/f*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I/d/f*ln(d*(1/d+f*x^2))*Pi*b*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*Pi*b*x^2*csgn(I*c)*csgn(I*c*x^n)^2+1/
4*I/d/f*Pi*b*csgn(I*c*x^n)^3-1/4*I*ln(d*(1/d+f*x^2))*Pi*x^2*b*csgn(I*c*x^n)
^3+1/2/f*b*n/d*ln(x)*ln(1+x*(-d*f)^(1/2))+1/2/f*b*n/d*ln(x)*ln(1-x*(-d*f)^(
1/2))+1/4*I*ln(d*(1/d+f*x^2))*Pi*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I/d/f*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log(d*f*x^2 + 1) - in
tegrate(1/2*(2*b*d*f*x^3*log(x^n) + (2*a*d*f - (d*f*n - 2*d*f*log(c))*b)*x^
3)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x*log(d*f*x^2 + 1)*log(c*x^n) + a*x*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

3.26 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}(a+b\log(cx^n))\,\mathrm{Li}_2\left(-dfx^2\right)+\frac{1}{4}bn\mathrm{Li}_3\left(-dfx^2\right)$$

```
[Out] -1/2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)+1/4*b*n*polylog(3,-d*f*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2421, 6724}

$$\frac{1}{4}bn\text{PolyLog}\left(3,-dfx^2\right)-\frac{1}{2}\text{PolyLog}\left(2,-dfx^2\right)(a+b\log(cx^n))$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -1/2*((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)]) + (b*n*PolyLog[3, -(d*f*x^
2)])/4
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx &= -\frac{1}{2}(a+b\log(cx^n))\,\mathrm{Li}_2\left(-dfx^2\right)+\frac{1}{2}(bn)\int \frac{\mathrm{Li}_2\left(-dfx^2\right)}{x}\,dx\\ &= -\frac{1}{2}(a+b\log(cx^n))\,\mathrm{Li}_2\left(-dfx^2\right)+\frac{1}{4}bn\mathrm{Li}_3\left(-dfx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.28

$$-\frac{1}{2}a\text{Li}_2\left(-dfx^2\right)-\frac{1}{2}b\log\left(cx^n\right)\text{Li}_2\left(-dfx^2\right)+\frac{1}{4}bn\text{Li}_3\left(-dfx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -1/2*(a*PolyLog[2, -(d*f*x^2)]) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^2)])/2 +
 (b*n*PolyLog[3, -(d*f*x^2)])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 1026, normalized size = 26.31

method	result	size
risch	Expression too large to display	1026

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*ln(1+x*(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1
/2*I*ln(1-x*(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/
2*I*ln(x)*ln(d*f*x^2+1)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(x
)*ln(d*f*x^2+1)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-b*dilog(1+x*(-d*f)^(1/2))*ln
(x^n)-b*dilog(1-x*(-d*f)^(1/2))*ln(x^n)-b*ln(1+x*(-d*f)^(1/2))*ln(x)*ln(x^n
)-b*ln(1-x*(-d*f)^(1/2))*ln(x)*ln(x^n)+b*ln(x)*ln(d*f*x^2+1)*ln(x^n)+1/2*I*
dilog(1-x*(-d*f)^(1/2))*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*ln(1
+x*(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+ln(x)*ln(d*f*x^2+1)
*a-ln(1+x*(-d*f)^(1/2))*ln(x)*a-ln(1-x*(-d*f)^(1/2))*ln(x)*a-1/2*I*ln(1-x*(
-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(x)*ln(d*f*x^2+
1)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*dilog(1+x*(-d*f)^(1/2))*Pi*b*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-dilog(1+x*(-d*f)^(1/2))*a-dilog(1-x*(-d*f)^
(1/2))*a-dilog(1+x*(-d*f)^(1/2))*ln(c)*b-dilog(1-x*(-d*f)^(1/2))*ln(c)*b-1/
2*I*ln(1+x*(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(1-x*
(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-ln(1+x*(-d*f)^(1/2))*ln(
x)*ln(c)*b-ln(1-x*(-d*f)^(1/2))*ln(x)*ln(c)*b+ln(x)*ln(d*f*x^2+1)*ln(c)*b+1
/2*I*dilog(1-x*(-d*f)^(1/2))*Pi*b*csgn(I*c*x^n)^3+1/2*I*dilog(1+x*(-d*f)^(1
/2))*Pi*b*csgn(I*c*x^n)^3+b*ln(1+x*(-d*f)^(1/2))*ln(x)^2*n+b*ln(1-x*(-d*f)^
(1/2))*ln(x)^2*n-b*ln(x)^2*ln(d*f*x^2+1)*n+b*dilog(1+x*(-d*f)^(1/2))*ln(x)*
n+b*dilog(1-x*(-d*f)^(1/2))*ln(x)*n-1/2*b*n*ln(x)*polylog(2,-d*f*x^2)+1/4*b
*n*polylog(3,-d*f*x^2)-1/2*I*dilog(1+x*(-d*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/2*I*dilog(1-x*(-d*f)^(1/2))*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1
/2*I*ln(1+x*(-d*f)^(1/2))*ln(x)*Pi*b*csgn(I*c*x^n)^3+1/2*I*ln(1-x*(-d*f)^(1
/2))*ln(x)*Pi*b*csgn(I*c*x^n)^3-1/2*I*ln(x)*ln(d*f*x^2+1)*Pi*b*csgn(I*c*x^n
```

```
)^3-1/2*I*dilog(1+x*(-d*f)^(1/2))*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*dilo
g(1-x*(-d*f)^(1/2))*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*f
*x^2 + 1) - integrate(-(b*d*f*n*x*log(x)^2 - 2*b*d*f*x*log(x)*log(x^n) - 2*
(b*d*f*log(c) + a*d*f)*x*log(x))/(d*f*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x, x)
```

3.27 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=141

$$\frac{1}{2}bdfn\log(x)-\frac{1}{2}bdfn\log^2(x)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bdfn\log\left(1+dfx^2\right)-\frac{bn\log\left(1+dfx^2\right)}{4x^2}-\frac{1}{2}df(a+$$

```
[Out] 1/2*b*d*f*n*ln(x)-1/2*b*d*f*n*ln(x)^2+d*f*ln(x)*(a+b*ln(c*x^n))-1/4*b*d*f*n
*ln(d*f*x^2+1)-1/4*b*n*ln(d*f*x^2+1)/x^2-1/2*d*f*(a+b*ln(c*x^n))*ln(d*f*x^2
+1)-1/2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/4*b*d*f*n*polylog(2,-d*f*x^2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2504, 2442, 36, 29, 31, 2423, 2338, 2438}

$$-\frac{1}{4}bdfn\text{PolyLog}(2,-dfx^2)+df\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}df\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{\log\left(dfx^2+1\right)\left(a+b\log\left(cx^n\right)\right)}{2x^2}-\frac{1}{4}bdfn\log\left(dfx^2+1\right)-\frac{bn\log\left(dfx^2+1\right)}{4x^2}-\frac{1}{2}bdfn\log^2(x)+\frac{1}{2}bdfn\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (b*d*f*n*Log[x])/2 - (b*d*f*n*Log[x]^2)/2 + d*f*Log[x]*(a + b*Log[c*x^n]) -
 (b*d*f*n*Log[1 + d*f*x^2])/4 - (b*n*Log[1 + d*f*x^2])/(4*x^2) - (d*f*(a +
b*Log[c*x^n])*Log[1 + d*f*x^2])/2 - ((a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(
2*x^2) - (b*d*f*n*PolyLog[2, -(d*f*x^2)])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = df\log(x)\,(a+b\log(cx^n)) - \frac{1}{2}df(a+b\log(cx^n))\log\left(1+dfx^2\right) \cdots$$

$$= df\log(x)\,(a+b\log(cx^n)) - \frac{1}{2}df(a+b\log(cx^n))\log\left(1+dfx^2\right) \cdots$$

$$= -\frac{1}{2}bdfn\log^2(x) + df\log(x)\,(a+b\log(cx^n)) - \frac{1}{2}df(a+b\log(cx \cdots$$

$$= -\frac{1}{2}bdfn\log^2(x) + df\log(x)\,(a+b\log(cx^n)) - \frac{bn\log\left(1+dfx^2\right)}{4x^2} \cdots$$

$$= -\frac{1}{2}bdfn\log^2(x) + df\log(x)\,(a+b\log(cx^n)) - \frac{bn\log\left(1+dfx^2\right)}{4x^2} \cdots$$

$$= \frac{1}{2}bdfn\log(x) - \frac{1}{2}bdfn\log^2(x) + df\log(x)\,(a+b\log(cx^n)) - \frac{1}{4}bd \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 241, normalized size = 1.71

$$adf\log(x)+\frac{1}{2}bdf\log(x)\left(n+2(-n\log(x)+\log\left(cx^n\right))\right)-\frac{1}{2}adf\log\left(1+dfx^2\right)-\frac{a\log\left(1+dfx^2\right)}{2x^2}-\frac{1}{4}bdf(n+2(-n\log(x)+\log\left(cx^n\right)))\log\left(1+dfx^2\right)-\frac{b(n+2n\log(x)+2(-n\log(x)+\log\left(cx^n\right)))\log\left(1+dfx^2\right)}{4x^2}+bdfn\left(\frac{\log^2(x)}{2}+\frac{1}{2}\left(-\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)-\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)\right)+\frac{1}{2}\left(-\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)-\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] a*d*f*Log[x] + (b*d*f*Log[x]*(n + 2*(-(n*Log[x]) + Log[c*x^n])))/2 - (a*d*f
*Log[1 + d*f*x^2])/2 - (a*Log[1 + d*f*x^2])/(2*x^2) - (b*d*f*(n + 2*(-(n*Lo
g[x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/4 - (b*(n + 2*n*Log[x] + 2*(-(n*Log[
x]) + Log[c*x^n]))*Log[1 + d*f*x^2])/(4*x^2) + b*d*f*n*(Log[x]^2/2 + (-(Log
[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/2 +
 (-(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])
/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 619, normalized size = 4.39

method	result
risch	$-\frac{ifd\ln(x)\pi b\,\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(ic\,x^n)}{2}+\frac{ifd\ln(x)\pi b\,\text{csgn}(ix^n)\text{csgn}(ic\,x^n)^2}{2}-\frac{ifd\ln(df\,x^2+1)\pi b\,\text{csgn}(ix^n)\text{csgn}(ic\,x^n)^2}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*f*d*ln(x)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*f*d*ln(d*f*x^2+1)*Pi
*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*f*d*ln(x)*Pi*b*csgn(I*c)*csgn(I*c*x^n)
^2-1/4*I*f*d*ln(d*f*x^2+1)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*f*d*b*n*dilog
(1-x*(-d*f)^(1/2))-1/2*a/x^2*ln(d*f*x^2+1)+1/4*I*Pi*b*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)/x^2*ln(d*f*x^2+1)+a*d*f*ln(x)-1/2*a*d*f*ln(d*f*x^2+1)-1/4*I*
Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2/x^2*ln(d*f*x^2+1)+1/2*n*b*ln(x)*ln(d*f*x^2
+1)*d*f+(-1/2*b/x^2*ln(d*(1/d+f*x^2))+b*f*d*ln(x)-1/2*b*f*d*ln(d*(1/d+f*x^2
)))*ln(x^n)-1/2*b*ln(c)/x^2*ln(d*f*x^2+1)-1/2*f*d*b*n*dilog(1+x*(-d*f)^(1/2
))-1/2*f*d*ln(d*f*x^2+1)*ln(c)*b+f*d*ln(x)*ln(c)*b-1/4*I*Pi*b*csgn(I*c)*csg
n(I*c*x^n)^2/x^2*ln(d*f*x^2+1)-1/4*b*n*ln(d*f*x^2+1)/x^2+1/4*I*f*d*ln(d*f*x
^2+1)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I*f*d*ln(d*f*x^2+1)*Pi*b
*csgn(I*c*x^n)^3-1/2*f*d*b*n*ln(x)*ln(1+x*(-d*f)^(1/2))-1/2*f*d*b*n*ln(x)*l
n(1-x*(-d*f)^(1/2))+1/2*b*d*f*n*ln(x)-1/2*b*d*f*n*ln(x)^2-1/4*b*d*f*n*ln(d*
f*x^2+1)-1/2*I*f*d*ln(x)*Pi*b*csgn(I*c*x^n)^3+1/4*I*Pi*b*csgn(I*c*x^n)^3/x^
2*ln(d*f*x^2+1)-1/2*I*f*d*ln(x)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log(d*f*x^2 + 1)/x^2 + integra
te(1/2*(2*b*d*f*log(x^n) + 2*a*d*f + (d*f*n + 2*d*f*log(c))*b)/(d*f*x^3 + x
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

3.28 $\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=241

$$-\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2bn\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)}{9d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n))-\frac{2\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)}{3d^{3/2}f^{3}}$$

```
[Out] -8/9*b*n*x/d/f+4/27*b*n*x^3+2/9*b*n*arctan(x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/
2)+2/3*x*(a+b*ln(c*x^n))/d/f-2/9*x^3*(a+b*ln(c*x^n))-2/3*arctan(x*d^(1/2)*f
^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/f^(3/2)-1/9*b*n*x^3*ln(d*f*x^2+1)+1/3*x^3*(
a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(3
/2)/f^(3/2)-1/3*I*b*n*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2505, 308, 211, 2423, 4940, 2438, 209}

$$\frac{ibn\text{PolyLog}\left(2,-i\sqrt{d}\ \sqrt{f}\ x\right)}{3d^{3/2}f^{3/2}}-\frac{ibn\text{PolyLog}\left(2,i\sqrt{d}\ \sqrt{f}\ x\right)}{3d^{3/2}f^{3/2}}-\frac{2\text{ArcTan}\left(\sqrt{d}\ \sqrt{f}\ x\right)(a+b\log(cx^n))}{3d^{3/2}f^{3/2}}+\frac{2x(a+b\log(cx^n))}{3df}+\frac{1}{3}x^3\log\left(dfx^2+1\right)(a+b\log(cx^n))-\frac{2}{9}x^3(a+b\log(cx^n))+\frac{2bn\text{ArcTan}\left(\sqrt{d}\ \sqrt{f}\ x\right)}{9d^{3/2}f^{3/2}}-\frac{1}{9}bnx^3\log\left(dfx^2+1\right)-\frac{8bnx}{9df}+\frac{4}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-8*b*n*x)/(9*d*f) + (4*b*n*x^3)/27 + (2*b*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/(9*
d^(3/2)*f^(3/2)) + (2*x*(a + b*Log[c*x^n]))/(3*d*f) - (2*x^3*(a + b*Log[c*x
^n]))/9 - (2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(3*d^(3/2)*f^(3/
2)) - (b*n*x^3*Log[1 + d*f*x^2])/9 + (x^3*(a + b*Log[c*x^n])*Log[1 + d*f*x^
2])/3 + ((I/3)*b*n*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) -
((I/3)*b*n*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n))-\frac{2\tan^{-1}\left(\sqrt{d}\ \sqrt{}\right.}{3}$$

$$=-\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n$$

$$=-\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n$$

$$=-\frac{2bnx}{3df}+\frac{2}{27}bnx^3+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n$$

$$=-\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2x(a+b\log(cx^n))}{3df}-\frac{2}{9}x^3(a+b\log(cx^n$$

$$=-\frac{8bnx}{9df}+\frac{4}{27}bnx^3+\frac{2bn\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)}{9d^{3/2}f^{3/2}}+\frac{2x(a+b\log(}{3df}$$

Mathematica [A]
time = 0.07, size = 364, normalized size = 1.51

$$\frac{2ax}{3df}-\frac{2ax^3}{9}-\frac{2a\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)}{3d^{3/2}f^{3/2}}+\frac{2bx(-n+3(-n\log(x)+\log(cx^n)))}{9df}-\frac{2}{27}bx^3(-n+3(-n\log(x)+\log(cx^n)))-\frac{2b\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)(-n+3(-n\log(x)+\log(cx^n)))}{9d^{3/2}f^{3/2}}+\frac{1}{3}ax^3\log\left(1+dfx^2\right)+\frac{1}{9}bx^3(-n+3n\log(x)+3(-n\log(x)+\log(cx^n)))\log\left(1+dfx^2\right)-\frac{2}{3}bdfn\left(-\frac{x(-1+\log(x))}{d^2f^2}+\frac{-\frac{x^3}{9}+\frac{1}{3}x^3\log(x)}{df}-\frac{i\left(\log(x)\log\left(1+i\sqrt{d}\ \sqrt{f}\ x\right)+\text{Li}_2\left(-i\sqrt{d}\ \sqrt{f}\ x\right)\right)}{2d^{5/2}f^{5/2}}+\frac{i\left(\log(x)\log\left(1-i\sqrt{d}\ \sqrt{f}\ x\right)+\text{Li}_2\left(i\sqrt{d}\ \sqrt{f}\ x\right)\right)}{2d^{5/2}f^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (2*a*x)/(3*d*f) - (2*a*x^3)/9 - (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(3*d^(3/2)*
f^(3/2)) + (2*b*x*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d*f) - (2*b*x^3*(
-n + 3*(-(n*Log[x]) + Log[c*x^n])))/27 - (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*(-n
 + 3*(-(n*Log[x]) + Log[c*x^n])))/(9*d^(3/2)*f^(3/2)) + (a*x^3*Log[1 + d*f*
x^2])/3 + (b*x^3*(-n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + d
*f*x^2])/9 - (2*b*d*f*n*(-((x*(-1 + Log[x]))/(d^2*f^2)) + (-1/9*x^3 + (x^3*
Log[x])/3)/(d*f) - ((I/2)*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2,
 (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/2)) + ((I/2)*(Log[x]*Log[1 - I*Sqr
t[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]))/(d^(5/2)*f^(5/2))))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 891, normalized size = 3.70

method	result

risch	$\frac{x^3\ln(df\,x^2+1)a}{3}-\frac{2\ln(c)b\,x^3}{9}+\frac{2bx\ln(x^n)}{3df}-\frac{2x^3a}{9}+\frac{2b\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln(x)}{3df\sqrt{df}}+\frac{bn\sqrt{-df}\,\ln(x)\ln\left(1+x\sqrt{-df}\right)}{3d^2f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(d*f*x^2+1)*a+1/3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2/d/f*x+1/3*I*
b*Pi*csgn(I*c*x^n)^3/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2/9*ln(c)*b*
x^3+1/6*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*x^3*ln(d*f*x^2+1)+2/3*b/d/f*x*ln(x
^n)-2/9*x^3*a+2/3*b/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*n*ln(x)+1/3*b
*n/d^2/f^2*(-d*f)^(1/2)*ln(x)*ln(1+x*(-d*f)^(1/2))-1/3*b*n/d^2/f^2*(-d*f)^(
1/2)*ln(x)*ln(1-x*(-d*f)^(1/2))+4/27*b*n*x^3+1/3*I*b*Pi*csgn(I*c)*csgn(I*c*
x^n)^2/d/f*x-1/6*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x^3*ln(d*f*x^2+
1)-2/3*a/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+2/3*a/d/f*x+1/3*b*ln(d*f
*x^2+1)*ln(x^n)*x^3+1/9*I*b*Pi*csgn(I*c*x^n)^3*x^3+1/3*b*ln(c)*x^3*ln(d*f*x
^2+1)+1/9*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x^3+1/6*I*b*Pi*csgn(I*
x^n)*csgn(I*c*x^n)^2*x^3*ln(d*f*x^2+1)-1/3*I*b*Pi*csgn(I*c*x^n)^3/d/f*x-2/9
*x^3*b*ln(x^n)-1/3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2/d/f/(d*f)^(1/2)*arctan(
x*d*f/(d*f)^(1/2))+2/3*b*ln(c)/d/f*x-1/3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)/d/f*x-8/9*b*n*x/d/f-1/3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2/d/f/(d*
f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2/3*b*ln(c)/d/f/(d*f)^(1/2)*arctan(x*d*f
/(d*f)^(1/2))-1/9*b*n*x^3*ln(d*f*x^2+1)-1/9*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2*x^3+2/9*b*n/d/f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+1/3*b*n/d^2/f^2*(
-d*f)^(1/2)*dilog(1+x*(-d*f)^(1/2))-1/3*b*n/d^2/f^2*(-d*f)^(1/2)*dilog(1-x*
(-d*f)^(1/2))-1/6*I*b*Pi*csgn(I*c*x^n)^3*x^3*ln(d*f*x^2+1)-2/3*b/d/f/(d*f)^
(1/2)*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)-1/9*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^
2*x^3+1/3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)/d/f/(d*f)^(1/2)*arctan
(x*d*f/(d*f)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log(d*f*x^2 + 1) - in
tegrate(2/9*(3*b*d*f*x^4*log(x^n) + (3*a*d*f - (d*f*n - 3*d*f*log(c))*b)*x^
4)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*log(d*f*x^2 + 1)*log(c*x^n) + a*x^2*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

3.29 $\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=182

$$4bnx-\frac{2bn\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}-2x(a+b\log(cx^n))+\frac{2\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{f}}-bnx\log(1+df$$

```
[Out] 4*b*n*x-2*x*(a+b*ln(c*x^n))-b*n*x*ln(d*f*x^2+1)+x*(a+b*ln(c*x^n))*ln(d*f*x^
2+1)-2*b*n*arctan(x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+2*arctan(x*d^(1/2)*f^(
1/2))*(a+b*ln(c*x^n))/d^(1/2)/f^(1/2)-I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))
/d^(1/2)/f^(1/2)+I*b*n*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2498, 327, 211, 2417, 4940, 2438, 209}

$$-\frac{ibn\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+\frac{ibn\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}+\frac{2\text{ArcTan}\left(\sqrt{d}\sqrt{f}\,x\right)(a+b\log(cx^n))}{\sqrt{d}\sqrt{f}}+x\log\left(dfx^2+1\right)(a+b\log(cx^n))-2x(a+b\log(cx^n))-\frac{2bn\text{ArcTan}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}}-bnx\log\left(dfx^2+1\right)+4bnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] 4*b*n*x - (2*b*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 2*x*(a + b*
Log[c*x^n]) + (2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/(Sqrt[d]*Sqr
t[f]) - b*n*x*Log[1 + d*f*x^2] + x*(a + b*Log[c*x^n])*Log[1 + d*f*x^2] - (I
*b*n*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + (I*b*n*PolyLog
[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int (a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = -2x(a+b\log(cx^n)) + \frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))}{\sqrt{d}\,\sqrt{f}}$$

$$= 2bnx - 2x(a+b\log(cx^n)) + \frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(}{\sqrt{d}\,\sqrt{f}}$$

$$= 2bnx - 2x(a+b\log(cx^n)) + \frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(}{\sqrt{d}\,\sqrt{f}}$$

$$= 4bnx - 2x(a+b\log(cx^n)) + \frac{2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(}{\sqrt{d}\,\sqrt{f}}$$

$$= 4bnx - \frac{2bn\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)}{\sqrt{d}\,\sqrt{f}} - 2x(a+b\log(cx^n)) + \frac{2\tan}{}$$

Mathematica [A]
time = 0.07, size = 254, normalized size = 1.40

$$-2ax+\frac{2a\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)}{\sqrt{d}\,\sqrt{f}}-2bx(-n-n\log(x)+\log(cx^n))+\frac{2b\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(-n-n\log(x)+\log(cx^n))}{\sqrt{d}\,\sqrt{f}}+ax\log(1+dfx^2)+bx(-n+\log(cx^n))\log(1+dfx^2)-2bdfn\left(\frac{x(-1+\log(x))}{df}+\frac{i\left(\log(x)\log\left(1+i\sqrt{d}\,\sqrt{f}\,x\right)+\text{Li}_2\left(-i\sqrt{d}\,\sqrt{f}\,x\right)\right)}{2d^{3/2}f^{3/2}}-\frac{i\left(\log(x)\log\left(1-i\sqrt{d}\,\sqrt{f}\,x\right)+\text{Li}_2\left(i\sqrt{d}\,\sqrt{f}\,x\right)\right)}{2d^{3/2}f^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -2*a*x + (2*a*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 2*b*x*(-n - n*
Log[x] + Log[c*x^n]) + (2*b*ArcTan[Sqrt[d]*Sqrt[f]*x]*(-n - n*Log[x] + Log[
c*x^n]))/(Sqrt[d]*Sqrt[f]) + a*x*Log[1 + d*f*x^2] + b*x*(-n + Log[c*x^n])*L
og[1 + d*f*x^2] - 2*b*d*f*n*((x*(-1 + Log[x]))/(d*f) + ((I/2)*(Log[x]*Log[1
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(d^(3/2)*f^(
3/2)) - ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]))/(d^(3/2)*f^(3/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 643, normalized size = 3.53

method	result
risch	$x\ln(df\,x^2+1)\,a-2\ln(c)\,bx+ib\pi\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)\,x+\frac{2a\arctan\left(\frac{xdf}{\sqrt{df}}\right)}{\sqrt{df}}+4bnx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*f*x^2+1)*a-2*ln(c)*b*x+2*a/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+4*b
*n*x+1/2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*x*ln(d*f*x^2+1)+I*b*Pi*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)*x+I*b*Pi*csgn(I*c*x^n)^3*x-b*n*(-d*f)^(1/2)/d/f
*ln(x)*ln(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)/d/f*ln(x)*ln(1-x*(-d*f)^(1/2))
+b*ln(c)*x*ln(d*f*x^2+1)+2*b*ln(c)/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+b*
ln(d*f*x^2+1)*ln(x^n)*x+2*b/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)+I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2*a
*x-1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x*ln(d*f*x^2+1)-I*b*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+I*b
*Pi*csgn(I*c)*csgn(I*c*x^n)^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2*b/(d*
f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*n*ln(x)-2*b*n/(d*f)^(1/2)*arctan(x*d*f/(
d*f)^(1/2))-2*b*x*ln(x^n)+1/2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*x*ln(d*f*x^2
+1)-b*n*(-d*f)^(1/2)/d/f*dilog(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)/d/f*dilog
(1-x*(-d*f)^(1/2))-b*n*x*ln(d*f*x^2+1)-I*b*Pi*csgn(I*c*x^n)^3/(d*f)^(1/2)*a
rctan(x*d*f/(d*f)^(1/2))-I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*x-1/2*I*b*Pi*cs
gn(I*c*x^n)^3*x*ln(d*f*x^2+1)-I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*x^2 + 1) - integrate(2*(b*d
*f*x^2*log(x^n) + (a*d*f - (d*f*n - d*f*log(c))*b)*x^2)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\log(cx^n))\log(dfx^2+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Integral((a + b*log(c*x**n))*log(d*f*x**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)), x)
```

3.30 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=169

$$2b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)+2\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))-\frac{bn\log(1+dfx^2)}{x}-\frac{(a+b\ldots}{}$$

```
[Out] -b*n*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x+2*b*n*arctan(x*d^(1/2)
*f^(1/2))*d^(1/2)*f^(1/2)+2*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))*d^(1/
2)*f^(1/2)-I*b*n*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+I*b*n*poly
log(2,I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2505, 211, 2423, 4940, 2438, 209}

$$-ib\sqrt{d}\,\sqrt{f}\,n\text{PolyLog}\left(2,-i\sqrt{d}\,\sqrt{f}\,x\right)+ib\sqrt{d}\,\sqrt{f}\,n\text{PolyLog}\left(2,i\sqrt{d}\,\sqrt{f}\,x\right)+2\sqrt{d}\,\sqrt{f}\,\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{x}+2b\sqrt{d}\,\sqrt{f}\,n\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{bn\log(dfx^2+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*b*Sqrt[d]*Sqrt[f]*n*ArcTan[Sqrt[d]*Sqrt[f]*x] + 2*Sqrt[d]*Sqrt[f]*ArcTan[
Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) - (b*n*Log[1 + d*f*x^2])/x - ((a + b*
Log[c*x^n])*Log[1 + d*f*x^2])/x - I*b*Sqrt[d]*Sqrt[f]*n*PolyLog[2, (-I)*Sqr
t[d]*Sqrt[f]*x] + I*b*Sqrt[d]*Sqrt[f]*n*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = 2\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n)) - \frac{(a+b\log(cx^n)}{}$$

$$= 2\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n)) - \frac{(a+b\log(cx^n)}{}$$

$$= 2\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n)) - \frac{bn\log(1+dfx}{x}$$

$$= 2b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 2\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right.$$

Mathematica [A]
time = 0.07, size = 221, normalized size = 1.31

$$2a\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)+2b\sqrt{d}\,\sqrt{f}\,\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(n-n\log(x)+\log(cx^n))-\frac{a\log(1+dfx^2)}{x}-\frac{b(n+\log(cx^n))\log(1+dfx^2)}{x}+2bdfn\left(-\frac{i\left(\log(x)\log\left(1+i\sqrt{d}\,\sqrt{f}\,x\right)+\text{Li}_2\left(-i\sqrt{d}\,\sqrt{f}\,x\right)\right)}{2\sqrt{d}\,\sqrt{f}}+\frac{i\left(\log(x)\log\left(1-i\sqrt{d}\,\sqrt{f}\,x\right)+\text{Li}_2\left(i\sqrt{d}\,\sqrt{f}\,x\right)\right)}{2\sqrt{d}\,\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*a*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x] + 2*b*Sqrt[d]*Sqrt[f]*ArcTan[
Sqrt[d]*Sqrt[f]*x]*(n - n*Log[x] + Log[c*x^n]) - (a*Log[1 + d*f*x^2])/x - (
b*(n + Log[c*x^n])*Log[1 + d*f*x^2])/x + 2*b*d*f*n*(((-1/2*I)*(Log[x]*Log[1
```

```
 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]))/(Sqrt[d]*Sqr
t[f]) + ((I/2)*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]))/(Sqrt[d]*Sqrt[f]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 547, normalized size = 3.24

method	result
risch	$-\frac{b\ln(df\,x^2+1)\ln(x^n)}{x}-\frac{2bdf\arctan\left(\frac{xdf}{\sqrt{df}}\right)n\ln(x)}{\sqrt{df}}+\frac{2bdf\arctan\left(\frac{xdf}{\sqrt{df}}\right)\ln(x^n)}{\sqrt{df}}-\frac{bn\ln(df\,x^2+1)}{x}+\frac{2bndf\arctan\left(\cdots\right.}{\sqrt{df}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(d*f*x^2+1)/x*ln(x^n)-2*b*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*n*
ln(x)+2*b*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)-b*n*ln(d*f*x^2+
1)/x+2*b*n*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-b*n*(-d*f)^(1/2)*ln(x)
*ln(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)*ln(x)*ln(1-x*(-d*f)^(1/2))-b*n*(-d*f
)^(1/2)*dilog(1+x*(-d*f)^(1/2))+b*n*(-d*f)^(1/2)*dilog(1-x*(-d*f)^(1/2))-1/
2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*ln(d*f*x^2+1)/x+I*b*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+I*b*Pi*csgn(
I*c)*csgn(I*c*x^n)^2*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-I*b*Pi*csgn(
I*c*x^n)^3*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-1/2*I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2*ln(d*f*x^2+1)/x+1/2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)*ln(d*f*x^2+1)/x+1/2*I*b*Pi*csgn(I*c*x^n)^3*ln(d*f*x^2+1)/x-b*ln(c)*ln(
d*f*x^2+1)/x+2*b*ln(c)*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-ln(d*f*x^2
+1)/x*a+2*a*d*f/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] -(b*(n + log(c)) + b*log(x^n) + a)*log(d*f*x^2 + 1)/x + integrate(2*(b*d*f*
log(x^n) + a*d*f + (d*f*n + d*f*log(c))*b)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log{(cx^n)})\log{(dfx^2 + 1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*log(d*f*x**2 + 1)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

3.31 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=211

$$-\frac{8bdfn}{9x}-\frac{2}{9}bd^{3/2}f^{3/2}n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2df(a+b\log(cx^n))}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx$$

```
[Out] -8/9*b*d*f*n/x-2/9*b*d^(3/2)*f^(3/2)*n*arctan(x*d^(1/2)*f^(1/2))-2/3*d*f*(a
+b*ln(c*x^n))/x-2/3*d^(3/2)*f^(3/2)*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n
))-1/9*b*n*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3+1/3*I*b*
d^(3/2)*f^(3/2)*n*polylog(2,-I*x*d^(1/2)*f^(1/2))-1/3*I*b*d^(3/2)*f^(3/2)*n
*polylog(2,I*x*d^(1/2)*f^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2505, 331, 211, 2423, 4940, 2438, 209}

$$\frac{1}{3}ibd^{3/2}f^{3/2}n\text{PolyLog}\left(2,-i\sqrt{d}\,\sqrt{f}\,x\right)-\frac{1}{3}ibd^{3/2}f^{3/2}n\text{PolyLog}\left(2,i\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2}{3}d^{3/2}f^{3/2}\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))-\frac{2df(a+b\log(cx^n))}{3x}-\frac{\log(dfx^2+1)(a+b\log(cx^n))}{3x^3}-\frac{2}{9}bd^{3/2}f^{3/2}n\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{bn\log(dfx^2+1)}{9x^3}-\frac{8bdfn}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-8*b*d*f*n)/(9*x) - (2*b*d^(3/2)*f^(3/2)*n*ArcTan[Sqrt[d]*Sqrt[f]*x])/9 -
(2*d*f*(a + b*Log[c*x^n]))/(3*x) - (2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f
]*x]*(a + b*Log[c*x^n]))/3 - (b*n*Log[1 + d*f*x^2])/(9*x^3) - ((a + b*Log[c
*x^n])*Log[1 + d*f*x^2])/(3*x^3) + (I/3)*b*d^(3/2)*f^(3/2)*n*PolyLog[2, (-I
)*Sqrt[d]*Sqrt[f]*x] - (I/3)*b*d^(3/2)*f^(3/2)*n*PolyLog[2, I*Sqrt[d]*Sqrt[
f]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = -\frac{2df(a+b\log(cx^n))}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log$$

$$= -\frac{2bdfn}{3x}-\frac{2df(a+b\log(cx^n))}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)$$

$$= -\frac{2bdfn}{3x}-\frac{2df(a+b\log(cx^n))}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)$$

$$= -\frac{8bdfn}{9x}-\frac{2df(a+b\log(cx^n))}{3x}-\frac{2}{3}d^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)$$

$$= -\frac{8bdfn}{9x}-\frac{2}{9}bd^{3/2}f^{3/2}n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2df(a+b\log(cx^n)}{3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 285, normalized size = 1.35

$$-\frac{2adf\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-dfx^2\right)}{3x}-\frac{2}{9}bd^{3/2}f^{3/2}\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)(n+3(-n\log(x)+\log(cx^n)))-\frac{2b(dfn+3df(-n\log(x)+\log(cx^n)))}{9x}-\frac{a\log(1+dfx^2)}{3x^3}-\frac{b(n+3n\log(x)+3(-n\log(x)+\log(cx^n)))\log(1+dfx^2)}{9x^3}+\frac{2}{3}bdfn\left(-\frac{1}{x}-\frac{\log(x)}{x}+\frac{1}{2}i\sqrt{d}\sqrt{f}\left(\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)\right)-\frac{1}{2}i\sqrt{d}\sqrt{f}\left(\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-2*a*d*f*Hypergeometric2F1[-1/2, 1, 1/2, -(d*f*x^2)])/(3*x) - (2*b*d^(3/2)
*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(n + 3*(-(n*Log[x]) + Log[c*x^n])))/9 -
(2*b*(d*f*n + 3*d*f*(-(n*Log[x]) + Log[c*x^n])))/(9*x) - (a*Log[1 + d*f*x^2
])/(3*x^3) - (b*(n + 3*n*Log[x] + 3*(-(n*Log[x]) + Log[c*x^n]))*Log[1 + d*f
*x^2])/(9*x^3) + (2*b*d*f*n*(-x^(-1) - Log[x]/x + (I/2)*Sqrt[d]*Sqrt[f]*(Lo
g[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) - (
I/2)*Sqrt[d]*Sqrt[f]*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sq
rt[d]*Sqrt[f]*x])))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 734, normalized size = 3.48

method	result
risch	$-\frac{\ln(df\,x^2+1)a}{3x^3}+\frac{bndf\sqrt{-df}\ \text{dilog}\left(1+x\sqrt{-df}\right)}{3}-\frac{bndf\sqrt{-df}\ \text{dilog}\left(1-x\sqrt{-df}\right)}{3}+\frac{ib\pi\text{csgn}(ic\,x^n)^3df}{3x}+\frac{2b\,d^2f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(d*f*x^2+1)/x^3*a+2/3*b*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2)
)*n*ln(x)+1/3*b*n*d*f*(-d*f)^(1/2)*ln(x)*ln(1+x*(-d*f)^(1/2))-1/3*b*n*d*f*(
-d*f)^(1/2)*ln(x)*ln(1-x*(-d*f)^(1/2))+1/3*I*b*Pi*csgn(I*c*x^n)^3*d^2*f^2/(
d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2/3*a*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f
/(d*f)^(1/2))+1/6*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(d*f*x^2+1)/
x^3-1/3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*d*f/x-1/6*I*b*Pi*csgn(I*c)*csgn(
I*c*x^n)^2*ln(d*f*x^2+1)/x^3-1/3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d*f/x-2/3
*a*d*f/x-1/3*b*ln(c)*ln(d*f*x^2+1)/x^3-1/6*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2*ln(d*f*x^2+1)/x^3+1/3*I*b*Pi*csgn(I*c*x^n)^3*d*f/x-2/3*b*ln(c)*d*f/x-1/3
*b*ln(d*f*x^2+1)/x^3*ln(x^n)-1/3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d^2*f^2/(
d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-2/3*b*d*f/x*ln(x^n)-1/3*I*b*Pi*csgn(I*
x^n)*csgn(I*c*x^n)^2*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-1/9*b*n*
ln(d*f*x^2+1)/x^3+1/3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*d*f/x-2/9*
b*n*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))+1/3*b*n*d*f*(-d*f)^(1/2)*
dilog(1+x*(-d*f)^(1/2))-1/3*b*n*d*f*(-d*f)^(1/2)*dilog(1-x*(-d*f)^(1/2))-2/
3*b*ln(c)*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))-8/9*b*d*f*n/x+1/3*I
```

```
*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/
(d*f)^(1/2))-2/3*b*d^2*f^2/(d*f)^(1/2)*arctan(x*d*f/(d*f)^(1/2))*ln(x^n)+1/
6*I*b*Pi*csgn(I*c*x^n)^3*ln(d*f*x^2+1)/x^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*(b*(n + 3*log(c)) + 3*b*log(x^n) + 3*a)*log(d*f*x^2 + 1)/x^3 + integra
te(2/9*(3*b*d*f*log(x^n) + 3*a*d*f + (d*f*n + 3*d*f*log(c))*b)/(d*f*x^4 + x
^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(d*f*x^2 + 1)*log(c*x^n) + a*log(d*f*x^2 + 1))/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**2))/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + 1/d)*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

3.32 $\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=367

$$\frac{7b^2n^2x^2}{32df}-\frac{3}{64}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a+b\log(cx^n))+\frac{x^2(a+b\log(cx^n))^2}{4df}-\frac{1}{8}x^4(a+b\log(c$$

```
[Out] 7/32*b^2*n^2*x^2/d/f-3/64*b^2*n^2*x^4-3/8*b*n*x^2*(a+b*ln(c*x^n))/d/f+1/8*b
*n*x^4*(a+b*ln(c*x^n))+1/4*x^2*(a+b*ln(c*x^n))^2/d/f-1/8*x^4*(a+b*ln(c*x^n)
)^2-1/32*b^2*n^2*ln(d*f*x^2+1)/d^2/f^2+1/32*b^2*n^2*x^4*ln(d*f*x^2+1)+1/8*b
*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2-1/8*b*n*x^4*(a+b*ln(c*x^n))*ln(d*f
*x^2+1)-1/4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))
^2*ln(d*f*x^2+1)+1/16*b^2*n^2*polylog(2,-d*f*x^2)/d^2/f^2-1/4*b*n*(a+b*ln(c
*x^n))*polylog(2,-d*f*x^2)/d^2/f^2+1/8*b^2*n^2*polylog(3,-d*f*x^2)/d^2/f^2
```

Rubi [A]
time = 0.25, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.321$, Rules used = {2504, 2442, 45, 2424, 2341, 2421, 6724, 2423, 2438}

$$-\frac{bn\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{4d^2f^2}+\frac{b^2n^2\text{PolyLog}(2,-dfx^2)}{16d^2f^2}+\frac{b^2n^2\text{PolyLog}(3,-dfx^2)}{8d^2f^2}-\frac{\log(dfx^2+1)(a+b\log(cx^n))^2}{4d^2f^2}+\frac{bn\log(dfx^2+1)(a+b\log(cx^n))}{8d^2f^2}+\frac{x^2(a+b\log(cx^n))^2}{4df}-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))^2-\frac{1}{8}bnx^4\log(dfx^2+1)(a+b\log(cx^n))-\frac{1}{8}x^4(a+b\log(cx^n))^2+\frac{1}{8}bnx^4(a+b\log(cx^n))-\frac{b^2n^2\log(dfx^2+1)}{32d^2f^2}+\frac{7b^2n^2x^2}{32df}+\frac{1}{32}b^2n^2x^4\log(dfx^2+1)-\frac{3}{64}b^2n^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (7*b^2*n^2*x^2)/(32*d*f) - (3*b^2*n^2*x^4)/64 - (3*b*n*x^2*(a + b*Log[c*x^n
]))/(8*d*f) + (b*n*x^4*(a + b*Log[c*x^n]))/8 + (x^2*(a + b*Log[c*x^n])^2)/(
4*d*f) - (x^4*(a + b*Log[c*x^n])^2)/8 - (b^2*n^2*Log[1 + d*f*x^2])/(32*d^2*
f^2) + (b^2*n^2*x^4*Log[1 + d*f*x^2])/32 + (b*n*(a + b*Log[c*x^n])*Log[1 +
d*f*x^2])/(8*d^2*f^2) - (b*n*x^4*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/8 - (
(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*Log[c*x^n]
)^2*Log[1 + d*f*x^2])/4 + (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(16*d^2*f^2) - (
b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(4*d^2*f^2) + (b^2*n^2*PolyL
og[3, -(d*f*x^2)])/(8*d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx &= \frac{x^2(a+b\log(cx^n))^2}{4df}-\frac{1}{8}x^4(a+b\log(cx^n))^2-\frac{(a+b\log(\cdots}{\cdots}\\
&= \frac{x^2(a+b\log(cx^n))^2}{4df}-\frac{1}{8}x^4(a+b\log(cx^n))^2-\frac{(a+b\log(\cdots}{\cdots}\\
&= \frac{b^2n^2x^2}{8df}-\frac{1}{64}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a+\cdots\\
&= \frac{3b^2n^2x^2}{16df}-\frac{1}{32}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a\cdots\\
&= \frac{3b^2n^2x^2}{16df}-\frac{1}{32}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a\cdots\\
&= \frac{3b^2n^2x^2}{16df}-\frac{1}{32}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a\cdots\\
&= \frac{3b^2n^2x^2}{16df}-\frac{1}{32}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a\cdots\\
&= \frac{7b^2n^2x^2}{32df}-\frac{3}{64}b^2n^2x^4-\frac{3bnx^2(a+b\log(cx^n))}{8df}+\frac{1}{8}bnx^4(a\cdots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 654, normalized size = 1.78

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (2*d*f*x^2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) + 1
6*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) - d^
2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n*Log[x] - Log[c*x^n]) + 16
```

```
*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[x]) + Log[c*x^n])^2) + 2*d
^2*f^2*x^4*(8*a^2 - 4*a*b*n + b^2*n^2 - 4*b*(-4*a + b*n)*Log[c*x^n] + 8*b^2
*Log[c*x^n]^2)*Log[1 + d*f*x^2] - 2*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*b^2*n*(n
*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(-(n*Log[
x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] + b*n*(-4*a + b*n + 4*b*n*Log[x] - 4*
b*Log[c*x^n])*(4*d*f*x^2 - d^2*f^2*x^4 - 8*d*f*x^2*Log[x] + 4*d^2*f^2*x^4*L
og[x] + 8*Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 8*Log[x]*Log[1 + I*Sqrt[d]*
Sqrt[f]*x] + 8*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 8*PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x]) + 32*b^2*n^2*((d*f*x^2*(1 - 2*Log[x] + 2*Log[x]^2))/4 - (d^2*f^
2*x^4*(1 - 4*Log[x] + 8*Log[x]^2))/32 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]
*x])/2 - (Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)
*Sqrt[d]*Sqrt[f]*x] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (
-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/(64*d^2*f^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3}(a+b\ln(c\,x^{n}))^{2}\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/32*(8*b^2*x^4*log(x^n)^2 - 4*(b^2*(n - 4*log(c)) - 4*a*b)*x^4*log(x^n) +
((n^2 - 4*n*log(c) + 8*log(c)^2)*b^2 - 4*a*b*(n - 4*log(c)) + 8*a^2)*x^4)*l
og(d*f*x^2 + 1) - integrate(1/16*(8*b^2*d*f*x^5*log(x^n)^2 + 4*(4*a*b*d*f -
 (d*f*n - 4*d*f*log(c))*b^2)*x^5*log(x^n) + (8*a^2*d*f - 4*(d*f*n - 4*d*f*l
og(c))*a*b + (d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*b^2)*x^5)/(d*f*x^2
 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^3*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^3*log(d*f*x^2 + 1)
*log(c*x^n) + a^2*x^3*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.33 $\int x(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=241

$$-\frac{3}{4}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2+\frac{b^2n^2(1+dfx^2)\log(1+dfx^2)}{4df}-\frac{bn(1+dfx^2)(a+b$$

```
[Out] -3/4*b^2*n^2*x^2+b*n*x^2*(a+b*ln(c*x^n))-1/2*x^2*(a+b*ln(c*x^n))^2+1/4*b^2*
n^2*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f-1/2*b*n*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*
f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f-1/4*b^2*n^
2*polylog(2,-d*f*x^2)/d/f+1/2*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)/d/f-1
/4*b^2*n^2*polylog(3,-d*f*x^2)/d/f
```

Rubi [A]
time = 0.35, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2504, 2436, 2332, 2424, 2341, 14, 2393, 2338, 6874, 2421, 6724, 2423, 2525, 2458, 45, 2352}

$$\frac{bn\,\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{2df}-\frac{b^2n^2\,\text{PolyLog}(2,-dfx^2)}{4df}-\frac{b^2n^2\,\text{PolyLog}(3,-dfx^2)}{4df}-\frac{bn(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{2df}+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^2}{2df}+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2+\frac{b^2n^2(dfx^2+1)\log(dfx^2+1)}{4df}-\frac{3}{4}b^2n^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-3*b^2*n^2*x^2)/4 + b*n*x^2*(a + b*Log[c*x^n]) - (x^2*(a + b*Log[c*x^n]))^2
)/2 + (b^2*n^2*(1 + d*f*x^2)*Log[1 + d*f*x^2])/(4*d*f) - (b*n*(1 + d*f*x^2)
*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(2*d*f) + ((1 + d*f*x^2)*(a + b*Log[c
*x^n])^2*Log[1 + d*f*x^2])/(2*d*f) - (b^2*n^2*PolyLog[2, -(d*f*x^2)])/(4*d*
f) + (b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(2*d*f) - (b^2*n^2*Pol
yLog[3, -(d*f*x^2)])/(4*d*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx=-\frac{1}{2}x^2(a+b\log(cx^n))^2+\frac{(1+dfx^2)(a+b\log(cx^n))^2\log(}{2df}$$

$$=-\frac{1}{2}x^2(a+b\log(cx^n))^2+\frac{(1+dfx^2)(a+b\log(cx^n))^2\log(}{2df}$$

$$=-\frac{1}{4}b^2n^2x^2+\frac{1}{2}bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2$$

$$=-\frac{1}{4}b^2n^2x^2+\frac{1}{2}bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2$$

$$=-\frac{1}{4}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{1}{2}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{1}{2}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{1}{2}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{1}{2}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{1}{2}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

$$=-\frac{3}{4}b^2n^2x^2+bnx^2(a+b\log(cx^n))-\frac{1}{2}x^2(a+b\log(cx^n))^2-$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 519, normalized size = 2.15

$$\frac{-dfx^2\left(2a^2-2abn+b^2n^2+2b^2n(n\log(x)-\log(cx^n))+4ab(-n\log(x)+\log(cx^n))+2b^2(-n\log(x)+\log(cx^n))^2\right)+dfx^2\left(2a^2-2abn+b^2n^2-2b(-2a+bn)\log(cx^n)+2b^2\log^2(cx^n)\right)\log(1+dfx^2)+\left(2a^2-2abn+b^2n^2+2b^2n(n\log(x)-\log(cx^n))+4ab(-n\log(x)+\log(cx^n))+2b^2(-n\log(x)+\log(cx^n))^2\right)\log(1+dfx^2)+2bn(2a-bn-2bn\log(x)+2b\log(cx^n))\left(\frac{1}{2}dfx^2-dfx^2\log(x)+\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)\right)-b^2n^2\left(dfx^2-2dfx^2\log(x)+2dfx^2\log^2(x)-2\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)-2\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)-4\log(x)\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)-4\log(x)\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)+4\text{Li}_3\left(-i\sqrt{d}\sqrt{f}x\right)+4\text{Li}_3\left(i\sqrt{d}\sqrt{f}x\right)\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-(d*f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[c*x^n]) + 4
*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)) + d*
f*x^2*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[
c*x^n]^2)*Log[1 + d*f*x^2] + (2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x]
 - Log[c*x^n]) + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Lo
g[c*x^n])^2)*Log[1 + d*f*x^2] + 2*b*n*(2*a - b*n - 2*b*n*Log[x] + 2*b*Log[c
*x^n])*((d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x]
+ Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]
+ PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(d*f*x^2 - 2*d*f*x^2*Log[x] +
2*d*f*x^2*Log[x]^2 - 2*Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^2*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] -
 4*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, (-I)*Sqrt[d]*Sqrt[
f]*x] + 4*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))/(4*d*f)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(a+b\ln (c\,x^{n}))^{2} \ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*x^2*log(x^n)^2 - 2*(b^2*(n - 2*log(c)) - 2*a*b)*x^2*log(x^n) + (
(n^2 - 2*n*log(c) + 2*log(c)^2)*b^2 - 2*a*b*(n - 2*log(c)) + 2*a^2)*x^2)*lo
g(d*f*x^2 + 1) - integrate(1/2*(2*b^2*d*f*x^3*log(x^n)^2 + 2*(2*a*b*d*f - (
d*f*n - 2*d*f*log(c))*b^2)*x^3*log(x^n) + (2*a^2*d*f - 2*(d*f*n - 2*d*f*log
(c))*a*b + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2)*x^3)/(d*f*x^2 +
 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x*log(d*f*x^2 + 1)*log
(c*x^n) + a^2*x*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.34 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2}(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^2)+\frac{1}{2}bn(a+b\log(cx^n))\operatorname{Li}_3(-dfx^2)-\frac{1}{4}b^2n^2\operatorname{Li}_4(-dfx^2)$$

```
[Out] -1/2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)+1/2*b*n*(a+b*ln(c*x^n))*polylog(
3,-d*f*x^2)-1/4*b^2*n^2*polylog(4,-d*f*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2421, 2430, 6724}

$$\frac{1}{2}bn\text{PolyLog}(3,-dfx^2)(a+b\log(cx^n))-\frac{1}{2}\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))^2-\frac{1}{4}b^2n^2\text{PolyLog}(4,-dfx^2)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -1/2*((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)]) + (b*n*(a + b*Log[c*x^n]
)*PolyLog[3, -(d*f*x^2)])/2 - (b^2*n^2*PolyLog[4, -(d*f*x^2)])/4
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^2)+(bn)\int \frac{(a+b\log(cx^n))\operatorname{Li}_2(}{x}$$
$$= -\frac{1}{2}(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^2)+\frac{1}{2}bn(a+b\log(cx^n))\operatorname{Li}_3(-d$$
$$= -\frac{1}{2}(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^2)+\frac{1}{2}bn(a+b\log(cx^n))\operatorname{Li}_3(-d$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 484, normalized size = 6.91

$$\frac{1}{3}\left(\log(x)\left(b^2n^2\log^2(x)-3bn\log(x)(a+b\log(cx^n))+3(a+b\log(cx^n))^2\right)\log(1+dfx^2)-3(a-bn\log(x)+b\log(cx^n))^2\left(\log(x)\left(\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log\left(1+i\sqrt{d}\sqrt{f}x\right)\right)+\operatorname{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+\operatorname{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)\right)+3bn(-a+bn\log(x)-b\log(cx^n))\left(\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+2\log(x)\operatorname{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+2\log(x)\operatorname{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)-2\operatorname{Li}_3\left(-i\sqrt{d}\sqrt{f}x\right)-2\operatorname{Li}_3\left(i\sqrt{d}\sqrt{f}x\right)\right)-b^2n^2\left(\log^3(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\log^3(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\operatorname{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)+3\log^2(x)\operatorname{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{Li}_3\left(-i\sqrt{d}\sqrt{f}x\right)-6\log(x)\operatorname{Li}_3\left(i\sqrt{d}\sqrt{f}x\right)+6\operatorname{Li}_4\left(-i\sqrt{d}\sqrt{f}x\right)+6\operatorname{Li}_4\left(i\sqrt{d}\sqrt{f}x\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] (Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*(a + b*Log[
c*x^n])^2)*Log[1 + d*f*x^2] - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[x]*(
Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2, (
-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b*n*(-a + b*n
*Log[x] - b*Log[c*x^n])*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^2*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[
f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) - b^2*n^2*(Log[x]^3*Log[1 - I*Sq
rt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyL
og[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
- 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, I*Sqrt[
d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[
d]*Sqrt[f]*x]))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 6180, normalized size = 88.29

method	result	size
risch	Expression too large to display	6180

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^2 + 1) - integrate(2/3*
(b^2*d*f*n^2*x*log(x)^3 + 3*b^2*d*f*x*log(x)*log(x^n)^2 - 3*(b^2*d*f*n*log(
c) + a*b*d*f*n)*x*log(x)^2 + 3*(b^2*d*f*log(c)^2 + 2*a*b*d*f*log(c) + a^2*d
*f)*x*log(x) - 3*(b^2*d*f*n*x*log(x)^2 - 2*(b^2*d*f*log(c) + a*b*d*f)*x*log
(x))*log(x^n))/(d*f*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

3.35 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=257

$$\frac{1}{2}b^2dfn^2\log(x)-\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))^2-\frac{1}{4}b^2dfn^2\log$$

```
[Out] 1/2*b^2*d*f*n^2*ln(x)-1/2*b*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-1/2*d*f*l
n(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/4*b^2*d*f*n^2*ln(d*f*x^2+1)-1/4*b^2*n^2*
ln(d*f*x^2+1)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-1/2*(a+b*ln(c*x
^n))^2*ln(d*f*x^2+1)/x^2+1/4*b^2*d*f*n^2*polylog(2,-1/d/f/x^2)+1/2*b*d*f*n*
(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+1/4*b^2*d*f*n^2*polylog(3,-1/d/f/x^2)
```

Rubi [A]
time = 0.23, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2342, 2341, 2425, 272, 36, 29, 31, 2379, 2438, 2421, 6724}

$$\frac{1}{2}bdfn\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)(a+b\log(cx^n))+\frac{1}{4}b^2dfn^2\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{1}{4}b^2dfn^2\text{PolyLog}\left(3,-\frac{1}{dfx^2}\right)-\frac{1}{2}bdfn\log\left(\frac{1}{dfx^2}+1\right)(a+b\log(cx^n))-\frac{bn\log(dfx^2+1)(a+b\log(cx^n))}{2x^2}-\frac{1}{2}df\log\left(\frac{1}{dfx^2}+1\right)(a+b\log(cx^n))^2-\frac{\log(dfx^2+1)(a+b\log(cx^n))^2}{2x^2}-\frac{1}{4}b^2dfn^2\log(dfx^2+1)-\frac{b^2n^2\log(dfx^2+1)}{4x^2}+\frac{1}{2}b^2dfn^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (b^2*d*f*n^2*Log[x])/2 - (b*d*f*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n]))/
2 - (d*f*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/2 - (b^2*d*f*n^2*Log[1
+ d*f*x^2])/4 - (b^2*n^2*Log[1 + d*f*x^2])/(4*x^2) - (b*n*(a + b*Log[c*x^n]
)*Log[1 + d*f*x^2])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(2*x^
2) + (b^2*d*f*n^2*PolyLog[2, -(1/(d*f*x^2))])/4 + (b*d*f*n*(a + b*Log[c*x^n
])*PolyLog[2, -(1/(d*f*x^2))])/2 + (b^2*d*f*n^2*PolyLog[3, -(1/(d*f*x^2))])
/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = -\frac{b^2n^2\log(1+dfx^2)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(1+dfx^2)}{2x^2} - \frac{(a+}{}$$

$$= -\frac{b^2n^2\log(1+dfx^2)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(1+dfx^2)}{2x^2} - \frac{(a+}{}$$

$$= -\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n)) - \frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)$$

$$= -\frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n)) - \frac{1}{2}df\log\left(1+\frac{1}{dfx^2}\right)$$

$$= \frac{1}{2}b^2dfn^2\log(x) - \frac{1}{2}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n)) - \frac{1}{2}df\,\mathrm{l}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 488, normalized size = 1.90

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (2*d*f*Log[x]*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*Log[x]) + Log[c*x^n])
 + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)
 - ((2*a^2 + 2*a*b*n + b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x
^n]^2)*Log[1 + d*f*x^2])/x^2 - d*f*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*
Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log
[x]) + Log[c*x^n])^2)*Log[1 + d*f*x^2] - 2*b*d*f*n*(-2*a - b*n + 2*b*n*Log[
x] - 2*b*Log[c*x^n])*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[1
 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - PolyLog[2,
I*Sqrt[d]*Sqrt[f]*x]) + (2*b^2*d*f*n^2*(2*Log[x]^3 - 3*Log[x]^2*Log[1 - I*S
qrt[d]*Sqrt[f]*x] - 3*Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*Poly
Log[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] +
 6*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]))
/3)/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 3493, normalized size = 13.59

method	result	size
risch	Expression too large to display	3493

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*f*ln(x)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+1/8*d*f*ln(d
*f*x^2+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-1/4*d*f*ln(d*f
*x^2+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+1/2*I*d*f*ln(d*f*x
^2+1)*Pi*ln(c)*b^2*csgn(I*c*x^n)^3+2*d*f*ln(x)*ln(c)*a*b+1/4*I*n*d*f*polylo
g(2,-d*f*x^2)*b^2*Pi*csgn(I*c*x^n)^3-1/2*I/x^2*ln(d*f*x^2+1)*ln(x^n)*b^2*Pi
*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*n*d*f*ln(d*f*x^2+1)*b^2*Pi*csgn(I*c*x^n)^3
-1/2*I*n*d*f*ln(x)*b^2*Pi*csgn(I*c*x^n)^3+1/2*I/x^2*ln(d*f*x^2+1)*Pi*ln(c)*
b^2*csgn(I*c*x^n)^3+1/2*I/x^2*ln(d*f*x^2+1)*Pi*a*b*csgn(I*c*x^n)^3+1/2*I/x^
2*ln(d*f*x^2+1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n)^3-1/2*b*n*d*f*polylog(2,-d*f*x
^2)*a+1/2/x^2*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-
1/2*I/x^2*ln(d*f*x^2+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I/x^2*ln
(d*f*x^2+1)*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*d*f*ln(d*f*x^2+1
)*Pi*a*b*csgn(I*c*x^n)^3-1/2*I/x^2*ln(d*f*x^2+1)*Pi*a*b*csgn(I*c)*csgn(I*c*
x^n)^2-1/2*I/x^2*ln(d*f*x^2+1)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*d*f*l
n(d*f*x^2+1)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-1/4*d*f*ln(x)*P
i^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-1/2*a^2/x^2*ln(d*f*x^2+1)
-1/2*b^2*n^2*d*f*ln(x)^2-1/4*b^2*n^2*d*f*polylog(2,-d*f*x^2)+1/3*b^2*n^2*d*
f*ln(x)^3+1/4*b^2*n^2*d*f*polylog(3,-d*f*x^2)+1/2*I/x^2*ln(d*f*x^2+1)*ln(x^
n)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*ln(d*f*x^2+1)*ln(x^n)*d
*f*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I/x^2*ln(d*f*x^2+1)*ln(x^n)*b^2*Pi*
csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(d*f*x^2+1)*ln(x^n)*d*f*b^2*Pi*csgn(I*c
*x^n)^3+1/2*I*n*d*f*ln(x)^2*b^2*Pi*csgn(I*c*x^n)^3-b*n*d*f*ln(x)^2*a-1/2*b*
n*d*f*ln(d*f*x^2+1)*a+b*n*d*f*ln(x)*a-1/2*b^2/x^2*ln(d*f*x^2+1)*ln(x^n)^2-1
/2*a^2*d*f*ln(d*f*x^2+1)+a^2*d*f*ln(x)-I*d*f*ln(x)*Pi*a*b*csgn(I*c*x^n)^3+d
*f*ln(x)*ln(c)^2*b^2-1/x^2*ln(d*f*x^2+1)*ln(c)*a*b-1/2*n/x^2*ln(d*f*x^2+1)*
b^2*ln(c)-1/2*d*f*ln(d*f*x^2+1)*ln(c)^2*b^2-I*d*f*ln(x)*Pi*ln(c)*b^2*csgn(I
*c*x^n)^3+1/2*d*f*ln(x)*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-d*
f*ln(x)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-1/4*d*f*ln(d*f*x^2+1
)*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-I*ln(x)*ln(x^n)*d*f*b^2*
Pi*csgn(I*c*x^n)^3-1/4*I*n/x^2*ln(d*f*x^2+1)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)
^2-1/4*I*n/x^2*ln(d*f*x^2+1)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2/x^2*ln(
d*f*x^2+1)*ln(c)^2*b^2-1/4*I*n*d*f*ln(d*f*x^2+1)*b^2*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-1/4*I*n*d*f*ln(d*f*x^2+1)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*d*f
*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+1/8*d*f*ln(d*f*x^2+1)*Pi^
2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-1/4*d*f*ln(x)*Pi^2*b^2*csgn(I*c)^2*csgn
```

```
(I*c*x^n)^4-1/2*b*n/x^2*ln(d*f*x^2+1)*a-1/4*I*n*d*f*polylog(2,-d*f*x^2)*b^2
*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*b^2*n/x^2*ln(d*f*x^2+1)*ln(x^n)-1/2*b^2
*ln(d*f*x^2+1)*ln(x^n)^2*d*f+b^2*ln(x)*ln(x^n)^2*d*f-b^2*ln(x)^2*ln(x^n)*d*
f*n-1/2*b^2*ln(d*f*x^2+1)*ln(x^n)*d*f*n+b^2*ln(x)*ln(x^n)*d*f*n-1/2*b^2*ln(
x^n)*polylog(2,-d*f*x^2)*d*f*n+1/8*d*f*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*c*x^n)
^6-1/4*d*f*ln(x)*Pi^2*b^2*csgn(I*c*x^n)^6+1/8/x^2*ln(d*f*x^2+1)*Pi^2*b^2*cs
gn(I*x^n)^2*csgn(I*c*x^n)^4-1/2*I*d*f*ln(d*f*x^2+1)*Pi*a*b*csgn(I*c)*csgn(I
*c*x^n)^2+1/8/x^2*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*c*x^n)^6-b/x^2*ln(d*f*x^2+1
)*ln(x^n)*a-1/x^2*ln(d*f*x^2+1)*ln(x^n)*b^2*ln(c)-1/2*I*d*f*ln(d*f*x^2+1)*P
i*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*n*d*f*polylog(2,-d*f*x^2)*b^2*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2-1/4*b^2*n^2*ln(d*f*x^2+1)/x^2+I*ln(x)*ln(x^n)*d*f*b
^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*d*f*ln(x)*Pi^2*b^2*csgn(I*x^n)^2*csgn(I
*c*x^n)^4+1/2*d*f*ln(x)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-ln(d*f*x^2+1)*
ln(x^n)*d*f*b^2*ln(c)+2*ln(x)*ln(x^n)*d*f*b^2*ln(c)+1/4*I*n*d*f*polylog(2,-
d*f*x^2)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(d*f*x^2+1)*ln(
x^n)*d*f*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*d*f*ln(x)*Pi*ln(c)*b^
2*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(d*f*x^2+1)*ln(x^n)*d*f*b^2*Pi*csgn(I*x
^n)*csgn(I*c*x^n)^2-1/4/x^2*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n
)^5-b*ln(d*f*x^2+1)*ln(x^n)*d*f*a+2*b*ln(x)*ln(x^n)*d*f*a+1/2*I*n*d*f*ln(x)
^2*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*d*f*ln(x)*Pi*a*b*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*d*f*ln(d*f*x^2+1)*Pi*ln(c)*b^2*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)-1/2*I*n*d*f*ln(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2+1/4*I*n/x^2*ln(d*f*x^2+1)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/
2*I*d*f*ln(d*f*x^2+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*x^n)^2+1/8/x^2*ln(d*f
*x^2+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-1/4/x^2*ln(d*f*x^2+1)*Pi^2*b^2
*csgn(I*c)*csgn(I*c*x^n)^5+I*ln(x)*ln(x^n)*d*f*b^2*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2+1/8*d*f*ln(d*f*x^2+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4+I*d*f*ln(
x)*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*n*d*f*ln(x)*b^2*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-n*d*f*ln(x)^2*b^2*ln(c)-1/2*n*d*f*ln(d*f*x^2+1)*b^2*ln(c)+n*d
*f*ln(x)*b^2*ln(c)-1/2*n*d*f*polylog(2,-d*f*x^2)*b^2*ln(c)-d*f*ln(d*f*x^2+1
)*ln(c)*a*b+I*d*f*ln(x)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*d*f*ln(d*f*x
^2+1)*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+1/2*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
[Out] -1/4*(2*b^2*log(x^n)^2 + (n^2 + 2*n*log(c) + 2*log(c)^2)*b^2 + 2*a*b*(n + 2
*log(c)) + 2*a^2 + 2*(b^2*(n + 2*log(c)) + 2*a*b)*log(x^n))*log(d*f*x^2 + 1
)/x^2 + integrate(1/2*(2*b^2*d*f*log(x^n)^2 + 2*a^2*d*f + 2*(d*f*n + 2*d*f*
log(c))*a*b + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^2 + 2*(2*a*b*d*
f + (d*f*n + 2*d*f*log(c))*b^2)*log(x^n))/(d*f*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

3.36 $\int x^2(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=612

$$-\frac{16abnx}{9df}+\frac{52b^2n^2x}{27df}-\frac{4}{27}b^2n^2x^3-\frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)}{27d^{3/2}f^{3/2}}-\frac{16b^2nx\log(cx^n)}{9df}+\frac{8}{27}bnx^3(a+b\log(cx^n))+\frac{4bn}{}$$

```
[Out] -16/9*a*b*n*x/d/f+52/27*b^2*n^2*x/d/f-4/27*b^2*n^2*x^3-4/27*b^2*n^2*arctan(
x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)-16/9*b^2*n*x*ln(c*x^n)/d/f+8/27*b*n*x^3*
(a+b*ln(c*x^n))+4/9*b*n*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))/d^(3/2)/f
^(3/2)+2/3*x*(a+b*ln(c*x^n))^2/d/f-2/9*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*n^2*x
^3*ln(d*f*x^2+1)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*f*x^2+1)+1/3*x^3*(a+b*ln(
c*x^n))^2*ln(d*f*x^2+1)-1/3*(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))/(-
d)^(3/2)/f^(3/2)+1/3*(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))/(-d)^(3/2
)/f^(3/2)+2/3*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))/(-d)^(3/
2)/f^(3/2)-2/3*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))/(-d)^(3/
2)/f^(3/2)-2/9*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)+2/
9*I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))/d^(3/2)/f^(3/2)-2/3*b^2*n^2*poly
log(3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(3/2)/f^(3/2)+2/3*b^2*n^2*polylog(3,x*(-d
)^(1/2)*f^(1/2))/(-d)^(3/2)/f^(3/2)
```

Rubi [A]
time = 0.69, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {2342, 2341, 2425, 308, 209, 2393, 2332, 2361, 12, 4940, 2438, 2395, 2333, 2367, 2354, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-16*a*b*n*x)/(9*d*f) + (52*b^2*n^2*x)/(27*d*f) - (4*b^2*n^2*x^3)/27 - (4*b
^2*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x])/(27*d^(3/2)*f^(3/2)) - (16*b^2*n*x*Log[c*
x^n])/(9*d*f) + (8*b*n*x^3*(a + b*Log[c*x^n]))/27 + (4*b*n*ArcTan[Sqrt[d]*S
qrt[f]*x]*(a + b*Log[c*x^n]))/(9*d^(3/2)*f^(3/2)) + (2*x*(a + b*Log[c*x^n])
^2)/(3*d*f) - (2*x^3*(a + b*Log[c*x^n])^2)/9 - ((a + b*Log[c*x^n])^2*Log[1
- Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) + ((a + b*Log[c*x^n])^2*Log[1
 + Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) + (2*b^2*n^2*x^3*Log[1 + d*f
*x^2])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/9 + (x^3*(a + b
*Log[c*x^n])^2*Log[1 + d*f*x^2])/3 + (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -
(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(3/2)) - (2*b*n*(a + b*Log[c*x^n])*P
olyLog[2, Sqrt[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2)) - (((2*I)/9)*b^2*n^2*
PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) + (((2*I)/9)*b^2*n^2*
```

```
PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(d^(3/2)*f^(3/2)) - (2*b^2*n^2*PolyLog[3,
-(Sqrt[-d]*Sqrt[f]*x)])/(3*(-d)^(3/2)*f^(3/2)) + (2*b^2*n^2*PolyLog[3, Sqrt
[-d]*Sqrt[f]*x])/(3*(-d)^(3/2)*f^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
```

```
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = \frac{2}{27}b^2n^2x^3\log(1+dfx^2) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log(1+d$$

$$= \frac{2}{27}b^2n^2x^3\log(1+dfx^2) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log(1+d$$

$$= \frac{2}{27}b^2n^2x^3\log(1+dfx^2) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log(1+d$$

$$= \frac{4b^2n^2x}{27df} - \frac{4}{81}b^2n^2x^3 + \frac{2}{27}b^2n^2x^3\log(1+dfx^2) - \frac{2}{9}bnx^3(a-$$

$$= -\frac{4abnx}{9df} + \frac{4b^2n^2x}{27df} - \frac{8}{81}b^2n^2x^3 - \frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{27d^{3/2}f^{3/2}}$$

$$= -\frac{16abnx}{9df} + \frac{16b^2n^2x}{27df} - \frac{4}{27}b^2n^2x^3 - \frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\right.}{27d^{3/2}f^{3/2}}$$

$$= -\frac{16abnx}{9df} + \frac{52b^2n^2x}{27df} - \frac{4}{27}b^2n^2x^3 - \frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\right.}{27d^{3/2}f^{3/2}}$$

$$= -\frac{16abnx}{9df} + \frac{52b^2n^2x}{27df} - \frac{4}{27}b^2n^2x^3 - \frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\right.}{27d^{3/2}f^{3/2}}$$

$$= -\frac{16abnx}{9df} + \frac{52b^2n^2x}{27df} - \frac{4}{27}b^2n^2x^3 - \frac{4b^2n^2\tan^{-1}\left(\sqrt{d}\sqrt{f}\right.}{27d^{3/2}f^{3/2}}$$

Mathematica [A]
time = 0.38, size = 703, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (6*Sqrt[d]*Sqrt[f]*x*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(n*Log[x] - Log
[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Log[x]) + Log[c*x
^n])^2) - 2*d^(3/2)*f^(3/2)*x^3*(9*a^2 - 6*a*b*n + 2*b^2*n^2 + 6*b^2*n*(n*L
og[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 9*b^2*(-(n*Log[x]
) + Log[c*x^n])^2) - 6*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 - 6*a*b*n + 2*b^2*n
^2 + 6*b^2*n*(n*Log[x] - Log[c*x^n]) + 18*a*b*(-(n*Log[x]) + Log[c*x^n]) +
```

```
9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) + 3*d^(3/2)*f^(3/2)*x^3*(9*a^2 - 6*a*b*
n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*Log[1 + d
*f*x^2] - 18*b*n*(3*a - b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(-2*Sqrt[d]*Sq
rt[f]*x*(-1 + Log[x]) + (2*d^(3/2)*f^(3/2)*x^3*(-1 + 3*Log[x]))/9 - I*(Log[
x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(
Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) + 5
4*b^2*n^2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) - (d^(3/2)*f^(3/2)*x
^3*(2 - 6*Log[x] + 9*Log[x]^2))/27 + (I/2)*(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt
[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sq
rt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]
*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x])))/(81
*d^(3/2)*f^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\ln (c\,x^{n}))^{2}\ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^2*x^3*log(x^n)^2 - 6*(b^2*(n - 3*log(c)) - 3*a*b)*x^3*log(x^n) +
((2*n^2 - 6*n*log(c) + 9*log(c)^2)*b^2 - 6*a*b*(n - 3*log(c)) + 9*a^2)*x^3)
*log(d*f*x^2 + 1) - integrate(2/27*(9*b^2*d*f*x^4*log(x^n)^2 + 6*(3*a*b*d*f
 - (d*f*n - 3*d*f*log(c))*b^2)*x^4*log(x^n) + (9*a^2*d*f - 6*(d*f*n - 3*d*f
*log(c))*a*b + (2*d*f*n^2 - 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^4)/(d*f
*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*x^2*log(d*f*x^2 + 1)
*log(c*x^n) + a^2*x^2*log(d*f*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.37 $\int (a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right) dx$

Optimal. Leaf size=519

$$4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+8b^2nx\log(cx^n)-\frac{4b^2n\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\mathrm{l}}{\sqrt{d}\sqrt{f}}$$

```
[Out] 4*a*b*n*x-8*b^2*n^2*x+4*b*n*(-b*n+a)*x+8*b^2*n*x*ln(c*x^n)-2*x*(a+b*ln(c*x^
n))^2-2*a*b*n*x*ln(d*f*x^2+1)+2*b^2*n^2*x*ln(d*f*x^2+1)-2*b^2*n*x*ln(c*x^n)
*ln(d*f*x^2+1)+x*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)-(a+b*ln(c*x^n))^2*ln(1-x*(
-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f
^(1/2))/(-d)^(1/2)/f^(1/2)+2*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^
(1/2))/(-d)^(1/2)/f^(1/2)-2*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1
/2))/(-d)^(1/2)/f^(1/2)-2*b^2*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/
2)/f^(1/2)+2*b^2*n^2*polylog(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-4*b
*n*(-b*n+a)*arctan(x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)-4*b^2*n*arctan(x*d^(1
/2)*f^(1/2))*ln(c*x^n)/d^(1/2)/f^(1/2)-2*I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^
(1/2))/d^(1/2)/f^(1/2)+2*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(1/2)/
f^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {2333, 2332, 2418, 6, 327, 209, 2393, 2361, 12, 4940, 2438, 2395, 2367, 2354, 2421, 6724}

$$\frac{2bn\text{PolyLog}\left(2,-\sqrt{-d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{f}}-\frac{2bn\text{PolyLog}\left(2,\sqrt{-d}\sqrt{f}x\right)(a+b\log(cx^n))}{\sqrt{-d}\sqrt{f}}+\frac{2ib^2n^2\text{PolyLog}\left(2,-i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-\frac{2ib^2n^2\text{PolyLog}\left(2,i\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-\frac{2b^2n^2\text{PolyLog}\left(3,-\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}}+\frac{2b^2n^2\text{PolyLog}\left(3,\sqrt{-d}\sqrt{f}x\right)}{\sqrt{-d}\sqrt{f}}-\frac{4bn(a-bn)\text{ArcTan}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-\frac{\log\left(1-\sqrt{-d}\sqrt{f}x\right)(a+b\log(cx^n))^2}{\sqrt{-d}\sqrt{f}}+\frac{\log\left(\sqrt{-d}\sqrt{f}x+1\right)(a+b\log(cx^n))^2}{\sqrt{-d}\sqrt{f}}+x\log\left(dfx^2+1\right)(a+b\log(cx^n))^2-2x(a+b\log(cx^n))^2-2abnx\log\left(dfx^2+1\right)+4abnx+4bnx(a-bn)-\frac{4b^2n\text{ArcTan}\left(\sqrt{d}\sqrt{f}x\right)\log(cx^n)}{\sqrt{d}\sqrt{f}}-2b^2nx\log(cx^n)\log\left(dfx^2+1\right)+8b^2nx\log(cx^n)+2b^2n^2x\log\left(dfx^2+1\right)-8b^2n^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] 4*a*b*n*x - 8*b^2*n^2*x + 4*b*n*(a - b*n)*x - (4*b*n*(a - b*n)*ArcTan[Sqrt[
d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + 8*b^2*n*x*Log[c*x^n] - (4*b^2*n*ArcTan[S
qrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*Sqrt[f]) - 2*x*(a + b*Log[c*x^n])^2
 - ((a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (
(a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - 2*a*
b*n*x*Log[1 + d*f*x^2] + 2*b^2*n^2*x*Log[1 + d*f*x^2] - 2*b^2*n*x*Log[c*x^n
]*Log[1 + d*f*x^2] + x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2] + (2*b*n*(a +
b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (2*b*
n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (
(2*I)*b^2*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - ((2*I
)*b^2*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - (2*b^2*n^2*P
olyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (2*b^2*n^2*PolyLog[3
, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
```

```
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)dx = -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log(c$$

$$= -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log(c$$

$$= -2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)-2b^2nx\log(c$$

$$= 4bn(a-bn)x-2abnx\log\left(1+dfx^2\right)+2b^2n^2x\log\left(1+dfx^2\right)$$

$$= 4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}-2abnx\log$$

$$= -4b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)}{\sqrt{d}\sqrt{f}}+$$

$$= 4abnx-4b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{\ }\right.}{\sqrt{d}\sqrt{f}}$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{\ }\right.}{\sqrt{d}\sqrt{f}}$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{\ }\right.}{\sqrt{d}\sqrt{f}}$$

$$= 4abnx-8b^2n^2x+4bn(a-bn)x-\frac{4bn(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{\ }\right.}{\sqrt{d}\sqrt{f}}$$

Mathematica [A]
time = 0.22, size = 544, normalized size = 1.05

$$\frac{-2\sqrt{d}\sqrt{f}x\left(a^2-2abn+2b^2n^2+2b^2n(n\log(x)-\log(cx^n))+2ab(-n\log(x)+\log(cx^n))+b^2(-n\log(x)+\log(cx^n))^2\right)+2\tan^{-1}\left(\sqrt{d}\sqrt{f}x\right)\left(a^2-2abn+2b^2n^2+2b^2n(n\log(x)-\log(cx^n))+2ab(-n\log(x)+\log(cx^n))+b^2(-n\log(x)+\log(cx^n))^2\right)+\sqrt{d}\sqrt{f}x\left(a^2-2abn+2b^2n^2+2b(a-bn)\log(cx^n)+b^2\log^2(cx^n)\right)\log\left(1+dfx^2\right)+2bn(a-bn-bn\log(x)+b\log(cx^n))\left(-2\sqrt{d}\sqrt{f}x(-1+\log(x))-i\left(\log(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)\right)+i\left(\log(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)\right)\right)-2b^2n^2\left(\sqrt{d}\sqrt{f}x\left(2-2\log(x)+\log^2(x)\right)+\frac{1}{2}i\left(\log^2(x)\log\left(1+i\sqrt{d}\sqrt{f}x\right)+2\log(x)\text{Li}_2\left(-i\sqrt{d}\sqrt{f}x\right)-2\text{Li}_3\left(-i\sqrt{d}\sqrt{f}x\right)\right)-\frac{1}{2}i\left(\log^2(x)\log\left(1-i\sqrt{d}\sqrt{f}x\right)+2\log(x)\text{Li}_2\left(i\sqrt{d}\sqrt{f}x\right)-2\text{Li}_3\left(i\sqrt{d}\sqrt{f}x\right)\right)\right)}{\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[
c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])
^2) + 2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*L
og[x] - Log[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) +
 Log[c*x^n])^2) + Sqrt[d]*Sqrt[f]*x*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b
*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2] + 2*b*n*(a - b*n - b*n*
Log[x] + b*Log[c*x^n])*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[x]*Log[
1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(Log[x]*
Log[1 - I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) - 2*b^2*n^
2*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) + (I/2)*(Log[x]^2*Log[1 + I*
Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLo
g[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x
] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[
f]*x])))/(Sqrt[d]*Sqrt[f])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\,x^{n}\right)\right)^{2} \ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] (b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*l
og(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*x^2 + 1) - int
egrate(2*(b^2*d*f*x^2*log(x^n)^2 + 2*(a*b*d*f - (d*f*n - d*f*log(c))*b^2)*x
```

```
^2*log(x^n) + (a^2*d*f - 2*(d*f*n - d*f*log(c))*a*b + (2*d*f*n^2 - 2*d*f*n*
log(c) + d*f*log(c)^2)*b^2)*x^2)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="fricas")
[Out] integral(b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*x
^n) + a^2*log(d*f*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \log (cx^n))^2 \log \left(dfx^2 + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2)),x)
[Out] Integral((a + b*log(c*x**n))**2*log(d*f*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln \left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2,x)
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.38 $\int \frac{(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2} dx$

Optimal. Leaf size=459

$$4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)+4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+\sqrt{-d}\,\sqrt{f}\,(a+b\log$$

```
[Out] -2*b^2*n^2*ln(d*f*x^2+1)/x-2*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x-(a+b*ln(c*
x^n))^2*ln(d*f*x^2+1)/x+(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(
1/2)*f^(1/2)-(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2
)-2*b*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)
+2*b*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+2
*b^2*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-2*b^2*n^2*poly
log(3,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+4*b^2*n^2*arctan(x*d^(1/2)*f
^(1/2))*d^(1/2)*f^(1/2)+4*b*n*arctan(x*d^(1/2)*f^(1/2))*(a+b*ln(c*x^n))*d^(
1/2)*f^(1/2)-2*I*b^2*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+2*
I*b^2*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2342, 2341, 2425, 209, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724}

$$-2b\sqrt{-d}\,\sqrt{f}\,n\text{PolyLog}\left(2,-\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+2b\sqrt{-d}\,\sqrt{f}\,n\text{PolyLog}\left(2,\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))-2ib^2\sqrt{d}\,\sqrt{f}\,n^2\text{PolyLog}\left(2,-i\sqrt{d}\,\sqrt{f}\,x\right)+2ib^2\sqrt{d}\,\sqrt{f}\,n^2\text{PolyLog}\left(2,i\sqrt{d}\,\sqrt{f}\,x\right)+2b^2\sqrt{-d}\,\sqrt{f}\,n^2\text{PolyLog}\left(3,-\sqrt{-d}\,\sqrt{f}\,x\right)-2b^2\sqrt{-d}\,\sqrt{f}\,n^2\text{PolyLog}\left(3,\sqrt{-d}\,\sqrt{f}\,x\right)+4b\sqrt{d}\,\sqrt{f}\,n\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+\sqrt{-d}\,\sqrt{f}\,\log\left(1-\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))^2-\sqrt{-d}\,\sqrt{f}\,\log\left(\sqrt{-d}\,\sqrt{f}\,x+1\right)(a+b\log(cx^n))^2-\frac{2bn\log(dfx^2+1)(a+b\log(cx^n))}{x}-\frac{\log(dfx^2+1)(a+b\log(cx^n))^2}{x}+4b^2\sqrt{d}\,\sqrt{f}\,n^2\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2b^2n^2\log(dfx^2+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 4*b^2*Sqrt[d]*Sqrt[f]*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x] + 4*b*Sqrt[d]*Sqrt[f]*n
*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) + Sqrt[-d]*Sqrt[f]*(a + b*Log
[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x] - Sqrt[-d]*Sqrt[f]*(a + b*Log[c*x^n]
)^2*Log[1 + Sqrt[-d]*Sqrt[f]*x] - (2*b^2*n^2*Log[1 + d*f*x^2])/x - (2*b*n*(
a + b*Log[c*x^n])*Log[1 + d*f*x^2])/x - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x
^2])/x - 2*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sq
rt[f]*x)] + 2*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*S
qrt[f]*x] - (2*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x
] + (2*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 2*b^2*S
qrt[-d]*Sqrt[f]*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] - 2*b^2*Sqrt[-d]*Sqrt
[f]*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = -\frac{2b^2n^2\log(1+dfx^2)}{x} - \frac{2bn(a+b\log(cx^n))\log(1+dfx^2)}{x} - \frac{(a}{}$$

$$= -\frac{2b^2n^2\log(1+dfx^2)}{x} - \frac{2bn(a+b\log(cx^n))\log(1+dfx^2)}{x} - \frac{(a}{}$$

$$= 4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{\right.$$

$$= 4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{\right.$$

$$= 4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{\right.$$

$$= 4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{\right.$$

$$= 4b^2\sqrt{d}\,\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) + 4b\sqrt{d}\,\sqrt{f}\,n\tan^{-1}\left(\sqrt{d}\,\sqrt{\right.$$

Mathematica [A]
time = 0.19, size = 414, normalized size = 0.90

$$2\sqrt{d}\,\sqrt{f}\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)\left(a^2+2abn+2b^2n^2+2ab(-n\log(x)+\log(cx^n))+2b^2n(-n\log(x)+\log(cx^n))+b^2(-n\log(x)+\log(cx^n))^2\right)-\frac{(a^2+2abn+2b^2n^2+2b(a+bn)\log(cx^n)+b^2\log^2(cx^n))\log(1+dfx^2)}{x}+2ib\sqrt{d}\,\sqrt{f}\,n(a+bn-bn\log(x)+b\log(cx^n))\left(\log(x)\left(\log\left(1-i\sqrt{d}\,\sqrt{f}\,x\right)-\log\left(1+i\sqrt{d}\,\sqrt{f}\,x\right)\right)-\text{Li}_2\left(-i\sqrt{d}\,\sqrt{f}\,x\right)+\text{Li}_2\left(i\sqrt{d}\,\sqrt{f}\,x\right)\right)+ib^2\sqrt{d}\,\sqrt{f}\,n^2\left(\log^2(x)\log\left(1-i\sqrt{d}\,\sqrt{f}\,x\right)-\log^2(x)\log\left(1+i\sqrt{d}\,\sqrt{f}\,x\right)-2\log(x)\text{Li}_2\left(-i\sqrt{d}\,\sqrt{f}\,x\right)+2\log(x)\text{Li}_2\left(i\sqrt{d}\,\sqrt{f}\,x\right)+2\text{Li}_3\left(-i\sqrt{d}\,\sqrt{f}\,x\right)-2\text{Li}_3\left(i\sqrt{d}\,\sqrt{f}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*
a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(
-(n*Log[x]) + Log[c*x^n])^2) - ((a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*
Log[c*x^n] + b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/x + (2*I)*b*Sqrt[d]*Sqrt[f
]*n*(a + b*n - b*n*Log[x] + b*Log[c*x^n])*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f
]*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
 PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + I*b^2*Sqrt[d]*Sqrt[f]*n^2*(Log[x]^2*Log
[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x
]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*Sqrt[f
]*x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*log(x^n)^2 + (2*n^2 + 2*n*log(c) + log(c)^2)*b^2 + 2*a*b*(n + log(c))
 + a^2 + 2*(b^2*(n + log(c)) + a*b)*log(x^n))*log(d*f*x^2 + 1)/x + integrat
e(2*(b^2*d*f*log(x^n)^2 + a^2*d*f + 2*(d*f*n + d*f*log(c))*a*b + (2*d*f*n^2
 + 2*d*f*n*log(c) + d*f*log(c)^2)*b^2 + 2*(a*b*d*f + (d*f*n + d*f*log(c))*b
^2)*log(x^n))/(d*f*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln \left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

3.39 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=543

$$-\frac{52b^2dfn^2}{27x}-\frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{16bdfn(a+b\log(cx^n))}{9x}-\frac{4}{9}bd^{3/2}f^{3/2}n\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)$$

```
[Out] -52/27*b^2*d*f*n^2/x-4/27*b^2*d^(3/2)*f^(3/2)*n^2*arctan(x*d^(1/2)*f^(1/2))
-16/9*b*d*f*n*(a+b*ln(c*x^n))/x-4/9*b*d^(3/2)*f^(3/2)*n*arctan(x*d^(1/2)*f^
(1/2))*(a+b*ln(c*x^n))-2/3*d*f*(a+b*ln(c*x^n))^2/x-2/27*b^2*n^2*ln(d*f*x^2+
1)/x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d
*f*x^2+1)/x^3+1/3*(-d)^(3/2)*f^(3/2)*(a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^
(1/2))-1/3*(-d)^(3/2)*f^(3/2)*(a+b*ln(c*x^n))^2*ln(1+x*(-d)^(1/2)*f^(1/2))-
2/3*b*(-d)^(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,-x*(-d)^(1/2)*f^(1/2))
+2/3*b*(-d)^(3/2)*f^(3/2)*n*(a+b*ln(c*x^n))*polylog(2,x*(-d)^(1/2)*f^(1/2))
-2/9*I*b^2*d^(3/2)*f^(3/2)*n^2*polylog(2,I*x*d^(1/2)*f^(1/2))+2/9*I*b^2*d^(
3/2)*f^(3/2)*n^2*polylog(2,-I*x*d^(1/2)*f^(1/2))+2/3*b^2*(-d)^(3/2)*f^(3/2)
*n^2*polylog(3,-x*(-d)^(1/2)*f^(1/2))-2/3*b^2*(-d)^(3/2)*f^(3/2)*n^2*polylo
g(3,x*(-d)^(1/2)*f^(1/2))
```

Rubi [A]
time = 0.53, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2342, 2341, 2425, 331, 209, 2380, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724}

$$-\frac{2}{3}b(-d)^{3/2}f^{3/2}n\text{PolyLog}\left(2,-\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+\frac{2}{3}b(-d)^{3/2}f^{3/2}n\text{PolyLog}\left(2,\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+\frac{2}{9}ib^2d^{3/2}f^{3/2}n^2\text{PolyLog}\left(2,-i\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2}{9}ib^2d^{3/2}f^{3/2}n^2\text{PolyLog}\left(2,i\sqrt{d}\,\sqrt{f}\,x\right)+\frac{2}{3}b^2(-d)^{3/2}f^{3/2}n^2\text{PolyLog}\left(3,-\sqrt{-d}\,\sqrt{f}\,x\right)-\frac{2}{3}b^2(-d)^{3/2}f^{3/2}n^2\text{PolyLog}\left(3,\sqrt{-d}\,\sqrt{f}\,x\right)-\frac{4}{9}bd^{3/2}f^{3/2}n\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))+\frac{1}{3}(-d)^{3/2}f^{3/2}\log\left(1-\sqrt{-d}\,\sqrt{f}\,x\right)(a+b\log(cx^n))^2-\frac{1}{3}(-d)^{3/2}f^{3/2}\log\left(\sqrt{-d}\,\sqrt{f}\,x+1\right)(a+b\log(cx^n))^2-\frac{16bdfn(a+b\log(cx^n))}{9x}-\frac{2df(a+b\log(cx^n))^2}{3x}-\frac{2bn\log(dfx^2+1)(a+b\log(cx^n))}{9x^3}-\frac{\log(dfx^2+1)(a+b\log(cx^n))^2}{3x^3}-\frac{4}{27}b^2d^{3/2}f^{3/2}n^2\text{ArcTan}\left(\sqrt{d}\,\sqrt{f}\,x\right)-\frac{2b^2n^2\log(dfx^2+1)}{27x^3}-\frac{52b^2dfn^2}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-52*b^2*d*f*n^2)/(27*x) - (4*b^2*d^(3/2)*f^(3/2)*n^2*ArcTan[Sqrt[d]*Sqrt[f
]*x])/27 - (16*b*d*f*n*(a + b*Log[c*x^n]))/(9*x) - (4*b*d^(3/2)*f^(3/2)*n*A
rcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]))/9 - (2*d*f*(a + b*Log[c*x^n])^
2)/(3*x) + ((-d)^(3/2)*f^(3/2)*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f
]*x])/3 - ((-d)^(3/2)*f^(3/2)*(a + b*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]
*x])/3 - (2*b^2*n^2*Log[1 + d*f*x^2])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*
Log[1 + d*f*x^2])/(9*x^3) - ((a + b*Log[c*x^n])^2*Log[1 + d*f*x^2])/(3*x^3)
 - (2*b*(-d)^(3/2)*f^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[
f]*x)])/3 + (2*b*(-d)^(3/2)*f^(3/2)*n*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d
]*Sqrt[f]*x])/3 + ((2*I)/9)*b^2*d^(3/2)*f^(3/2)*n^2*PolyLog[2, (-I)*Sqrt[d]
*Sqrt[f]*x] - ((2*I)/9)*b^2*d^(3/2)*f^(3/2)*n^2*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + (2*b^2*(-d)^(3/2)*f^(3/2)*n^2*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/3 -
 (2*b^2*(-d)^(3/2)*f^(3/2)*n^2*PolyLog[3, Sqrt[-d]*Sqrt[f]*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^4}\,dx = -\frac{2b^2n^2\log(1+dfx^2)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(1+dfx^2)}{9x^3} - (a$$

$$= -\frac{2b^2n^2\log(1+dfx^2)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(1+dfx^2)}{9x^3} - (a$$

$$= -\frac{4b^2dfn^2}{27x} - \frac{2b^2n^2\log(1+dfx^2)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(1+}{9x^3}$$

$$= -\frac{4b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{2b^2n^2\log(1+}{27x^3}$$

$$= -\frac{16b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{4bdfn(a+b}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{16bdfn(a+b}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{16bdfn(a+b}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{16bdfn(a+b}{9x}$$

$$= -\frac{52b^2dfn^2}{27x} - \frac{4}{27}b^2d^{3/2}f^{3/2}n^2\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right) - \frac{16bdfn(a+b}{9x}$$

Mathematica [A]
time = 0.35, size = 585, normalized size = 1.08

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^2)])/x^4,x]
```

```
[Out] (-2*d^(3/2)*f^(3/2)*ArcTan[Sqrt[d]*Sqrt[f]*x]*(9*a^2 + 6*a*b*n + 2*b^2*n^2
+ 18*a*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^2*n*(-(n*Log[x]) + Log[c*x^n]) +
9*b^2*(-(n*Log[x]) + Log[c*x^n])^2) - (2*d*f*(9*a^2 + 6*a*b*n + 2*b^2*n^2 +
 9*b^2*n^2*Log[x]^2 + 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2 - 6*b
*n*Log[x]*(3*a + b*n + 3*b*Log[c*x^n])))/x - ((9*a^2 + 6*a*b*n + 2*b^2*n^2
+ 6*b*(3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c*x^n]^2)*Log[1 + d*f*x^2])/x^3 +
((6*I)*b*d*f*n*(3*a + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n])*(2*I + (2*I)*Log
[x] + Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (
```

```
-I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]*Log[1 - I*Sqrt[d]*Sqrt[
f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])))/x + ((9*I)*b^2*d*f*n^2*(4*I + (4
*I)*Log[x] + (2*I)*Log[x]^2 + Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1 + I*Sqrt[d]
*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, (-
I)*Sqrt[d]*Sqrt[f]*x]) - Sqrt[d]*Sqrt[f]*x*(Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt
[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d]*
Sqrt[f]*x])))/x)/27
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^4,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^2))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^2*log(x^n)^2 + (2*n^2 + 6*n*log(c) + 9*log(c)^2)*b^2 + 6*a*b*(n
+ 3*log(c)) + 9*a^2 + 6*(b^2*(n + 3*log(c)) + 3*a*b)*log(x^n))*log(d*f*x^2
+ 1)/x^3 + integrate(2/27*(9*b^2*d*f*log(x^n)^2 + 9*a^2*d*f + 6*(d*f*n + 3*
d*f*log(c))*a*b + (2*d*f*n^2 + 6*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2 + 6*(3*
a*b*d*f + (d*f*n + 3*d*f*log(c))*b^2)*log(x^n))/(d*f*x^4 + x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(d*f*x^2 + 1)*log(c*x^n)^2 + 2*a*b*log(d*f*x^2 + 1)*log(c*
x^n) + a^2*log(d*f*x^2 + 1))/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**2))/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^2))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + 1/d)*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^2)/x^4, x)
```

3.40 $\int x^3(a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=591

$$-\frac{45b^3n^3x^2}{128df}+\frac{3}{64}b^3n^3x^4+\frac{21b^2n^2x^2(a+b\log(cx^n))}{32df}-\frac{9}{64}b^2n^2x^4(a+b\log(cx^n))-\frac{9bnx^2(a+b\log(cx^n))^2}{16df}+\frac{3}{16}$$

```
[Out] -45/128*b^3*n^3*x^2/d/f+3/64*b^3*n^3*x^4+21/32*b^2*n^2*x^2*(a+b*ln(c*x^n))/
d/f-9/64*b^2*n^2*x^4*(a+b*ln(c*x^n))-9/16*b*n*x^2*(a+b*ln(c*x^n))^2/d/f+3/1
6*b*n*x^4*(a+b*ln(c*x^n))^2+1/4*x^2*(a+b*ln(c*x^n))^3/d/f-1/8*x^4*(a+b*ln(c
*x^n))^3+3/128*b^3*n^3*ln(d*f*x^2+1)/d^2/f^2-3/128*b^3*n^3*x^4*ln(d*f*x^2+1
)-3/32*b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d^2/f^2+3/32*b^2*n^2*x^4*(a+b*
ln(c*x^n))*ln(d*f*x^2+1)+3/16*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d^2/f^2-3
/16*b*n*x^4*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)-1/4*(a+b*ln(c*x^n))^3*ln(d*f*x^
2+1)/d^2/f^2+1/4*x^4*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)-3/64*b^3*n^3*polylog(2
,-d*f*x^2)/d^2/f^2+3/16*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^2)/d^2/f^2
-3/8*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)/d^2/f^2-3/32*b^3*n^3*polylog
(3,-d*f*x^2)/d^2/f^2+3/8*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^2)/d^2/f^
2-3/16*b^3*n^3*polylog(4,-d*f*x^2)/d^2/f^2
```

Rubi [A]
time = 0.50, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2504, 2442, 45, 2424, 2342, 2341, 2421, 2430, 6724, 2423, 2438}

$$\frac{3b^2n^2\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{16d^2f^2}+\frac{3b^2n^2\text{PolyLog}(3,-dfx^2)(a+b\log(cx^n))}{8d^2f^2}-\frac{3bn\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))^2}{8d^2f^2}-\frac{3b^3n^3\text{PolyLog}(2,-dfx^2)}{64d^2f^2}-\frac{3b^3n^3\text{PolyLog}(3,-dfx^2)}{32d^2f^2}-\frac{3b^3n^3\text{PolyLog}(4,-dfx^2)}{16d^2f^2}-\frac{3b^2n^2\log(dfx^2+1)(a+b\log(cx^n))}{32d^2f^2}+\frac{21b^2n^2x^2(a+b\log(cx^n))}{32df}+\frac{3}{32}b^2n^2x^4\log(dfx^2+1)(a+b\log(cx^n))-\frac{9}{64}b^2n^2x^4(a+b\log(cx^n))-\frac{\log(dfx^2+1)(a+b\log(cx^n))^3}{4d^2f^2}+\frac{3bn\log(dfx^2+1)(a+b\log(cx^n))^2}{16d^2f^2}+\frac{x^2(a+b\log(cx^n))^3}{4df}-\frac{9bnx^2(a+b\log(cx^n))^2}{16df}+\frac{1}{4}x^4\log(dfx^2+1)(a+b\log(cx^n))^3-\frac{3}{16}bnx^4\log(dfx^2+1)(a+b\log(cx^n))^2-\frac{1}{8}x^4(a+b\log(cx^n))^3+\frac{3}{16}bnx^4(a+b\log(cx^n))^2+\frac{3b^3n^3\log(dfx^2+1)}{128d^2f^2}-\frac{45b^3n^3x^2}{128df}-\frac{3}{128}b^3n^3x^4\log(dfx^2+1)+\frac{3}{64}b^3n^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-45*b^3*n^3*x^2)/(128*d*f) + (3*b^3*n^3*x^4)/64 + (21*b^2*n^2*x^2*(a + b*L
og[c*x^n]))/(32*d*f) - (9*b^2*n^2*x^4*(a + b*Log[c*x^n]))/64 - (9*b*n*x^2*(
a + b*Log[c*x^n])^2)/(16*d*f) + (3*b*n*x^4*(a + b*Log[c*x^n])^2)/16 + (x^2*
(a + b*Log[c*x^n])^3)/(4*d*f) - (x^4*(a + b*Log[c*x^n])^3)/8 + (3*b^3*n^3*L
og[1 + d*f*x^2])/(128*d^2*f^2) - (3*b^3*n^3*x^4*Log[1 + d*f*x^2])/128 - (3*
b^2*n^2*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/(32*d^2*f^2) + (3*b^2*n^2*x^4*
(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/32 + (3*b*n*(a + b*Log[c*x^n])^2*Log[1
 + d*f*x^2])/(16*d^2*f^2) - (3*b*n*x^4*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2
])/16 - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(4*d^2*f^2) + (x^4*(a + b*L
og[c*x^n])^3*Log[1 + d*f*x^2])/4 - (3*b^3*n^3*PolyLog[2, -(d*f*x^2)])/(64*d
^2*f^2) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^2)])/(16*d^2*f^2
) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^2)])/(8*d^2*f^2) - (3*b^
3*n^3*PolyLog[3, -(d*f*x^2)])/(32*d^2*f^2) + (3*b^2*n^2*(a + b*Log[c*x^n])*
PolyLog[3, -(d*f*x^2)])/(8*d^2*f^2) - (3*b^3*n^3*PolyLog[4, -(d*f*x^2)])/(1
6*d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = \frac{x^2(a+b\log(cx^n))^3}{4df} - \frac{1}{8}x^4(a+b\log(cx^n))^3 - \frac{(a+b\log(c$$

$$= \frac{x^2(a+b\log(cx^n))^3}{4df} - \frac{1}{8}x^4(a+b\log(cx^n))^3 - \frac{(a+b\log(c$$

$$= -\frac{9bnx^2(a+b\log(cx^n))^2}{16df} + \frac{3}{16}bnx^4(a+b\log(cx^n))^2 + \frac{x^2(}{}$$

$$= -\frac{3b^3n^3x^2}{16df} + \frac{3}{256}b^3n^3x^4 + \frac{3b^2n^2x^2(a+b\log(cx^n))}{8df} - \frac{3}{64}b^2n$$

$$= -\frac{9b^3n^3x^2}{32df} + \frac{3}{128}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b^2$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b$$

$$= -\frac{21b^3n^3x^2}{64df} + \frac{9}{256}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b$$

$$= -\frac{45b^3n^3x^2}{128df} + \frac{3}{64}b^3n^3x^4 + \frac{21b^2n^2x^2(a+b\log(cx^n))}{32df} - \frac{9}{64}b^2$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 1234, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -1/256*(-2*d*f*x^2*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*b
^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b^3
*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 2
4*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n])^3)
 + d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 48*a*b^2*n
*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*b^3*n^2
*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 24*b^
3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - 2
```

```
*d^2*f^2*x^4*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3*b^3*n^3 + 12*b*(8*a^2
- 4*a*b*n + b^2*n^2)*Log[c*x^n] - 24*b^2*(-4*a + b*n)*Log[c*x^n]^2 + 32*b^3
*Log[c*x^n]^3)*Log[1 + d*f*x^2] + 2*(32*a^3 - 24*a^2*b*n + 12*a*b^2*n^2 - 3
*b^3*n^3 + 48*a*b^2*n*(n*Log[x] - Log[c*x^n]) + 96*a^2*b*(-(n*Log[x]) + Log
[c*x^n]) + 12*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 96*a*b^2*(-(n*Log[x]) +
Log[c*x^n])^2 - 24*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 32*b^3*(-(n*Log[x])
 + Log[c*x^n])^3)*Log[1 + d*f*x^2] + 24*b*n*(8*a^2 - 4*a*b*n + b^2*n^2 + 4*
b^2*n*(n*Log[x] - Log[c*x^n]) + 16*a*b*(-(n*Log[x]) + Log[c*x^n]) + 8*b^2*(
-(n*Log[x]) + Log[c*x^n])^2)*((d*f*x^2)/2 - (d^2*f^2*x^4)/8 - d*f*x^2*Log[x
] + (d^2*f^2*x^4*Log[x])/2 + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]*L
og[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog[
2, I*Sqrt[d]*Sqrt[f]*x]) - 96*b^2*n^2*(4*a - b*n - 4*b*n*Log[x] + 4*b*Log[c
*x^n])*((d*f*x^2*(1 - 2*Log[x] + 2*Log[x]^2))/4 - (d^2*f^2*x^4*(1 - 4*Log[x
] + 8*Log[x]^2))/32 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (Log[x]^2
*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x
] - Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]*Sqrt[f
]*x] + PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + b^3*n^3*(-16*d*f*x^2*(-3 + 6*Log[
x] - 6*Log[x]^2 + 4*Log[x]^3) + d^2*f^2*x^4*(-3 + 12*Log[x] - 24*Log[x]^2 +
 32*Log[x]^3) + 64*(Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*Poly
Log[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x
] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]) + 64*(Log[x]^3*Log[1 - I*Sqrt[d]*
Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[
3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x])))/(d^2*f^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3}(a+b\ln(c\,x^{n}))^{3}\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int(x^3*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/128*(32*b^3*x^4*log(x^n)^3 - 24*(b^3*(n - 4*log(c)) - 4*a*b^2)*x^4*log(x^
n)^2 + 12*((n^2 - 4*n*log(c) + 8*log(c)^2)*b^3 - 4*a*b^2*(n - 4*log(c)) + 8
*a^2*b)*x^4*log(x^n) + (12*(n^2 - 4*n*log(c) + 8*log(c)^2)*a*b^2 - (3*n^3 -
```

```
 12*n^2*log(c) + 24*n*log(c)^2 - 32*log(c)^3)*b^3 - 24*a^2*b*(n - 4*log(c))
 + 32*a^3)*x^4)*log(d*f*x^2 + 1) - integrate(1/64*(32*b^3*d*f*x^5*log(x^n)^
3 + 24*(4*a*b^2*d*f - (d*f*n - 4*d*f*log(c))*b^3)*x^5*log(x^n)^2 + 12*(8*a^
2*b*d*f - 4*(d*f*n - 4*d*f*log(c))*a*b^2 + (d*f*n^2 - 4*d*f*n*log(c) + 8*d*
f*log(c)^2)*b^3)*x^5*log(x^n) + (32*a^3*d*f - 24*(d*f*n - 4*d*f*log(c))*a^2
*b + 12*(d*f*n^2 - 4*d*f*n*log(c) + 8*d*f*log(c)^2)*a*b^2 - (3*d*f*n^3 - 12
*d*f*n^2*log(c) + 24*d*f*n*log(c)^2 - 32*d*f*log(c)^3)*b^3)*x^5)/(d*f*x^2 +
 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x^3*log(d*f*x^2 +
1)*log(c*x^n)^2 + 3*a^2*b*x^3*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x^3*log(d*f
*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
[Out] int(x^3*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

3.41 $\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=411

$$\frac{3}{2}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))^2-\frac{1}{2}x^2(a+b\log(cx^n))^3-\frac{3b^3n^3(1+dfx^2)\log(1+}{8df}$$

```
[Out] 3/2*b^3*n^3*x^2-9/4*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*n*x^2*(a+b*ln(c*x^n))
^2-1/2*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*n^3*(d*f*x^2+1)*ln(d*f*x^2+1)/d/f+3/4*
b^2*n^2*(d*f*x^2+1)*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/d/f-3/4*b*n*(d*f*x^2+1)*(
a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/d/f+1/2*(d*f*x^2+1)*(a+b*ln(c*x^n))^3*ln(d*f
*x^2+1)/d/f+3/8*b^3*n^3*polylog(2,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*x^n))
*polylog(2,-d*f*x^2)/d/f+3/4*b*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^2)/d/f+
3/8*b^3*n^3*polylog(3,-d*f*x^2)/d/f-3/4*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-
d*f*x^2)/d/f+3/8*b^3*n^3*polylog(4,-d*f*x^2)/d/f
```

Rubi [A]
time = 0.72, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 21, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {2504, 2436, 2332, 2424, 2342, 2341, 2395, 2339, 30, 6874, 2421, 2430, 6724, 14, 2393, 2338, 2423, 2525, 2458, 45, 2352}

$$-\frac{3b^2n^2\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))}{4df}-\frac{3b^2n^2\text{PolyLog}(3,-dfx^2)(a+b\log(cx^n))}{4df}+\frac{3bn\text{PolyLog}(2,-dfx^2)(a+b\log(cx^n))^2}{4df}+\frac{3b^3n^3\text{PolyLog}(2,-dfx^2)}{8df}+\frac{3b^3n^3\text{PolyLog}(3,-dfx^2)}{8df}+\frac{3b^3n^3\text{PolyLog}(4,-dfx^2)}{8df}+\frac{3b^2n^2(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))}{4df}-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))-\frac{3bn(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^2}{4df}+\frac{(dfx^2+1)\log(dfx^2+1)(a+b\log(cx^n))^3}{2df}+\frac{3}{2}bnx^2(a+b\log(cx^n))^2-\frac{1}{2}x^2(a+b\log(cx^n))^3-\frac{3b^3n^3(dfx^2+1)\log(dfx^2+1)}{8df}+\frac{3}{2}b^3n^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (3*b^3*n^3*x^2)/2 - (9*b^2*n^2*x^2*(a + b*Log[c*x^n]))/4 + (3*b*n*x^2*(a +
b*Log[c*x^n])^2)/2 - (x^2*(a + b*Log[c*x^n])^3)/2 - (3*b^3*n^3*(1 + d*f*x^2
)*Log[1 + d*f*x^2])/(8*d*f) + (3*b^2*n^2*(1 + d*f*x^2)*(a + b*Log[c*x^n])*L
og[1 + d*f*x^2])/(4*d*f) - (3*b*n*(1 + d*f*x^2)*(a + b*Log[c*x^n])^2*Log[1
+ d*f*x^2])/(4*d*f) + ((1 + d*f*x^2)*(a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])
/(2*d*f) + (3*b^3*n^3*PolyLog[2, -(d*f*x^2)])/(8*d*f) - (3*b^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -(d*f*x^2)])/(4*d*f) + (3*b*n*(a + b*Log[c*x^n])^2*Po
lyLog[2, -(d*f*x^2)])/(4*d*f) + (3*b^3*n^3*PolyLog[3, -(d*f*x^2)])/(8*d*f)
- (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*x^2)])/(4*d*f) + (3*b^3*n^
3*PolyLog[4, -(d*f*x^2)])/(8*d*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
```

```
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx = -\frac{1}{2}x^2(a+b\log(cx^n))^3+\frac{(1+dfx^2)(a+b\log(cx^n))^3\log(1}{2df}$$

$$= -\frac{1}{2}x^2(a+b\log(cx^n))^3+\frac{(1+dfx^2)(a+b\log(cx^n))^3\log(1}{2df}$$

$$= \frac{3}{4}bnx^2(a+b\log(cx^n))^2-\frac{1}{2}x^2(a+b\log(cx^n))^3+\frac{(1+dfx^2)}{}$$

$$= \frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{4}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{8}b^3n^3x^2-\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{4}b^3n^3x^2-\frac{3}{2}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{4}b^3n^3x^2-\frac{3}{2}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{4}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{9}{8}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

$$= \frac{3}{2}b^3n^3x^2-\frac{9}{4}b^2n^2x^2(a+b\log(cx^n))+\frac{3}{2}bnx^2(a+b\log(cx^n))$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.40, size = 1004, normalized size = 2.44

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-(d*f*x^2*(4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2*n*(n*Log
[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Lo
g[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*b^3*n*(-(n*
Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)) + d*f*x^2*(4
*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 6*b*(2*a^2 - 2*a*b*n + b^2*n^2
)*Log[c*x^n] - 6*b^2*(-2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*Log[1
+ d*f*x^2] + (4*a^3 - 6*a^2*b*n + 6*a*b^2*n^2 - 3*b^3*n^3 + 12*a*b^2*n*(n*L
og[x] - Log[c*x^n]) + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*
Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 6*b^3*n*(-(
n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)*Log[1 + d*f
*x^2] + 6*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 + 2*b^2*n*(n*Log[x] - Log[c*x^n])
+ 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)*((
d*f*x^2)/2 - d*f*x^2*Log[x] + Log[x]*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]*
Log[1 + I*Sqrt[d]*Sqrt[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyLog
[2, I*Sqrt[d]*Sqrt[f]*x]) + 3*b^2*n^2*(-2*a + b*n + 2*b*n*Log[x] - 2*b*Log[
c*x^n])*(d*f*x^2 - 2*d*f*x^2*Log[x] + 2*d*f*x^2*Log[x]^2 - 2*Log[x]^2*Log[1
 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x
]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f
]*x] + 4*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 4*PolyLog[3, I*Sqrt[d]*Sqrt[f
]*x]) - b^3*n^3*(-3*d*f*x^2 + 6*d*f*x^2*Log[x] - 6*d*f*x^2*Log[x]^2 + 4*d*f
*x^2*Log[x]^3 - 4*Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - 4*Log[x]^3*Log[1
+ I*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 1
2*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[3, (-I)*Sqrt
[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[4,
(-I)*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x]))/(8*d*f)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^3\ln\left(d\left(\frac{1}{d}+f\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^3*x^2*log(x^n)^3 - 6*(b^3*(n - 2*log(c)) - 2*a*b^2)*x^2*log(x^n)^2
 + 6*((n^2 - 2*n*log(c) + 2*log(c)^2)*b^3 - 2*a*b^2*(n - 2*log(c)) + 2*a^2*
b)*x^2*log(x^n) + (6*(n^2 - 2*n*log(c) + 2*log(c)^2)*a*b^2 - (3*n^3 - 6*n^2
*log(c) + 6*n*log(c)^2 - 4*log(c)^3)*b^3 - 6*a^2*b*(n - 2*log(c)) + 4*a^3)*
x^2)*log(d*f*x^2 + 1) - integrate(1/4*(4*b^3*d*f*x^3*log(x^n)^3 + 6*(2*a*b^
2*d*f - (d*f*n - 2*d*f*log(c))*b^3)*x^3*log(x^n)^2 + 6*(2*a^2*b*d*f - 2*(d*
f*n - 2*d*f*log(c))*a*b^2 + (d*f*n^2 - 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^3
)*x^3*log(x^n) + (4*a^3*d*f - 6*(d*f*n - 2*d*f*log(c))*a^2*b + 6*(d*f*n^2 -
 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^2 - (3*d*f*n^3 - 6*d*f*n^2*log(c) + 6
*d*f*n*log(c)^2 - 4*d*f*log(c)^3)*b^3)*x^3)/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*x*log(d*f*x^2 + 1)*l
og(c*x^n)^2 + 3*a^2*b*x*log(d*f*x^2 + 1)*log(c*x^n) + a^3*x*log(d*f*x^2 + 1
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln \left(d \left(f x^2 + \frac{1}{d}\right)\right) (a + b \ln (c x^n))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
[Out] int(x*log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

3.42 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2}(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^2)+\frac{3}{4}bn(a+b\log(cx^n))^2\operatorname{Li}_3(-dfx^2)-\frac{3}{4}b^2n^2(a+b\log(cx^n))\operatorname{Li}_4(-dfx^2)+\frac{3}{8}b^3$$

```
[Out] -1/2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^2)+3/4*b*n*(a+b*ln(c*x^n))^2*polylo
g(3,-d*f*x^2)-3/4*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^2)+3/8*b^3*n^3*p
olylog(5,-d*f*x^2)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2421, 2430, 6724}

$$-\frac{3}{4}b^2n^2\text{PolyLog}(4,-dfx^2)\,(a+b\log(cx^n))+\frac{3}{4}bn\text{PolyLog}(3,-dfx^2)\,(a+b\log(cx^n))^2-\frac{1}{2}\text{PolyLog}(2,-dfx^2)\,(a+b\log(cx^n))^3+\frac{3}{8}b^3n^3\text{PolyLog}(5,-dfx^2)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] -1/2*((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^2)]) + (3*b*n*(a + b*Log[c*x^
n])^2*PolyLog[3, -(d*f*x^2)])/4 - (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4,
-(d*f*x^2)])/4 + (3*b^3*n^3*PolyLog[5, -(d*f*x^2)])/8
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x}\,dx = -\frac{1}{2}(a+b\log(cx^n))^3 \operatorname{Li}_2(-dfx^2) + \frac{1}{2}(3bn)\int \frac{(a+b\log(cx^n))^2}{x}$$

$$= -\frac{1}{2}(a+b\log(cx^n))^3 \operatorname{Li}_2(-dfx^2) + \frac{3}{4}bn(a+b\log(cx^n))^2 \operatorname{Li}_3(-$$

$$= -\frac{1}{2}(a+b\log(cx^n))^3 \operatorname{Li}_2(-dfx^2) + \frac{3}{4}bn(a+b\log(cx^n))^2 \operatorname{Li}_3(-$$

$$= -\frac{1}{2}(a+b\log(cx^n))^3 \operatorname{Li}_2(-dfx^2) + \frac{3}{4}bn(a+b\log(cx^n))^2 \operatorname{Li}_3(-$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 754, normalized size = 7.47

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x,x]
```

```
[Out] (-(Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(a + b*Log[c*x^n]) + 6*b*n
*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n])^3)*Log[1 + d*f*x^2]) -
4*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f]*x] +
 Log[1 + I*Sqrt[d]*Sqrt[f]*x]) + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + PolyL
og[2, I*Sqrt[d]*Sqrt[f]*x]) - 6*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Log[
x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] +
 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d
]*Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 2*PolyLog[3, I*Sqrt[d
]*Sqrt[f]*x]) + 4*b^2*n^2*(-a + b*n*Log[x] - b*Log[c*x^n])*(Log[x]^3*Log[1
- I*Sqrt[d]*Sqrt[f]*x] + Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2
*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[
f]*x] - 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, I
*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I
*Sqrt[d]*Sqrt[f]*x]) - b^3*n^3*(Log[x]^4*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + Log
[x]^4*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 4*Log[x]^3*PolyLog[2, (-I)*Sqrt[d]*Sqr
t[f]*x] + 4*Log[x]^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[
3, (-I)*Sqrt[d]*Sqrt[f]*x] - 12*Log[x]^2*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] +
24*Log[x]*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] + 24*Log[x]*PolyLog[4, I*Sqrt[
d]*Sqrt[f]*x] - 24*PolyLog[5, (-I)*Sqrt[d]*Sqrt[f]*x] - 24*PolyLog[5, I*Sqr
t[d]*Sqrt[f]*x]))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 23414, normalized size = 231.82

method	result	size
risch	Expression too large to display	23414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^2 + 1) - integrate(-1/2*(b^3*d*f*n^3
*x*log(x)^4 - 4*b^3*d*f*x*log(x)*log(x^n)^3 - 4*(b^3*d*f*n^2*log(c) + a*b^2
*d*f*n^2)*x*log(x)^3 + 6*(b^3*d*f*n*log(c)^2 + 2*a*b^2*d*f*n*log(c) + a^2*b
*d*f*n)*x*log(x)^2 - 4*(b^3*d*f*log(c)^3 + 3*a*b^2*d*f*log(c)^2 + 3*a^2*b*d
*f*log(c) + a^3*d*f)*x*log(x) + 6*(b^3*d*f*n*x*log(x)^2 - 2*(b^3*d*f*log(c)
 + a*b^2*d*f)*x*log(x))*log(x^n)^2 - 4*(b^3*d*f*n^2*x*log(x)^3 - 3*(b^3*d*f
*n*log(c) + a*b^2*d*f*n)*x*log(x)^2 + 3*(b^3*d*f*log(c)^2 + 2*a*b^2*d*f*log
(c) + a^2*b*d*f)*x*log(x))*log(x^n))/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x, x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

3.43 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=425

$$\frac{3}{4}b^3dfn^3\log(x)-\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{3}{4}bdfn\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))^2-\frac{1}{2}df\log$$

```
[Out] 3/4*b^3*d*f*n^3*ln(x)-3/4*b^2*d*f*n^2*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))-3/4*b
*d*f*n*ln(1+1/d/f/x^2)*(a+b*ln(c*x^n))^2-1/2*d*f*ln(1+1/d/f/x^2)*(a+b*ln(c*
x^n))^3-3/8*b^3*d*f*n^3*ln(d*f*x^2+1)-3/8*b^3*n^3*ln(d*f*x^2+1)/x^2-3/4*b^2
*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*f*x^2
+1)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)/x^2+3/8*b^3*d*f*n^3*polylog(2,-
1/d/f/x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(2,-1/d/f/x^2)+3/4*b*d*f*
n*(a+b*ln(c*x^n))^2*polylog(2,-1/d/f/x^2)+3/8*b^3*d*f*n^3*polylog(3,-1/d/f/
x^2)+3/4*b^2*d*f*n^2*(a+b*ln(c*x^n))*polylog(3,-1/d/f/x^2)+3/8*b^3*d*f*n^3*
polylog(4,-1/d/f/x^2)
```

Rubi [A]

time = 0.38, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2342, 2341, 2425, 272, 36, 29, 31, 2379, 2438, 2421, 6724, 2430}

$$\frac{3}{4}b^2dfn^2\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)(a+b\log(cx^n))+\frac{3}{4}b^2dfn^2\text{PolyLog}\left(3,-\frac{1}{dfx^2}\right)(a+b\log(cx^n))+\frac{3}{4}bdfn\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)(a+b\log(cx^n))^2+\frac{3}{8}b^3dfn^3\text{PolyLog}\left(2,-\frac{1}{dfx^2}\right)+\frac{3}{8}b^3dfn^3\text{PolyLog}\left(3,-\frac{1}{dfx^2}\right)+\frac{3}{8}b^3dfn^3\text{PolyLog}\left(4,-\frac{1}{dfx^2}\right)-\frac{3}{4}b^2dfn^2\log\left(\frac{1}{dfx^2}+1\right)(a+b\log(cx^n))-\frac{3b^2n^2\log(dfx^2+1)(a+b\log(cx^n))}{4x^2}-\frac{3}{4}bdfn\log\left(\frac{1}{dfx^2}+1\right)(a+b\log(cx^n))^2-\frac{3bn\log(dfx^2+1)(a+b\log(cx^n))^2}{4x^2}-\frac{1}{2}df\log\left(\frac{1}{dfx^2}+1\right)(a+b\log(cx^n))^3-\frac{\log(dfx^2+1)(a+b\log(cx^n))^3}{2x^2}-\frac{3}{8}b^3dfn^3\log(dfx^2+1)-\frac{3b^3n^3\log(dfx^2+1)}{8x^2}+\frac{3}{4}b^3dfn^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (3*b^3*d*f*n^3*Log[x])/4 - (3*b^2*d*f*n^2*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c
*x^n]))/4 - (3*b*d*f*n*Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^2)/4 - (d*f*
Log[1 + 1/(d*f*x^2)]*(a + b*Log[c*x^n])^3)/2 - (3*b^3*d*f*n^3*Log[1 + d*f*x
^2])/8 - (3*b^3*n^3*Log[1 + d*f*x^2])/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n
])*Log[1 + d*f*x^2])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2]
)/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/(2*x^2) + (3*b^3*d*f*n^
3*PolyLog[2, -(1/(d*f*x^2))])/8 + (3*b^2*d*f*n^2*(a + b*Log[c*x^n])*PolyLog
[2, -(1/(d*f*x^2))])/4 + (3*b*d*f*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(1/(d*
f*x^2))])/4 + (3*b^3*d*f*n^3*PolyLog[3, -(1/(d*f*x^2))])/8 + (3*b^2*d*f*n^2
*(a + b*Log[c*x^n])*PolyLog[3, -(1/(d*f*x^2))])/4 + (3*b^3*d*f*n^3*PolyLog[
4, -(1/(d*f*x^2))])/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
```

```
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^3}\,dx = -\frac{3b^3n^3\log(1+dfx^2)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{4x^2}-$$

$$= -\frac{3b^3n^3\log(1+dfx^2)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{4x^2}-$$

$$= -\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{3}{4}bdfn\log\left(1+\frac{1}{df}\right.$$

$$= -\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{3}{4}bdfn\log\left(1+\frac{1}{df}\right.$$

$$= \frac{3}{4}b^3dfn^3\log(x)-\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{3}{4}b$$

$$= \frac{3}{4}b^3dfn^3\log(x)-\frac{3}{4}b^2dfn^2\log\left(1+\frac{1}{dfx^2}\right)(a+b\log(cx^n))-\frac{3}{4}b$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 940, normalized size = 2.21

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^3,x]
```

```
[Out] (2*d*f*Log[x]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*
Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(
-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n
*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3) - ((4*a
^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*n^2)*
Log[c*x^n] + 6*b^2*(2*a + b*n)*Log[c*x^n]^2 + 4*b^3*Log[c*x^n]^3)*Log[1 + d
*f*x^2])/x^2 - d*f*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*
(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*
n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*
b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Log[x]) + Log[c*x^n])^3)*Lo
g[1 + d*f*x^2] + 6*b*d*f*n*(2*a^2 + 2*a*b*n + b^2*n^2 + 4*a*b*(-(n*Log[x])
+ Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*(-(n*Log[x]) + L
og[c*x^n])^2)*(Log[x]*(Log[x] - Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[1 + I*Sq
rt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - PolyLog[2, I*Sqrt[
d]*Sqrt[f]*x]) + 12*b^2*d*f*n^2*(2*a + b*n - 2*b*n*Log[x] + 2*b*Log[c*x^n])
*(Log[x]^3/3 - (Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (Log[x]^2*Log[1
+ I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - Log
[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] +
PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + 2*b^3*d*f*n^3*(Log[x]^4 - 2*Log[x]^3*Log
[1 - I*Sqrt[d]*Sqrt[f]*x] - 2*Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x] - 6*Log
[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]^2*PolyLog[2, I*Sqrt[d]*
Sqrt[f]*x] + 12*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] + 12*Log[x]*PolyL
og[3, I*Sqrt[d]*Sqrt[f]*x] - 12*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x] - 12*Pol
yLog[4, I*Sqrt[d]*Sqrt[f]*x]))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 13973, normalized size = 32.88

method	result	size
risch	Expression too large to display	13973

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*log(x^n)^3 + 6*(n^2 + 2*n*log(c) + 2*log(c)^2)*a*b^2 + (3*n^3 +
 6*n^2*log(c) + 6*n*log(c)^2 + 4*log(c)^3)*b^3 + 6*a^2*b*(n + 2*log(c)) + 4
*a^3 + 6*(b^3*(n + 2*log(c)) + 2*a*b^2)*log(x^n)^2 + 6*((n^2 + 2*n*log(c) +
 2*log(c)^2)*b^3 + 2*a*b^2*(n + 2*log(c)) + 2*a^2*b)*log(x^n))*log(d*f*x^2
+ 1)/x^2 + integrate(1/4*(4*b^3*d*f*log(x^n)^3 + 4*a^3*d*f + 6*(d*f*n + 2*d
*f*log(c))*a^2*b + 6*(d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*a*b^2 + (3
*d*f*n^3 + 6*d*f*n^2*log(c) + 6*d*f*n*log(c)^2 + 4*d*f*log(c)^3)*b^3 + 6*(2
*a*b^2*d*f + (d*f*n + 2*d*f*log(c))*b^3)*log(x^n)^2 + 6*(2*a^2*b*d*f + 2*(d
*f*n + 2*d*f*log(c))*a*b^2 + (d*f*n^2 + 2*d*f*n*log(c) + 2*d*f*log(c)^2)*b^
3)*log(x^n))/(d*f*x^3 + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^3,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

3.44 $\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\,dx$

Optimal. Leaf size=938

$$-24ab^2n^2x+36b^3n^3x-12b^2n^2(a-bn)x+\frac{12b^2n^2(a-bn)\tan^{-1}\left(\sqrt{d}\,\sqrt{f}\,x\right)}{\sqrt{d}\,\sqrt{f}}-36b^3n^2x\log(cx^n)+\frac{12b^3n^2\tan^{-1}}{}$$

```
[Out] -12*b^2*n^2*(-b*n+a)*x-24*a*b^2*n^2*x-6*b^3*n^3*x*ln(d*f*x^2+1)-(a+b*ln(c*x
^n))^3*ln(1-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+(a+b*ln(c*x^n))^3*ln(1
+x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-36*b^3*n^2*x*ln(c*x^n)+12*b*n*x*(
a+b*ln(c*x^n))^2+x*(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)+6*a*b^2*n^2*x*ln(d*f*x^2
+1)+6*b^3*n^2*x*ln(c*x^n)*ln(d*f*x^2+1)+36*b^3*n^3*x-2*x*(a+b*ln(c*x^n))^3-
3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)+6*b^3*n^3*polylog(3,-x*(-d)^(1/2)*f
^(1/2))/(-d)^(1/2)/f^(1/2)-6*b^3*n^3*polylog(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(
1/2)/f^(1/2)+6*b^3*n^3*polylog(4,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-
6*b^3*n^3*polylog(4,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+6*I*b^3*n^3*po
lylog(2,I*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+3*b*n*(a+b*ln(c*x^n))^2*ln(1-x
*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2*ln(1+x*(-d)
^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-x*(
-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-x*
(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,
x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2*polylog(2,
x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(
3,-x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polyl
og(3,x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+12*b^2*n^2*(-b*n+a)*arctan(x*
d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+12*b^3*n^2*arctan(x*d^(1/2)*f^(1/2))*ln(c*
x^n)/d^(1/2)/f^(1/2)-6*I*b^3*n^3*polylog(2,-I*x*d^(1/2)*f^(1/2))/d^(1/2)/f^
(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {2333, 2332, 2418, 6, 327, 209, 2393, 2361, 12, 4940, 2438, 2395, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] -24*a*b^2*n^2*x + 36*b^3*n^3*x - 12*b^2*n^2*(a - b*n)*x + (12*b^2*n^2*(a -
b*n)*ArcTan[Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) - 36*b^3*n^2*x*Log[c*x^n]
 + (12*b^3*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*Log[c*x^n])/(Sqrt[d]*Sqrt[f]) + 12
*b*n*x*(a + b*Log[c*x^n])^2 - 2*x*(a + b*Log[c*x^n])^3 + (3*b*n*(a + b*Log[
```

```
c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - ((a + b*Log[c*x
^n])^3*Log[1 - Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - (3*b*n*(a + b*Log[
c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + ((a + b*Log[c*x
^n])^3*Log[1 + Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + 6*a*b^2*n^2*x*Log[
1 + d*f*x^2] - 6*b^3*n^3*x*Log[1 + d*f*x^2] + 6*b^3*n^2*x*Log[c*x^n]*Log[1
+ d*f*x^2] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[1 + d*f*x^2] + x*(a + b*Log[c
*x^n])^3*Log[1 + d*f*x^2] - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt
[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[
2, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) + (6*b^2*n^2*(a + b*Log[c*x^n
])*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - (3*b*n*(a + b*Log[c
*x^n])^2*PolyLog[2, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) - ((6*I)*b^3*n^
3*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + ((6*I)*b^3*n^3*Po
lyLog[2, I*Sqrt[d]*Sqrt[f]*x])/(Sqrt[d]*Sqrt[f]) + (6*b^3*n^3*PolyLog[3, -(
Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^2*n^2*(a + b*Log[c*x^n])*Po
lyLog[3, -(Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^3*n^3*PolyLog[3,
 Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (6*b^2*n^2*(a + b*Log[c*x^n])*Po
lyLog[3, Sqrt[-d]*Sqrt[f]*x])/(Sqrt[-d]*Sqrt[f]) + (6*b^3*n^3*PolyLog[4, -(
Sqrt[-d]*Sqrt[f]*x)])/(Sqrt[-d]*Sqrt[f]) - (6*b^3*n^3*PolyLog[4, Sqrt[-d]*S
qrt[f]*x])/(Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^3 \log\left(d\left(\frac{1}{d}+fx^2\right)\right)\, dx = 6ab^2n^2x\log\left(1+dfx^2\right) - 6b^3n^3x\log\left(1+dfx^2\right) + 6b^3n^2x\log(\ldots$$

$$= 6ab^2n^2x\log\left(1+dfx^2\right) - 6b^3n^3x\log\left(1+dfx^2\right) + 6b^3n^2x\log(\ldots$$

$$= 6ab^2n^2x\log\left(1+dfx^2\right) - 6b^3n^3x\log\left(1+dfx^2\right) + 6b^3n^2x\log(\ldots$$

$$= -12b^2n^2(a-bn)x + 6ab^2n^2x\log\left(1+dfx^2\right) - 6b^3n^3x\log(1+\ldots$$

$$= -12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right)}{\sqrt{d}\sqrt{f}} + 6a\ldots$$

$$= 12b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\tan^{-1}\left(\sqrt{d}\sqrt{f}\ldots\right.}{\sqrt{d}\sqrt{f}}\ldots$$

$$= -12ab^2n^2x + 12b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\,\mathrm{ta}\ldots}{\sqrt{d}\ldots}$$

$$= -24ab^2n^2x + 24b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\,\mathrm{ta}\ldots}{\sqrt{d}\ldots}$$

$$= -24ab^2n^2x + 36b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\,\mathrm{ta}\ldots}{\sqrt{d}\ldots}$$

$$= -24ab^2n^2x + 36b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\,\mathrm{ta}\ldots}{\sqrt{d}\ldots}$$

$$= -24ab^2n^2x + 36b^3n^3x - 12b^2n^2(a-bn)x + \frac{12b^2n^2(a-bn)\,\mathrm{ta}\ldots}{\sqrt{d}\ldots}$$

Mathematica [A]
time = 0.45, size = 1027, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)],x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*b^2*
n*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*
(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^3*n
*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 2*ArcTa
n[Sqrt[d]*Sqrt[f]*x]*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 6*a*b^2*n
*(n*Log[x] - Log[c*x^n]) + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(
-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 - 3*b^3*n*
(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) + Sqrt[d]*
Sqrt[f]*x*(a^3 - 3*a^2*b*n + 6*a*b^2*n^2 - 6*b^3*n^3 + 3*b*(a^2 - 2*a*b*n +
 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a - b*n)*Log[c*x^n]^2 + b^3*Log[c*x^n]^3)*L
og[1 + d*f*x^2] + 3*b*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b^2*n*(n*Log[x] - Lo
g[c*x^n]) + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n
])^2)*(-2*Sqrt[d]*Sqrt[f]*x*(-1 + Log[x]) - I*(Log[x]*Log[1 + I*Sqrt[d]*Sqr
t[f]*x] + PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]) + I*(Log[x]*Log[1 - I*Sqrt[d]
*Sqrt[f]*x] + PolyLog[2, I*Sqrt[d]*Sqrt[f]*x])) - 6*b^2*n^2*(a - b*n - b*n*
Log[x] + b*Log[c*x^n])*(Sqrt[d]*Sqrt[f]*x*(2 - 2*Log[x] + Log[x]^2) + (I/2)
*(Log[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, (-I)*Sqrt[d]*
Sqrt[f]*x] - 2*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x]) - (I/2)*(Log[x]^2*Log[1
- I*Sqrt[d]*Sqrt[f]*x] + 2*Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] - 2*PolyL
og[3, I*Sqrt[d]*Sqrt[f]*x])) + 2*b^3*n^3*(-(Sqrt[d]*Sqrt[f]*x*(-6 + 6*Log[x
] - 3*Log[x]^2 + Log[x]^3)) - (I/2)*(Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]*x]
+ 3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log[x]*PolyLog[3, (-I)*
Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]) + (I/2)*(Log[x]^
3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]
 - 6*Log[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] + 6*PolyLog[4, I*Sqrt[d]*Sqrt[f
]*x])))/(Sqrt[d]*Sqrt[f])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\,x^{n}\right)\right)^{3} \ln \left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="maxima")
```

```
[Out] (b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2 -
 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) + (3
*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log(c)
^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*x^2 + 1) - inte
grate(2*(b^3*d*f*x^2*log(x^n)^3 + 3*(a*b^2*d*f - (d*f*n - d*f*log(c))*b^3)*
x^2*log(x^n)^2 + 3*(a^2*b*d*f - 2*(d*f*n - d*f*log(c))*a*b^2 + (2*d*f*n^2 -
 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*x^2*log(x^n) + (a^3*d*f - 3*(d*f*n - d
*f*log(c))*a^2*b + 3*(2*d*f*n^2 - 2*d*f*n*log(c) + d*f*log(c)^2)*a*b^2 - (6
*d*f*n^3 - 6*d*f*n^2*log(c) + 3*d*f*n*log(c)^2 - d*f*log(c)^3)*b^3)*x^2)/(d
*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="fricas")
```

```
[Out] integral(b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(c
*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3, x)
```

3.45 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=849

$12b^3\sqrt{d}\ \sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)+12b^2\sqrt{d}\ \sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\ \sqrt{f}\ x\right)(a+b\log(cx^n))+3b\sqrt{-d}\ \sqrt{f}\,n($

```
[Out] -6*b^3*n^3*ln(d*f*x^2+1)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*f*x^2+1)/x-3*b*n*
(a+b*ln(c*x^n))^2*ln(d*f*x^2+1)/x-(a+b*ln(c*x^n))^3*ln(d*f*x^2+1)/x+3*b*n*(
a+b*ln(c*x^n))^2*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+(a+b*ln(c*x^
n))^3*ln(1-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2
*ln(1+x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-(a+b*ln(c*x^n))^3*ln(1+x*(-d
)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-x*
(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-3*b*n*(a+b*ln(c*x^n))^2*polylog(2,-x
*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(2
,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+3*b*n*(a+b*ln(c*x^n))^2*polylog(2
,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^3*n^3*polylog(3,-x*(-d)^(1/2)
*f^(1/2))*(-d)^(1/2)*f^(1/2)+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-x*(-d)^(1
/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)-6*b^3*n^3*polylog(3,x*(-d)^(1/2)*f^(1/2))*(
-d)^(1/2)*f^(1/2)-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,x*(-d)^(1/2)*f^(1/2))
*(-d)^(1/2)*f^(1/2)-6*b^3*n^3*polylog(4,-x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f
^(1/2)+6*b^3*n^3*polylog(4,x*(-d)^(1/2)*f^(1/2))*(-d)^(1/2)*f^(1/2)+12*b^3*
n^3*arctan(x*d^(1/2)*f^(1/2))*d^(1/2)*f^(1/2)+12*b^2*n^2*arctan(x*d^(1/2)*f
^(1/2))*(a+b*ln(c*x^n))*d^(1/2)*f^(1/2)+6*I*b^3*n^3*polylog(2,I*x*d^(1/2)*f
^(1/2))*d^(1/2)*f^(1/2)-6*I*b^3*n^3*polylog(2,-I*x*d^(1/2)*f^(1/2))*d^(1/2)
*f^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2342, 2341, 2425, 209, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 12*b^3*Sqrt[d]*Sqrt[f]*n^3*ArcTan[Sqrt[d]*Sqrt[f]*x] + 12*b^2*Sqrt[d]*Sqrt[
f]*n^2*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a + b*Log[c*x^n]) + 3*b*Sqrt[-d]*Sqrt[f]*
n*(a + b*Log[c*x^n])^2*Log[1 - Sqrt[-d]*Sqrt[f]*x] + Sqrt[-d]*Sqrt[f]*(a +
b*Log[c*x^n])^3*Log[1 - Sqrt[-d]*Sqrt[f]*x] - 3*b*Sqrt[-d]*Sqrt[f]*n*(a + b
*Log[c*x^n])^2*Log[1 + Sqrt[-d]*Sqrt[f]*x] - Sqrt[-d]*Sqrt[f]*(a + b*Log[c*
x^n])^3*Log[1 + Sqrt[-d]*Sqrt[f]*x] - (6*b^3*n^3*Log[1 + d*f*x^2])/x - (6*b
^2*n^2*(a + b*Log[c*x^n])*Log[1 + d*f*x^2])/x - (3*b*n*(a + b*Log[c*x^n])^2
```

```
*Log[1 + d*f*x^2])/x - ((a + b*Log[c*x^n])^3*Log[1 + d*f*x^2])/x - 6*b^2*Sq
rt[-d]*Sqrt[f]*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)] - 3
*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(Sqrt[-d]*Sqrt[f]*x)
] + 6*b^2*Sqrt[-d]*Sqrt[f]*n^2*(a + b*Log[c*x^n])*PolyLog[2, Sqrt[-d]*Sqrt[
f]*x] + 3*b*Sqrt[-d]*Sqrt[f]*n*(a + b*Log[c*x^n])^2*PolyLog[2, Sqrt[-d]*Sqr
t[f]*x] - (6*I)*b^3*Sqrt[d]*Sqrt[f]*n^3*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x]
+ (6*I)*b^3*Sqrt[d]*Sqrt[f]*n^3*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + 6*b^3*Sqr
t[-d]*Sqrt[f]*n^3*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] + 6*b^2*Sqrt[-d]*Sqrt[f
]*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(Sqrt[-d]*Sqrt[f]*x)] - 6*b^3*Sqrt[-d]
*Sqrt[f]*n^3*PolyLog[3, Sqrt[-d]*Sqrt[f]*x] - 6*b^2*Sqrt[-d]*Sqrt[f]*n^2*(a
 + b*Log[c*x^n])*PolyLog[3, Sqrt[-d]*Sqrt[f]*x] - 6*b^3*Sqrt[-d]*Sqrt[f]*n^
3*PolyLog[4, -(Sqrt[-d]*Sqrt[f]*x)] + 6*b^3*Sqrt[-d]*Sqrt[f]*n^3*PolyLog[4,
 Sqrt[-d]*Sqrt[f]*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^2\right)\right)}{x^2}\,dx = -\frac{6b^3n^3\log(1+dfx^2)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{x} -$$

$$= -\frac{6b^3n^3\log(1+dfx^2)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(1+dfx^2)}{x} -$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

$$= 12b^3\sqrt{d}\sqrt{f}\,n^3\tan^{-1}\left(\sqrt{d}\sqrt{f}\,x\right) + 12b^2\sqrt{d}\sqrt{f}\,n^2\tan^{-1}\left(\sqrt{d}\right.$$

Mathematica [A]
time = 0.21, size = 794, normalized size = 0.94

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^2)])/x^2,x]
```

```
[Out] 2*Sqrt[d]*Sqrt[f]*ArcTan[Sqrt[d]*Sqrt[f]*x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2
+ 6*b^3*n^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) +
 Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x])
+ Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) +
 Log[c*x^n])^3) - ((a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*b*(a^2 +
```

```
2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2 + b^3*Log[c*
x^n]^3)*Log[1 + d*f*x^2])/x + (3*I)*b*Sqrt[d]*Sqrt[f]*n*(a^2 + 2*a*b*n + 2*
b^2*n^2 + 2*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x
^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(Log[x]*(Log[1 - I*Sqrt[d]*Sqrt[f]
*x] - Log[1 + I*Sqrt[d]*Sqrt[f]*x]) - PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] +
PolyLog[2, I*Sqrt[d]*Sqrt[f]*x]) + (6*I)*b^2*Sqrt[d]*Sqrt[f]*n^2*(a + b*n -
 b*n*Log[x] + b*Log[c*x^n])*((Log[x]^2*Log[1 - I*Sqrt[d]*Sqrt[f]*x])/2 - (L
og[x]^2*Log[1 + I*Sqrt[d]*Sqrt[f]*x])/2 - Log[x]*PolyLog[2, (-I)*Sqrt[d]*Sq
rt[f]*x] + Log[x]*PolyLog[2, I*Sqrt[d]*Sqrt[f]*x] + PolyLog[3, (-I)*Sqrt[d]
*Sqrt[f]*x] - PolyLog[3, I*Sqrt[d]*Sqrt[f]*x]) + I*b^3*Sqrt[d]*Sqrt[f]*n^3*
(Log[x]^3*Log[1 - I*Sqrt[d]*Sqrt[f]*x] - Log[x]^3*Log[1 + I*Sqrt[d]*Sqrt[f]
*x] - 3*Log[x]^2*PolyLog[2, (-I)*Sqrt[d]*Sqrt[f]*x] + 3*Log[x]^2*PolyLog[2,
 I*Sqrt[d]*Sqrt[f]*x] + 6*Log[x]*PolyLog[3, (-I)*Sqrt[d]*Sqrt[f]*x] - 6*Log
[x]*PolyLog[3, I*Sqrt[d]*Sqrt[f]*x] - 6*PolyLog[4, (-I)*Sqrt[d]*Sqrt[f]*x]
+ 6*PolyLog[4, I*Sqrt[d]*Sqrt[f]*x])
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(\frac{1}{d}+f\,x^{2}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^2))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] -(b^3*log(x^n)^3 + 3*(2*n^2 + 2*n*log(c) + log(c)^2)*a*b^2 + (6*n^3 + 6*n^2
*log(c) + 3*n*log(c)^2 + log(c)^3)*b^3 + 3*a^2*b*(n + log(c)) + a^3 + 3*(b^
3*(n + log(c)) + a*b^2)*log(x^n)^2 + 3*((2*n^2 + 2*n*log(c) + log(c)^2)*b^3
 + 2*a*b^2*(n + log(c)) + a^2*b)*log(x^n))*log(d*f*x^2 + 1)/x + integrate(2
*(b^3*d*f*log(x^n)^3 + a^3*d*f + 3*(d*f*n + d*f*log(c))*a^2*b + 3*(2*d*f*n^
2 + 2*d*f*n*log(c) + d*f*log(c)^2)*a*b^2 + (6*d*f*n^3 + 6*d*f*n^2*log(c) +
3*d*f*n*log(c)^2 + d*f*log(c)^3)*b^3 + 3*(a*b^2*d*f + (d*f*n + d*f*log(c))*
b^3)*log(x^n)^2 + 3*(a^2*b*d*f + 2*(d*f*n + d*f*log(c))*a*b^2 + (2*d*f*n^2
+ 2*d*f*n*log(c) + d*f*log(c)^2)*b^3)*log(x^n))/(d*f*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="fricas")
[Out] integral((b^3*log(d*f*x^2 + 1)*log(c*x^n)^3 + 3*a*b^2*log(d*f*x^2 + 1)*log(
c*x^n)^2 + 3*a^2*b*log(d*f*x^2 + 1)*log(c*x^n) + a^3*log(d*f*x^2 + 1))/x^2,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**2))/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^2))/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + 1/d)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
[Out] int((log(d*(f*x^2 + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

3.46 $\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=350

$$-\frac{7bn\sqrt{x}}{9d^5f^5}+\frac{2bnx}{9d^4f^4}-\frac{bnx^{3/2}}{9d^3f^3}+\frac{5bnx^2}{72d^2f^2}-\frac{11bnx^{5/2}}{225df}+\frac{1}{27}bnx^3+\frac{bn\log\left(1+df\sqrt{x}\right)}{9d^6f^6}-\frac{1}{9}bnx^3\log\left(1+df\sqrt{x}\right)+\sqrt{}$$

```
[Out] 2/9*b*n*x/d^4/f^4-1/9*b*n*x^(3/2)/d^3/f^3+5/72*b*n*x^2/d^2/f^2-11/225*b*n*x
^(5/2)/d/f+1/27*b*n*x^3-1/6*x*(a+b*ln(c*x^n))/d^4/f^4+1/9*x^(3/2)*(a+b*ln(c
*x^n))/d^3/f^3-1/12*x^2*(a+b*ln(c*x^n))/d^2/f^2+1/15*x^(5/2)*(a+b*ln(c*x^n)
)/d/f-1/18*x^3*(a+b*ln(c*x^n))+1/9*b*n*ln(1+d*f*x^(1/2))/d^6/f^6-1/9*b*n*x^
3*ln(1+d*f*x^(1/2))-1/3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^6/f^6+1/3*x^3*(
a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-2/3*b*n*polylog(2,-d*f*x^(1/2))/d^6/f^6-7/
9*b*n*x^(1/2)/d^5/f^5+1/3*(a+b*ln(c*x^n))*x^(1/2)/d^5/f^5
```

Rubi [A]
time = 0.19, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2504, 2442, 45, 2423, 2438}

$$-\frac{2bn\text{PolyLog}\left(2,-df\sqrt{x}\right)}{3d^6f^6}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{3d^6f^6}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{3d^5f^5}-\frac{x\left(a+b\log\left(cx^n\right)\right)}{6d^4f^4}+\frac{x^{3/2}\left(a+b\log\left(cx^n\right)\right)}{9d^3f^3}-\frac{x^2\left(a+b\log\left(cx^n\right)\right)}{12d^2f^2}+\frac{x^{5/2}\left(a+b\log\left(cx^n\right)\right)}{15df}+\frac{1}{3}x^3\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{18}x^3\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log\left(df\sqrt{x}+1\right)}{9d^6f^6}-\frac{7bn\sqrt{x}}{9d^5f^5}+\frac{2bnx}{9d^4f^4}-\frac{bnx^{3/2}}{9d^3f^3}+\frac{5bnx^2}{72d^2f^2}-\frac{11bnx^{5/2}}{225df}-\frac{1}{9}bnx^3\log\left(df\sqrt{x}+1\right)+\frac{1}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-7*b*n*Sqrt[x])/(9*d^5*f^5) + (2*b*n*x)/(9*d^4*f^4) - (b*n*x^(3/2))/(9*d^3
*f^3) + (5*b*n*x^2)/(72*d^2*f^2) - (11*b*n*x^(5/2))/(225*d*f) + (b*n*x^3)/2
7 + (b*n*Log[1 + d*f*Sqrt[x]])/(9*d^6*f^6) - (b*n*x^3*Log[1 + d*f*Sqrt[x]])
/9 + (Sqrt[x]*(a + b*Log[c*x^n]))/(3*d^5*f^5) - (x*(a + b*Log[c*x^n]))/(6*d
^4*f^4) + (x^(3/2)*(a + b*Log[c*x^n]))/(9*d^3*f^3) - (x^2*(a + b*Log[c*x^n]
))/(12*d^2*f^2) + (x^(5/2)*(a + b*Log[c*x^n]))/(15*d*f) - (x^3*(a + b*Log[c
*x^n]))/18 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*d^6*f^6) + (x^3*L
og[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 - (2*b*n*PolyLog[2, -(d*f*Sqrt[x]
)])/(3*d^6*f^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))}{3d^5f^5} - \frac{x(a+b\log(cx^n))}{6d^4f^4} + \frac{x^{3/2}(a+b\log}{9d^3f^3}$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5} + \frac{bnx}{6d^4f^4} - \frac{2bnx^{3/2}}{27d^3f^3} + \frac{bnx^2}{24d^2f^2} - \frac{2bnx^{5/2}}{75df} + \frac{1}{54}b$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5} + \frac{bnx}{6d^4f^4} - \frac{2bnx^{3/2}}{27d^3f^3} + \frac{bnx^2}{24d^2f^2} - \frac{2bnx^{5/2}}{75df} + \frac{1}{54}b$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5} + \frac{bnx}{6d^4f^4} - \frac{2bnx^{3/2}}{27d^3f^3} + \frac{bnx^2}{24d^2f^2} - \frac{2bnx^{5/2}}{75df} + \frac{1}{54}b$$

$$= -\frac{2bn\sqrt{x}}{3d^5f^5} + \frac{bnx}{6d^4f^4} - \frac{2bnx^{3/2}}{27d^3f^3} + \frac{bnx^2}{24d^2f^2} - \frac{2bnx^{5/2}}{75df} + \frac{1}{54}b$$

$$= -\frac{7bn\sqrt{x}}{9d^5f^5} + \frac{2bnx}{9d^4f^4} - \frac{bnx^{3/2}}{9d^3f^3} + \frac{5bnx^2}{72d^2f^2} - \frac{11bnx^{5/2}}{225df} + \frac{1}{27}b$$

Mathematica [A]
time = 0.22, size = 263, normalized size = 0.75

$$\frac{600(-1+d^6f^6x^3)\log\left(1+df\sqrt{x}\right)(3a-bn+3b\log(cx^n))+df\sqrt{x}\left(-30a\left(-60+30df\sqrt{x}-20d^2f^2x+15d^3f^3x^{3/2}-12d^4f^4x^2+10d^5f^5x^{5/2}\right)+bn\left(-4200+1200df\sqrt{x}-600d^2f^2x+375d^3f^3x^{3/2}-264d^4f^4x^2+200d^5f^5x^{5/2}\right)-30b\left(-60+30df\sqrt{x}-20d^2f^2x+15d^3f^3x^{3/2}-12d^4f^4x^2+10d^5f^5x^{5/2}\right)\log(cx^n)\right)-3600bn\text{Li}_2\left(-df\sqrt{x}\right)}{5400d^6f^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (600*(-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a - b*n + 3*b*Log[c*x^n]) +
 d*f*Sqrt[x]*(-30*a*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(3/
2) - 12*d^4*f^4*x^2 + 10*d^5*f^5*x^(5/2)) + b*n*(-4200 + 1200*d*f*Sqrt[x] -
 600*d^2*f^2*x + 375*d^3*f^3*x^(3/2) - 264*d^4*f^4*x^2 + 200*d^5*f^5*x^(5/2
)) - 30*b*(-60 + 30*d*f*Sqrt[x] - 20*d^2*f^2*x + 15*d^3*f^3*x^(3/2) - 12*d^
4*f^4*x^2 + 10*d^5*f^5*x^(5/2))*Log[c*x^n]) - 3600*b*n*PolyLog[2, -(d*f*Sqr
t[x])])/(5400*d^6*f^6)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

3.47 $\int x \log \left(d\left(\frac{1}{d}+f \sqrt{x}\right)\right) (a+b \log (c x^n)) \, dx$

Optimal. Leaf size=268

$$-\frac{5bn\sqrt{x}}{4d^3f^3}+\frac{3bnx}{8d^2f^2}-\frac{7bnx^{3/2}}{36df}+\frac{1}{8}bnx^2+\frac{bn\log\left(1+df\sqrt{x}\right)}{4d^4f^4}-\frac{1}{4}bnx^2\log\left(1+df\sqrt{x}\right)+\frac{\sqrt{x}\,(a+b\log(cx^n))}{2d^3f^3}-$$

```
[Out] 3/8*b*n*x/d^2/f^2-7/36*b*n*x^(3/2)/d/f+1/8*b*n*x^2-1/4*x*(a+b*ln(c*x^n))/d^
2/f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))/d/f-1/8*x^2*(a+b*ln(c*x^n))+1/4*b*n*ln(1+
d*f*x^(1/2))/d^4/f^4-1/4*b*n*x^2*ln(1+d*f*x^(1/2))-1/2*(a+b*ln(c*x^n))*ln(1
+d*f*x^(1/2))/d^4/f^4+1/2*x^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-b*n*polylog
(2,-d*f*x^(1/2))/d^4/f^4-5/4*b*n*x^(1/2)/d^3/f^3+1/2*(a+b*ln(c*x^n))*x^(1/2
)/d^3/f^3
```

Rubi [A]
time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2504, 2442, 45, 2423, 2438}

$$-\frac{bn\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^4f^4}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{2d^4f^4}+\frac{\sqrt{x}\,(a+b\log(cx^n))}{2d^3f^3}-\frac{x(a+b\log(cx^n))}{4d^2f^2}+\frac{x^{3/2}(a+b\log(cx^n))}{6df}+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log\left(df\sqrt{x}+1\right)}{4d^4f^4}-\frac{5bn\sqrt{x}}{4d^3f^3}+\frac{3bnx}{8d^2f^2}-\frac{7bnx^{3/2}}{36df}-\frac{1}{4}bnx^2\log\left(df\sqrt{x}+1\right)+\frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-5*b*n*Sqrt[x])/(4*d^3*f^3) + (3*b*n*x)/(8*d^2*f^2) - (7*b*n*x^(3/2))/(36*
d*f) + (b*n*x^2)/8 + (b*n*Log[1 + d*f*Sqrt[x]])/(4*d^4*f^4) - (b*n*x^2*Log[
1 + d*f*Sqrt[x]])/4 + (Sqrt[x]*(a + b*Log[c*x^n]))/(2*d^3*f^3) - (x*(a + b*
Log[c*x^n]))/(4*d^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n]))/(6*d*f) - (x^2*(a +
 b*Log[c*x^n]))/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*d^4*f^4) +
 (x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/2 - (b*n*PolyLog[2, -(d*f*Sq
rt[x])])/(d^4*f^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx=\frac{\sqrt{x}\,(a+b\log(cx^n))}{2d^3f^3}-\frac{x(a+b\log(cx^n))}{4d^2f^2}+\frac{x^{3/2}(a+b\log}{6df}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2+\frac{\sqrt{x}\,(a+b\log(cx}{2d^3f^3}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2+\frac{\sqrt{x}\,(a+b\log(cx}{2d^3f^3}$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2-\frac{1}{4}bnx^2\log\left(1+df\right.$$

$$=-\frac{bn\sqrt{x}}{d^3f^3}+\frac{bnx}{4d^2f^2}-\frac{bnx^{3/2}}{9df}+\frac{1}{16}bnx^2-\frac{1}{4}bnx^2\log\left(1+df\right.$$

$$=-\frac{5bn\sqrt{x}}{4d^3f^3}+\frac{3bnx}{8d^2f^2}-\frac{7bnx^{3/2}}{36df}+\frac{1}{8}bnx^2+\frac{bn\log\left(1+df\sqrt{\right.}}{4d^4f^4}$$

Mathematica [A]
time = 0.16, size = 191, normalized size = 0.71

$$\frac{18(-1+d^4f^4x^2)\log\left(1+df\sqrt{x}\right)(2a-bn+2b\log(cx^n))+df\sqrt{x}\left(-3a\left(-12+6df\sqrt{x}-4d^2f^2x+3d^3f^3x^{3/2}\right)+bn\left(-90+27df\sqrt{x}-14d^2f^2x+9d^3f^3x^{3/2}\right)-3b\left(-12+6df\sqrt{x}-4d^2f^2x+3d^3f^3x^{3/2}\right)\log(cx^n)\right)-72bn\mathrm{Li}_2\left(-df\sqrt{x}\right)}{72d^4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (18*(-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a - b*n + 2*b*Log[c*x^n]) +
d*f*Sqrt[x]*(-3*a*(-12 + 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2)) +
 b*n*(-90 + 27*d*f*Sqrt[x] - 14*d^2*f^2*x + 9*d^3*f^3*x^(3/2)) - 3*b*(-12 +
 6*d*f*Sqrt[x] - 4*d^2*f^2*x + 3*d^3*f^3*x^(3/2))*Log[c*x^n]) - 72*b*n*Poly
Log[2, -(d*f*Sqrt[x])])/(72*d^4*f^4)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log(d*f*sqrt(x) + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

3.48 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=172

$$-\frac{3bn\sqrt{x}}{df}+bnx-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{bn\log\left(1+df\sqrt{x}\right)}{d^2f^2}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{1}{2}x(a+b\log\left(cx^n\right.$$

```
[Out] b*n*x-1/2*x*(a+b*ln(c*x^n))-b*n*x*ln(d*(1/d+f*x^(1/2)))+x*(a+b*ln(c*x^n))*l
n(d*(1/d+f*x^(1/2)))+b*n*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*x^n))*ln(1+d*f
*x^(1/2))/d^2/f^2-2*b*n*polylog(2,-d*f*x^(1/2))/d^2/f^2-3*b*n*x^(1/2)/d/f+(
a+b*ln(c*x^n))*x^(1/2)/d/f
```

Rubi [A]

time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2498, 272, 45, 2417, 2438}

$$-\frac{2bn\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{d^2f^2}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{df}-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)+\frac{bn\log\left(df\sqrt{x}+1\right)}{d^2f^2}-\frac{3bn\sqrt{x}}{df}-bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+bnx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-3*b*n*Sqrt[x])/(d*f) + b*n*x - b*n*x*Log[d*(d^(-1) + f*Sqrt[x])] + (b*n*L
og[1 + d*f*Sqrt[x]])/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - (x*(a
 + b*Log[c*x^n]))/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]) - (L
og[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2) - (2*b*n*PolyLog[2, -(d*f
*Sqrt[x])])/(d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
```

```
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))}{df} - \frac{1}{2}x(a+b\log(cx^n)) + x\log\left(d\left(\frac{1}{d}+\right.\right.$$

$$= -\frac{2bn\sqrt{x}}{df} + \frac{bnx}{2} + \frac{\sqrt{x}\,(a+b\log(cx^n))}{df} - \frac{1}{2}x(a+b\log(cx$$

$$= -\frac{2bn\sqrt{x}}{df} + \frac{bnx}{2} - bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + \frac{\sqrt{x}\,(a+b}{df}$$

$$= -\frac{2bn\sqrt{x}}{df} + \frac{bnx}{2} - bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + \frac{\sqrt{x}\,(a+b}{df}$$

$$= -\frac{2bn\sqrt{x}}{df} + \frac{bnx}{2} - bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + \frac{\sqrt{x}\,(a+b}{df}$$

$$= -\frac{3bn\sqrt{x}}{df} + bnx - bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) + \frac{bn\log(1+}{d^2f^2}$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 0.68

$$-\frac{-2(-1+d^2f^2x)\log\left(1+df\sqrt{x}\right)(a-bn+b\log(cx^n))+df\sqrt{x}\left(-2a+6bn+adf\sqrt{x}-2bdfn\sqrt{x}+b\left(-2+df\sqrt{x}\right)\log(cx^n)\right)+4bn\mathrm{Li}_2\left(-df\sqrt{x}\right)}{2d^2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/2*(-2*(-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a - b*n + b*Log[c*x^n]) + d
*f*Sqrt[x]*(-2*a + 6*b*n + a*d*f*Sqrt[x] - 2*b*d*f*n*Sqrt[x] + b*(-2 + d*f*
Sqrt[x])*Log[c*x^n]) + 4*b*n*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log(d*f*sqrt(x) + 1) - 1/9*(3*b*d*f
*x^2*log(x^n) + (3*a*d*f - (5*d*f*n - 3*d*f*log(c))*b)*x^2)/sqrt(x) + integ
rate(1/2*(b*d^2*f^2*x*log(x^n) + (a*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*
b)*x)/(d*f*sqrt(x) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)),x)
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)), x)
```

3.49 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$-2(a+b\log(cx^n))\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+4bn\mathrm{Li}_3\left(-df\sqrt{x}\right)$$

```
[Out] -2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+4*b*n*polylog(3,-d*f*x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2421, 6724}

$$4bn\text{PolyLog}\left(3,-df\sqrt{x}\right)-2\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*PolyLog[3, -(d*f*S
qrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}\,dx&=-2(a+b\log(cx^n))\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+(2bn)\int \frac{\mathrm{Li}_2\left(-df\sqrt{x}\right)}{x}\,dx\\&=-2(a+b\log(cx^n))\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+4bn\mathrm{Li}_3\left(-df\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.28

$$-2a\mathrm{Li}_2\left(-df\sqrt{x}\right)-2b\log(cx^n)\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+4bn\mathrm{Li}_3\left(-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] -2*a*PolyLog[2, -(d*f*Sqrt[x])] - 2*b*Log[c*x^n]*PolyLog[2, -(d*f*Sqrt[x])]
 + 4*b*n*PolyLog[3, -(d*f*Sqrt[x])]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a + b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d} + f\sqrt{x}\,\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x, x)
```

3.50 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=196

$$-\frac{3bdfn}{\sqrt{x}}+bd^2f^2n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{x}-\frac{1}{2}bd^2f^2n\log(x)+\frac{1}{4}bd^2f^2n\log^2(x)-\frac{df\left(a+b\log\left(cx^n\right.\right.}{\sqrt{x}}$$

```
[Out] -1/2*b*d^2*f^2*n*ln(x)+1/4*b*d^2*f^2*n*ln(x)^2-1/2*d^2*f^2*ln(x)*(a+b*ln(c*
x^n))+b*d^2*f^2*n*ln(1+d*f*x^(1/2))-b*n*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln
(c*x^n))*ln(1+d*f*x^(1/2))-(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x+2*b*d^2*f^2*
n*polylog(2,-d*f*x^(1/2))-3*b*d*f*n/x^(1/2)-d*f*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2442, 46, 2423, 2438, 2338}

$$2bd^2f^2n\text{PolyLog}\left(2,-df\sqrt{x}\right)+d^2f^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}d^2f^2\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{x}+\frac{1}{4}bd^2f^2n\log^2(x)+bd^2f^2n\log\left(df\sqrt{x}+1\right)-\frac{1}{2}bd^2f^2n\log(x)-\frac{3bdfn}{\sqrt{x}}-\frac{bn\log\left(df\sqrt{x}+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (-3*b*d*f*n)/Sqrt[x] + b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]] - (b*n*Log[1 + d*f*
Sqrt[x]])/x - (b*d^2*f^2*n*Log[x])/2 + (b*d^2*f^2*n*Log[x]^2)/4 - (d*f*(a +
 b*Log[c*x^n]))/Sqrt[x] + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) -
 (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/x - (d^2*f^2*Log[x]*(a + b*Log[c
*x^n]))/2 + 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
```

```
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}+d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)$$

$$= -\frac{2bdfn}{\sqrt{x}}-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}+d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\,\mathrm{lo}\right.$$

$$= -\frac{2bdfn}{\sqrt{x}}+\frac{1}{4}bd^2f^2n\log^2(x)-\frac{df\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}}+d^2f^2\log\left(1\right.$$

$$= -\frac{2bdfn}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{x}+\frac{1}{4}bd^2f^2n\log^2(x)-\frac{df(a+b}{\sqrt{}}$$

$$= -\frac{2bdfn}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{x}+\frac{1}{4}bd^2f^2n\log^2(x)-\frac{df(a+b}{\sqrt{}}$$

$$= -\frac{3bdfn}{\sqrt{x}}+bd^2f^2n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{x}-\frac{1}{2}$$

Mathematica [A]

time = 0.14, size = 124, normalized size = 0.63

$$\frac{1}{4}bd^2f^2n\log^2(x)+\frac{(-1+d^2f^2x)\log\left(1+df\sqrt{x}\right)(a+bn+b\log(cx^n))}{x}-\frac{1}{2}d^2f^2\log(x)(a+bn+b\log(cx^n))-\frac{df(a+3bn+b\log(cx^n))}{\sqrt{x}}+2bd^2f^2n\mathrm{Li}_2\left(-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (b*d^2*f^2*n*Log[x]^2)/4 + ((-1 + d^2*f^2*x)*Log[1 + d*f*Sqrt[x]]*(a + b*n
+ b*Log[c*x^n]))/x - (d^2*f^2*Log[x]*(a + b*n + b*Log[c*x^n]))/2 - (d*f*(a
+ 3*b*n + b*Log[c*x^n]))/Sqrt[x] + 2*b*d^2*f^2*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^2, x)
```

3.51 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=289

$$-\frac{7bdfn}{36x^{3/2}}+\frac{3bd^2f^2n}{8x}-\frac{5bd^3f^3n}{4\sqrt{x}}+\frac{1}{4}bd^4f^4n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}-\frac{1}{8}bd^4f^4n\log(x)+\frac{1}{8}bd^4f^4n\log$$

```
[Out] -7/36*b*d*f*n/x^(3/2)+3/8*b*d^2*f^2*n/x-1/8*b*d^4*f^4*n*ln(x)+1/8*b*d^4*f^4
*n*ln(x)^2-1/6*d*f*(a+b*ln(c*x^n))/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))/x-1/
4*d^4*f^4*ln(x)*(a+b*ln(c*x^n))+1/4*b*d^4*f^4*n*ln(1+d*f*x^(1/2))-1/4*b*n*l
n(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/2*(a+b
*ln(c*x^n))*ln(1+d*f*x^(1/2))/x^2+b*d^4*f^4*n*polylog(2,-d*f*x^(1/2))-5/4*b
*d^3*f^3*n/x^(1/2)-1/2*d^3*f^3*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2442, 46, 2423, 2438, 2338}

$$bd^4f^4n\text{PolyLog}(2,-df\sqrt{x})+\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-\frac{1}{4}d^4f^4\log(x)(a+b\log(cx^n))-\frac{d^3f^3(a+b\log(cx^n))}{2\sqrt{x}}+\frac{d^2f^2(a+b\log(cx^n))}{4x}-\frac{df(a+b\log(cx^n))}{6x^{3/2}}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{2x^2}+\frac{1}{8}bd^4f^4n\log^2(x)+\frac{1}{4}bd^4f^4n\log\left(df\sqrt{x}+1\right)-\frac{1}{8}bd^4f^4n\log(x)-\frac{5bd^3f^3n}{4\sqrt{x}}+\frac{3bd^2f^2n}{8x}-\frac{7bdfn}{36x^{3/2}}-\frac{bn\log\left(df\sqrt{x}+1\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-7*b*d*f*n)/(36*x^(3/2)) + (3*b*d^2*f^2*n)/(8*x) - (5*b*d^3*f^3*n)/(4*Sqrt
[x]) + (b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]])/4 - (b*n*Log[1 + d*f*Sqrt[x]])/(4
*x^2) - (b*d^4*f^4*n*Log[x])/8 + (b*d^4*f^4*n*Log[x]^2)/8 - (d*f*(a + b*Log
[c*x^n]))/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*x^n]))/(4*x) - (d^3*f^3*(a +
b*Log[c*x^n]))/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n
]))/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*x^2) - (d^4*f^4*Log[x]
*(a + b*Log[c*x^n]))/4 + b*d^4*f^4*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^3}\,dx = -\frac{df(a+b\log(cx^n))}{6x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))}{4x}-\frac{d^3f^3(a+b\log}{2\sqrt{x}}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{df(a+b\log(cx^n))}{6x^{3/2}}+\frac{d^2f^2(a+}{}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}+\frac{1}{8}bd^4f^4n\log^2(x)-\frac{df(a+b\log}{6x^{3/2}}$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}+\frac{1}{8}bd^4f^4n$$

$$= -\frac{bdfn}{9x^{3/2}}+\frac{bd^2f^2n}{4x}-\frac{bd^3f^3n}{\sqrt{x}}-\frac{bn\log\left(1+df\sqrt{x}\right)}{4x^2}+\frac{1}{8}bd^4f^4n$$

$$= -\frac{7bdfn}{36x^{3/2}}+\frac{3bd^2f^2n}{8x}-\frac{5bd^3f^3n}{4\sqrt{x}}+\frac{1}{4}bd^4f^4n\log\left(1+df\sqrt{x}\right)-$$

Mathematica [A]
time = 0.18, size = 207, normalized size = 0.72

$$\frac{(-1+d^4f^4x^2)\log\left(1+df\sqrt{x}\right)(2a+bn+2b\log(cx^n))}{4x^2}-\frac{df\left(12a+14bn-18adf\sqrt{x}-27bdfn\sqrt{x}+36ad^2f^2x+90bd^2f^2nx-9bd^3f^3nx^{3/2}\log^2(x)+6b\left(2-3df\sqrt{x}+6d^2f^2x\right)\log(cx^n)+9d^3f^3x^{3/2}\log(x)\left(2a+bn+2b\log(cx^n)\right)\right)}{72x^{3/2}}+bd^4f^4n\mathrm{Li}_2\left(-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] ((-1 + d^4*f^4*x^2)*Log[1 + d*f*Sqrt[x]]*(2*a + b*n + 2*b*Log[c*x^n]))/(4*x
^2) - (d*f*(12*a + 14*b*n - 18*a*d*f*Sqrt[x] - 27*b*d*f*n*Sqrt[x] + 36*a*d^
2*f^2*x + 90*b*d^2*f^2*n*x - 9*b*d^3*f^3*n*x^(3/2)*Log[x]^2 + 6*b*(2 - 3*d*
f*Sqrt[x] + 6*d^2*f^2*x)*Log[c*x^n] + 9*d^3*f^3*x^(3/2)*Log[x]*(2*a + b*n +
 2*b*Log[c*x^n])))/(72*x^(3/2)) + b*d^4*f^4*n*PolyLog[2, -(d*f*Sqrt[x])]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^3, x)
```

3.52 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx$

Optimal. Leaf size=372

$$-\frac{11bdfn}{225x^{5/2}}+\frac{5bd^2f^2n}{72x^2}-\frac{bd^3f^3n}{9x^{3/2}}+\frac{2bd^4f^4n}{9x}-\frac{7bd^5f^5n}{9\sqrt{x}}+\frac{1}{9}bd^6f^6n\log\left(1+df\sqrt{x}\right)-\frac{bn\log\left(1+df\sqrt{x}\right)}{9x^3}-\frac{1}{18}bd^6f$$

```
[Out] -11/225*b*d*f*n/x^(5/2)+5/72*b*d^2*f^2*n/x^2-1/9*b*d^3*f^3*n/x^(3/2)+2/9*b*
d^4*f^4*n/x-1/18*b*d^6*f^6*n*ln(x)+1/12*b*d^6*f^6*n*ln(x)^2-1/15*d*f*(a+b*l
n(c*x^n))/x^(5/2)+1/12*d^2*f^2*(a+b*ln(c*x^n))/x^2-1/9*d^3*f^3*(a+b*ln(c*x^
n))/x^(3/2)+1/6*d^4*f^4*(a+b*ln(c*x^n))/x-1/6*d^6*f^6*ln(x)*(a+b*ln(c*x^n))
+1/9*b*d^6*f^6*n*ln(1+d*f*x^(1/2))-1/9*b*n*ln(1+d*f*x^(1/2))/x^3+1/3*d^6*f^
6*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x
^3+2/3*b*d^6*f^6*n*polylog(2,-d*f*x^(1/2))-7/9*b*d^5*f^5*n/x^(1/2)-1/3*d^5*
f^5*(a+b*ln(c*x^n))/x^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2442, 46, 2423, 2438, 2338}

$$\frac{2}{3}bd^6f^6n\text{PolyLog}(2,-df\sqrt{x})+\frac{1}{3}d^6f^6\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-\frac{1}{6}d^6f^6\log(x)(a+b\log(cx^n))-\frac{d^5f^5(a+b\log(cx^n))}{3\sqrt{x}}+\frac{d^4f^4(a+b\log(cx^n))}{6x}-\frac{d^3f^3(a+b\log(cx^n))}{9x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))}{12x^2}-\frac{df(a+b\log(cx^n))}{15x^{5/2}}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{3x^3}+\frac{1}{12}bd^6f^6n\log^2(x)+\frac{1}{9}bd^6f^6n\log\left(df\sqrt{x}+1\right)-\frac{1}{18}bd^6f^6n\log(x)-\frac{7bd^5f^5n}{9\sqrt{x}}+\frac{2bd^4f^4n}{9x}-\frac{bd^3f^3n}{9x^{3/2}}+\frac{5bd^2f^2n}{72x^2}-\frac{11bdfn}{225x^{5/2}}-\frac{bn\log\left(df\sqrt{x}+1\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (-11*b*d*f*n)/(225*x^(5/2)) + (5*b*d^2*f^2*n)/(72*x^2) - (b*d^3*f^3*n)/(9*x
^(3/2)) + (2*b*d^4*f^4*n)/(9*x) - (7*b*d^5*f^5*n)/(9*Sqrt[x]) + (b*d^6*f^6*
n*Log[1 + d*f*Sqrt[x]])/9 - (b*n*Log[1 + d*f*Sqrt[x]])/(9*x^3) - (b*d^6*f^6
*n*Log[x])/18 + (b*d^6*f^6*n*Log[x]^2)/12 - (d*f*(a + b*Log[c*x^n]))/(15*x^
(5/2)) + (d^2*f^2*(a + b*Log[c*x^n]))/(12*x^2) - (d^3*f^3*(a + b*Log[c*x^n]
))/(9*x^(3/2)) + (d^4*f^4*(a + b*Log[c*x^n]))/(6*x) - (d^5*f^5*(a + b*Log[c
*x^n]))/(3*Sqrt[x]) + (d^6*f^6*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/3 -
 (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(3*x^3) - (d^6*f^6*Log[x]*(a + b
*Log[c*x^n]))/6 + (2*b*d^6*f^6*n*PolyLog[2, -(d*f*Sqrt[x])])/3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x^4}\,dx = -\frac{df(a+b\log(cx^n))}{15x^{5/2}} + \frac{d^2f^2(a+b\log(cx^n))}{12x^2} - \frac{d^3f^3(a+b\log(c}{9x^{3/2}}$$

$$= -\frac{2bdfn}{75x^{5/2}} + \frac{bd^2f^2n}{24x^2} - \frac{2bd^3f^3n}{27x^{3/2}} + \frac{bd^4f^4n}{6x} - \frac{2bd^5f^5n}{3\sqrt{x}} - \frac{df(a+b}{15}$$

$$= -\frac{2bdfn}{75x^{5/2}} + \frac{bd^2f^2n}{24x^2} - \frac{2bd^3f^3n}{27x^{3/2}} + \frac{bd^4f^4n}{6x} - \frac{2bd^5f^5n}{3\sqrt{x}} + \frac{1}{12}bd^6f^6$$

$$= -\frac{2bdfn}{75x^{5/2}} + \frac{bd^2f^2n}{24x^2} - \frac{2bd^3f^3n}{27x^{3/2}} + \frac{bd^4f^4n}{6x} - \frac{2bd^5f^5n}{3\sqrt{x}} - \frac{bn\log(1}{}$$

$$= -\frac{2bdfn}{75x^{5/2}} + \frac{bd^2f^2n}{24x^2} - \frac{2bd^3f^3n}{27x^{3/2}} + \frac{bd^4f^4n}{6x} - \frac{2bd^5f^5n}{3\sqrt{x}} - \frac{bn\log(1}{}$$

$$= -\frac{11bdfn}{225x^{5/2}} + \frac{5bd^2f^2n}{72x^2} - \frac{bd^3f^3n}{9x^{3/2}} + \frac{2bd^4f^4n}{9x} - \frac{7bd^5f^5n}{9\sqrt{x}} + \frac{1}{9}bd^6f$$

Mathematica [A]
time = 0.25, size = 288, normalized size = 0.77

$$\frac{(-1+d^6f^6x^3)\log(1+df\sqrt{x})(3a+bn+3b\log(cx^n))}{9x^3} - \frac{df\left(120a+88bn-150adf\sqrt{x}-125bdfn\sqrt{x}+200ad^2f^2x+200bd^2f^2nx-300ad^3f^3x^{3/2}-400bd^3f^3nx^{3/2}+600ad^4f^4x^2+1400bd^4f^4nx^2-150bd^5f^5nx^{5/2}\log^2(x)+10b\left(12-15df\sqrt{x}+20d^2f^2x-30d^3f^3x^{3/2}+60d^4f^4x^2\right)\log(cx^n)+100d^5f^5x^{5/2}\log(x)(3a+bn+3b\log(cx^n))\right)}{1800x^{5/2}} + \frac{2}{3}bd^6f^6n\mathrm{Li}_2(-df\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] ((-1 + d^6*f^6*x^3)*Log[1 + d*f*Sqrt[x]]*(3*a + b*n + 3*b*Log[c*x^n]))/(9*x
^3) - (d*f*(120*a + 88*b*n - 150*a*d*f*Sqrt[x] - 125*b*d*f*n*Sqrt[x] + 200*
a*d^2*f^2*x + 200*b*d^2*f^2*n*x - 300*a*d^3*f^3*x^(3/2) - 400*b*d^3*f^3*n*x
^(3/2) + 600*a*d^4*f^4*x^2 + 1400*b*d^4*f^4*n*x^2 - 150*b*d^5*f^5*n*x^(5/2)
*Log[x]^2 + 10*b*(12 - 15*d*f*Sqrt[x] + 20*d^2*f^2*x - 30*d^3*f^3*x^(3/2) +
 60*d^4*f^4*x^2)*Log[c*x^n] + 100*d^5*f^5*x^(5/2)*Log[x]*(3*a + b*n + 3*b*L
og[c*x^n])))/(1800*x^(5/2)) + (2*b*d^6*f^6*n*PolyLog[2, -(d*f*Sqrt[x])])/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^4,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="maxima"
)
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="fricas"
)
[Out] integral((b*log(c*x^n) + a)*log(d*f*sqrt(x) + 1)/x^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**(1/2)))/x**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^(1/2)))/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + 1/d)*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n)))/x^4, x)
```

3.53 $\int x^2 \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx$

Optimal. Leaf size=708

$$\frac{86b^2n^2\sqrt{x}}{27d^5f^5}+\frac{abnx}{3d^4f^4}-\frac{13b^2n^2x}{27d^4f^4}+\frac{14b^2n^2x^{3/2}}{81d^3f^3}-\frac{19b^2n^2x^2}{216d^2f^2}+\frac{182b^2n^2x^{5/2}}{3375df}-\frac{1}{27}b^2n^2x^3-\frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{27d^6f^6}+\frac{2}{27}$$

```
[Out] -1/6*x*(a+b*ln(c*x^n))^2/d^4/f^4+1/9*x^(3/2)*(a+b*ln(c*x^n))^2/d^3/f^3-1/12
*x^2*(a+b*ln(c*x^n))^2/d^2/f^2+1/15*x^(5/2)*(a+b*ln(c*x^n))^2/d/f+2/27*b^2*
n^2*x^3*ln(1+d*f*x^(1/2))-1/3*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^6/f^6+1
/3*(a+b*ln(c*x^n))^2*x^(1/2)/d^5/f^5+2/27*b*n*x^3*(a+b*ln(c*x^n))-13/27*b^2
*n^2*x/d^4/f^4+14/81*b^2*n^2*x^(3/2)/d^3/f^3-19/216*b^2*n^2*x^2/d^2/f^2+182
/3375*b^2*n^2*x^(5/2)/d/f+1/3*a*b*n*x/d^4/f^4+1/3*x^3*(a+b*ln(c*x^n))^2*ln(
1+d*f*x^(1/2))-1/18*x^3*(a+b*ln(c*x^n))^2-1/27*b^2*n^2*x^3-2/27*b^2*n^2*ln(
1+d*f*x^(1/2))/d^6/f^6-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))+4/9*b^
2*n^2*polylog(2,-d*f*x^(1/2))/d^6/f^6+8/3*b^2*n^2*polylog(3,-d*f*x^(1/2))/d
^6/f^6+86/27*b^2*n^2*x^(1/2)/d^5/f^5+1/3*b^2*n*x*ln(c*x^n)/d^4/f^4+1/9*b*n*
x*(a+b*ln(c*x^n))/d^4/f^4-2/9*b*n*x^(3/2)*(a+b*ln(c*x^n))/d^3/f^3+5/36*b*n*
x^2*(a+b*ln(c*x^n))/d^2/f^2-22/225*b*n*x^(5/2)*(a+b*ln(c*x^n))/d/f+2/9*b*n*
(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^6/f^6-4/3*b*n*(a+b*ln(c*x^n))*polylog(2
,-d*f*x^(1/2))/d^6/f^6-14/9*b*n*(a+b*ln(c*x^n))*x^(1/2)/d^5/f^5
```

Rubi [A]
time = 0.45, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2442, 45, 2424, 2332, 2341, 2421, 6724, 2423, 2438}

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (86*b^2*n^2*Sqrt[x])/(27*d^5*f^5) + (a*b*n*x)/(3*d^4*f^4) - (13*b^2*n^2*x)/
(27*d^4*f^4) + (14*b^2*n^2*x^(3/2))/(81*d^3*f^3) - (19*b^2*n^2*x^2)/(216*d^
2*f^2) + (182*b^2*n^2*x^(5/2))/(3375*d*f) - (b^2*n^2*x^3)/27 - (2*b^2*n^2*L
og[1 + d*f*Sqrt[x]])/(27*d^6*f^6) + (2*b^2*n^2*x^3*Log[1 + d*f*Sqrt[x]])/27
 + (b^2*n*x*Log[c*x^n])/(3*d^4*f^4) - (14*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(
9*d^5*f^5) + (b*n*x*(a + b*Log[c*x^n]))/(9*d^4*f^4) - (2*b*n*x^(3/2)*(a + b
*Log[c*x^n]))/(9*d^3*f^3) + (5*b*n*x^2*(a + b*Log[c*x^n]))/(36*d^2*f^2) - (
22*b*n*x^(5/2)*(a + b*Log[c*x^n]))/(225*d*f) + (2*b*n*x^3*(a + b*Log[c*x^n]
))/27 + (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*d^6*f^6) - (2*b*
n*x^3*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/9 + (Sqrt[x]*(a + b*Log[c*x^
n])^2)/(3*d^5*f^5) - (x*(a + b*Log[c*x^n])^2)/(6*d^4*f^4) + (x^(3/2)*(a + b
*Log[c*x^n])^2)/(9*d^3*f^3) - (x^2*(a + b*Log[c*x^n])^2)/(12*d^2*f^2) + (x^
(5/2)*(a + b*Log[c*x^n])^2)/(15*d*f) - (x^3*(a + b*Log[c*x^n])^2)/18 - (Log
```

```
[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(3*d^6*f^6) + (x^3*Log[1 + d*f*Sqrt
[x]]*(a + b*Log[c*x^n])^2)/3 + (4*b^2*n^2*PolyLog[2, -(d*f*Sqrt[x])])/(9*d^
6*f^6) - (4*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(3*d^6*f^6)
+ (8*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(3*d^6*f^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
```

```
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{3d^5f^5} - \frac{x(a+b\log(cx^n))^2}{6d^4f^4} + \frac{x^{3/2}(a+}{9}$$

$$= \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{3d^5f^5} - \frac{x(a+b\log(cx^n))^2}{6d^4f^4} + \frac{x^{3/2}(a+}{9}$$

$$= \frac{8b^2n^2\sqrt{x}}{3d^5f^5} + \frac{abnx}{3d^4f^4} + \frac{8b^2n^2x^{3/2}}{81d^3f^3} - \frac{b^2n^2x^2}{24d^2f^2} + \frac{8b^2n^2x^{5/2}}{375df}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5} + \frac{abnx}{3d^4f^4} - \frac{4b^2n^2x}{9d^4f^4} + \frac{4b^2n^2x^{3/2}}{27d^3f^3} - \frac{5b^2n^2x^2}{72d^2f^2}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5} + \frac{abnx}{3d^4f^4} - \frac{4b^2n^2x}{9d^4f^4} + \frac{4b^2n^2x^{3/2}}{27d^3f^3} - \frac{5b^2n^2x^2}{72d^2f^2}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5} + \frac{abnx}{3d^4f^4} - \frac{4b^2n^2x}{9d^4f^4} + \frac{4b^2n^2x^{3/2}}{27d^3f^3} - \frac{5b^2n^2x^2}{72d^2f^2}$$

$$= \frac{28b^2n^2\sqrt{x}}{9d^5f^5} + \frac{abnx}{3d^4f^4} - \frac{4b^2n^2x}{9d^4f^4} + \frac{4b^2n^2x^{3/2}}{27d^3f^3} - \frac{5b^2n^2x^2}{72d^2f^2}$$

$$= \frac{86b^2n^2\sqrt{x}}{27d^5f^5} + \frac{abnx}{3d^4f^4} - \frac{13b^2n^2x}{27d^4f^4} + \frac{14b^2n^2x^{3/2}}{81d^3f^3} - \frac{19b^2n^2}{216d^2}$$

Mathematica [A]
time = 0.34, size = 995, normalized size = 1.41

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (27000*a^2*d*f*Sqrt[x] - 126000*a*b*d*f*n*Sqrt[x] + 258000*b^2*d*f*n^2*Sqrt
[x] - 13500*a^2*d^2*f^2*x + 36000*a*b*d^2*f^2*n*x - 39000*b^2*d^2*f^2*n^2*x
 + 9000*a^2*d^3*f^3*x^(3/2) - 18000*a*b*d^3*f^3*n*x^(3/2) + 14000*b^2*d^3*f
^3*n^2*x^(3/2) - 6750*a^2*d^4*f^4*x^2 + 11250*a*b*d^4*f^4*n*x^2 - 7125*b^2*
d^4*f^4*n^2*x^2 + 5400*a^2*d^5*f^5*x^(5/2) - 7920*a*b*d^5*f^5*n*x^(5/2) + 4
368*b^2*d^5*f^5*n^2*x^(5/2) - 4500*a^2*d^6*f^6*x^3 + 6000*a*b*d^6*f^6*n*x^3
 - 3000*b^2*d^6*f^6*n^2*x^3 - 27000*a^2*Log[1 + d*f*Sqrt[x]] + 18000*a*b*n*
Log[1 + d*f*Sqrt[x]] - 6000*b^2*n^2*Log[1 + d*f*Sqrt[x]] + 27000*a^2*d^6*f^
6*x^3*Log[1 + d*f*Sqrt[x]] - 18000*a*b*d^6*f^6*n*x^3*Log[1 + d*f*Sqrt[x]] +
 6000*b^2*d^6*f^6*n^2*x^3*Log[1 + d*f*Sqrt[x]] + 54000*a*b*d*f*Sqrt[x]*Log[
c*x^n] - 126000*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 27000*a*b*d^2*f^2*x*Log[c*x^
```

```
n] + 36000*b^2*d^2*f^2*n*x*Log[c*x^n] + 18000*a*b*d^3*f^3*x^(3/2)*Log[c*x^n
] - 18000*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 13500*a*b*d^4*f^4*x^2*Log[c*x^
n] + 11250*b^2*d^4*f^4*n*x^2*Log[c*x^n] + 10800*a*b*d^5*f^5*x^(5/2)*Log[c*x
^n] - 7920*b^2*d^5*f^5*n*x^(5/2)*Log[c*x^n] - 9000*a*b*d^6*f^6*x^3*Log[c*x^
n] + 6000*b^2*d^6*f^6*n*x^3*Log[c*x^n] - 54000*a*b*Log[1 + d*f*Sqrt[x]]*Log
[c*x^n] + 18000*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 54000*a*b*d^6*f^6*x
^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 18000*b^2*d^6*f^6*n*x^3*Log[1 + d*f*Sq
rt[x]]*Log[c*x^n] + 27000*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 13500*b^2*d^2*f^2*
x*Log[c*x^n]^2 + 9000*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 6750*b^2*d^4*f^4*x
^2*Log[c*x^n]^2 + 5400*b^2*d^5*f^5*x^(5/2)*Log[c*x^n]^2 - 4500*b^2*d^6*f^6*
x^3*Log[c*x^n]^2 - 27000*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 27000*b^2*
d^6*f^6*x^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 36000*b*n*(-3*a + b*n - 3*b
*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 216000*b^2*n^2*PolyLog[3, -(d*f*S
qrt[x])])/(81000*d^6*f^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*sq
rt(x) + 1), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.54 $\int x \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=557

$$\frac{21b^2n^2\sqrt{x}}{4d^3f^3}+\frac{abnx}{2d^2f^2}-\frac{7b^2n^2x}{8d^2f^2}+\frac{37b^2n^2x^{3/2}}{108df}-\frac{3}{16}b^2n^2x^2-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4d^4f^4}+\frac{1}{4}b^2n^2x^2\log\left(1+df\sqrt{x}\right)+\frac{b^2n}{}$$

```
[Out] 1/2*a*b*n*x/d^2/f^2-7/8*b^2*n^2*x/d^2/f^2+37/108*b^2*n^2*x^(3/2)/d/f-3/16*b
^2*n^2*x^2+1/2*b^2*n*x*ln(c*x^n)/d^2/f^2+1/4*b*n*x*(a+b*ln(c*x^n))/d^2/f^2-
7/18*b*n*x^(3/2)*(a+b*ln(c*x^n))/d/f+1/4*b*n*x^2*(a+b*ln(c*x^n))-1/4*x*(a+b
*ln(c*x^n))^2/d^2/f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))^2/d/f-1/8*x^2*(a+b*ln(c*x
^n))^2-1/4*b^2*n^2*ln(1+d*f*x^(1/2))/d^4/f^4+1/4*b^2*n^2*x^2*ln(1+d*f*x^(1/
2))+1/2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^4/f^4-1/2*b*n*x^2*(a+b*ln(c
*x^n))*ln(1+d*f*x^(1/2))-1/2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^4/f^4+1/
2*x^2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))+b^2*n^2*polylog(2,-d*f*x^(1/2))/d
^4/f^4-2*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^4/f^4+4*b^2*n^2*poly
log(3,-d*f*x^(1/2))/d^4/f^4+21/4*b^2*n^2*x^(1/2)/d^3/f^3-5/2*b*n*(a+b*ln(c*
x^n))*x^(1/2)/d^3/f^3+1/2*(a+b*ln(c*x^n))^2*x^(1/2)/d^3/f^3
```

Rubi [A]
time = 0.32, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2504, 2442, 45, 2424, 2332, 2341, 2421, 6724, 2423, 2438}

$$-\frac{2bn\,\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{d^4f^4}+\frac{b^2n^2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^4f^4}+\frac{4b^2n^2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^4f^4}+\frac{bn\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2d^4f^4}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2d^4f^4}-\frac{5bn\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2d^3f^3}+\frac{\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{2d^3f^3}+\frac{bnx\left(a+b\log\left(cx^n\right)\right)}{4d^2f^2}-\frac{x\left(a+b\log\left(cx^n\right)\right)^2}{4d^2f^2}-\frac{7bnx^{3/2}\left(a+b\log\left(cx^n\right)\right)}{18df}+\frac{x^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}{6df}-\frac{1}{2}bnx^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)+\frac{1}{2}x^2\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{4}bnx^2\left(a+b\log\left(cx^n\right)\right)-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{abnx}{2d^2f^2}+\frac{b^2nx\log\left(cx^n\right)}{2d^2f^2}-\frac{b^2n^2\log\left(df\sqrt{x}+1\right)}{4d^4f^4}+\frac{21b^2n^2\sqrt{x}}{4d^3f^3}-\frac{7b^2n^2x}{8d^2f^2}+\frac{37b^2n^2x^{3/2}}{108df}+\frac{1}{4}b^2n^2x^2\log\left(df\sqrt{x}+1\right)-\frac{3}{16}b^2n^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (21*b^2*n^2*Sqrt[x])/(4*d^3*f^3) + (a*b*n*x)/(2*d^2*f^2) - (7*b^2*n^2*x)/(8
*d^2*f^2) + (37*b^2*n^2*x^(3/2))/(108*d*f) - (3*b^2*n^2*x^2)/16 - (b^2*n^2*
Log[1 + d*f*Sqrt[x]])/(4*d^4*f^4) + (b^2*n^2*x^2*Log[1 + d*f*Sqrt[x]])/4 +
(b^2*n*x*Log[c*x^n])/(2*d^2*f^2) - (5*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(2*d^
3*f^3) + (b*n*x*(a + b*Log[c*x^n]))/(4*d^2*f^2) - (7*b*n*x^(3/2)*(a + b*Log
[c*x^n]))/(18*d*f) + (b*n*x^2*(a + b*Log[c*x^n]))/4 + (b*n*Log[1 + d*f*Sqrt
[x]]*(a + b*Log[c*x^n]))/(2*d^4*f^4) - (b*n*x^2*Log[1 + d*f*Sqrt[x]]*(a + b
*Log[c*x^n]))/2 + (Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*d^3*f^3) - (x*(a + b*Lo
g[c*x^n])^2)/(4*d^2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^2)/(6*d*f) - (x^2*(a
 + b*Log[c*x^n])^2)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(2*d^4*
f^4) + (x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/2 + (b^2*n^2*PolyLog
[2, -(d*f*Sqrt[x])])/(d^4*f^4) - (2*b*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f
*Sqrt[x])])/(d^4*f^4) + (4*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{2d^3f^3} - \frac{x(a+b\log(cx^n))^2}{4d^2f^2} + \frac{x^{3/2}(a+b}{6}$$

$$= \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{2d^3f^3} - \frac{x(a+b\log(cx^n))^2}{4d^2f^2} + \frac{x^{3/2}(a+b}{6}$$

$$= \frac{4b^2n^2\sqrt{x}}{d^3f^3} + \frac{abnx}{2d^2f^2} + \frac{4b^2n^2x^{3/2}}{27df} - \frac{1}{16}b^2n^2x^2 - \frac{5bn\sqrt{x}\,(}{}$$

$$= \frac{5b^2n^2\sqrt{x}}{d^3f^3} + \frac{abnx}{2d^2f^2} - \frac{3b^2n^2x}{4d^2f^2} + \frac{7b^2n^2x^{3/2}}{27df} - \frac{1}{8}b^2n^2x^2 +$$

$$= \frac{5b^2n^2\sqrt{x}}{d^3f^3} + \frac{abnx}{2d^2f^2} - \frac{3b^2n^2x}{4d^2f^2} + \frac{7b^2n^2x^{3/2}}{27df} - \frac{1}{8}b^2n^2x^2 +$$

$$= \frac{5b^2n^2\sqrt{x}}{d^3f^3} + \frac{abnx}{2d^2f^2} - \frac{3b^2n^2x}{4d^2f^2} + \frac{7b^2n^2x^{3/2}}{27df} - \frac{1}{8}b^2n^2x^2 +$$

$$= \frac{5b^2n^2\sqrt{x}}{d^3f^3} + \frac{abnx}{2d^2f^2} - \frac{3b^2n^2x}{4d^2f^2} + \frac{7b^2n^2x^{3/2}}{27df} - \frac{1}{8}b^2n^2x^2 +$$

$$= \frac{21b^2n^2\sqrt{x}}{4d^3f^3} + \frac{abnx}{2d^2f^2} - \frac{7b^2n^2x}{8d^2f^2} + \frac{37b^2n^2x^{3/2}}{108df} - \frac{3}{16}b^2n^2x$$

Mathematica [A]
time = 0.24, size = 769, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (216*a^2*d*f*Sqrt[x] - 1080*a*b*d*f*n*Sqrt[x] + 2268*b^2*d*f*n^2*Sqrt[x] -
108*a^2*d^2*f^2*x + 324*a*b*d^2*f^2*n*x - 378*b^2*d^2*f^2*n^2*x + 72*a^2*d^
3*f^3*x^(3/2) - 168*a*b*d^3*f^3*n*x^(3/2) + 148*b^2*d^3*f^3*n^2*x^(3/2) - 5
4*a^2*d^4*f^4*x^2 + 108*a*b*d^4*f^4*n*x^2 - 81*b^2*d^4*f^4*n^2*x^2 - 216*a^
2*Log[1 + d*f*Sqrt[x]] + 216*a*b*n*Log[1 + d*f*Sqrt[x]] - 108*b^2*n^2*Log[1
 + d*f*Sqrt[x]] + 216*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 216*a*b*d^4*f^
4*n*x^2*Log[1 + d*f*Sqrt[x]] + 108*b^2*d^4*f^4*n^2*x^2*Log[1 + d*f*Sqrt[x]]
 + 432*a*b*d*f*Sqrt[x]*Log[c*x^n] - 1080*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 216
*a*b*d^2*f^2*x*Log[c*x^n] + 324*b^2*d^2*f^2*n*x*Log[c*x^n] + 144*a*b*d^3*f^
3*x^(3/2)*Log[c*x^n] - 168*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] - 108*a*b*d^4*f
^4*x^2*Log[c*x^n] + 108*b^2*d^4*f^4*n*x^2*Log[c*x^n] - 432*a*b*Log[1 + d*f*
```

```
Sqrt[x]]*Log[c*x^n] + 216*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 432*a*b*d
^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*b^2*d^4*f^4*n*x^2*Log[1 +
d*f*Sqrt[x]]*Log[c*x^n] + 216*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 108*b^2*d^2*f^
2*x*Log[c*x^n]^2 + 72*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 54*b^2*d^4*f^4*x^2
*Log[c*x^n]^2 - 216*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^2*d^4*f^4
*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 432*b*n*(-2*a + b*n - 2*b*Log[c*x^
n])*PolyLog[2, -(d*f*Sqrt[x])] + 1728*b^2*n^2*PolyLog[3, -(d*f*Sqrt[x])])/(
432*d^4*f^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
[Out] int(x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
+ 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(f\,\sqrt{x}\,+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.55 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right) (a+b\log (cx^n))^2 \, dx$

Optimal. Leaf size=374

$$\frac{14b^2n^2\sqrt{x}}{df}+abnx-3b^2n^2x+2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-\frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{d^2f^2}+b^2nx\log\left(cx^n\right)-\frac{6bn\sqrt{x}}{}$$

```
[Out] a*b*n*x-3*b^2*n^2*x+b^2*n*x*ln(c*x^n)+b*n*x*(a+b*ln(c*x^n))-1/2*x*(a+b*ln(c
*x^n))^2+2*b^2*n^2*x*ln(d*(1/d+f*x^(1/2)))-2*b*n*x*(a+b*ln(c*x^n))*ln(d*(1/
d+f*x^(1/2)))+x*(a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))-2*b^2*n^2*ln(1+d*f*
x^(1/2))/d^2/f^2+2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*
x^n))^2*ln(1+d*f*x^(1/2))/d^2/f^2+4*b^2*n^2*polylog(2,-d*f*x^(1/2))/d^2/f^2
-4*b*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2+8*b^2*n^2*polylog(3,
-d*f*x^(1/2))/d^2/f^2+14*b^2*n^2*x^(1/2)/d/f-6*b*n*(a+b*ln(c*x^n))*x^(1/2)/
d/f+(a+b*ln(c*x^n))^2*x^(1/2)/d/f
```

Rubi [A]
time = 0.19, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2498, 272, 45, 2417, 2332, 2341, 2438, 2421, 6724}

$$-\frac{4bn\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))}{d^2f^2}+\frac{4b^2n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)}{d^2f^2}+\frac{8b^2n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)}{d^2f^2}+\frac{2bn\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{d^2f^2}-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{d^2f^2}-2bnx\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))-\frac{6bn\sqrt{x}(a+b\log(cx^n))}{df}+x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2+\frac{\sqrt{x}(a+b\log(cx^n))^2}{df}+bnx(a+b\log(cx^n))-\frac{1}{2}x(a+b\log(cx^n))^2+abnx+b^2nx\log(cx^n)-\frac{2b^2n^2\log\left(df\sqrt{x}+1\right)}{d^2f^2}+\frac{14b^2n^2\sqrt{x}}{df}+2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)-3b^2n^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (14*b^2*n^2*Sqrt[x])/(d*f) + a*b*n*x - 3*b^2*n^2*x + 2*b^2*n^2*x*Log[d*(d^(
-1) + f*Sqrt[x])] - (2*b^2*n^2*Log[1 + d*f*Sqrt[x]])/(d^2*f^2) + b^2*n*x*Lo
g[c*x^n] - (6*b*n*Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) + b*n*x*(a + b*Log[c*x^
n]) - 2*b*n*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n]) + (2*b*n*Log[1
 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^n])
^2)/(d*f) - (x*(a + b*Log[c*x^n])^2)/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(a +
 b*Log[c*x^n])^2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(d^2*f^2) +
(4*b^2*n^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (4*b*n*(a + b*Log[c*x^n]
)*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) + (8*b^2*n^2*PolyLog[3, -(d*f*Sqrt[
x])])/(d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{df} - \frac{1}{2}x(a+b\log(cx^n))^2 + x\log\left(d\left(\frac{1}{d}\right.\right.$$

$$= \frac{\sqrt{x}\,(a+b\log(cx^n))^2}{df} - \frac{1}{2}x(a+b\log(cx^n))^2 + x\log\left(d\left(\frac{1}{d}\right.\right.$$

$$= \frac{8b^2n^2\sqrt{x}}{df} + abnx - \frac{6bn\sqrt{x}\,(a+b\log(cx^n))}{df} + bnx(a+b\,\mathrm{lo}$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + b^2nx\log(cx^n) - \frac{6bn\sqrt{x}\,(a}{}$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right.$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right.$$

$$= \frac{12b^2n^2\sqrt{x}}{df} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right.$$

$$= \frac{14b^2n^2\sqrt{x}}{df} + abnx - 3b^2n^2x + 2b^2n^2x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right.$$

Mathematica [A]
time = 0.19, size = 527, normalized size = 1.41

$$-\frac{-2a^2df\sqrt{x}+12abdfn\sqrt{x}-28b^2dfn^2\sqrt{x}+a^2d^2f^2x-4abd^2f^2nx+6b^2d^2f^2n^2x+2a^2\log\left(1+df\sqrt{x}\right)-4abn\log\left(1+df\sqrt{x}\right)+4b^2n^2\log\left(1+df\sqrt{x}\right)-2a^2d^2f^2x\log\left(1+df\sqrt{x}\right)+4abd^2f^2nx\log\left(1+df\sqrt{x}\right)-4b^2d^2f^2n^2x\log\left(1+df\sqrt{x}\right)-4abdf\sqrt{x}\log(cx^n)+12b^2dfn\sqrt{x}\log(cx^n)+2abd^2f^2x\log(cx^n)-4b^2d^2f^2nx\log(cx^n)+4ab\log\left(1+df\sqrt{x}\right)\log(cx^n)-4b^2n\log\left(1+df\sqrt{x}\right)\log(cx^n)-4abd^2f^2x\log\left(1+df\sqrt{x}\right)\log(cx^n)+4b^2d^2f^2nx\log\left(1+df\sqrt{x}\right)\log(cx^n)-2b^2df\sqrt{x}\log^2(cx^n)+b^2d^2f^2x\log^2(cx^n)+2b^2\log\left(1+df\sqrt{x}\right)\log^2(cx^n)-2b^2d^2f^2x\log\left(1+df\sqrt{x}\right)\log^2(cx^n)+8bn(a-bn+b\log(cx^n))\,\mathrm{Li}_2\left(-df\sqrt{x}\right)-16b^2n^2\,\mathrm{Li}_3\left(-df\sqrt{x}\right)}{2d^2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] -1/2*(-2*a^2*d*f*Sqrt[x] + 12*a*b*d*f*n*Sqrt[x] - 28*b^2*d*f*n^2*Sqrt[x] +
a^2*d^2*f^2*x - 4*a*b*d^2*f^2*n*x + 6*b^2*d^2*f^2*n^2*x + 2*a^2*Log[1 + d*f
*Sqrt[x]] - 4*a*b*n*Log[1 + d*f*Sqrt[x]] + 4*b^2*n^2*Log[1 + d*f*Sqrt[x]] -
 2*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 4*a*b*d^2*f^2*n*x*Log[1 + d*f*Sqrt[
x]] - 4*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] - 4*a*b*d*f*Sqrt[x]*Log[c*x^
n] + 12*b^2*d*f*n*Sqrt[x]*Log[c*x^n] + 2*a*b*d^2*f^2*x*Log[c*x^n] - 4*b^2*d
```

```
^2*f^2*n*x*Log[c*x^n] + 4*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*b^2*n*Log
[1 + d*f*Sqrt[x]]*Log[c*x^n] - 4*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x
^n] + 4*b^2*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 2*b^2*d*f*Sqrt[x]
*Log[c*x^n]^2 + b^2*d^2*f^2*x*Log[c*x^n]^2 + 2*b^2*Log[1 + d*f*Sqrt[x]]*Log
[c*x^n]^2 - 2*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 8*b*n*(a -
b*n + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 16*b^2*n^2*PolyLog[3, -(d*
f*Sqrt[x])])/(d^2*f^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\,x^{n}\right)\right)^{2} \ln \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
[Out] (b^2*x*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*log(x^n) + ((2*n^2 - 2*n*l
og(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x)*log(d*f*sqrt(x) + 1) -
 1/27*(9*b^2*d*f*x^2*log(x^n)^2 + 6*(3*a*b*d*f - (5*d*f*n - 3*d*f*log(c))*b
^2)*x^2*log(x^n) + (9*a^2*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b + (38*d*f*n^
2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^2)*x^2)/sqrt(x) + integrate(1/2*(b^
2*d^2*f^2*x*log(x^n)^2 + 2*(a*b*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*b^2)
*x*log(x^n) + (a^2*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b + (2*d^2*f^
2*n^2 - 2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^2)*x)/(d*f*sqrt(x) + 1), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2, x)
```

3.56 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$

Optimal. Leaf size=70

$$-2(a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+8bn(a+b\log(cx^n))\,\mathrm{Li}_3\left(-df\sqrt{x}\right)-16b^2n^2\mathrm{Li}_4\left(-df\sqrt{x}\right)$$

```
[Out] -2*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))+8*b*n*(a+b*ln(c*x^n))*polylog(
3,-d*f*x^(1/2))-16*b^2*n^2*polylog(4,-d*f*x^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2421, 2430, 6724}

$$8bn\text{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log(cx^n))-2\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^2-16b^2n^2\text{PolyLog}\left(4,-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 8*b*n*(a + b*Log[c*x^n
])*PolyLog[3, -(d*f*Sqrt[x])] - 16*b^2*n^2*PolyLog[4, -(d*f*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx = -2(a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+(4bn)\int \frac{(a+b\log(cx^n))}{x}$$

$$= -2(a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+8bn(a+b\log(cx^n))\,\mathrm{Li}_3\left(-\right.$$

$$= -2(a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+8bn(a+b\log(cx^n))\,\mathrm{Li}_3\left(-\right.$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 1.00

$$-2\left((a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+4bn\left(-\left((a+b\log(cx^n))\,\mathrm{Li}_3\left(-df\sqrt{x}\right)\right)+2bn\mathrm{Li}_4\left(-df\sqrt{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] -2*((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[c
*x^n])*PolyLog[3, -(d*f*Sqrt[x])]) + 2*b*n*PolyLog[4, -(d*f*Sqrt[x])]))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^2\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

3.57 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal. Leaf size=389

$$-\frac{14b^2dfn^2}{\sqrt{x}}+2b^2d^2f^2n^2\log\left(1+df\sqrt{x}\right)-\frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x}-b^2d^2f^2n^2\log(x)+\frac{1}{2}b^2d^2f^2n^2\log^2(x)-\frac{6bdf}{}$$

[Out] -b^2*d^2*f^2*n^2*ln(x)+1/2*b^2*d^2*f^2*n^2*ln(x)^2-b*d^2*f^2*n*ln(x)*(a+b*ln(c*x^n))-1/6*d^2*f^2*(a+b*ln(c*x^n))^3/b/n+2*b^2*d^2*f^2*n^2*ln(1+d*f*x^(1/2))-2*b^2*n^2*ln(1+d*f*x^(1/2))/x+2*b*d^2*f^2*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-2*b*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))-(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x+4*b^2*d^2*f^2*n^2*polylog(2,-d*f*x^(1/2))+4*b*d^2*f^2*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))-8*b^2*d^2*f^2*n^2*polylog(3,-d*f*x^(1/2))-14*b^2*d*f*n^2/x^(1/2)-6*b*d*f*n*(a+b*ln(c*x^n))/x^(1/2)-d*f*(a+b*ln(c*x^n))^2/x^(1/2)

Rubi [A]

time = 0.28, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2504, 2442, 46, 2424, 2341, 2423, 2438, 2338, 2421, 6724, 2413, 12, 2339, 30}

$$4bd^2f^2n\,\text{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))+4b^2d^2f^2n^2\,\text{PolyLog}\left(2,-df\sqrt{x}\right)-8b^2d^2f^2n^2\,\text{PolyLog}\left(3,-df\sqrt{x}\right)-\frac{d^2f^2(a+b\log(cx^n))^3}{6bn}+d^2f^2\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2+2bd^2f^2n\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))-bd^2f^2n\log(x)(a+b\log(cx^n))-\frac{\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))^2}{x}-\frac{df(a+b\log(cx^n))^2}{\sqrt{x}}-\frac{2bn\log\left(df\sqrt{x}+1\right)(a+b\log(cx^n))}{x}-\frac{6bdfn(a+b\log(cx^n))}{\sqrt{x}}+\frac{1}{2}b^2d^2f^2n^2\log^2(x)+2b^2d^2f^2n^2\log\left(df\sqrt{x}+1\right)-b^2d^2f^2n^2\log(x)-\frac{14b^2dfn^2}{\sqrt{x}}-\frac{2b^2n^2\log\left(df\sqrt{x}+1\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]

[Out] (-14*b^2*d*f*n^2)/Sqrt[x] + 2*b^2*d^2*f^2*n^2*Log[1 + d*f*Sqrt[x]] - (2*b^2*n^2*Log[1 + d*f*Sqrt[x]])/x - b^2*d^2*f^2*n^2*Log[x] + (b^2*d^2*f^2*n^2*Log[x]^2)/2 - (6*b*d*f*n*(a + b*Log[c*x^n]))/Sqrt[x] + 2*b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) - (2*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/x - b*d^2*f^2*n*Log[x]*(a + b*Log[c*x^n]) - (d*f*(a + b*Log[c*x^n])^2)/Sqrt[x] + d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/x - (d^2*f^2*(a + b*Log[c*x^n])^3)/(6*b*n) + 4*b^2*d^2*f^2*n^2*PolyLog[2, -(d*f*Sqrt[x])] + 4*b*d^2*f^2*n*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] - 8*b^2*d^2*f^2*n^2*PolyLog[3, -(d*f*Sqrt[x])]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= -\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right.$$

$$= -\frac{8b^2dfn^2}{\sqrt{x}} - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 2bd^2f^2n\log\left(1+df\sqrt{x}\right.$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + 2bd^2f^2n\log\left(1+df\sqrt{\cdot}\right.$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \frac{6bdfn\left(a+b\log\left(cx^n\right)\right)}{\sqrt{x}} + \cdots$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \cdots$$

$$= -\frac{12b^2dfn^2}{\sqrt{x}} - \frac{2b^2n^2\log\left(1+df\sqrt{x}\right)}{x} + \frac{1}{2}b^2d^2f^2n^2\log^2(x) - \cdots$$

$$= -\frac{14b^2dfn^2}{\sqrt{x}} + 2b^2d^2f^2n^2\log\left(1+df\sqrt{x}\right) - \frac{2b^2n^2\log\left(1+df\cdots\right.}{x}$$

Mathematica [A]
time = 0.25, size = 627, normalized size = 1.61

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] -1/6*(6*a^2*d*f*Sqrt[x] + 36*a*b*d*f*n*Sqrt[x] + 84*b^2*d*f*n^2*Sqrt[x] + 6
*a^2*Log[1 + d*f*Sqrt[x]] + 12*a*b*n*Log[1 + d*f*Sqrt[x]] + 12*b^2*n^2*Log[
1 + d*f*Sqrt[x]] - 6*a^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] - 12*a*b*d^2*f^2*n*
x*Log[1 + d*f*Sqrt[x]] - 12*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] + 3*a^2*
d^2*f^2*x*Log[x] + 6*a*b*d^2*f^2*n*x*Log[x] + 6*b^2*d^2*f^2*n^2*x*Log[x] -
3*a*b*d^2*f^2*n*x*Log[x]^2 - 3*b^2*d^2*f^2*n^2*x*Log[x]^2 + b^2*d^2*f^2*n^2
*x*Log[x]^3 + 12*a*b*d*f*Sqrt[x]*Log[c*x^n] + 36*b^2*d*f*n*Sqrt[x]*Log[c*x^
n] + 12*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^2*n*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n] - 12*a*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*b^2*d^2
*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 6*a*b*d^2*f^2*x*Log[x]*Log[c*x^n
] + 6*b^2*d^2*f^2*n*x*Log[x]*Log[c*x^n] - 3*b^2*d^2*f^2*n*x*Log[x]^2*Log[c*
x^n] + 6*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 6*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^
n]^2 - 6*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 3*b^2*d^2*f^2*x*
```

```
Log[x]*Log[c*x^n]^2 - 24*b*d^2*f^2*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2,
-(d*f*Sqrt[x])] + 48*b^2*d^2*f^2*n^2*x*PolyLog[3, -(d*f*Sqrt[x])])/x
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^2 \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\,\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x^2,x)
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxim
a")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="frica
s")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^2, x)
```

3.58 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal. Leaf size=555

$$-\frac{37b^2dfn^2}{108x^{3/2}}+\frac{7b^2d^2f^2n^2}{8x}-\frac{21b^2d^3f^3n^2}{4\sqrt{x}}+\frac{1}{4}b^2d^4f^4n^2\log\left(1+df\sqrt{x}\right)-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}-\frac{1}{8}b^2d^4f^4n^2\log(x)$$

```
[Out] -37/108*b^2*d*f*n^2/x^(3/2)+7/8*b^2*d^2*f^2*n^2/x-1/8*b^2*d^4*f^4*n^2*ln(x)
+1/8*b^2*d^4*f^4*n^2*ln(x)^2-7/18*b*d*f*n*(a+b*ln(c*x^n))/x^(3/2)+3/4*b*d^2
*f^2*n*(a+b*ln(c*x^n))/x-1/4*b*d^4*f^4*n*ln(x)*(a+b*ln(c*x^n))-1/6*d*f*(a+b
*ln(c*x^n))^2/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^2/x-1/12*d^4*f^4*(a+b*ln(
c*x^n))^3/b/n+1/4*b^2*d^4*f^4*n^2*ln(1+d*f*x^(1/2))-1/4*b^2*n^2*ln(1+d*f*x^
(1/2))/x^2+1/2*b*d^4*f^4*n*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-1/2*b*n*(a+b*l
n(c*x^n))*ln(1+d*f*x^(1/2))/x^2+1/2*d^4*f^4*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1
/2))-1/2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x^2+b^2*d^4*f^4*n^2*polylog(2,
-d*f*x^(1/2))+2*b*d^4*f^4*n*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))-4*b^2*d
^4*f^4*n^2*polylog(3,-d*f*x^(1/2))-21/4*b^2*d^3*f^3*n^2/x^(1/2)-5/2*b*d^3*f
^3*n*(a+b*ln(c*x^n))/x^(1/2)-1/2*d^3*f^3*(a+b*ln(c*x^n))^2/x^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2504, 2442, 46, 2424, 2341, 2423, 2438, 2338, 2421, 6724, 2413, 12, 2339, 30}

$$2bd^4f^4n\text{PolyLog}\left(2,-df\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)+b^2d^4f^4n^2\text{PolyLog}\left(2,-df\sqrt{x}\right)-4b^2d^4f^4n^2\text{PolyLog}\left(3,-df\sqrt{x}\right)-\frac{d^4f^4\left(a+b\log\left(cx^n\right)\right)^3}{12bn}+\frac{1}{2}d^4f^4\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{2}bd^4f^4n\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)-\frac{1}{4}bd^4f^4n\log(x)\left(a+b\log\left(cx^n\right)\right)-\frac{d^3f^3\left(a+b\log\left(cx^n\right)\right)^2}{2\sqrt{x}}-\frac{5bd^3f^3n\left(a+b\log\left(cx^n\right)\right)}{2\sqrt{x}}+\frac{d^2f^2\left(a+b\log\left(cx^n\right)\right)^2}{4x}+\frac{3bd^2f^2n\left(a+b\log\left(cx^n\right)\right)}{4x}-\frac{df\left(a+b\log\left(cx^n\right)\right)^2}{6x^{3/2}}-\frac{7bdfn\left(a+b\log\left(cx^n\right)\right)}{18x^{3/2}}-\frac{\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2x^2}-\frac{bn\log\left(df\sqrt{x}+1\right)\left(a+b\log\left(cx^n\right)\right)}{2x^2}+\frac{1}{8}b^2d^4f^4n^2\log^2(x)+\frac{1}{4}b^2d^4f^4n^2\log\left(df\sqrt{x}+1\right)-\frac{1}{8}b^2d^4f^4n^2\log(x)-\frac{21b^2d^3f^3n^2}{4\sqrt{x}}+\frac{7b^2d^2f^2n^2}{8x}-\frac{37b^2dfn^2}{108x^{3/2}}-\frac{b^2n^2\log\left(df\sqrt{x}+1\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] (-37*b^2*d*f*n^2)/(108*x^(3/2)) + (7*b^2*d^2*f^2*n^2)/(8*x) - (21*b^2*d^3*f
^3*n^2)/(4*Sqrt[x]) + (b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/4 - (b^2*n^2*L
og[1 + d*f*Sqrt[x]])/(4*x^2) - (b^2*d^4*f^4*n^2*Log[x])/8 + (b^2*d^4*f^4*n^
2*Log[x]^2)/8 - (7*b*d*f*n*(a + b*Log[c*x^n]))/(18*x^(3/2)) + (3*b*d^2*f^2*
n*(a + b*Log[c*x^n]))/(4*x) - (5*b*d^3*f^3*n*(a + b*Log[c*x^n]))/(2*Sqrt[x]
) + (b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/2 - (b*n*Log[1 +
d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*x^2) - (b*d^4*f^4*n*Log[x]*(a + b*Log[c
*x^n]))/4 - (d*f*(a + b*Log[c*x^n])^2)/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*
x^n])^2)/(4*x) - (d^3*f^3*(a + b*Log[c*x^n])^2)/(2*Sqrt[x]) + (d^4*f^4*Log[
1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log
[c*x^n])^2)/(2*x^2) - (d^4*f^4*(a + b*Log[c*x^n])^3)/(12*b*n) + b^2*d^4*f^4
*n^2*PolyLog[2, -(d*f*Sqrt[x])] + 2*b*d^4*f^4*n*(a + b*Log[c*x^n])*PolyLog[
2, -(d*f*Sqrt[x])] - 4*b^2*d^4*f^4*n^2*PolyLog[3, -(d*f*Sqrt[x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{df(a+b\log(cx^n))^2}{6x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))^2}{4x}-\frac{d^3f^3(a+b\mathrm{l}}{2\sqrt{}}$$

$$= -\frac{df(a+b\log(cx^n))^2}{6x^{3/2}}+\frac{d^2f^2(a+b\log(cx^n))^2}{4x}-\frac{d^3f^3(a+b\mathrm{l}}{2\sqrt{}}$$

$$= -\frac{4b^2dfn^2}{27x^{3/2}}+\frac{b^2d^2f^2n^2}{2x}-\frac{4b^2d^3f^3n^2}{\sqrt{x}}-\frac{7bdfn(a+b\log(cx^n))}{18x^{3/2}}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{7bdfn(a+b\log(cx^n)}{18x^{3/2}}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}+\frac{1}{8}b^2d^4f^4n^2\log^2(x)-$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}$$

$$= -\frac{7b^2dfn^2}{27x^{3/2}}+\frac{3b^2d^2f^2n^2}{4x}-\frac{5b^2d^3f^3n^2}{\sqrt{x}}-\frac{b^2n^2\log\left(1+df\sqrt{x}\right)}{4x^2}$$

$$= -\frac{37b^2dfn^2}{108x^{3/2}}+\frac{7b^2d^2f^2n^2}{8x}-\frac{21b^2d^3f^3n^2}{4\sqrt{x}}+\frac{1}{4}b^2d^4f^4n^2\log(1+$$

Mathematica [A]
time = 0.34, size = 881, normalized size = 1.59

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/216*(36*a^2*d*f*Sqrt[x] + 84*a*b*d*f*n*Sqrt[x] + 74*b^2*d*f*n^2*Sqrt[x]
- 54*a^2*d^2*f^2*x - 162*a*b*d^2*f^2*n*x - 189*b^2*d^2*f^2*n^2*x + 108*a^2*
d^3*f^3*x^(3/2) + 540*a*b*d^3*f^3*n*x^(3/2) + 1134*b^2*d^3*f^3*n^2*x^(3/2)
+ 108*a^2*Log[1 + d*f*Sqrt[x]] + 108*a*b*n*Log[1 + d*f*Sqrt[x]] + 54*b^2*n^
2*Log[1 + d*f*Sqrt[x]] - 108*a^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]] - 108*a*b
*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] - 54*b^2*d^4*f^4*n^2*x^2*Log[1 + d*f*Sq
rt[x]] + 54*a^2*d^4*f^4*x^2*Log[x] + 54*a*b*d^4*f^4*n*x^2*Log[x] + 27*b^2*d
^4*f^4*n^2*x^2*Log[x] - 54*a*b*d^4*f^4*n*x^2*Log[x]^2 - 27*b^2*d^4*f^4*n^2*
x^2*Log[x]^2 + 18*b^2*d^4*f^4*n^2*x^2*Log[x]^3 + 72*a*b*d*f*Sqrt[x]*Log[c*x
^n] + 84*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 108*a*b*d^2*f^2*x*Log[c*x^n] - 162*
b^2*d^2*f^2*n*x*Log[c*x^n] + 216*a*b*d^3*f^3*x^(3/2)*Log[c*x^n] + 540*b^2*d
```

```
^3*f^3*n*x^(3/2)*Log[c*x^n] + 216*a*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 108
*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 216*a*b*d^4*f^4*x^2*Log[1 + d*f*Sq
rt[x]]*Log[c*x^n] - 108*b^2*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] +
 108*a*b*d^4*f^4*x^2*Log[x]*Log[c*x^n] + 54*b^2*d^4*f^4*n*x^2*Log[x]*Log[c*
x^n] - 54*b^2*d^4*f^4*n*x^2*Log[x]^2*Log[c*x^n] + 36*b^2*d*f*Sqrt[x]*Log[c*
x^n]^2 - 54*b^2*d^2*f^2*x*Log[c*x^n]^2 + 108*b^2*d^3*f^3*x^(3/2)*Log[c*x^n]
^2 + 108*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 108*b^2*d^4*f^4*x^2*Log[1
+ d*f*Sqrt[x]]*Log[c*x^n]^2 + 54*b^2*d^4*f^4*x^2*Log[x]*Log[c*x^n]^2 - 216*
b*d^4*f^4*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])] + 8
64*b^2*d^4*f^4*n^2*x^2*PolyLog[3, -(d*f*Sqrt[x])])/x^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\,\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="frica
s")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + 1)/x
^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**(1/2)))/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^2)/x^3, x)
```

3.59 $\int x \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx$

Optimal. Leaf size=858

$$-\frac{255b^3n^3\sqrt{x}}{8d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{45b^3n^3x}{16d^2f^2}-\frac{175b^3n^3x^{3/2}}{216df}+\frac{3}{8}b^3n^3x^2+\frac{3b^3n^3\log\left(1+df\sqrt{x}\right)}{8d^4f^4}-\frac{3}{8}b^3n^3x^2\log\left(1+df\sqrt{x}\right.$$

```
[Out] -9/16*b^2*n^2*x^2*(a+b*ln(c*x^n))+3/8*b*n*x^2*(a+b*ln(c*x^n))^2+45/16*b^3*n
^3*x/d^2/f^2-175/216*b^3*n^3*x^(3/2)/d/f-3/8*b^3*n^3*x^2*ln(1+d*f*x^(1/2))-
1/2*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))/d^4/f^4+1/2*(a+b*ln(c*x^n))^3*x^(1/
2)/d^3/f^3-1/4*x*(a+b*ln(c*x^n))^3/d^2/f^2+1/6*x^(3/2)*(a+b*ln(c*x^n))^3/d/
f+3/8*b^3*n^3*x^2+1/2*x^2*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))-1/8*x^2*(a+b*
ln(c*x^n))^3+3/8*b^3*n^3*ln(1+d*f*x^(1/2))/d^4/f^4+3/4*b^2*n^2*x^2*(a+b*ln(
c*x^n))*ln(1+d*f*x^(1/2))-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))-3
/2*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^4/f^4-6*b^3*n^3*polylog(3,-d*f*x^(1/2)
)/d^4/f^4-24*b^3*n^3*polylog(4,-d*f*x^(1/2))/d^4/f^4-255/8*b^3*n^3*x^(1/2)/
d^3/f^3-9/4*a*b^2*n^2*x/d^2/f^2-3/4*b^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2
))/d^4/f^4+3/4*b*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^4/f^4+3*b^2*n^2*(a
+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^4/f^4-3*b*n*(a+b*ln(c*x^n))^2*polyl
og(2,-d*f*x^(1/2))/d^4/f^4+12*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2
))/d^4/f^4+63/4*b^2*n^2*(a+b*ln(c*x^n))*x^(1/2)/d^3/f^3-15/4*b*n*(a+b*ln(c*
x^n))^2*x^(1/2)/d^3/f^3-9/4*b^3*n^2*x*ln(c*x^n)/d^2/f^2-3/8*b^2*n^2*x*(a+b*
ln(c*x^n))/d^2/f^2+37/36*b^2*n^2*x^(3/2)*(a+b*ln(c*x^n))/d/f+9/8*b*n*x*(a+b
*ln(c*x^n))^2/d^2/f^2-7/12*b*n*x^(3/2)*(a+b*ln(c*x^n))^2/d/f
```

Rubi [A]
time = 0.63, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2504, 2442, 45, 2424, 2333, 2332, 2342, 2341, 2421, 2430, 6724, 2423, 2438}

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-255*b^3*n^3*Sqrt[x])/(8*d^3*f^3) - (9*a*b^2*n^2*x)/(4*d^2*f^2) + (45*b^3*
n^3*x)/(16*d^2*f^2) - (175*b^3*n^3*x^(3/2))/(216*d*f) + (3*b^3*n^3*x^2)/8 +
 (3*b^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*d^4*f^4) - (3*b^3*n^3*x^2*Log[1 + d*f*
Sqrt[x]])/8 - (9*b^3*n^2*x*Log[c*x^n])/(4*d^2*f^2) + (63*b^2*n^2*Sqrt[x]*(a
 + b*Log[c*x^n]))/(4*d^3*f^3) - (3*b^2*n^2*x*(a + b*Log[c*x^n]))/(8*d^2*f^2
) + (37*b^2*n^2*x^(3/2)*(a + b*Log[c*x^n]))/(36*d*f) - (9*b^2*n^2*x^2*(a +
b*Log[c*x^n]))/16 - (3*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*
d^4*f^4) + (3*b^2*n^2*x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/4 - (15*
b*n*Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*d^3*f^3) + (9*b*n*x*(a + b*Log[c*x^n])
^2)/(8*d^2*f^2) - (7*b*n*x^(3/2)*(a + b*Log[c*x^n])^2)/(12*d*f) + (3*b*n*x^
```

```
2*(a + b*Log[c*x^n])^2)/8 + (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^
2)/(4*d^4*f^4) - (3*b*n*x^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 +
(Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*d^3*f^3) - (x*(a + b*Log[c*x^n])^3)/(4*d^
2*f^2) + (x^(3/2)*(a + b*Log[c*x^n])^3)/(6*d*f) - (x^2*(a + b*Log[c*x^n])^3
)/8 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*d^4*f^4) + (x^2*Log[1
+ d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/2 - (3*b^3*n^3*PolyLog[2, -(d*f*Sqrt[x
])])/(2*d^4*f^4) + (3*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])]
)/(d^4*f^4) - (3*b*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^4*
f^4) - (6*b^3*n^3*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) + (12*b^2*n^2*(a +
b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])])/(d^4*f^4) - (24*b^3*n^3*PolyLog[4
, -(d*f*Sqrt[x])])/(d^4*f^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx=\frac{\sqrt{x}\,(a+b\log(cx^n))^3}{2d^3f^3}-\frac{x(a+b\log(cx^n))^3}{4d^2f^2}+\frac{x^{3/2}(a+b}{6}$$

$$=\frac{\sqrt{x}\,(a+b\log(cx^n))^3}{2d^3f^3}-\frac{x(a+b\log(cx^n))^3}{4d^2f^2}+\frac{x^{3/2}(a+b}{6}$$

$$=-\frac{15bn\sqrt{x}\,(a+b\log(cx^n))^2}{4d^3f^3}+\frac{9bnx(a+b\log(cx^n))^2}{8d^2f^2}-$$

$$=-\frac{24b^3n^3\sqrt{x}}{d^3f^3}-\frac{3ab^2n^2x}{2d^2f^2}-\frac{8b^3n^3x^{3/2}}{27df}+\frac{3}{32}b^3n^3x^2+\frac{12b}{}$$

$$=-\frac{30b^3n^3\sqrt{x}}{d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{3b^3n^3x}{2d^2f^2}-\frac{14b^3n^3x^{3/2}}{27df}+\frac{3}{16}b^3$$

$$=-\frac{63b^3n^3\sqrt{x}}{2d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{21b^3n^3x}{8d^2f^2}-\frac{37b^3n^3x^{3/2}}{54df}+\frac{9}{32}b$$

$$=-\frac{63b^3n^3\sqrt{x}}{2d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{21b^3n^3x}{8d^2f^2}-\frac{37b^3n^3x^{3/2}}{54df}+\frac{9}{32}b$$

$$=-\frac{63b^3n^3\sqrt{x}}{2d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{21b^3n^3x}{8d^2f^2}-\frac{37b^3n^3x^{3/2}}{54df}+\frac{9}{32}b$$

$$=-\frac{63b^3n^3\sqrt{x}}{2d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{21b^3n^3x}{8d^2f^2}-\frac{37b^3n^3x^{3/2}}{54df}+\frac{9}{32}b$$

$$=-\frac{255b^3n^3\sqrt{x}}{8d^3f^3}-\frac{9ab^2n^2x}{4d^2f^2}+\frac{45b^3n^3x}{16d^2f^2}-\frac{175b^3n^3x^{3/2}}{216df}+\frac{3}{8}$$

Mathematica [A]

time = 0.37, size = 1432, normalized size = 1.67

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (216*a^3*d*f*Sqrt[x] - 1620*a^2*b*d*f*n*Sqrt[x] + 6804*a*b^2*d*f*n^2*Sqrt[x
] - 13770*b^3*d*f*n^3*Sqrt[x] - 108*a^3*d^2*f^2*x + 486*a^2*b*d^2*f^2*n*x -
 1134*a*b^2*d^2*f^2*n^2*x + 1215*b^3*d^2*f^2*n^3*x + 72*a^3*d^3*f^3*x^(3/2)
 - 252*a^2*b*d^3*f^3*n*x^(3/2) + 444*a*b^2*d^3*f^3*n^2*x^(3/2) - 350*b^3*d^
3*f^3*n^3*x^(3/2) - 54*a^3*d^4*f^4*x^2 + 162*a^2*b*d^4*f^4*n*x^2 - 243*a*b^
2*d^4*f^4*n^2*x^2 + 162*b^3*d^4*f^4*n^3*x^2 - 216*a^3*Log[1 + d*f*Sqrt[x]]
+ 324*a^2*b*n*Log[1 + d*f*Sqrt[x]] - 324*a*b^2*n^2*Log[1 + d*f*Sqrt[x]] + 1
62*b^3*n^3*Log[1 + d*f*Sqrt[x]] + 216*a^3*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]
- 324*a^2*b*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]] + 324*a*b^2*d^4*f^4*n^2*x^2*
Log[1 + d*f*Sqrt[x]] - 162*b^3*d^4*f^4*n^3*x^2*Log[1 + d*f*Sqrt[x]] + 648*a
^2*b*d*f*Sqrt[x]*Log[c*x^n] - 3240*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n] + 6804*b^
3*d*f*n^2*Sqrt[x]*Log[c*x^n] - 324*a^2*b*d^2*f^2*x*Log[c*x^n] + 972*a*b^2*d
^2*f^2*n*x*Log[c*x^n] - 1134*b^3*d^2*f^2*n^2*x*Log[c*x^n] + 216*a^2*b*d^3*f
^3*x^(3/2)*Log[c*x^n] - 504*a*b^2*d^3*f^3*n*x^(3/2)*Log[c*x^n] + 444*b^3*d^
3*f^3*n^2*x^(3/2)*Log[c*x^n] - 162*a^2*b*d^4*f^4*x^2*Log[c*x^n] + 324*a*b^2
*d^4*f^4*n*x^2*Log[c*x^n] - 243*b^3*d^4*f^4*n^2*x^2*Log[c*x^n] - 648*a^2*b*
Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^
n] - 324*b^3*n^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 648*a^2*b*d^4*f^4*x^2*Lo
g[1 + d*f*Sqrt[x]]*Log[c*x^n] - 648*a*b^2*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]
]*Log[c*x^n] + 324*b^3*d^4*f^4*n^2*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 64
8*a*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 - 1620*b^3*d*f*n*Sqrt[x]*Log[c*x^n]^2 - 32
4*a*b^2*d^2*f^2*x*Log[c*x^n]^2 + 486*b^3*d^2*f^2*n*x*Log[c*x^n]^2 + 216*a*b
^2*d^3*f^3*x^(3/2)*Log[c*x^n]^2 - 252*b^3*d^3*f^3*n*x^(3/2)*Log[c*x^n]^2 -
162*a*b^2*d^4*f^4*x^2*Log[c*x^n]^2 + 162*b^3*d^4*f^4*n*x^2*Log[c*x^n]^2 - 6
48*a*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 324*b^3*n*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n]^2 + 648*a*b^2*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 3
24*b^3*d^4*f^4*n*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^3*d*f*Sqrt[x
]*Log[c*x^n]^3 - 108*b^3*d^2*f^2*x*Log[c*x^n]^3 + 72*b^3*d^3*f^3*x^(3/2)*Lo
g[c*x^n]^3 - 54*b^3*d^4*f^4*x^2*Log[c*x^n]^3 - 216*b^3*Log[1 + d*f*Sqrt[x]]
*Log[c*x^n]^3 + 216*b^3*d^4*f^4*x^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3 - 648
*b*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c
*x^n]^2)*PolyLog[2, -(d*f*Sqrt[x])] + 2592*b^2*n^2*(2*a - b*n + 2*b*Log[c*x
^n])*PolyLog[3, -(d*f*Sqrt[x])] - 10368*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])
/(432*d^4*f^4)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^3\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log(d*f*sqrt(x) + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2))),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
[Out] int(x*log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

3.60 $\int \log \left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log (cx^n))^3\,dx$

Optimal. Leaf size=604

$$-\frac{90b^3n^3\sqrt{x}}{df}-6ab^2n^2x+12b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)+\frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{d^2f^2}-6b^3n^2x\log\left(cx^n\right)$$

```
[Out] -6*a*b^2*n^2*x+12*b^3*n^3*x-6*b^3*n^2*x*ln(c*x^n)-3*b^2*n^2*x*(a+b*ln(c*x^n
))+3*b*n*x*(a+b*ln(c*x^n))^2-1/2*x*(a+b*ln(c*x^n))^3-6*b^3*n^3*x*ln(d*(1/d+
f*x^(1/2)))+6*b^2*n^2*x*(a+b*ln(c*x^n))*ln(d*(1/d+f*x^(1/2)))-3*b*n*x*(a+b*
ln(c*x^n))^2*ln(d*(1/d+f*x^(1/2)))+x*(a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))
)+6*b^3*n^3*ln(1+d*f*x^(1/2))/d^2/f^2-6*b^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^
(1/2))/d^2/f^2+3*b*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/d^2/f^2-(a+b*ln(c*
x^n))^3*ln(1+d*f*x^(1/2))/d^2/f^2-12*b^3*n^3*polylog(2,-d*f*x^(1/2))/d^2/f^
2+12*b^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))/d^2/f^2-6*b*n*(a+b*ln(
c*x^n))^2*polylog(2,-d*f*x^(1/2))/d^2/f^2-24*b^3*n^3*polylog(3,-d*f*x^(1/2)
)/d^2/f^2+24*b^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2))/d^2/f^2-48*b^3
*n^3*polylog(4,-d*f*x^(1/2))/d^2/f^2-90*b^3*n^3*x^(1/2)/d/f+42*b^2*n^2*(a+b
*ln(c*x^n))*x^(1/2)/d/f-9*b*n*(a+b*ln(c*x^n))^2*x^(1/2)/d/f+(a+b*ln(c*x^n))
^3*x^(1/2)/d/f
```

Rubi [A]
time = 0.38, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2498, 272, 45, 2417, 2333, 2332, 2342, 2341, 2438, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-90*b^3*n^3*Sqrt[x])/(d*f) - 6*a*b^2*n^2*x + 12*b^3*n^3*x - 6*b^3*n^3*x*Lo
g[d*(d^(-1) + f*Sqrt[x])] + (6*b^3*n^3*Log[1 + d*f*Sqrt[x]])/(d^2*f^2) - 6*
b^3*n^2*x*Log[c*x^n] + (42*b^2*n^2*Sqrt[x]*(a + b*Log[c*x^n]))/(d*f) - 3*b^
2*n^2*x*(a + b*Log[c*x^n]) + 6*b^2*n^2*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b
*Log[c*x^n]) - (6*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(d^2*f^2
) - (9*b*n*Sqrt[x]*(a + b*Log[c*x^n])^2)/(d*f) + 3*b*n*x*(a + b*Log[c*x^n])
^2 - 3*b*n*x*Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 + (3*b*n*Log[
1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/(d^2*f^2) + (Sqrt[x]*(a + b*Log[c*x^
n])^3)/(d*f) - (x*(a + b*Log[c*x^n])^3)/2 + x*Log[d*(d^(-1) + f*Sqrt[x])]*(
a + b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(d^2*f^2)
 - (12*b^3*n^3*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) + (12*b^2*n^2*(a + b*L
og[c*x^n])*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (6*b*n*(a + b*Log[c*x^n]
)^2*PolyLog[2, -(d*f*Sqrt[x])])/(d^2*f^2) - (24*b^3*n^3*PolyLog[3, -(d*f*Sq
```

```
rt[x])])/(d^2*f^2) + (24*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x
])])/(d^2*f^2) - (48*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
```

```
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \frac{\sqrt{x}\,(a+b\log(cx^n))^3}{df}-\frac{1}{2}x(a+b\log(cx^n))^3+x\log\left(d\left(\frac{1}{d}\right.\right.$$

$$= \frac{\sqrt{x}\,(a+b\log(cx^n))^3}{df}-\frac{1}{2}x(a+b\log(cx^n))^3+x\log\left(d\left(\frac{1}{d}\right.\right.$$

$$= -\frac{9bn\sqrt{x}\,(a+b\log(cx^n))^2}{df}+3bnx(a+b\log(cx^n))^2-3bnx$$

$$= -\frac{48b^3n^3\sqrt{x}}{df}-3ab^2n^2x+\frac{24b^2n^2\sqrt{x}\,(a+b\log(cx^n))}{df}-\frac{9b}{}$$

$$= -\frac{72b^3n^3\sqrt{x}}{df}-6ab^2n^2x+3b^3n^3x-3b^3n^2x\log(cx^n)+\frac{42b}{}$$

$$= -\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^2x\log(cx^n)+\frac{42b}{}$$

$$= -\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{}\right.\right.$$

$$= -\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{}\right.\right.$$

$$= -\frac{84b^3n^3\sqrt{x}}{df}-6ab^2n^2x+9b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\sqrt{}\right.\right.$$

$$= -\frac{90b^3n^3\sqrt{x}}{df}-6ab^2n^2x+12b^3n^3x-6b^3n^3x\log\left(d\left(\frac{1}{d}+f\right.\right.$$

Mathematica [A]
time = 0.31, size = 986, normalized size = 1.63

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] -1/2*(-2*a^3*d*f*Sqrt[x] + 18*a^2*b*d*f*n*Sqrt[x] - 84*a*b^2*d*f*n^2*Sqrt[x
] + 180*b^3*d*f*n^3*Sqrt[x] + a^3*d^2*f^2*x - 6*a^2*b*d^2*f^2*n*x + 18*a*b^
2*d^2*f^2*n^2*x - 24*b^3*d^2*f^2*n^3*x + 2*a^3*Log[1 + d*f*Sqrt[x]] - 6*a^2
*b*n*Log[1 + d*f*Sqrt[x]] + 12*a*b^2*n^2*Log[1 + d*f*Sqrt[x]] - 12*b^3*n^3*
```

```
Log[1 + d*f*Sqrt[x]] - 2*a^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]] + 6*a^2*b*d^2*f
^2*n*x*Log[1 + d*f*Sqrt[x]] - 12*a*b^2*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt[x]] +
 12*b^3*d^2*f^2*n^3*x*Log[1 + d*f*Sqrt[x]] - 6*a^2*b*d*f*Sqrt[x]*Log[c*x^n]
 + 36*a*b^2*d*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*d*f*n^2*Sqrt[x]*Log[c*x^n] +
3*a^2*b*d^2*f^2*x*Log[c*x^n] - 12*a*b^2*d^2*f^2*n*x*Log[c*x^n] + 18*b^3*d^2
*f^2*n^2*x*Log[c*x^n] + 6*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*a*b^2*
n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*b^3*n^2*Log[1 + d*f*Sqrt[x]]*Log[c*x
^n] - 6*a^2*b*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] + 12*a*b^2*d^2*f^2*
n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n] - 12*b^3*d^2*f^2*n^2*x*Log[1 + d*f*Sqrt
[x]]*Log[c*x^n] - 6*a*b^2*d*f*Sqrt[x]*Log[c*x^n]^2 + 18*b^3*d*f*n*Sqrt[x]*L
og[c*x^n]^2 + 3*a*b^2*d^2*f^2*x*Log[c*x^n]^2 - 6*b^3*d^2*f^2*n*x*Log[c*x^n]
^2 + 6*a*b^2*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 6*b^3*n*Log[1 + d*f*Sqrt[x
]]*Log[c*x^n]^2 - 6*a*b^2*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 + 6*b
^3*d^2*f^2*n*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2 - 2*b^3*d*f*Sqrt[x]*Log[c*
x^n]^3 + b^3*d^2*f^2*x*Log[c*x^n]^3 + 2*b^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]
^3 - 2*b^3*d^2*f^2*x*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3 + 12*b*n*(a^2 - 2*a*
b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -
(d*f*Sqrt[x])] - 48*b^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[
x])] + 96*b^3*n^3*PolyLog[4, -(d*f*Sqrt[x])])/(d^2*f^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))),x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] (b^3*x*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*log(x^n)^2 + 3*((2*n^2 -
 2*n*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*log(x^n) + (3
*(2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log(c)
^2 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x)*log(d*f*sqrt(x) + 1) -
1/27*(9*b^3*d*f*x^2*log(x^n)^3 + 9*(3*a*b^2*d*f - (5*d*f*n - 3*d*f*log(c))*
b^3)*x^2*log(x^n)^2 + 3*(9*a^2*b*d*f - 6*(5*d*f*n - 3*d*f*log(c))*a*b^2 + (
38*d*f*n^2 - 30*d*f*n*log(c) + 9*d*f*log(c)^2)*b^3)*x^2*log(x^n) + (9*a^3*d
```

```
*f - 9*(5*d*f*n - 3*d*f*log(c))*a^2*b + 3*(38*d*f*n^2 - 30*d*f*n*log(c) + 9
*d*f*log(c)^2)*a*b^2 - (130*d*f*n^3 - 114*d*f*n^2*log(c) + 45*d*f*n*log(c)^
2 - 9*d*f*log(c)^3)*b^3)*x^2)/sqrt(x) + integrate(1/2*(b^3*d^2*f^2*x*log(x^
n)^3 + 3*(a*b^2*d^2*f^2 - (d^2*f^2*n - d^2*f^2*log(c))*b^3)*x*log(x^n)^2 +
3*(a^2*b*d^2*f^2 - 2*(d^2*f^2*n - d^2*f^2*log(c))*a*b^2 + (2*d^2*f^2*n^2 -
2*d^2*f^2*n*log(c) + d^2*f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*d^2*f^2 - 3*(
d^2*f^2*n - d^2*f^2*log(c))*a^2*b + 3*(2*d^2*f^2*n^2 - 2*d^2*f^2*n*log(c) +
 d^2*f^2*log(c)^2)*a*b^2 - (6*d^2*f^2*n^3 - 6*d^2*f^2*n^2*log(c) + 3*d^2*f^
2*n*log(c)^2 - d^2*f^2*log(c)^3)*b^3)*x)/(d*f*sqrt(x) + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2))),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3,x)
[Out] int(log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3, x)
```

3.61 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x}\,dx$

Optimal. Leaf size=101

$$-2(a+b\log(cx^n))^3\,\mathrm{Li}_2\left(-df\sqrt{x}\right)+12bn(a+b\log(cx^n))^2\,\mathrm{Li}_3\left(-df\sqrt{x}\right)-48b^2n^2(a+b\log(cx^n))\,\mathrm{Li}_4\left(-df\sqrt{x}\right)$$

```
[Out] -2*(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^(1/2))+12*b*n*(a+b*ln(c*x^n))^2*polyl
og(3,-d*f*x^(1/2))-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^(1/2))+96*b^
3*n^3*polylog(5,-d*f*x^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2421, 2430, 6724}

$$-48b^2n^2\mathrm{PolyLog}\left(4,-df\sqrt{x}\right)(a+b\log(cx^n))+12bn\mathrm{PolyLog}\left(3,-df\sqrt{x}\right)(a+b\log(cx^n))^2-2\mathrm{PolyLog}\left(2,-df\sqrt{x}\right)(a+b\log(cx^n))^3+96b^3n^3\mathrm{PolyLog}\left(5,-df\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b*n*(a + b*Log[c*x^
n])^2*PolyLog[3, -(d*f*Sqrt[x])] - 48*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4,
 -(d*f*Sqrt[x])] + 96*b^3*n^3*PolyLog[5, -(d*f*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x}\,dx = -2(a+b\log(cx^n))^3\operatorname{Li}_2\left(-df\sqrt{x}\right)+(6bn)\int \frac{(a+b\log(cx^n))^2}{x}$$

$$= -2(a+b\log(cx^n))^3\operatorname{Li}_2\left(-df\sqrt{x}\right)+12bn(a+b\log(cx^n))^2\operatorname{Li}_3$$

$$= -2(a+b\log(cx^n))^3\operatorname{Li}_2\left(-df\sqrt{x}\right)+12bn(a+b\log(cx^n))^2\operatorname{Li}_3$$

$$= -2(a+b\log(cx^n))^3\operatorname{Li}_2\left(-df\sqrt{x}\right)+12bn(a+b\log(cx^n))^2\operatorname{Li}_3$$

Mathematica [A]
time = 0.12, size = 98, normalized size = 0.97

$$-2(a+b\log(cx^n))^3\operatorname{Li}_2\left(-df\sqrt{x}\right)+12bn\left((a+b\log(cx^n))^2\operatorname{Li}_3\left(-df\sqrt{x}\right)+4bn\left(-\left((a+b\log(cx^n))\operatorname{Li}_4\left(-df\sqrt{x}\right)\right)+2bn\operatorname{Li}_5\left(-df\sqrt{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] -2*(a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b*n*((a + b*Log[c*x
^n])^2*PolyLog[3, -(d*f*Sqrt[x])] + 4*b*n*(-((a + b*Log[c*x^n])*PolyLog[4,
-(d*f*Sqrt[x])]) + 2*b*n*PolyLog[5, -(d*f*Sqrt[x])]))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^3\ln\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

3.62 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$

Optimal. Leaf size=610

$$-\frac{90b^3dfn^3}{\sqrt{x}}+6b^3d^2f^2n^3\log\left(1+df\sqrt{x}\right)-\frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x}-3b^3d^2f^2n^3\log(x)+\frac{3}{2}b^3d^2f^2n^3\log^2(x)-\frac{42b^2}{}$$

```
[Out] -3*b^3*d^2*f^2*n^3*ln(x)+3/2*b^3*d^2*f^2*n^3*ln(x)^2-3*b^2*d^2*f^2*n^2*ln(x
)*(a+b*ln(c*x^n))-1/2*d^2*f^2*(a+b*ln(c*x^n))^3-1/8*d^2*f^2*(a+b*ln(c*x^n))
^4/b/n+6*b^3*d^2*f^2*n^3*ln(1+d*f*x^(1/2))-6*b^3*n^3*ln(1+d*f*x^(1/2))/x+6*
b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))-6*b^2*n^2*(a+b*ln(c*x^n))
*ln(1+d*f*x^(1/2))/x+3*b*d^2*f^2*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))-3*b*
n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))/x+d^2*f^2*(a+b*ln(c*x^n))^3*ln(1+d*f*
x^(1/2))-(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))/x+12*b^3*d^2*f^2*n^3*polylog(2
,-d*f*x^(1/2))+12*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+6
*b*d^2*f^2*n*(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^(1/2))-24*b^3*d^2*f^2*n^3*p
olylog(3,-d*f*x^(1/2))-24*b^2*d^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^
(1/2))+48*b^3*d^2*f^2*n^3*polylog(4,-d*f*x^(1/2))-90*b^3*d*f*n^3/x^(1/2)-42
*b^2*d*f*n^2*(a+b*ln(c*x^n))/x^(1/2)-9*b*d*f*n*(a+b*ln(c*x^n))^2/x^(1/2)-d*
f*(a+b*ln(c*x^n))^3/x^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2504, 2442, 46, 2424, 2342, 2341, 2423, 2438, 2338, 2421, 6724, 2413, 12, 2339, 30, 2430}

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] (-90*b^3*d*f*n^3)/Sqrt[x] + 6*b^3*d^2*f^2*n^3*Log[1 + d*f*Sqrt[x]] - (6*b^3
*n^3*Log[1 + d*f*Sqrt[x]])/x - 3*b^3*d^2*f^2*n^3*Log[x] + (3*b^3*d^2*f^2*n^
3*Log[x]^2)/2 - (42*b^2*d*f*n^2*(a + b*Log[c*x^n]))/Sqrt[x] + 6*b^2*d^2*f^2
*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]) - (6*b^2*n^2*Log[1 + d*f*Sqrt[
x]]*(a + b*Log[c*x^n]))/x - 3*b^2*d^2*f^2*n^2*Log[x]*(a + b*Log[c*x^n]) - (
9*b*d*f*n*(a + b*Log[c*x^n])^2)/Sqrt[x] + 3*b*d^2*f^2*n*Log[1 + d*f*Sqrt[x]
]*(a + b*Log[c*x^n])^2 - (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/
x - (d^2*f^2*(a + b*Log[c*x^n])^3)/2 - (d*f*(a + b*Log[c*x^n])^3)/Sqrt[x] +
 d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3 - (Log[1 + d*f*Sqrt[x]]*
(a + b*Log[c*x^n])^3)/x - (d^2*f^2*(a + b*Log[c*x^n])^4)/(8*b*n) + 12*b^3*d
^2*f^2*n^3*PolyLog[2, -(d*f*Sqrt[x])] + 12*b^2*d^2*f^2*n^2*(a + b*Log[c*x^n
])*PolyLog[2, -(d*f*Sqrt[x])] + 6*b*d^2*f^2*n*(a + b*Log[c*x^n])^2*PolyLog[
```

```
2, -(d*f*Sqrt[x])] - 24*b^3*d^2*f^2*n^3*PolyLog[3, -(d*f*Sqrt[x])] - 24*b^2
*d^2*f^2*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(d*f*Sqrt[x])] + 48*b^3*d^2*f^2
*n^3*PolyLog[4, -(d*f*Sqrt[x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{df(a+b\log(cx^n))^3}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^3$$

$$= -\frac{df(a+b\log(cx^n))^3}{\sqrt{x}} + d^2f^2\log\left(1+df\sqrt{x}\right)(a+b\log(cx^n))^3$$

$$= -\frac{9bdfn(a+b\log(cx^n))^2}{\sqrt{x}} + 3bd^2f^2n\log\left(1+df\sqrt{x}\right)(a+b\log$$

$$= -\frac{48b^3dfn^3}{\sqrt{x}} - \frac{24b^2dfn^2(a+b\log(cx^n))}{\sqrt{x}} - \frac{9bdfn(a+b\log(cx^n$$

$$= -\frac{72b^3dfn^3}{\sqrt{x}} - \frac{42b^2dfn^2(a+b\log(cx^n))}{\sqrt{x}} + 6b^2d^2f^2n^2\log\left(1+d\right.$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{42b^2dfn^2(a+b\log(cx^n))}{\sqrt{x}} + 6b^2d^2f^2n^2\log\left(1+d\right.$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42b^2dfn^2(a+b\log(cx^n))}{\sqrt{x}} +$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42}{}$$

$$= -\frac{84b^3dfn^3}{\sqrt{x}} - \frac{6b^3n^3\log\left(1+df\sqrt{x}\right)}{x} + \frac{3}{2}b^3d^2f^2n^3\log^2(x) - \frac{42}{}$$

$$= -\frac{90b^3dfn^3}{\sqrt{x}} + 6b^3d^2f^2n^3\log\left(1+df\sqrt{x}\right) - \frac{6b^3n^3\log\left(1+df\sqrt{\right.}}{x}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1455 vs. $2(610) = 1220$.
time = 0.56, size = 1455, normalized size = 2.39

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] d^2*f^2*Log[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3
*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) +
6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2
 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3)
 - d^2*f^2*Log[Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*
b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3
*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 3*
```

```
b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3) - (L
og[1 + d*f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*b*n*
Log[x] + 6*a*b^2*n^2*Log[x] + 6*b^3*n^3*Log[x] + 3*a*b^2*n^2*Log[x]^2 + 3*b
^3*n^3*Log[x]^2 + b^3*n^3*Log[x]^3 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6
*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n])
+ 6*a*b^2*n*Log[x]*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*Log[x]*(-(n*Log[x
]) + Log[c*x^n]) + 3*b^3*n^2*Log[x]^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*
(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3
*n*Log[x]*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3))
/x + (-(a^3*d*f) - 3*a^2*b*d*f*n - 6*a*b^2*d*f*n^2 - 6*b^3*d*f*n^3 - 3*a^2*
b*d*f*(-(n*Log[x]) + Log[c*x^n]) - 6*a*b^2*d*f*n*(-(n*Log[x]) + Log[c*x^n])
 - 6*b^3*d*f*n^2*(-(n*Log[x]) + Log[c*x^n]) - 3*a*b^2*d*f*(-(n*Log[x]) + Lo
g[c*x^n])^2 - 3*b^3*d*f*n*(-(n*Log[x]) + Log[c*x^n])^2 - b^3*d*f*(-(n*Log[x
]) + Log[c*x^n])^3)/Sqrt[x] + 3*b*d*f*n*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*a*b*
(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*
Log[x]) + Log[c*x^n])^2)*((-(1/Sqrt[x]) + d*f*Log[1 + d*f*Sqrt[x]] - d*f*Lo
g[Sqrt[x]])*(-2*Log[Sqrt[x]] + Log[x]) + 2*(-(1/Sqrt[x]) - Log[Sqrt[x]]/Sqr
t[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x]] - (d*f*Log[Sqrt[x]]^2)/2 + d*f
*PolyLog[2, -(d*f*Sqrt[x])])) + 3*b^2*d*f*n^2*(a + b*n + b*(-(n*Log[x]) + L
og[c*x^n]))*((-(1/Sqrt[x]) + d*f*Log[1 + d*f*Sqrt[x]] - d*f*Log[Sqrt[x]])*(
-2*Log[Sqrt[x]] + Log[x])^2 + 4*(-2*Log[Sqrt[x]] + Log[x])*(-(1/Sqrt[x]) -
Log[Sqrt[x]]/Sqrt[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x]] - (d*f*Log[Sqr
t[x]]^2)/2 + d*f*PolyLog[2, -(d*f*Sqrt[x])]) + 4*(-2/Sqrt[x] - (2*Log[Sqrt[
x]])/Sqrt[x] - Log[Sqrt[x]]^2/Sqrt[x] + d*f*Log[1 + d*f*Sqrt[x]]*Log[Sqrt[x
]]^2 - (d*f*Log[Sqrt[x]]^3)/3 + 2*d*f*Log[Sqrt[x]]*PolyLog[2, -(d*f*Sqrt[x]
)] - 2*d*f*PolyLog[3, -(d*f*Sqrt[x])])) + (b^3*d*f*n^3*(1 + 1/(d*f*Sqrt[x])
)*(2*(-(d*f*Sqrt[x]) + d^2*f^2*x*Log[1 + 1/(d*f*Sqrt[x])])*Log[x]^3 - 12*d*
f*Sqrt[x]*Log[x]^2*(1 + d*f*Sqrt[x]*PolyLog[2, -(1/(d*f*Sqrt[x]))]) - 48*d*
f*Sqrt[x]*Log[x]*(1 + d*f*Sqrt[x]*PolyLog[3, -(1/(d*f*Sqrt[x]))]) - 96*d*f*
Sqrt[x]*(1 + d*f*Sqrt[x]*PolyLog[4, -(1/(d*f*Sqrt[x]))])))/(2*(1 + d*f*Sqrt
[x])*Sqrt[x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln \left(c\,x^{n}\right)\right)^{3}\ln \left(d\left(\frac{1}{d}+f\sqrt{x}\,\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="frica
s")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^2, x)
```

3.63 $\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$

Optimal. Leaf size=849

$$-\frac{175b^3dfn^3}{216x^{3/2}}+\frac{45b^3d^2f^2n^3}{16x}-\frac{255b^3d^3f^3n^3}{8\sqrt{x}}+\frac{3}{8}b^3d^4f^4n^3\log\left(1+df\sqrt{x}\right)-\frac{3b^3n^3\log\left(1+df\sqrt{x}\right)}{8x^2}-\frac{3}{16}b^3d^4f^4n^3$$

```
[Out] -175/216*b^3*d*f*n^3/x^(3/2)+45/16*b^3*d^2*f^2*n^3/x-1/6*d*f*(a+b*ln(c*x^n)
)^3/x^(3/2)+1/4*d^2*f^2*(a+b*ln(c*x^n))^3/x-3/8*b^3*n^3*ln(1+d*f*x^(1/2))/x
^2+1/2*d^4*f^4*(a+b*ln(c*x^n))^3*ln(1+d*f*x^(1/2))-1/2*d^3*f^3*(a+b*ln(c*x^
n))^3/x^(1/2)-1/8*d^4*f^4*(a+b*ln(c*x^n))^3-1/2*(a+b*ln(c*x^n))^3*ln(1+d*f*
x^(1/2))/x^2-3/16*b^3*d^4*f^4*n^3*ln(x)+3/16*b^3*d^4*f^4*n^3*ln(x)^2-1/16*d
^4*f^4*(a+b*ln(c*x^n))^4/b/n+3/8*b^3*d^4*f^4*n^3*ln(1+d*f*x^(1/2))-3/4*b^2*
n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2))/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(1+d*
f*x^(1/2))/x^2+3/2*b^3*d^4*f^4*n^3*polylog(2,-d*f*x^(1/2))-6*b^3*d^4*f^4*n^
3*polylog(3,-d*f*x^(1/2))+24*b^3*d^4*f^4*n^3*polylog(4,-d*f*x^(1/2))-255/8*
b^3*d^3*f^3*n^3/x^(1/2)+3/4*b^2*d^4*f^4*n^2*(a+b*ln(c*x^n))*ln(1+d*f*x^(1/2
))+3/4*b*d^4*f^4*n*(a+b*ln(c*x^n))^2*ln(1+d*f*x^(1/2))+3*b^2*d^4*f^4*n^2*(a
+b*ln(c*x^n))*polylog(2,-d*f*x^(1/2))+3*b*d^4*f^4*n*(a+b*ln(c*x^n))^2*polyl
og(2,-d*f*x^(1/2))-12*b^2*d^4*f^4*n^2*(a+b*ln(c*x^n))*polylog(3,-d*f*x^(1/2
))-63/4*b^2*d^3*f^3*n^2*(a+b*ln(c*x^n))/x^(1/2)-15/4*b*d^3*f^3*n*(a+b*ln(c*
x^n))^2/x^(1/2)-37/36*b^2*d*f*n^2*(a+b*ln(c*x^n))/x^(3/2)+21/8*b^2*d^2*f^2*
n^2*(a+b*ln(c*x^n))/x-3/8*b^2*d^4*f^4*n^2*ln(x)*(a+b*ln(c*x^n))-7/12*b*d*f*
n*(a+b*ln(c*x^n))^2/x^(3/2)+9/8*b*d^2*f^2*n*(a+b*ln(c*x^n))^2/x
```

Rubi [A]
time = 0.79, antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2504, 2442, 46, 2424, 2342, 2341, 2423, 2438, 2338, 2421, 6724, 2413, 12, 2339, 30, 2430}

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] (-175*b^3*d*f*n^3)/(216*x^(3/2)) + (45*b^3*d^2*f^2*n^3)/(16*x) - (255*b^3*d
^3*f^3*n^3)/(8*Sqrt[x]) + (3*b^3*d^4*f^4*n^3*Log[1 + d*f*Sqrt[x]])/8 - (3*b
^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*x^2) - (3*b^3*d^4*f^4*n^3*Log[x])/16 + (3*b
^3*d^4*f^4*n^3*Log[x]^2)/16 - (37*b^2*d*f*n^2*(a + b*Log[c*x^n]))/(36*x^(3/
2)) + (21*b^2*d^2*f^2*n^2*(a + b*Log[c*x^n]))/(8*x) - (63*b^2*d^3*f^3*n^2*(
a + b*Log[c*x^n]))/(4*Sqrt[x]) + (3*b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]]*(a
 + b*Log[c*x^n]))/4 - (3*b^2*n^2*Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n]))/(
4*x^2) - (3*b^2*d^4*f^4*n^2*Log[x]*(a + b*Log[c*x^n]))/8 - (7*b*d*f*n*(a +
```

```
b*Log[c*x^n])^2)/(12*x^(3/2)) + (9*b*d^2*f^2*n*(a + b*Log[c*x^n])^2)/(8*x)
- (15*b*d^3*f^3*n*(a + b*Log[c*x^n])^2)/(4*Sqrt[x]) + (3*b*d^4*f^4*n*Log[1
+ d*f*Sqrt[x]]*(a + b*Log[c*x^n])^2)/4 - (3*b*n*Log[1 + d*f*Sqrt[x]]*(a + b
*Log[c*x^n])^2)/(4*x^2) - (d^4*f^4*(a + b*Log[c*x^n])^3)/8 - (d*f*(a + b*Lo
g[c*x^n])^3)/(6*x^(3/2)) + (d^2*f^2*(a + b*Log[c*x^n])^3)/(4*x) - (d^3*f^3*
(a + b*Log[c*x^n])^3)/(2*Sqrt[x]) + (d^4*f^4*Log[1 + d*f*Sqrt[x]]*(a + b*Lo
g[c*x^n])^3)/2 - (Log[1 + d*f*Sqrt[x]]*(a + b*Log[c*x^n])^3)/(2*x^2) - (d^4
*f^4*(a + b*Log[c*x^n])^4)/(16*b*n) + (3*b^3*d^4*f^4*n^3*PolyLog[2, -(d*f*S
qrt[x])])/2 + 3*b^2*d^4*f^4*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(d*f*Sqrt[x]
)] + 3*b*d^4*f^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*Sqrt[x])] - 6*b^3*
d^4*f^4*n^3*PolyLog[3, -(d*f*Sqrt[x])] - 12*b^2*d^4*f^4*n^2*(a + b*Log[c*x^
n])*PolyLog[3, -(d*f*Sqrt[x])] + 24*b^3*d^4*f^4*n^3*PolyLog[4, -(d*f*Sqrt[x
])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
```

```
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{df(a+b\log(cx^n))^3}{6x^{3/2}} + \frac{d^2f^2(a+b\log(cx^n))^3}{4x} - \frac{d^3f^3(a+b\,\mathrm{l}}{2\sqrt{}}$$

$$= -\frac{df(a+b\log(cx^n))^3}{6x^{3/2}} + \frac{d^2f^2(a+b\log(cx^n))^3}{4x} - \frac{d^3f^3(a+b\,\mathrm{l}}{2\sqrt{}}$$

$$= -\frac{7bdfn(a+b\log(cx^n))^2}{12x^{3/2}} + \frac{9bd^2f^2n(a+b\log(cx^n))^2}{8x} - \frac{15bd}{}$$

$$= -\frac{8b^3dfn^3}{27x^{3/2}} + \frac{3b^3d^2f^2n^3}{2x} - \frac{24b^3d^3f^3n^3}{\sqrt{x}} - \frac{4b^2dfn^2(a+b\log(c}{9x^{3/2}}$$

$$= -\frac{14b^3dfn^3}{27x^{3/2}} + \frac{9b^3d^2f^2n^3}{4x} - \frac{30b^3d^3f^3n^3}{\sqrt{x}} - \frac{37b^2dfn^2(a+b\log}{36x^{3/2}}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{37b^2dfn^2(a+b\log}{36x^{3/2}}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} + \frac{3}{16}b^3d^4f^4n^3\log^2(x$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{3b^3n^3\log\left(1+df\right.}{8x^2}$$

$$= -\frac{37b^3dfn^3}{54x^{3/2}} + \frac{21b^3d^2f^2n^3}{8x} - \frac{63b^3d^3f^3n^3}{2\sqrt{x}} - \frac{3b^3n^3\log\left(1+df\right.}{8x^2}$$

$$= -\frac{175b^3dfn^3}{216x^{3/2}} + \frac{45b^3d^2f^2n^3}{16x} - \frac{255b^3d^3f^3n^3}{8\sqrt{x}} + \frac{3}{8}b^3d^4f^4n^3\log($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2009 vs. $2(849) = 1698$.
time = 0.67, size = 2009, normalized size = 2.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(d^(-1) + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] -1/6*(a^3*d*f)/x^(3/2) - (7*a^2*b*d*f*n)/(12*x^(3/2)) - (37*a*b^2*d*f*n^2)/
(36*x^(3/2)) - (175*b^3*d*f*n^3)/(216*x^(3/2)) + (a^3*d^2*f^2)/(4*x) + (9*a
^2*b*d^2*f^2*n)/(8*x) + (21*a*b^2*d^2*f^2*n^2)/(8*x) + (45*b^3*d^2*f^2*n^3)
/(16*x) - (a^3*d^3*f^3)/(2*Sqrt[x]) - (15*a^2*b*d^3*f^3*n)/(4*Sqrt[x]) - (6
3*a*b^2*d^3*f^3*n^2)/(4*Sqrt[x]) - (255*b^3*d^3*f^3*n^3)/(8*Sqrt[x]) + (a^3
*d^4*f^4*Log[1 + d*f*Sqrt[x]])/2 + (3*a^2*b*d^4*f^4*n*Log[1 + d*f*Sqrt[x]])
/4 + (3*a*b^2*d^4*f^4*n^2*Log[1 + d*f*Sqrt[x]])/4 + (3*b^3*d^4*f^4*n^3*Log[
```

```
1 + d*f*Sqrt[x]])/8 - (a^3*Log[1 + d*f*Sqrt[x]])/(2*x^2) - (3*a^2*b*n*Log[1
 + d*f*Sqrt[x]])/(4*x^2) - (3*a*b^2*n^2*Log[1 + d*f*Sqrt[x]])/(4*x^2) - (3*
b^3*n^3*Log[1 + d*f*Sqrt[x]])/(8*x^2) - (a^3*d^4*f^4*Log[x])/4 - (3*a^2*b*d
^4*f^4*n*Log[x])/8 - (3*a*b^2*d^4*f^4*n^2*Log[x])/8 - (3*b^3*d^4*f^4*n^3*Lo
g[x])/16 + (3*a^2*b*d^4*f^4*n*Log[x]^2)/8 + (3*a*b^2*d^4*f^4*n^2*Log[x]^2)/
8 + (3*b^3*d^4*f^4*n^3*Log[x]^2)/16 - (a*b^2*d^4*f^4*n^2*Log[x]^3)/4 - (b^3
*d^4*f^4*n^3*Log[x]^3)/8 + (b^3*d^4*f^4*n^3*Log[1 + 1/(d*f*Sqrt[x])]*Log[x]
^3)/2 - (b^3*d^4*f^4*n^3*Log[1 + d*f*Sqrt[x]]*Log[x]^3)/2 + (b^3*d^4*f^4*n^
3*Log[x]^4)/8 - (a^2*b*d*f*Log[c*x^n])/(2*x^(3/2)) - (7*a*b^2*d*f*n*Log[c*x
^n])/(6*x^(3/2)) - (37*b^3*d*f*n^2*Log[c*x^n])/(36*x^(3/2)) + (3*a^2*b*d^2*
f^2*Log[c*x^n])/(4*x) + (9*a*b^2*d^2*f^2*n*Log[c*x^n])/(4*x) + (21*b^3*d^2*
f^2*n^2*Log[c*x^n])/(8*x) - (3*a^2*b*d^3*f^3*Log[c*x^n])/(2*Sqrt[x]) - (15*
a*b^2*d^3*f^3*n*Log[c*x^n])/(2*Sqrt[x]) - (63*b^3*d^3*f^3*n^2*Log[c*x^n])/(
4*Sqrt[x]) + (3*a^2*b*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/2 + (3*a*b^2
*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/2 + (3*b^3*d^4*f^4*n^2*Log[1 +
d*f*Sqrt[x]]*Log[c*x^n])/4 - (3*a^2*b*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/(2*x
^2) - (3*a*b^2*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n])/(2*x^2) - (3*b^3*n^2*Log[
1 + d*f*Sqrt[x]]*Log[c*x^n])/(4*x^2) - (3*a^2*b*d^4*f^4*Log[x]*Log[c*x^n])/
4 - (3*a*b^2*d^4*f^4*n*Log[x]*Log[c*x^n])/4 - (3*b^3*d^4*f^4*n^2*Log[x]*Log
[c*x^n])/8 + (3*a*b^2*d^4*f^4*n*Log[x]^2*Log[c*x^n])/4 + (3*b^3*d^4*f^4*n^2
*Log[x]^2*Log[c*x^n])/8 - (b^3*d^4*f^4*n^2*Log[x]^3*Log[c*x^n])/4 - (a*b^2*
d*f*Log[c*x^n]^2)/(2*x^(3/2)) - (7*b^3*d*f*n*Log[c*x^n]^2)/(12*x^(3/2)) + (
3*a*b^2*d^2*f^2*Log[c*x^n]^2)/(4*x) + (9*b^3*d^2*f^2*n*Log[c*x^n]^2)/(8*x)
- (3*a*b^2*d^3*f^3*Log[c*x^n]^2)/(2*Sqrt[x]) - (15*b^3*d^3*f^3*n*Log[c*x^n]
^2)/(4*Sqrt[x]) + (3*a*b^2*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2)/2 + (
3*b^3*d^4*f^4*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^2)/4 - (3*a*b^2*Log[1 + d*f
*Sqrt[x]]*Log[c*x^n]^2)/(2*x^2) - (3*b^3*n*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^
2)/(4*x^2) - (3*a*b^2*d^4*f^4*Log[x]*Log[c*x^n]^2)/4 - (3*b^3*d^4*f^4*n*Log
[x]*Log[c*x^n]^2)/8 + (3*b^3*d^4*f^4*n*Log[x]^2*Log[c*x^n]^2)/8 - (b^3*d*f*
Log[c*x^n]^3)/(6*x^(3/2)) + (b^3*d^2*f^2*Log[c*x^n]^3)/(4*x) - (b^3*d^3*f^3
*Log[c*x^n]^3)/(2*Sqrt[x]) + (b^3*d^4*f^4*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3
)/2 - (b^3*Log[1 + d*f*Sqrt[x]]*Log[c*x^n]^3)/(2*x^2) - (b^3*d^4*f^4*Log[x]
*Log[c*x^n]^3)/4 - 3*b^3*d^4*f^4*n^3*Log[x]^2*PolyLog[2, -(1/(d*f*Sqrt[x]))
] + (3*b*d^4*f^4*n*(2*a^2 + 2*a*b*n + b^2*n^2 - 2*b^2*n^2*Log[x]^2 + 2*b*(2
*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -(d*f*Sqrt[x])])/2 -
12*b^3*d^4*f^4*n^3*Log[x]*PolyLog[3, -(1/(d*f*Sqrt[x]))] - 12*a*b^2*d^4*f^4
*n^2*PolyLog[3, -(d*f*Sqrt[x])] - 6*b^3*d^4*f^4*n^3*PolyLog[3, -(d*f*Sqrt[x
])] + 12*b^3*d^4*f^4*n^3*Log[x]*PolyLog[3, -(d*f*Sqrt[x])] - 12*b^3*d^4*f^4
*n^2*Log[c*x^n]*PolyLog[3, -(d*f*Sqrt[x])] - 24*b^3*d^4*f^4*n^3*PolyLog[4,
-(1/(d*f*Sqrt[x]))]
```

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^3 \ln\left(d\left(\frac{1}{d}+f\sqrt{x}\,\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^(1/2)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="frica
s")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + 1)/x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**(1/2)))/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^(1/2)))/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + 1/d)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\sqrt{x}+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3,x)
[Out] int((log(d*(f*x^(1/2) + 1/d))*(a + b*log(c*x^n))^3)/x^3, x)
```

3.64 $\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=137

$$-\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{Li}_3(-dfx^m)}{m^2}-\frac{12b^2n^2(a+b\log(cx^n))^2\operatorname{Li}_4(-dfx^m)}{m^3}+$$

```
[Out] -(a+b*ln(c*x^n))^4*polylog(2,-d*f*x^m)/m+4*b*n*(a+b*ln(c*x^n))^3*polylog(3,
-d*f*x^m)/m^2-12*b^2*n^2*(a+b*ln(c*x^n))^2*polylog(4,-d*f*x^m)/m^3+24*b^3*n
^3*(a+b*ln(c*x^n))*polylog(5,-d*f*x^m)/m^4-24*b^4*n^4*polylog(6,-d*f*x^m)/m
^5
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2421, 2430, 6724}

$$\frac{24b^3n^3\text{PolyLog}(5,-dfx^m)(a+b\log(cx^n))}{m^4}-\frac{12b^2n^2\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))^2}{m^3}+\frac{4bn\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^3}{m^2}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^4}{m}-\frac{24b^4n^4\text{PolyLog}(6,-dfx^m)}{m^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^4*PolyLog[2, -(d*f*x^m)])/m) + (4*b*n*(a + b*Log[c*x^
n])^3*PolyLog[3, -(d*f*x^m)])/m^2 - (12*b^2*n^2*(a + b*Log[c*x^n])^2*PolyLo
g[4, -(d*f*x^m)])/m^3 + (24*b^3*n^3*(a + b*Log[c*x^n])*PolyLog[5, -(d*f*x^m
)])/m^4 - (24*b^4*n^4*PolyLog[6, -(d*f*x^m)])/m^5
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^4\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{(4bn)\int\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{x}}{m}$$
$$= -\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{Li}_3(-df}{m^2}$$
$$= -\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{Li}_3(-df}{m^2}$$
$$= -\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{Li}_3(-df}{m^2}$$
$$= -\frac{(a+b\log(cx^n))^4\operatorname{Li}_2(-dfx^m)}{m}+\frac{4bn(a+b\log(cx^n))^3\operatorname{Li}_3(-df}{m^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1700 vs. $2(137) = 274$.
time = 0.44, size = 1700, normalized size = 12.41

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^4*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] (-2*a^3*b*m*n*Log[x]^3)/3 + (3*a^2*b^2*m*n^2*Log[x]^4)/2 - (6*a*b^3*m*n^3*L
og[x]^5)/5 + (b^4*m*n^4*Log[x]^6)/3 - 2*a^2*b^2*m*n*Log[x]^3*Log[c*x^n] + 3
*a*b^3*m*n^2*Log[x]^4*Log[c*x^n] - (6*b^4*m*n^3*Log[x]^5*Log[c*x^n])/5 - 2*
a*b^3*m*n*Log[x]^3*Log[c*x^n]^2 + (3*b^4*m*n^2*Log[x]^4*Log[c*x^n]^2)/2 - (
2*b^4*m*n*Log[x]^3*Log[c*x^n]^3)/3 - 2*a^3*b*n*Log[x]^2*Log[1 + 1/(d*f*x^m)
] + 4*a^2*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - 3*a*b^3*n^3*Log[x]^4*Log[
1 + 1/(d*f*x^m)] + (4*b^4*n^4*Log[x]^5*Log[1 + 1/(d*f*x^m)])/5 - 6*a^2*b^2*
n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + 8*a*b^3*n^2*Log[x]^3*Log[c*x^n
]*Log[1 + 1/(d*f*x^m)] - 3*b^4*n^3*Log[x]^4*Log[c*x^n]*Log[1 + 1/(d*f*x^m)]
 - 6*a*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] + 4*b^4*n^2*Log[x]^
3*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)] - 2*b^4*n*Log[x]^2*Log[c*x^n]^3*Log[1 +
 1/(d*f*x^m)] + 2*a^3*b*n*Log[x]^2*Log[1 + d*f*x^m] - 4*a^2*b^2*n^2*Log[x]^
3*Log[1 + d*f*x^m] + 3*a*b^3*n^3*Log[x]^4*Log[1 + d*f*x^m] - (4*b^4*n^4*Log
[x]^5*Log[1 + d*f*x^m])/5 + (a^4*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (4*a
^3*b*n*Log[x]*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (6*a^2*b^2*n^2*Log[x]^2
*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (4*a*b^3*n^3*Log[x]^3*Log[-(d*f*x^m)
```

```
]*Log[1 + d*f*x^m])/m + (b^4*n^4*Log[x]^4*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])
/m + 6*a^2*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 8*a*b^3*n^2*Log[x]^
3*Log[c*x^n]*Log[1 + d*f*x^m] + 3*b^4*n^3*Log[x]^4*Log[c*x^n]*Log[1 + d*f*x
^m] + (4*a^3*b*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m - (12*a^2*b^2
*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + (12*a*b^3*n^2*Lo
g[x]^2*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m - (4*b^4*n^3*Log[x]^3
*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + 6*a*b^3*n*Log[x]^2*Log[c*
x^n]^2*Log[1 + d*f*x^m] - 4*b^4*n^2*Log[x]^3*Log[c*x^n]^2*Log[1 + d*f*x^m]
+ (6*a^2*b^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m - (12*a*b^3*n
*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + (6*b^4*n^2*Log[x
]^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + 2*b^4*n*Log[x]^2*Log
[c*x^n]^3*Log[1 + d*f*x^m] + (4*a*b^3*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 +
d*f*x^m])/m - (4*b^4*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 + d*f*x^m]
)/m + (b^4*Log[-(d*f*x^m)]*Log[c*x^n]^4*Log[1 + d*f*x^m])/m + (b*n*Log[x]*(
-(b^3*n^3*Log[x]^3) + 4*b^2*n^2*Log[x]^2*(a + b*Log[c*x^n]) - 6*b*n*Log[x]*
(a + b*Log[c*x^n])^2 + 4*(a + b*Log[c*x^n])^3)*PolyLog[2, -(1/(d*f*x^m))])/
m + ((a - b*n*Log[x] + b*Log[c*x^n])^4*PolyLog[2, 1 + d*f*x^m])/m + (4*a^3*
b*n*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (12*a^2*b^2*n*Log[c*x^n]*PolyLog[3, -
(1/(d*f*x^m))])/m^2 + (12*a*b^3*n*Log[c*x^n]^2*PolyLog[3, -(1/(d*f*x^m))])/
m^2 + (4*b^4*n*Log[c*x^n]^3*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (12*a^2*b^2*n
^2*PolyLog[4, -(1/(d*f*x^m))])/m^3 + (24*a*b^3*n^2*Log[c*x^n]*PolyLog[4, -(
1/(d*f*x^m))])/m^3 + (12*b^4*n^2*Log[c*x^n]^2*PolyLog[4, -(1/(d*f*x^m))])/m
^3 + (24*a*b^3*n^3*PolyLog[5, -(1/(d*f*x^m))])/m^4 + (24*b^4*n^3*Log[c*x^n]
*PolyLog[5, -(1/(d*f*x^m))])/m^4 + (24*b^4*n^4*PolyLog[6, -(1/(d*f*x^m))])/
m^5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 38574, normalized size = 281.56

method	result	size
risch	Expression too large to display	38574

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^4*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*n^4*log(x)^5 + 5*b^4*log(x)*log(x^n)^4 - 5*(b^4*n^3*log(c) + a*b^3
*n^3)*log(x)^4 + 10*(b^4*n^2*log(c)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2)*l
og(x)^3 - 10*(b^4*n*log(x)^2 - 2*(b^4*log(c) + a*b^3)*log(x))*log(x^n)^3 +
10*(b^4*n^2*log(x)^3 - 3*(b^4*n*log(c) + a*b^3*n)*log(x)^2 + 3*(b^4*log(c)^
2 + 2*a*b^3*log(c) + a^2*b^2)*log(x))*log(x^n)^2 - 10*(b^4*n*log(c)^3 + 3*a
*b^3*n*log(c)^2 + 3*a^2*b^2*n*log(c) + a^3*b*n)*log(x)^2 - 5*(b^4*n^3*log(x
)^4 - 4*(b^4*n^2*log(c) + a*b^3*n^2)*log(x)^3 + 6*(b^4*n*log(c)^2 + 2*a*b^3
*n*log(c) + a^2*b^2*n)*log(x)^2 - 4*(b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3*a^
2*b^2*log(c) + a^3*b)*log(x))*log(x^n) + 5*(b^4*log(c)^4 + 4*a*b^3*log(c)^3
 + 6*a^2*b^2*log(c)^2 + 4*a^3*b*log(c) + a^4)*log(x))*log(d*f*x^m + 1) - in
tegrate(1/5*(5*b^4*d*f*m*x^m*log(x)*log(x^n)^4 - 10*(b^4*d*f*m*n*log(x)^2 -
 2*(b^4*d*f*m*log(c) + a*b^3*d*f*m)*log(x))*x^m*log(x^n)^3 + 10*(b^4*d*f*m*
n^2*log(x)^3 - 3*(b^4*d*f*m*n*log(c) + a*b^3*d*f*m*n)*log(x)^2 + 3*(b^4*d*f
*m*log(c)^2 + 2*a*b^3*d*f*m*log(c) + a^2*b^2*d*f*m)*log(x))*x^m*log(x^n)^2
- 5*(b^4*d*f*m*n^3*log(x)^4 - 4*(b^4*d*f*m*n^2*log(c) + a*b^3*d*f*m*n^2)*lo
g(x)^3 + 6*(b^4*d*f*m*n*log(c)^2 + 2*a*b^3*d*f*m*n*log(c) + a^2*b^2*d*f*m*n
)*log(x)^2 - 4*(b^4*d*f*m*log(c)^3 + 3*a*b^3*d*f*m*log(c)^2 + 3*a^2*b^2*d*f
*m*log(c) + a^3*b*d*f*m)*log(x))*x^m*log(x^n) + (b^4*d*f*m*n^4*log(x)^5 - 5
*(b^4*d*f*m*n^3*log(c) + a*b^3*d*f*m*n^3)*log(x)^4 + 10*(b^4*d*f*m*n^2*log(
c)^2 + 2*a*b^3*d*f*m*n^2*log(c) + a^2*b^2*d*f*m*n^2)*log(x)^3 - 10*(b^4*d*f
*m*n*log(c)^3 + 3*a*b^3*d*f*m*n*log(c)^2 + 3*a^2*b^2*d*f*m*n*log(c) + a^3*b
*d*f*m*n)*log(x)^2 + 5*(b^4*d*f*m*log(c)^4 + 4*a*b^3*d*f*m*log(c)^3 + 6*a^2
*b^2*d*f*m*log(c)^2 + 4*a^3*b*d*f*m*log(c) + a^4*d*f*m)*log(x))*x^m)/(d*f*x
*x^m + x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(136) = 272$.
time = 0.36, size = 523, normalized size = 3.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] -(24*b^4*n^4*polylog(6, -d*f*x^m) + (b^4*m^4*n^4*log(x)^4 + b^4*m^4*log(c)^
4 + 4*a*b^3*m^4*log(c)^3 + 6*a^2*b^2*m^4*log(c)^2 + 4*a^3*b*m^4*log(c) + a^
4*m^4 + 4*(b^4*m^4*n^3*log(c) + a*b^3*m^4*n^3)*log(x)^3 + 6*(b^4*m^4*n^2*lo
g(c)^2 + 2*a*b^3*m^4*n^2*log(c) + a^2*b^2*m^4*n^2)*log(x)^2 + 4*(b^4*m^4*n*
log(c)^3 + 3*a*b^3*m^4*n*log(c)^2 + 3*a^2*b^2*m^4*n*log(c) + a^3*b*m^4*n)*l
og(x))*dilog(-d*f*x^m) - 24*(b^4*m*n^4*log(x) + b^4*m*n^3*log(c) + a*b^3*m*
n^3)*polylog(5, -d*f*x^m) + 12*(b^4*m^2*n^4*log(x)^2 + b^4*m^2*n^2*log(c)^2
 + 2*a*b^3*m^2*n^2*log(c) + a^2*b^2*m^2*n^2 + 2*(b^4*m^2*n^3*log(c) + a*b^3
*m^2*n^3)*log(x))*polylog(4, -d*f*x^m) - 4*(b^4*m^3*n^4*log(x)^3 + b^4*m^3*
n*log(c)^3 + 3*a*b^3*m^3*n*log(c)^2 + 3*a^2*b^2*m^3*n*log(c) + a^3*b*m^3*n
+ 3*(b^4*m^3*n^3*log(c) + a*b^3*m^3*n^3)*log(x)^2 + 3*(b^4*m^3*n^2*log(c)^2
```

```
 + 2*a*b^3*m^3*n^2*log(c) + a^2*b^2*m^3*n^2)*log(x))*polylog(3, -d*f*x^m))/
m^5
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**4*ln(d*(1/d+f*x**m))/x,x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^4*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^4*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x,x)
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^4)/x, x)
```

3.65 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=105

$$-\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{m}+\frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_3(-dfx^m)}{m^2}-\frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_4(-dfx^m)}{m^3}+\frac{6b^3}{}$$

```
[Out] -(a+b*ln(c*x^n))^3*polylog(2,-d*f*x^m)/m+3*b*n*(a+b*ln(c*x^n))^2*polylog(3,
-d*f*x^m)/m^2-6*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-d*f*x^m)/m^3+6*b^3*n^3*p
olylog(5,-d*f*x^m)/m^4
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2421, 2430, 6724}

$$-\frac{6b^2n^2\text{PolyLog}(4,-dfx^m)(a+b\log(cx^n))}{m^3}+\frac{3bn\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))^2}{m^2}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^3}{m}+\frac{6b^3n^3\text{PolyLog}(5,-dfx^m)}{m^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^3*PolyLog[2, -(d*f*x^m)])/m) + (3*b*n*(a + b*Log[c*x^
n])^2*PolyLog[3, -(d*f*x^m)])/m^2 - (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4
, -(d*f*x^m)])/m^3 + (6*b^3*n^3*PolyLog[5, -(d*f*x^m)])/m^4
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{m}+\frac{(3bn)\int\frac{(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^{m}}{x}}{m}$$
$$= -\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{m}+\frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_3(-d}{m^2}$$
$$= -\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{m}+\frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_3(-d}{m^2}$$
$$= -\frac{(a+b\log(cx^n))^3\operatorname{Li}_2(-dfx^m)}{m}+\frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_3(-d}{m^2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1035 vs. $2(105) = 210$.
time = 0.24, size = 1035, normalized size = 9.86

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -1/2*(a^2*b*m*n*Log[x]^3) + (3*a*b^2*m*n^2*Log[x]^4)/4 - (3*b^3*m*n^3*Log[x
]^5)/10 - a*b^2*m*n*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*Log[x]^4*Log[c*x^n])
/4 - (b^3*m*n*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*Log[x]^2*Log[1 + 1/(d*f
*x^m)])/2 + 2*a*b^2*n^2*Log[x]^3*Log[1 + 1/(d*f*x^m)] - (3*b^3*n^3*Log[x]^4
*Log[1 + 1/(d*f*x^m)])/4 - 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m
)] + 2*b^3*n^2*Log[x]^3*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] - (3*b^3*n*Log[x]^2
*Log[c*x^n]^2*Log[1 + 1/(d*f*x^m)])/2 + (3*a^2*b*n*Log[x]^2*Log[1 + d*f*x^m
])/2 - 2*a*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m] + (3*b^3*n^3*Log[x]^4*Log[1 +
d*f*x^m])/4 + (a^3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (3*a^2*b*n*Log[x]*
Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + (3*a*b^2*n^2*Log[x]^2*Log[-(d*f*x^m)]
*Log[1 + d*f*x^m])/m - (b^3*n^3*Log[x]^3*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/
m + 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] - 2*b^3*n^2*Log[x]^3*Log
[c*x^n]*Log[1 + d*f*x^m] + (3*a^2*b*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*
x^m])/m - (6*a*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m
+ (3*b^3*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d*f*x^m])/m + (3*b
^3*n*Log[x]^2*Log[c*x^n]^2*Log[1 + d*f*x^m])/2 + (3*a*b^2*Log[-(d*f*x^m)]*L
og[c*x^n]^2*Log[1 + d*f*x^m])/m - (3*b^3*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n
]^2*Log[1 + d*f*x^m])/m + (b^3*Log[-(d*f*x^m)]*Log[c*x^n]^3*Log[1 + d*f*x^m
])/m + (b*n*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*
(a + b*Log[c*x^n])^2)*PolyLog[2, -(1/(d*f*x^m))])/m + ((a - b*n*Log[x] + b*
Log[c*x^n])^3*PolyLog[2, 1 + d*f*x^m])/m + (3*a^2*b*n*PolyLog[3, -(1/(d*f*x
```

```
^m))])/m^2 + (6*a*b^2*n*Log[c*x^n]*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (3*b^3
*n*Log[c*x^n]^2*PolyLog[3, -(1/(d*f*x^m))])/m^2 + (6*a*b^2*n^2*PolyLog[4, -
(1/(d*f*x^m))])/m^3 + (6*b^3*n^2*Log[c*x^n]*PolyLog[4, -(1/(d*f*x^m))])/m^3
 + (6*b^3*n^3*PolyLog[5, -(1/(d*f*x^m))])/m^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 11734, normalized size = 111.75

method	result	size
risch	Expression too large to display	11734

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log(d*f*x^m + 1) - integrate(1/4*(4*b^3*d*f*m*
x^m*log(x)*log(x^n)^3 - 6*(b^3*d*f*m*n*log(x)^2 - 2*(b^3*d*f*m*log(c) + a*b
^2*d*f*m)*log(x))*x^m*log(x^n)^2 + 4*(b^3*d*f*m*n^2*log(x)^3 - 3*(b^3*d*f*m
*n*log(c) + a*b^2*d*f*m*n)*log(x)^2 + 3*(b^3*d*f*m*log(c)^2 + 2*a*b^2*d*f*m
*log(c) + a^2*b*d*f*m)*log(x))*x^m*log(x^n) - (b^3*d*f*m*n^3*log(x)^4 - 4*(
b^3*d*f*m*n^2*log(c) + a*b^2*d*f*m*n^2)*log(x)^3 + 6*(b^3*d*f*m*n*log(c)^2
+ 2*a*b^2*d*f*m*n*log(c) + a^2*b*d*f*m*n)*log(x)^2 - 4*(b^3*d*f*m*log(c)^3
+ 3*a*b^2*d*f*m*log(c)^2 + 3*a^2*b*d*f*m*log(c) + a^3*d*f*m)*log(x))*x^m)/(
d*f*x*x^m + x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(104) = 208.
time = 0.36, size = 285, normalized size = 2.71

$$\frac{6\,b^3n^3\mathrm{polylog}(5,-dfx^m)-\left(b^3m^3n^3\log\left(x\right)^3+b^3m^3\log\left(c\right)^3+3\,ab^2m^3\log\left(c\right)^2+3\,a^2bm^3\log\left(c\right)+a^3m^3+3\left(b^3m^3n^2\log\left(c\right)+ab^2m^3n^2\right)\log\left(x\right)^2+3\left(b^3m^3n\log\left(c\right)^2+2\,ab^2m^3n\log\left(c\right)+a^2bm^3n\right)\log\left(x\right)\right)\mathrm{Li}_2(-dfx^m)-6\left(b^3mn^3\log\left(x\right)+b^3mn^2\log\left(c\right)+ab^2mn^2\right)\mathrm{polylog}(4,-dfx^m)+3\left(b^3m^2n^3\log\left(x\right)^2+b^3m^2n\log\left(c\right)^2+2\,ab^2m^2n\log\left(c\right)+a^2bm^2n+2\left(b^3m^2n^2\log\left(c\right)+ab^2m^2n^2\right)\log\left(x\right)\right)\mathrm{polylog}(3,-dfx^m)}{m^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] (6*b^3*n^3*polylog(5, -d*f*x^m) - (b^3*m^3*n^3*log(x)^3 + b^3*m^3*log(c)^3
+ 3*a*b^2*m^3*log(c)^2 + 3*a^2*b*m^3*log(c) + a^3*m^3 + 3*(b^3*m^3*n^2*log(
c) + a*b^2*m^3*n^2)*log(x)^2 + 3*(b^3*m^3*n*log(c)^2 + 2*a*b^2*m^3*n*log(c)
 + a^2*b*m^3*n)*log(x))*dilog(-d*f*x^m) - 6*(b^3*m*n^3*log(x) + b^3*m*n^2*l
og(c) + a*b^2*m*n^2)*polylog(4, -d*f*x^m) + 3*(b^3*m^2*n^3*log(x)^2 + b^3*m
^2*n*log(c)^2 + 2*a*b^2*m^2*n*log(c) + a^2*b*m^2*n + 2*(b^3*m^2*n^2*log(c)
+ a*b^2*m^2*n^2)*log(x))*polylog(3, -d*f*x^m))/m^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^3)/x, x)
```

3.66 $\int \frac{(a+b\log(cx^n))^2 \log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x} \, dx$

Optimal. Leaf size=73

$$-\frac{(a+b\log(cx^n))^2 \operatorname{Li}_2(-dfx^m)}{m}+\frac{2bn(a+b\log(cx^n))\operatorname{Li}_3(-dfx^m)}{m^2}-\frac{2b^2n^2\operatorname{Li}_4(-dfx^m)}{m^3}$$

```
[Out] -(a+b*ln(c*x^n))^2*polylog(2,-d*f*x^m)/m+2*b*n*(a+b*ln(c*x^n))*polylog(3,-d
*f*x^m)/m^2-2*b^2*n^2*polylog(4,-d*f*x^m)/m^3
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2421, 2430, 6724}

$$\frac{2bn\text{PolyLog}(3,-dfx^m)(a+b\log(cx^n))}{m^2}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))^2}{m}-\frac{2b^2n^2\text{PolyLog}(4,-dfx^m)}{m^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])^2*PolyLog[2, -(d*f*x^m)])/m) + (2*b*n*(a + b*Log[c*x^
n])*PolyLog[3, -(d*f*x^m)])/m^2 - (2*b^2*n^2*PolyLog[4, -(d*f*x^m)])/m^3
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx = -\frac{(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^m)}{m} + \frac{(2bn)\int \frac{(a+b\log(cx^n))\operatorname{Li}_2(-dfx^m}{x}}{m}$$

$$= -\frac{(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^m)}{m} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_3(-df}{m^2}$$

$$= -\frac{(a+b\log(cx^n))^2\operatorname{Li}_2(-dfx^m)}{m} + \frac{2bn(a+b\log(cx^n))\operatorname{Li}_3(-df}{m^2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 526 vs. 2(73) = 146.
time = 0.15, size = 526, normalized size = 7.21

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -1/3*(a*b*m*n*Log[x]^3) + (b^2*m*n^2*Log[x]^4)/4 - (b^2*m*n*Log[x]^3*Log[c*
x^n])/3 - a*b*n*Log[x]^2*Log[1 + 1/(d*f*x^m)] + (2*b^2*n^2*Log[x]^3*Log[1 +
 1/(d*f*x^m)])/3 - b^2*n*Log[x]^2*Log[c*x^n]*Log[1 + 1/(d*f*x^m)] + a*b*n*L
og[x]^2*Log[1 + d*f*x^m] - (2*b^2*n^2*Log[x]^3*Log[1 + d*f*x^m])/3 + (a^2*L
og[-(d*f*x^m)]*Log[1 + d*f*x^m])/m - (2*a*b*n*Log[x]*Log[-(d*f*x^m)]*Log[1
+ d*f*x^m])/m + (b^2*n^2*Log[x]^2*Log[-(d*f*x^m)]*Log[1 + d*f*x^m])/m + b^2
*n*Log[x]^2*Log[c*x^n]*Log[1 + d*f*x^m] + (2*a*b*Log[-(d*f*x^m)]*Log[c*x^n]
*Log[1 + d*f*x^m])/m - (2*b^2*n*Log[x]*Log[-(d*f*x^m)]*Log[c*x^n]*Log[1 + d
*f*x^m])/m + (b^2*Log[-(d*f*x^m)]*Log[c*x^n]^2*Log[1 + d*f*x^m])/m + (b*n*L
og[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]))*PolyLog[2, -(1/(d*f*x^m))])/m
+ ((a - b*n*Log[x] + b*Log[c*x^n])^2*PolyLog[2, 1 + d*f*x^m])/m + (2*a*b*n*
PolyLog[3, -(1/(d*f*x^m))])/m^2 + (2*b^2*n*Log[c*x^n]*PolyLog[3, -(1/(d*f*x
^m))])/m^2 + (2*b^2*n^2*PolyLog[4, -(1/(d*f*x^m))])/m^3
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 2578, normalized size = 35.32

method	result	size
risch	Expression too large to display	2578

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] I/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-2*b^2*n^2*p
olylog(4,-d*f*x^m)/m^3-I*n/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*c)*csg
```

n(I*c*x^n)^2-I*ln(x)*ln(d*(1/d+f*x^m))*ln(x^n)*b^2*Pi*csgn(I*c*x^n)^3-a^2/m
*dilog(d*f*x^m+1)+2*b^2/m*dilog(d*f*x^m+1)*ln(x)*ln(x^n)*n-2*b^2*n/m*ln(x)*
polylog(2,-d*f*x^m)*ln(x^n)-2*b*n/m*ln(x)*polylog(2,-d*f*x^m)*a+2*b/m*dilog
(d*f*x^m+1)*n*ln(x)*a-I/m*dilog(d*f*x^m+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*c*
x^n)^2-I/m*dilog(d*f*x^m+1)*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-I/m*dilog(d*f*
x^m+1)*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-2*n/m*ln(x)*polylog(2,-d*f*
x^m)*b^2*ln(c)+2/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*ln(c)-b^2*n*ln(1/d+f*x^m)*l
n(x)^2*ln(x^n)-1/m*dilog(d*f*x^m+1)*ln(c)^2*b^2-2/m*dilog(d*f*x^m+1)*ln(c)*
a*b+2*b*n/m^2*polylog(3,-d*f*x^m)*a-2*b/m*dilog(d*f*x^m+1)*ln(x^n)*a-2*b*ln
(x)*ln(d*f*x^m+1)*ln(x^n)*a+n*ln(x)^2*ln(d*f*x^m+1)*b^2*ln(c)+2*n/m^2*polyl
og(3,-d*f*x^m)*b^2*ln(c)-2/m*dilog(d*f*x^m+1)*ln(x^n)*b^2*ln(c)-2*ln(x)*ln(
d*f*x^m+1)*ln(x^n)*b^2*ln(c)-n*ln(d*(1/d+f*x^m))*ln(x)^2*b^2*ln(c)+2*ln(x)*
ln(d*(1/d+f*x^m))*ln(x^n)*b^2*ln(c)+b^2*ln(1/d+f*x^m)*ln(x)*ln(x^n)^2+1/3*b
^2/n*ln(d*(1/d+f*x^m))*ln(x^n)^3+1/3*b^2*n^2*ln(1/d+f*x^m)*ln(x)^3-1/3*b^2/
n*ln(1/d+f*x^m)*ln(x^n)^3-b^2/m*dilog(d*f*x^m+1)*ln(x^n)^2-b^2*ln(x)*ln(d*f
*x^m+1)*ln(x^n)^2-1/3*b^2*n^2*ln(x)^3*ln(d*f*x^m+1)-b*n*ln(d*(1/d+f*x^m))*l
n(x)^2*a+2*b*ln(x)*ln(d*(1/d+f*x^m))*ln(x^n)*a+b*n*ln(x)^2*ln(d*f*x^m+1)*a+
b^2*n^2/m*ln(x)^2*polylog(2,-d*f*x^m)-b^2/m*dilog(d*f*x^m+1)*ln(x)^2*n^2+2*
b^2*n/m^2*polylog(3,-d*f*x^m)*ln(x^n)+b^2*ln(x)^2*ln(d*f*x^m+1)*ln(x^n)*n+1
/2*I*n*ln(x)^2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I/m*dilog(d
*f*x^m+1)*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*ln(x)*ln(d*(1/d+f*x^m))*ln(x
^n)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*
csgn(I*c*x^n)^3-I/m*dilog(d*f*x^m+1)*Pi*ln(c)*b^2*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/2*I*n*ln(d*(1/d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4
/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-1/2/m*dilog(d*f*x^
m+1)*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+1/4/m*dilog(d*f*x^m+1)*Pi^2*b^2*csg
n(I*x^n)^2*csgn(I*c*x^n)^4-1/2/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*x^n)*csgn
(I*c*x^n)^5-1/2*I*n*ln(d*(1/d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*c)*csgn(I*c*x^n
)^2+1/2*I*n*ln(x)^2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/4/m*di
log(d*f*x^m+1)*Pi^2*b^2*csgn(I*c*x^n)^6+I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*
csgn(I*c)*csgn(I*c*x^n)^2+I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2+I*ln(x)*ln(d*(1/d+f*x^m))*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x
^n)^2+I*n/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n)^3-I/m*dilog(d*f*
x^m+1)*ln(x^n)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*ln(x)*ln(d*f*x^m+1)*ln(
x^n)*b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2-I*ln(x)*ln(d*f*x^m+1)*ln(x^n)*b^2*Pi*
csgn(I*x^n)*csgn(I*c*x^n)^2-I*n/m^2*polylog(3,-d*f*x^m)*b^2*Pi*csgn(I*c*x^n
)^3+1/2*I*n*ln(d*(1/d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*c*x^n)^3-1/2*I*n*ln(x)^
2*ln(d*f*x^m+1)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*n*ln(d*(1/
d+f*x^m))*ln(x)^2*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*ln(x)*ln(d*(
1/d+f*x^m))*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*n/m^2*poly
log(3,-d*f*x^m)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*ln(x)*ln(d*f*x
^m+1)*ln(x^n)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*n/m*ln(x)*polylo
g(2,-d*f*x^m)*b^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+I/m*dilog(d*f*x^m+1)*ln(x^
n)*b^2*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I/m*dilog(d*f*x^m+1)*n*ln(x)*
b^2*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I/m*dilog(d*f*x^m+1)*Pi*a*b*csgn(I*c*x^n)^

```
3+I/m*dilog(d*f*x^m+1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n)^3-1/2*I*n*ln(x)^2*ln(d*
f*x^m+1)*b^2*Pi*csgn(I*c*x^n)^3+1/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)^4+I*n/m*ln(x)*polylog(2,-d*f*x^m)*b^2*Pi*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)-I/m*dilog(d*f*x^m+1)*n*ln(x)*b^2*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I/m*dilog(d*f*x^m+1)*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)+I/m*dilog(d*f*x^m+1)*Pi*ln(c)*b^2*csgn(I*c*x^n)^3+I/m*dilog(d*f*
x^m+1)*Pi*ln(c)*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*ln(x)*ln(d*f*x^m+
1)*ln(x^n)*b^2*Pi*csgn(I*c*x^n)^3+1/4/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c)
^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-1/2/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c)*
csgn(I*x^n)^2*csgn(I*c*x^n)^3-1/2/m*dilog(d*f*x^m+1)*Pi^2*b^2*csgn(I*c)^2*c
sgn(I*x^n)*csgn(I*c*x^n)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log(d*f*x^m + 1) - integrate(1/3*
(3*b^2*d*f*m*x^m*log(x)*log(x^n)^2 - 3*(b^2*d*f*m*n*log(x)^2 - 2*(b^2*d*f*m
*log(c) + a*b*d*f*m)*log(x))*x^m*log(x^n) + (b^2*d*f*m*n^2*log(x)^3 - 3*(b^
2*d*f*m*n*log(c) + a*b*d*f*m*n)*log(x)^2 + 3*(b^2*d*f*m*log(c)^2 + 2*a*b*d*
f*m*log(c) + a^2*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

Fricas [A]
time = 0.35, size = 131, normalized size = 1.79

$$-\frac{2\,b^2n^2\mathrm{polylog}(4,-dfx^m)+\left(b^2m^2n^2\log\left(x\right)^2+b^2m^2\log\left(c\right)^2+2\,abm^2\log\left(c\right)+a^2m^2+2\left(b^2m^2n\log\left(c\right)+abm^2n\right)\log\left(x\right)\right)\mathrm{Li}_2(-dfx^m)-2\left(b^2mn^2\log\left(x\right)+b^2mn\log\left(c\right)+abmn\right)\mathrm{polylog}(3,-dfx^m)}{m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n^2*polylog(4, -d*f*x^m) + (b^2*m^2*n^2*log(x)^2 + b^2*m^2*log(c)^2
 + 2*a*b*m^2*log(c) + a^2*m^2 + 2*(b^2*m^2*n*log(c) + a*b*m^2*n)*log(x))*di
log(-d*f*x^m) - 2*(b^2*m*n^2*log(x) + b^2*m*n*log(c) + a*b*m*n)*polylog(3,
-d*f*x^m))/m^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n))^2)/x, x)
```

3.67 $\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=40

$$-\frac{(a+b\log(cx^n))\operatorname{Li}_2(-dfx^m)}{m}+\frac{bn\operatorname{Li}_3(-dfx^m)}{m^2}$$

```
[Out] -(a+b*ln(c*x^n))*polylog(2,-d*f*x^m)/m+b*n*polylog(3,-d*f*x^m)/m^2
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2421, 6724}

$$\frac{bn\text{PolyLog}(3,-dfx^m)}{m^2}-\frac{\text{PolyLog}(2,-dfx^m)(a+b\log(cx^n))}{m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -(((a + b*Log[c*x^n])*PolyLog[2, -(d*f*x^m)])/m) + (b*n*PolyLog[3, -(d*f*x^
m)])/m^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x}\,dx &= -\frac{(a+b\log(cx^n))\operatorname{Li}_2(-dfx^m)}{m}+\frac{(bn)\int\frac{\operatorname{Li}_2(-dfx^m)}{x}\,dx}{m}\\ &= -\frac{(a+b\log(cx^n))\operatorname{Li}_2(-dfx^m)}{m}+\frac{bn\operatorname{Li}_3(-dfx^m)}{m^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.30

$$-\frac{a\mathrm{Li}_2(-dfx^m)}{m}-\frac{b\log(cx^n)\,\mathrm{Li}_2(-dfx^m)}{m}+\frac{bn\mathrm{Li}_3(-dfx^m)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(d^(-1) + f*x^m)])/x,x]
```

```
[Out] -((a*PolyLog[2, -(d*f*x^m)])/m) - (b*Log[c*x^n]*PolyLog[2, -(d*f*x^m)])/m +
 (b*n*PolyLog[3, -(d*f*x^m)])/m^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 308, normalized size = 7.70

method	result
risch	$-\frac{b\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)n\ln(x)^2}{2}+b\ln(x)\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)\ln(x^n)+\frac{bn\ln(x)^2\ln(df\,x^m+1)}{2}-\frac{bn\ln(x)\,\mathrm{polylog}(2,-df\,x^m)}{m}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(1/d+f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*ln(d*(1/d+f*x^m))*n*ln(x)^2+b*ln(x)*ln(d*(1/d+f*x^m))*ln(x^n)+1/2*b*
n*ln(x)^2*ln(d*f*x^m+1)-b*n/m*ln(x)*polylog(2,-d*f*x^m)+b*n*polylog(3,-d*f*
x^m)/m^2+b/m*dilog(d*f*x^m+1)*n*ln(x)-b/m*dilog(d*f*x^m+1)*ln(x^n)-b*ln(x)*
ln(d*f*x^m+1)*ln(x^n)+1/2*I/m*dilog(d*f*x^m+1)*b*Pi*csgn(I*c)*csgn(I*x^n)*c
sgn(I*c*x^n)-1/2*I/m*dilog(d*f*x^m+1)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I/
m*dilog(d*f*x^m+1)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I/m*dilog(d*f*x^m+1
)*b*Pi*csgn(I*c*x^n)^3-1/m*dilog(d*f*x^m+1)*b*ln(c)-a/m*dilog(d*f*x^m+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log(d*f
*x^m + 1) - integrate(1/2*(2*b*d*f*m*x^m*log(x)*log(x^n) - (b*d*f*m*n*log(x
)^2 - 2*(b*d*f*m*log(c) + a*d*f*m)*log(x))*x^m)/(d*f*x*x^m + x), x)
```

Fricas [A]
time = 0.38, size = 42, normalized size = 1.05

$$\frac{bn\mathrm{polylog}(3,-dfx^m)-(bmn\log(x)+bm\log(c)+am)\mathrm{Li}_2(-dfx^m)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="fricas")
```

```
[Out] (b*n*polylog(3, -d*f*x^m) - (b*m*n*log(x) + b*m*log(c) + a*m)*dilog(-d*f*x^
m))/m^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(1/d+f*x**m))/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(1/d+f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + 1/d)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(f*x^m + 1/d))*(a + b*log(c*x^n)))/x, x)
```

3.68 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]
```

```
[Out] Defer[Int][Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])),x]
```

```
[Out] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)}{x\left(a+b\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

```
[Out] int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(log(d*f*x^m + 1)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))),x)
[Out] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))), x)
```

3.69 $\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x(a+b\log(cx^n))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Defer[Int][Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d\left(\frac{1}{d}+fx^m\right)\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Integrate[Log[d*(d^(-1) + f*x^m)]/(x*(a + b*Log[c*x^n])^2), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d\left(\frac{1}{d}+f\,x^m\right)\right)}{x\left(a+b\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(ln(d*(1/d+f*x^m))/x/(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] d*f*m*integrate(x^m/((b^2*d*f*n*log(c) + a*b*d*f*n)*x*x^m + (b^2*n*log(c) +
 a*b*n)*x + (b^2*d*f*n*x*x^m + b^2*n*x)*log(x^n)), x) - log(d*f*x^m + 1)/(b
^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log(d*f*x^m + 1)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(1/d+f*x**m))/x/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(1/d+f*x^m))/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((f*x^m + 1/d)*d)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\left(f\,x^m+\frac{1}{d}\right)\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2),x)
[Out] int(log(d*(f*x^m + 1/d))/(x*(a + b*log(c*x^n))^2), x)
```

3.70 $\int x^3(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=283

$$-\frac{5be^3mnx}{16f^3}+\frac{3be^2mnx^2}{32f^2}-\frac{7bemnx^3}{144f}+\frac{1}{32}bmnx^4+\frac{e^3mx(a+b\log(cx^n))}{4f^3}-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b}{12}$$

```
[Out] -5/16*b*e^3*m*n*x/f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*n*x^3/f+1/32*b*m*n
*x^4+1/4*e^3*m*x*(a+b*ln(c*x^n))/f^3-1/8*e^2*m*x^2*(a+b*ln(c*x^n))/f^2+1/12
*e*m*x^3*(a+b*ln(c*x^n))/f-1/16*m*x^4*(a+b*ln(c*x^n))+1/16*b*e^4*m*n*ln(f*x
+e)/f^4+1/4*b*e^4*m*n*ln(-f*x/e)*ln(f*x+e)/f^4-1/4*e^4*m*(a+b*ln(c*x^n))*ln
(f*x+e)/f^4-1/16*b*n*x^4*ln(d*(f*x+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*(f*x+
e)^m)+1/4*b*e^4*m*n*polylog(2,1+f*x/e)/f^4
```

Rubi [A]
time = 0.14, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2442, 45, 2423, 2441, 2352}

$$\frac{be^4mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{4f^4}+\frac{1}{4}x^4(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{e^4m\log(e+fx)(a+b\log(cx^n))}{4f^4}+\frac{e^3mx(a+b\log(cx^n))}{4f^3}-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b\log(cx^n))}{12f}-\frac{1}{16}mx^4(a+b\log(cx^n))-\frac{1}{16}bnx^4\log(d(e+fx)^m)+\frac{be^4mn\log(e+fx)}{16f^4}+\frac{be^4mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{4f^4}-\frac{5be^3mnx}{16f^3}+\frac{3be^2mnx^2}{32f^2}-\frac{7bemnx^3}{144f}+\frac{1}{32}bmnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-5*b*e^3*m*n*x)/(16*f^3) + (3*b*e^2*m*n*x^2)/(32*f^2) - (7*b*e*m*n*x^3)/(1
44*f) + (b*m*n*x^4)/32 + (e^3*m*x*(a + b*Log[c*x^n]))/(4*f^3) - (e^2*m*x^2*
(a + b*Log[c*x^n]))/(8*f^2) + (e*m*x^3*(a + b*Log[c*x^n]))/(12*f) - (m*x^4*
(a + b*Log[c*x^n]))/16 + (b*e^4*m*n*Log[e + f*x])/(16*f^4) + (b*e^4*m*n*Log
[-((f*x)/e)]*Log[e + f*x])/(4*f^4) - (e^4*m*(a + b*Log[c*x^n])*Log[e + f*x]
)/(4*f^4) - (b*n*x^4*Log[d*(e + f*x)^m])/16 + (x^4*(a + b*Log[c*x^n])*Log[d
*(e + f*x)^m])/4 + (b*e^4*m*n*PolyLog[2, 1 + (f*x)/e])/(4*f^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x^3(a+b\log(cx^n))\log\left(d(e+fx)^m\right)\,dx=\frac{e^3mx(a+b\log(cx^n))}{4f^3}-\frac{e^2mx^2(a+b\log(cx^n))}{8f^2}+\frac{emx^3(a+b}{12}$$

$$=-\frac{be^3mnx}{4f^3}+\frac{be^2mnx^2}{16f^2}-\frac{bemnx^3}{36f}+\frac{1}{64}bmnx^4+\frac{e^3mx(a+b\,\mathrm{lo}}{4f^3}$$

$$=-\frac{be^3mnx}{4f^3}+\frac{be^2mnx^2}{16f^2}-\frac{bemnx^3}{36f}+\frac{1}{64}bmnx^4+\frac{e^3mx(a+b\,\mathrm{lo}}{4f^3}$$

$$=-\frac{be^3mnx}{4f^3}+\frac{be^2mnx^2}{16f^2}-\frac{bemnx^3}{36f}+\frac{1}{64}bmnx^4+\frac{e^3mx(a+b\,\mathrm{lo}}{4f^3}$$

$$=-\frac{5be^3mnx}{16f^3}+\frac{3be^2mnx^2}{32f^2}-\frac{7bemnx^3}{144f}+\frac{1}{32}bmnx^4+\frac{e^3mx(a+}{4}$$

Mathematica [A]
time = 0.16, size = 290, normalized size = 1.02

$$-\frac{-72ae^3fmx+90be^3fmnx+36ae^2f^2mx^2-27be^2f^2mnx^2-24aef^3mx^3+14bef^3mnx^3+18af^4mx^4-9bf^4mnx^4+72ae^4m\log(e+fx)-18be^4mn\log(e+fx)-72be^4mn\log(x)\log(e+fx)-72af^4x^4\log\left(d(e+fx)^m\right)+18bf^4nx^4\log\left(d(e+fx)^m\right)+6b\log(cx^n)\left(fmx(-12e^3+6e^2fx-4ef^2x^2+3f^3x^3)+12e^4m\log(e+fx)-12f^4x^4\log\left(d(e+fx)^m\right)\right)+72be^4mn\log(x)\log\left(1+\frac{fx}{e}\right)+72be^4mn\mathrm{Li}_2\left(-\frac{fx}{e}\right)}{288f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] -1/288*(-72*a*e^3*f*m*x + 90*b*e^3*f*m*n*x + 36*a*e^2*f^2*m*x^2 - 27*b*e^2*
f^2*m*n*x^2 - 24*a*e*f^3*m*x^3 + 14*b*e*f^3*m*n*x^3 + 18*a*f^4*m*x^4 - 9*b*
f^4*m*n*x^4 + 72*a*e^4*m*Log[e + f*x] - 18*b*e^4*m*n*Log[e + f*x] - 72*b*e^
4*m*n*Log[x]*Log[e + f*x] - 72*a*f^4*x^4*Log[d*(e + f*x)^m] + 18*b*f^4*n*x^
4*Log[d*(e + f*x)^m] + 6*b*Log[c*x^n]*(f*m*x*(-12*e^3 + 6*e^2*f*x - 4*e*f^2
*x^2 + 3*f^3*x^3) + 12*e^4*m*Log[e + f*x] - 12*f^4*x^4*Log[d*(e + f*x)^m])
+ 72*b*e^4*m*n*Log[x]*Log[1 + (f*x)/e] + 72*b*e^4*m*n*PolyLog[2, -((f*x)/e)
])/f^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 2403, normalized size = 8.49

method	result	size
risch	Expression too large to display	2403

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*x^4*ln(x^n)-1/8*I*x^4*Pi*a*csgn(
I*d*(f*x+e)^m)^3-1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*b*csgn(I*c*x^n)^3-1/16
*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4
*b*e^4*m*n*ln(-f*x/e)*ln(f*x+e)/f^4+1/16*b*e^4*m*n*ln(f*x+e)/f^4+1/32*b*m*n
*x^4+1/8*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*x^4*ln(x^n)-1/16*Pi
^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*c*x^n)^3-1/
8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I/f^3*Pi*b*e^3
*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x+1/8*I/f^3*Pi*b*e^3*m*csgn(I*c)*csgn(I*c*x^
n)^2*x+1/8*I*x^4*ln(c)*Pi*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/8*I*x
^4*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-205/576*b*e^4*m*n/f
^4+(1/4*x^4*b*ln(x^n)+1/16*x^4*(-2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)+2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-
2*I*b*Pi*csgn(I*c*x^n)^3+4*b*ln(c)-b*n+4*a))*ln((f*x+e)^m)+1/24*I/f*Pi*x^3*
b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/16*I/f^2*Pi*x^2*b*e^2*m*csgn(I*c)*csgn(
I*c*x^n)^2-1/16*I/f^2*Pi*x^2*b*e^2*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/32*I*x^4
*Pi*b*m*csgn(I*c)*csgn(I*c*x^n)^2-1/32*I*x^4*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^
n)^2+1/12/f*x^3*a*e*m-1/8/f^2*x^2*a*e^2*m+1/4/f^3*a*e^3*m*x-1/4/f^4*e^4*m*l
n(f*x+e)*a-1/16*x^4*a*m-1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x
^4*b*csgn(I*c)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4
*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*x^4*ln(d)*a+1/12*m/f*b*ln(x^n)*e
*x^3-1/8*m/f^2*b*ln(x^n)*x^2*e^2+1/4*m/f^3*b*ln(x^n)*x*e^3-1/4*m/f^4*b*ln(x
^n)*e^4*ln(f*x+e)+1/4/f^3*ln(c)*b*e^3*m*x+1/12/f*ln(c)*x^3*b*e*m-1/8/f^2*ln
(c)*x^2*b*e^2*m-1/4/f^4*e^4*m*ln(f*x+e)*b*ln(c)-1/24*I/f*Pi*x^3*b*e*m*csgn(
I*c*x^n)^3+1/32*I*x^4*Pi*b*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/16*I/f^2
*Pi*x^2*b*e^2*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I/f^3*Pi*b*e^3*m*cs
```

```
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x+1/8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b
*csgn(I*c*x^n)^3+1/8*I*x^4*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*I*x^4*P
i*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/8*I*x^4*ln(d)*Pi*b*csgn(I*c*x
^n)^3-1/8*I*x^4*ln(d)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*x^4*ln
(c)*Pi*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/8*I/f^3*Pi*b*e^3
*m*csgn(I*c*x^n)^3*x+1/8*I/f^4*e^4*m*ln(f*x+e)*Pi*b*csgn(I*c*x^n)^3+1/24*I/
f*Pi*x^3*b*e*m*csgn(I*c)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x+e)
^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I/f^4
*e^4*m*ln(f*x+e)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*(f*x+e)^m)
*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*Pi*c
sgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*x^4*ln(x^n)+1/16*Pi^2*csgn
(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*Pi^2*csgn(I*
d*(f*x+e)^m)^3*x^4*b*csgn(I*c)*csgn(I*c*x^n)^2+1/4*x^4*ln(d)*ln(c)*b-1/16*x
^4*ln(c)*b*m-1/16*ln(d)*b*n*x^4-1/24*I/f*Pi*x^3*b*e*m*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)+1/8*I*x^4*ln(d)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/32*I*Pi*b*n
*x^4*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/32*I*Pi*b*n*x^4*csgn(I*(f*x+e)^m)*cs
gn(I*d*(f*x+e)^m)^2-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*
c)*csgn(I*c*x^n)^2-5/16*b*e^3*m*n*x/f^3+3/32*b*e^2*m*n*x^2/f^2-7/144*b*e*m*
n*x^3/f-1/16*m*b*ln(x^n)*x^4+1/4*ln(d)*b*x^4*ln(x^n)+1/16*Pi^2*csgn(I*d)*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*c)*csgn(I*c*x^n)^2+1/16*Pi
^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^4*b*csgn(I*x^n)*csgn(I
*c*x^n)^2+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^4*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)+1/16*I/f^2*Pi*x^2*b*e^2*m*csgn(I*c*x^n)^3+1/32*I*Pi*b*n*
x^4*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/8*I*x^4*ln(d)*Pi*b*cs
gn(I*x^n)*csgn(I*c*x^n)^2+1/8*I*x^4*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x+e)^m
)^2+1/4*n*b/f^4*e^4*m*dilog(-f*x/e)+1/16*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^4*b*c
sgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*x^4*ln(c)*Pi*b*csgn(I*d*(f*x+e)^m)^3+1/32*
I*Pi*b*n*x^4*csgn(I*d*(f*x+e)^m)^3+1/32*I*x^4*Pi*b*m*csgn(I*c*x^n)^3-1/8*I*
Pi*csgn(I*d*(f*x+e)^m)^3*b*x^4*ln(x^n)
```

Maxima [A]
time = 0.39, size = 356, normalized size = 1.26

$$-\frac{(\log(fxe^{(-1)}+1)\log(x)+\mathrm{Li}_2(-fxe^{(-1)}))bmne^4}{4f^4}+\frac{((mn-4m\log(c))b-4am)e^4\log(fx+e)}{16f^4}+\frac{72bmne^4\log(fx+e)\log(x)-9(2(f^4m-4f^4\log(d))a-(f^4mn-2f^4n\log(d)-2(f^4m-4f^4\log(d))\log(c))b)x^4+2(12af^3m-(7f^3mn-12f^3m\log(c))b)x^3e-9(4af^2m-(3f^2mn-4f^2m\log(c))b)x^2e^2+18(4afm-(5fmn-4fm\log(c))b)xe^3+18(4bf^4x^4\log(x^n)+(4af^4-(f^4n-4f^4\log(c))b)x^4)\log((fx+e)^m)+6(4bf^3mx^3e-6bf^2mx^2e^2-3(f^4m-4f^4\log(d))bx^4+12bfmxe^3-12bme^4\log(fx+e))\log(x^n)}{288f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] -1/4*(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*m*n*e^4/f^4 + 1/16
*((m*n - 4*m*log(c))*b - 4*a*m)*e^4*log(f*x + e)/f^4 + 1/288*(72*b*m*n*e^4*
log(f*x + e)*log(x) - 9*(2*(f^4*m - 4*f^4*log(d))*a - (f^4*m*n - 2*f^4*n*lo
g(d) - 2*(f^4*m - 4*f^4*log(d))*log(c))*b)*x^4 + 2*(12*a*f^3*m - (7*f^3*m*n
 - 12*f^3*m*log(c))*b)*x^3*e - 9*(4*a*f^2*m - (3*f^2*m*n - 4*f^2*m*log(c))*
b)*x^2*e^2 + 18*(4*a*f*m - (5*f*m*n - 4*f*m*log(c))*b)*x*e^3 + 18*(4*b*f^4*
```

```
x^4*log(x^n) + (4*a*f^4 - (f^4*n - 4*f^4*log(c))*b)*x^4)*log((f*x + e)^m) +
 6*(4*b*f^3*m*x^3*e - 6*b*f^2*m*x^2*e^2 - 3*(f^4*m - 4*f^4*log(d))*b*x^4 +
12*b*f*m*x*e^3 - 12*b*m*e^4*log(f*x + e))*log(x^n))/f^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
[Out] integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^3*log((f*x + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
[Out] int(x^3*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

3.71 $\int x^2(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=243

$$\frac{4be^2mnx}{9f^2}-\frac{5bemnx^2}{36f}+\frac{2}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}-\frac{1}{9}mx^3(a+b\log(cx^n))-\frac{b}{}$$

```
[Out] 4/9*b*e^2*m*n*x/f^2-5/36*b*e*m*n*x^2/f+2/27*b*m*n*x^3-1/3*e^2*m*x*(a+b*ln(c
*x^n))/f^2+1/6*e*m*x^2*(a+b*ln(c*x^n))/f-1/9*m*x^3*(a+b*ln(c*x^n))-1/9*b*e^
3*m*n*ln(f*x+e)/f^3-1/3*b*e^3*m*n*ln(-f*x/e)*ln(f*x+e)/f^3+1/3*e^3*m*(a+b*l
n(c*x^n))*ln(f*x+e)/f^3-1/9*b*n*x^3*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x^n))
*ln(d*(f*x+e)^m)-1/3*b*e^3*m*n*polylog(2,1+f*x/e)/f^3
```

Rubi [A]
time = 0.12, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2442, 45, 2423, 2441, 2352}

$$-\frac{be^3mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3f^3}+\frac{1}{3}x^3(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{e^3m\log(e+fx)(a+b\log(cx^n))}{3f^3}-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}-\frac{1}{9}mx^3(a+b\log(cx^n))-\frac{1}{9}bnx^3\log(d(e+fx)^m)-\frac{be^3mn\log(e+fx)}{9f^3}-\frac{be^3mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{3f^3}+\frac{4be^2mnx}{9f^2}-\frac{5bemnx^2}{36f}+\frac{2}{27}bmnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*b*e^2*m*n*x)/(9*f^2) - (5*b*e*m*n*x^2)/(36*f) + (2*b*m*n*x^3)/27 - (e^2*
m*x*(a + b*Log[c*x^n]))/(3*f^2) + (e*m*x^2*(a + b*Log[c*x^n]))/(6*f) - (m*x
^3*(a + b*Log[c*x^n]))/9 - (b*e^3*m*n*Log[e + f*x])/(9*f^3) - (b*e^3*m*n*Lo
g[-((f*x)/e)]*Log[e + f*x])/(3*f^3) + (e^3*m*(a + b*Log[c*x^n])*Log[e + f*x
])/(3*f^3) - (b*n*x^3*Log[d*(e + f*x)^m])/9 + (x^3*(a + b*Log[c*x^n])*Log[d
*(e + f*x)^m])/3 - (b*e^3*m*n*PolyLog[2, 1 + (f*x)/e])/(3*f^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x^2(a+b\log(cx^n))\log\left(d(e+fx)^m\right)\,dx &= -\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{emx^2(a+b\log(cx^n))}{6f}-\frac{1}{9}mx^3(a+b\\ &= \frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{em}{}\\ &= \frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{em}{}\\ &= \frac{be^2mnx}{3f^2}-\frac{bemnx^2}{12f}+\frac{1}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{em}{}\\ &= \frac{4be^2mnx}{9f^2}-\frac{5bemnx^2}{36f}+\frac{2}{27}bmnx^3-\frac{e^2mx(a+b\log(cx^n))}{3f^2}+\frac{e}{}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 252, normalized size = 1.04

$$\frac{-36ae^2fmx+48be^2fmnx+18aef^2mx^2-15bef^2mnx^2-12af^3mx^3+8bf^3mnx^3+36ae^3m\log(e+fx)-12be^3mn\log(e+fx)-36be^3mn\log(x)\log(e+fx)+36af^3x^3\log\left(d(e+fx)^m\right)-12bf^3nx^3\log\left(d(e+fx)^m\right)-6b\log(cx^n)\left(fmx(6e^2-3efx+2f^2x^2)-6e^3m\log(e+fx)-6f^3x^3\log\left(d(e+fx)^m\right)\right)+36be^3mn\log(x)\log\left(1+\frac{fx}{e}\right)+36be^3mn\text{Li}_2\left(-\frac{fx}{e}\right)}{108f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-36*a*e^2*f*m*x + 48*b*e^2*f*m*n*x + 18*a*e*f^2*m*x^2 - 15*b*e*f^2*m*n*x^2
 - 12*a*f^3*m*x^3 + 8*b*f^3*m*n*x^3 + 36*a*e^3*m*Log[e + f*x] - 12*b*e^3*m*
n*Log[e + f*x] - 36*b*e^3*m*n*Log[x]*Log[e + f*x] + 36*a*f^3*x^3*Log[d*(e +
 f*x)^m] - 12*b*f^3*n*x^3*Log[d*(e + f*x)^m] - 6*b*Log[c*x^n]*(f*m*x*(6*e^2
 - 3*e*f*x + 2*f^2*x^2) - 6*e^3*m*Log[e + f*x] - 6*f^3*x^3*Log[d*(e + f*x)^
m]) + 36*b*e^3*m*n*Log[x]*Log[1 + (f*x)/e] + 36*b*e^3*m*n*PolyLog[2, -((f*x
)/e)])/(108*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 2222, normalized size = 9.14

method	result	size
risch	Expression too large to display	2222

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b*e^3*m*n*ln(-f*x/e)*ln(f*x+e)/f^3-1/9*b*e^3*m*n*ln(f*x+e)/f^3+2/27*b*
m*n*x^3-1/18*I*Pi*b*n*x^3*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/12*Pi^2
*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^3*b*csgn(I*c*x^n)^3-1/12
*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*c)*csgn(I*c*x^n)^2+1/3*x
^3*ln(d)*a+1/3/f^3*e^3*m*ln(f*x+e)*a-1/3/f^2*a*e^2*m*x-1/12*Pi^2*csgn(I*d)*
csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*c)*csgn(I*c*x^n)^2-1/12*Pi^2*cs
gn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6
/f*x^2*a*e*m+1/6/f*ln(c)*x^2*b*e*m+1/3/f^3*e^3*m*ln(f*x+e)*b*ln(c)-1/3/f^2*
ln(c)*b*e^2*m*x-1/9*x^3*a*m+49/108*b*e^3*m*n/f^3-1/12*Pi^2*csgn(I*d)*csgn(I
*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1
/6*I/f^2*Pi*b*e^2*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x+1/6*I/f^3*e^3*m*ln(f*x+e)
*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+1/3*x^3*ln(d)*ln(c)*b-1/9*x^3*ln(c)*b*m-1/9
*ln(d)*b*n*x^3-1/6*I*x^3*Pi*ln(c)*b*csgn(I*d*(f*x+e)^m)^3+1/6*I*x^3*Pi*a*cs
gn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/18*I*x^3*Pi*b*m*csgn(I*c)*csgn(I*c*x^n)^2-1
/18*I*x^3*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/18*I*Pi*b*n*x^3*csgn(I*d)*cs
gn(I*d*(f*x+e)^m)^2+1/6*I/f^2*Pi*b*e^2*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)*x-1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/12
*I/f*Pi*x^2*b*e*m*csgn(I*c*x^n)^3-1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*c*x
^n)^3+1/12*I/f*Pi*x^2*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I/f^2*Pi*b*e^2*
m*csgn(I*c)*csgn(I*c*x^n)^2*x+1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^
m)^2*x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*c)*
csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*x^n)*csgn(I*c*
x^n)^2+1/6*I/f^3*e^3*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*I/f*
Pi*x^2*b*e*m*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I*x^3*Pi*ln(d)*b*csgn(I*x^n)*csg
n(I*c*x^n)^2+1/6*I*x^3*Pi*ln(c)*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/6*I*x^3
*Pi*ln(c)*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+(1/3*x^3*b*ln(x^n)+1/18
*x^3*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn
```

```
(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6
*b*ln(c)-2*b*n+6*a))*ln((f*x+e)^m)+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*cs
gn(I*d*(f*x+e)^m)*x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/18*I*Pi*b*n*x^3*csgn(
I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/18*I*x^3*Pi*b*m*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+1/6*I/f^2*Pi*b*e^2*m*csgn(I*c*x^n)^3*x-1/3*n*b/f^3*e^
3*m*dilog(-f*x/e)-1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^3*b*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)-1/6*I*x^3*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)+1/6*I*x^3*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^2-1/6*I*x^3*Pi*ln(d)*b*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/6*I*x^3*Pi*ln(c)*b*csgn(I*d)*csgn(I*(f*
x+e)^m)*csgn(I*d*(f*x+e)^m)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^3*b
*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(
f*x+e)^m)^2*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/12*Pi^2*csgn(I*d)*c
sgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^3*b*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I*
Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*x^3*ln(x^n)+1/6*I*Pi*csgn(I*(f*x+e)^m)
*csgn(I*d*(f*x+e)^m)^2*b*x^3*ln(x^n)-1/6*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*x^3*l
n(x^n)+1/18*I*Pi*b*n*x^3*csgn(I*d*(f*x+e)^m)^3+1/6*I*x^3*Pi*a*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)^2+1/18*I*x^3*Pi*b*m*csgn(I*c*x^n)^3-1/6*I*x^3*Pi*l
n(d)*b*csgn(I*c*x^n)^3-1/6*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e
)^m)*b*x^3*ln(x^n)-1/6*I*x^3*Pi*a*csgn(I*d*(f*x+e)^m)^3-1/12*Pi^2*csgn(I*d*
(f*x+e)^m)^3*x^3*b*csgn(I*c*x^n)^3+4/9*b*e^2*m*n*x/f^2-5/36*b*e*m*n*x^2/f-1
/9*m*b*ln(x^n)*x^3+1/3*ln(d)*b*x^3*ln(x^n)-1/12*I/f*Pi*x^2*b*e*m*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^3*b*c
sgn(I*c*x^n)^3+1/3*m/f^3*b*ln(x^n)*e^3*ln(f*x+e)+1/6*m/f*b*ln(x^n)*e*x^2-1/
3*m/f^2*b*ln(x^n)*x*e^2
```

Maxima [A]
time = 0.39, size = 311, normalized size = 1.28

$$\frac{\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bmne^3}{3\,f^3}-\frac{\left((mn-3\,m\log\left(c\right))b-3\,am\right)e^3\log\left(fx+e\right)}{9\,f^3}-\frac{36\,bmne^3\log\left(fx+e\right)\log\left(x\right)+4\left(3\left(f^3m-3\,f^3\log\left(d\right)\right)a-\left(2\,f^3mn-3\,f^3n\log\left(d\right)-3\left(f^3m-3\,f^3\log\left(d\right)\right)\log\left(c\right)\right)b\right)x^3-3\left(6\,af^2m-\left(5\,f^2mn-6\,f^2m\log\left(c\right)\right)b\right)x^2e+12\left(3\,afm-\left(4\,fmn-3\,fm\log\left(c\right)\right)b\right)xe^2-12\left(3\,bf^3x^3\log\left(x^n\right)+\left(3\,af^3-\left(f^3n-3\,f^3\log\left(c\right)\right)b\right)x^3\right)\log\left(\left(fx+e\right)^m\right)-6\left(3\,bf^2mx^2e-2\left(f^3m-3\,f^3\log\left(d\right)\right)bx^3-6\,bfmxe^2+6\,bme^3\log\left(fx+e\right)\right)\log\left(x^n\right)}{108\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/3*(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*m*n*e^3/f^3 - 1/9*(
(m*n - 3*m*log(c))*b - 3*a*m)*e^3*log(f*x + e)/f^3 - 1/108*(36*b*m*n*e^3*lo
g(f*x + e)*log(x) + 4*(3*(f^3*m - 3*f^3*log(d))*a - (2*f^3*m*n - 3*f^3*n*lo
g(d) - 3*(f^3*m - 3*f^3*log(d))*log(c))*b)*x^3 - 3*(6*a*f^2*m - (5*f^2*m*n
- 6*f^2*m*log(c))*b)*x^2*e + 12*(3*a*f*m - (4*f*m*n - 3*f*m*log(c))*b)*x*e^
2 - 12*(3*b*f^3*x^3*log(x^n) + (3*a*f^3 - (f^3*n - 3*f^3*log(c))*b)*x^3)*lo
g((f*x + e)^m) - 6*(3*b*f^2*m*x^2*e - 2*(f^3*m - 3*f^3*log(d))*b*x^3 - 6*b*
f*m*x*e^2 + 6*b*m*e^3*log(f*x + e))*log(x^n))/f^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

3.72 $\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=203

$$-\frac{3bemnx}{4f}+\frac{1}{4}bmnx^2+\frac{emx(a+b\log(cx^n))}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))+\frac{be^2mn\log(e+fx)}{4f^2}+\frac{be^2mn\log\left(-\frac{fx}{e}\right)\text{l}}{2f^2}$$

```
[Out] -3/4*b*e*m*n*x/f+1/4*b*m*n*x^2+1/2*e*m*x*(a+b*ln(c*x^n))/f-1/4*m*x^2*(a+b*l
n(c*x^n))+1/4*b*e^2*m*n*ln(f*x+e)/f^2+1/2*b*e^2*m*n*ln(-f*x/e)*ln(f*x+e)/f^
2-1/2*e^2*m*(a+b*ln(c*x^n))*ln(f*x+e)/f^2-1/4*b*n*x^2*ln(d*(f*x+e)^m)+1/2*x
^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*polylog(2,1+f*x/e)/f^2
```

Rubi [A]

time = 0.09, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2442, 45, 2423, 2441, 2352}

$$\frac{be^2mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2f^2}+\frac{1}{2}x^2(a+b\log(cx^n))\log(d(e+fx)^m)-\frac{e^2m\log(e+fx)(a+b\log(cx^n))}{2f^2}+\frac{emx(a+b\log(cx^n))}{2f}-\frac{1}{4}mx^2(a+b\log(cx^n))-\frac{1}{4}bnx^2\log(d(e+fx)^m)+\frac{be^2mn\log(e+fx)}{4f^2}+\frac{be^2mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{2f^2}-\frac{3bemnx}{4f}+\frac{1}{4}bmnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-3*b*e*m*n*x)/(4*f) + (b*m*n*x^2)/4 + (e*m*x*(a + b*Log[c*x^n]))/(2*f) - (
m*x^2*(a + b*Log[c*x^n]))/4 + (b*e^2*m*n*Log[e + f*x])/(4*f^2) + (b*e^2*m*n
*Log[-((f*x)/e)]*Log[e + f*x])/(2*f^2) - (e^2*m*(a + b*Log[c*x^n])*Log[e +
f*x])/(2*f^2) - (b*n*x^2*Log[d*(e + f*x)^m])/4 + (x^2*(a + b*Log[c*x^n])*Lo
g[d*(e + f*x)^m])/2 + (b*e^2*m*n*PolyLog[2, 1 + (f*x)/e])/(2*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x(a+b\log(cx^n))\log(d(e+fx)^m)\,dx = \frac{emx(a+b\log(cx^n))}{2f} - \frac{1}{4}mx^2(a+b\log(cx^n)) - \frac{e^2m(a+b\log$$

$$= -\frac{bemnx}{2f} + \frac{1}{8}bmnx^2 + \frac{emx(a+b\log(cx^n))}{2f} - \frac{1}{4}mx^2(a+b\log$$

$$= -\frac{bemnx}{2f} + \frac{1}{8}bmnx^2 + \frac{emx(a+b\log(cx^n))}{2f} - \frac{1}{4}mx^2(a+b\log$$

$$= -\frac{bemnx}{2f} + \frac{1}{8}bmnx^2 + \frac{emx(a+b\log(cx^n))}{2f} - \frac{1}{4}mx^2(a+b\log$$

$$= -\frac{3bemnx}{4f} + \frac{1}{4}bmnx^2 + \frac{emx(a+b\log(cx^n))}{2f} - \frac{1}{4}mx^2(a+b\log$$

Mathematica [A]
time = 0.09, size = 208, normalized size = 1.02

$$\frac{2aefmx - 3befmnx - af^2mx^2 + bf^2mnx^2 - 2ae^2m\log(e+fx) + be^2mn\log(e+fx) + 2be^2mn\log(x)\log(e+fx) + 2af^2x^2\log\left(d(e+fx)^m\right) - bf^2nx^2\log\left(d(e+fx)^m\right) + b\log\left(cx^n\right)\left(-2e^2m\log(e+fx) + fx\left(2em - fmx + 2fx\log\left(d(e+fx)^m\right)\right)\right) - 2be^2mn\log(x)\log\left(1+\frac{fx}{e}\right) - 2be^2mn\text{Li}_2\left(-\frac{fx}{e}\right)}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (2*a*e*f*m*x - 3*b*e*f*m*n*x - a*f^2*m*x^2 + b*f^2*m*n*x^2 - 2*a*e^2*m*Log[
e + f*x] + b*e^2*m*n*Log[e + f*x] + 2*b*e^2*m*n*Log[x]*Log[e + f*x] + 2*a*f
```

```
^2*x^2*Log[d*(e + f*x)^m] - b*f^2*n*x^2*Log[d*(e + f*x)^m] + b*Log[c*x^n]*(
-2*e^2*m*Log[e + f*x] + f*x*(2*e*m - f*m*x + 2*f*x*Log[d*(e + f*x)^m])) - 2
*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b*e^2*m*n*PolyLog[2, -((f*x)/e)])/(4
*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 2041, normalized size = 10.05

method	result	size
risch	Expression too large to display	2041

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/4*I*x^2*Pi*a*csgn(I*d)*
csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/2*b*e^2*m*n*ln(-f*x/e)*ln(f*x+e)/f^
2+1/4*b*e^2*m*n*ln(f*x+e)/f^2+1/4*b*m*n*x^2-1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn
(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d*(f*x+
e)^m)^3*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-5/8*b*e^2*m*n/f^2-1/2*m*a
*e^2/f^2*ln(f*x+e)+1/2*a*e*m/f*x-1/8*I*x^2*Pi*b*m*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/4*x^2*a*m+1/8*I*Pi*b*n*x^2*csgn(I*d*(f*x+e)^m)^3-1/4*I*x^2*Pi*ln(c)*b*
csgn(I*d*(f*x+e)^m)^3-1/4*I*x^2*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/8*I*x^2*Pi*b*m
*csgn(I*c*x^n)^3+1/4*I*x^2*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/2*x^2*ln(
d)*a-1/4*I*x^2*Pi*a*csgn(I*d*(f*x+e)^m)^3-1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^
2*b*csgn(I*c*x^n)^3+(1/2*x^2*b*ln(x^n)+1/4*x^2*(-I*b*Pi*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I
*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)-b*n+2*a))*ln((f*x+e)^m)+1/4*I/f*
Pi*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x-1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(
I*c)*csgn(I*c*x^n)^2-1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^
n)^2-1/4*ln(d)*b*n*x^2+1/2*x^2*ln(c)*ln(d)*b-1/4*x^2*ln(c)*b*m-1/8*I*x^2*Pi
*b*m*csgn(I*c)*csgn(I*c*x^n)^2-1/8*I*Pi*b*n*x^2*csgn(I*(f*x+e)^m)*csgn(I*d*
(f*x+e)^m)^2+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^2*b
*csgn(I*c)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f
*x+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I/f*
Pi*b*e*m*csgn(I*c)*csgn(I*c*x^n)^2*x+1/4*I*x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(
I*c*x^n)^2+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b*x^2*ln(x^n)+1/4*I*Pi*
csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b*x^2*ln(x^n)-1/2/f^2*e^2*m*ln(f*x+
e)*b*ln(c)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)-1/4*I*x^2*Pi*ln(c)*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*
d*(f*x+e)^m)-1/4*I*x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/8*I
*x^2*Pi*b*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*n*b/f^2*e^2*m*dilog(-f*
x/e)+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c*x^n)^3
+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*Pi^2*cs
gn(I*d*(f*x+e)^m)^3*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d)*cs
```

```
gn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3*b*
x^2*ln(x^n)+1/4*I*x^2*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/2*m/f*
b*ln(x^n)*x*e+1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)-1/4*I/f*Pi*b*e*m*csgn(I*c*x^n)^3*x+1/4*I/f^2*e^2*m*ln(f*x+e)*Pi*b*csgn(
I*c*x^n)^3-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b*x^2*l
n(x^n)-3/4*b*e*m*n*x/f+1/4*I*x^2*Pi*ln(c)*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2
+1/4*I*x^2*Pi*ln(c)*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/4*I*x^2*Pi*
ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*m*b*ln(x^n)*x^2+1/2*ln(d)*b*x^2*ln(x^
n)-1/2*m/f^2*b*ln(x^n)*e^2*ln(f*x+e)+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f
*x+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2/f*ln(c)*b*e*m*x-1/
8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*x^2*b*csgn(I*c*x^n)^
3-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2-
1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*c*x
^n)^2-1/4*I/f*Pi*b*e*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x+1/4*I/f^2*e^2*
m*ln(f*x+e)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
```

Maxima [A]
time = 0.39, size = 260, normalized size = 1.28

$$-\frac{\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bmne^2}{2\,f^2}+\frac{\left((mn-2\,m\log\left(c\right))b-2\,am\right)e^2\log\left(fx+e\right)}{4\,f^2}+\frac{2\,bmne^2\log\left(fx+e\right)\log\left(x\right)-\left(\left(f^2m-2\,f^2\log\left(d\right)\right)a-\left(f^2mn-f^2n\log\left(d\right)-\left(f^2m-2\,f^2\log\left(d\right)\right)\log\left(c\right)\right)b\right)x^2+\left(2\,afm-\left(3\,fmn-2\,fm\log\left(c\right)\right)b\right)xe+\left(2\,bf^2x^2\log\left(x^n\right)+\left(2\,af^2-\left(f^2n-2\,f^2\log\left(c\right)\right)b\right)x^2\right)\log\left(\left(fx+e\right)^m\right)+\left(2\,bfmxe-\left(f^2m-2\,f^2\log\left(d\right)\right)bx^2-2\,bme^2\log\left(fx+e\right)\right)\log\left(x^n\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] -1/2*(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*m*n*e^2/f^2 + 1/4*
((m*n - 2*m*log(c))*b - 2*a*m)*e^2*log(f*x + e)/f^2 + 1/4*(2*b*m*n*e^2*log(
f*x + e)*log(x) - ((f^2*m - 2*f^2*log(d))*a - (f^2*m*n - f^2*n*log(d) - (f^
2*m - 2*f^2*log(d))*log(c))*b)*x^2 + (2*a*f*m - (3*f*m*n - 2*f*m*log(c))*b)
*x*e + (2*b*f^2*x^2*log(x^n) + (2*a*f^2 - (f^2*n - 2*f^2*log(c))*b)*x^2)*lo
g((f*x + e)^m) + (2*b*f*m*x*e - (f^2*m - 2*f^2*log(d))*b*x^2 - 2*b*m*e^2*lo
g(f*x + e))*log(x^n))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

3.73 $\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=117

$$2bmnx - mx(a+b\log(cx^n)) - \frac{bn(e+fx)\log(d(e+fx)^m)}{f} - \frac{ben\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m)}{f} + \frac{(e+fx)(a+$$

```
[Out] 2*b*m*n*x-m*x*(a+b*ln(c*x^n))-b*n*(f*x+e)*ln(d*(f*x+e)^m)/f-b*e*n*ln(-f*x/e
)*ln(d*(f*x+e)^m)/f+(f*x+e)*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/f-b*e*m*n*polyl
og(2,1+f*x/e)/f
```

Rubi [A]

time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2436, 2332, 2417, 2458, 45, 2393, 2354, 2438}

$$-\frac{bemn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{f} + \frac{(e+fx)(a+b\log(cx^n))\log(d(e+fx)^m)}{f} - mx(a+b\log(cx^n)) - \frac{bn(e+fx)\log(d(e+fx)^m)}{f} - \frac{ben\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m)}{f} + 2bmnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] 2*b*m*n*x - m*x*(a + b*Log[c*x^n]) - (b*n*(e + f*x)*Log[d*(e + f*x)^m])/f -
 (b*e*n*Log[-((f*x)/e)]*Log[d*(e + f*x)^m])/f + ((e + f*x)*(a + b*Log[c*x^n
])*Log[d*(e + f*x)^m])/f - (b*e*m*n*PolyLog[2, 1 + (f*x)/e])/f
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$\int (a+b\log(cx^n))\log(d(e+fx)^m)\,dx = -mx(a+b\log(cx^n)) + \frac{(e+fx)(a+b\log(cx^n))\log(d(e+fx)^{m}}{f}$$
$$= bmnx - mx(a+b\log(cx^n)) + \frac{(e+fx)(a+b\log(cx^n))\log(d(e}{f}$$
$$= bmnx - mx(a+b\log(cx^n)) + \frac{(e+fx)(a+b\log(cx^n))\log(d(e}{f}$$
$$= bmnx - mx(a+b\log(cx^n)) + \frac{(e+fx)(a+b\log(cx^n))\log(d(e}{f}$$
$$= bmnx - mx(a+b\log(cx^n)) + \frac{(e+fx)(a+b\log(cx^n))\log(d(e}{f}$$
$$= 2bmnx - mx(a+b\log(cx^n)) - \frac{bn(e+fx)\log(d(e+fx)^m)}{f} - \frac{b}{}$$
$$= 2bmnx - mx(a+b\log(cx^n)) - \frac{bn(e+fx)\log(d(e+fx)^m)}{f} - \frac{b}{}$$

Mathematica [A]
time = 0.05, size = 152, normalized size = 1.30

$$\frac{-afmx+2bfmnx-bemn\log(e+fx)-bemn\log(x)\log(e+fx)+ae\log(d(e+fx)^m)+afx\log(d(e+fx)^m)-bfnx\log(d(e+fx)^m)+b\log(cx^n)(em\log(e+fx)+fx(-m+\log(d(e+fx)^m)))+bemn\log(x)\log\left(1+\frac{fx}{e}\right)+bemn\text{Li}_2\left(-\frac{fx}{e}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a*f*m*x) + 2*b*f*m*n*x - b*e*m*n*Log[e + f*x] - b*e*m*n*Log[x]*Log[e + f
*x] + a*e*Log[d*(e + f*x)^m] + a*f*x*Log[d*(e + f*x)^m] - b*f*n*x*Log[d*(e
+ f*x)^m] + b*Log[c*x^n]*(e*m*Log[e + f*x] + f*x*(-m + Log[d*(e + f*x)^m]))
 + b*e*m*n*Log[x]*Log[1 + (f*x)/e] + b*e*m*n*PolyLog[2, -((f*x)/e)])/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 1762, normalized size = 15.06

method	result	size
risch	Expression too large to display	1762

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1
/4*Pi^2*x*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*csgn(I*c)*csgn(I*c*x^n)
^2-x*a*m+2*b*m*n*x+m/f*e*ln(f*x+e)*b*ln(c)-m*b*ln(x^n)*x+ln(x^n)*ln(d)*x*b-
m*ln(c)*b*x+ln(d)*ln(c)*b*x-ln(d)*b*n*x+ln(d)*a*x+b*e*m*n/f+a*m/f*e*ln(f*x+
e)+(b*x*ln(x^n)+1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csg
n(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x
^n)^3+2*b*ln(c)-2*b*n+2*a)*x)*ln((f*x+e)^m)+m/f*b*ln(x^n)*e*ln(f*x+e)-b*e*m
*n/f*ln(f*x+e)-1/2*I*Pi*a*x*csgn(I*d*(f*x+e)^m)^3+1/4*Pi^2*x*b*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)^2*csgn(I*c*x^n)^3-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(
f*x+e)^m)*csgn(I*d*(f*x+e)^m)*csgn(I*c*x^n)^3-1/4*Pi^2*x*b*csgn(I*d)*csgn(I
*d*(f*x+e)^m)^2*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*m/f*e*ln(f*x+e)*Pi*b*csgn(I
*c)*csgn(I*c*x^n)^2+1/2*I*m/f*e*ln(f*x+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-
1/4*Pi^2*x*b*csgn(I*d*(f*x+e)^m)^3*csgn(I*c*x^n)^3+1/4*Pi^2*x*b*csgn(I*d)*c
sgn(I*d*(f*x+e)^m)^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*x*b*csgn(
I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-n*b*
e*m/f*ln(f*x+e)*ln(-f*x/e)-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*
d*(f*x+e)^m)*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*ln(c)*Pi*b*x*csgn(I*
d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)-1/2*I*m/f*e*ln(f*x+e)*Pi*b*csgn(I*
c*x^n)^3+1/2*I*m*Pi*b*x*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(c)*Pi*
b*x*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/2*I*ln(x^n)*Pi*x*b*csgn(I*d)*
csgn(I*d*(f*x+e)^m)^2+1/2*I*ln(x^n)*Pi*x*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+
e)^m)^2-1/2*I*Pi*a*x*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/2*I*
ln(d)*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(c)*Pi*b*x*csgn(I*d)*csgn(
I*d*(f*x+e)^m)^2-1/2*I*ln(d)*Pi*b*x*csgn(I*c*x^n)^3-1/2*I*ln(c)*Pi*b*x*csgn
(I*d*(f*x+e)^m)^3+1/2*I*Pi*a*x*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-n*b*
e*m/f*dilog(-f*x/e)+1/4*Pi^2*x*b*csgn(I*d*(f*x+e)^m)^3*csgn(I*c)*csgn(I*c*x
^n)^2+1/4*Pi^2*x*b*csgn(I*d*(f*x+e)^m)^3*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*
Pi*a*x*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I*ln(x^n)*Pi*x*b*csgn(I*d*(f*x+e
)^m)^3+1/2*I*Pi*b*n*x*csgn(I*d*(f*x+e)^m)^3+1/2*I*m*Pi*b*x*csgn(I*c*x^n)^3-
1/2*I*m*Pi*b*x*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*m*Pi*b*x*csgn(I*x^n)*csgn(I*
c*x^n)^2+1/2*I*ln(d)*Pi*b*x*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(d)*Pi*b*x*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x+e)^m
)^2*csgn(I*c*x^n)^3-1/2*I*ln(x^n)*Pi*x*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I
*d*(f*x+e)^m)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*
csgn(I*c)*csgn(I*c*x^n)^2+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d
*(f*x+e)^m)*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*b*n*x*csgn(I*d)*csgn(I*(f*
x+e)^m)*csgn(I*d*(f*x+e)^m)-1/4*Pi^2*x*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)
^m)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*x*b*csgn(I*d*(f*x+e)^m)^3*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*Pi*b*n*x*csgn(I*d)*csgn(I*d*(f*x+e)^m)^
2-1/2*I*Pi*b*n*x*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/2*I*m/f*e*ln(f*x
+e)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
```

Maxima [A]
time = 0.38, size = 193, normalized size = 1.65

$$\frac{\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bmne}{f}-\frac{((mn-m\log\left(c\right))b-am)e\log\left(fx+e\right)}{f}-\frac{bmne\log\left(fx+e\right)\log\left(x\right)+((fm-f\log\left(d\right))a-(2\,fmn-fn\log\left(d\right)-(fm-f\log\left(d\right))\log\left(c\right))b)x-(bfx\log\left(x^n\right)-((fn-f\log\left(c\right))b-af)x)\log\left(\left(fx+e\right)^m\right)-(bme\log\left(fx+e\right)-(fm-f\log\left(d\right))bx)\log\left(x^n\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] (log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*m*n*e/f - ((m*n - m*log
(c))*b - a*m)*e*log(f*x + e)/f - (b*m*n*e*log(f*x + e)*log(x) + ((f*m - f*l
og(d))*a - (2*f*m*n - f*n*log(d) - (f*m - f*log(d))*log(c))*b)*x - (b*f*x*l
og(x^n) - ((f*n - f*log(c))*b - a*f)*x)*log((f*x + e)^m) - (b*m*e*log(f*x +
 e) - (f*m - f*log(d))*b*x)*log(x^n))/f
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n)), x)
```

3.74 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx$

Optimal. Leaf size=100

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{m(a+b\log(cx^n))^2\log\left(1+\frac{fx}{e}\right)}{2bn}-m(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{fx}{e}\right)+bmn\mathrm{L}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))*polylog(2,-f*x/e)+b*m*n*polylog(3,-f*x/e)
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2422, 2354, 2421, 6724}

$$-m\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))+bmn\text{PolyLog}\left(3,-\frac{fx}{e}\right)+\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn}-\frac{m\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(2*b*n) - (m*(a + b*Log[c*x^n])^2
*Log[1 + (f*x)/e])/(2*b*n) - m*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] +
b*m*n*PolyLog[3, -((f*x)/e)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x}\,dx &= \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn} - \frac{(fm)\int\frac{(a+b\log(cx^n))^2}{e+fx}\,dx}{2bn}\\ &= \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}\\ &= \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}\\ &= \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2bn} - \frac{m(a+b\log(cx^n))^2\log(1+}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 147, normalized size = 1.47

$$-\frac{1}{2}bn\log^2(x)\log(d(e+fx)^m)+a\log\left(-\frac{fx}{e}\right)\log(d(e+fx)^m)+b\log(x)\log(cx^n)\log(d(e+fx)^m)+\frac{1}{2}bmn\log^2(x)\log\left(1+\frac{fx}{e}\right)-bm\log(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right)-bm\log(cx^n)\operatorname{Li}_2\left(-\frac{fx}{e}\right)+am\operatorname{Li}_2\left(1+\frac{fx}{e}\right)+bmn\operatorname{Li}_3\left(-\frac{fx}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] -1/2*(b*n*Log[x]^2*Log[d*(e + f*x)^m]) + a*Log[-((f*x)/e)]*Log[d*(e + f*x)^
m] + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] + (b*m*n*Log[x]^2*Log[1 + (f*x)
/e])/2 - b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - b*m*Log[c*x^n]*PolyLog[2,
 -((f*x)/e)] + a*m*PolyLog[2, 1 + (f*x)/e] + b*m*n*PolyLog[3, -((f*x)/e)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 1795, normalized size = 17.95

method	result	size
risch	Expression too large to display	1795

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*c*x^n)^3+1/4*Pi^2*c
sgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*c*x^n)^3+1/4*Pi^2*csg
n(I*d*(f*x+e)^m)^3*ln(x)*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*m*dilog((f*x+e)/
e)*b*Pi*csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3*b/n*ln(x^n)^2-m*dilo
g((f*x+e)/e)*b*ln(x^n)+1/2*ln(d)*b/n*ln(x^n)^2+(b*ln(x)*ln(x^n)-1/2*b*n*ln(
x)^2-1/2*I*Pi*ln(x)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*ln(x)*b*
csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*
I*Pi*ln(x)*b*csgn(I*c*x^n)^3+ln(c)*ln(x)*b+ln(x)*a)*ln((f*x+e)^m)-1/2*I*Pi*
csgn(I*d*(f*x+e)^m)^3*a*ln(x)-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3*ln(x)*b*csgn(I
*c*x^n)^3+1/2*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*a*ln(x)-1/2*I*Pi
*csgn(I*d*(f*x+e)^m)^3*ln(x)*b*ln(c)-1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*c*x^n)^3
+1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*a*ln(x)+1/4*Pi^2*csgn(I*d*(f*x+e)
^m)^3*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+m*ln(x)^2*ln((f*x+e)/e)*b*n-m*b*n
*ln(x)*polylog(2,-f*x/e)+m*dilog((f*x+e)/e)*b*n*ln(x)-m*ln(x)*ln((f*x+e)/e)
*b*ln(c)-m*dilog((f*x+e)/e)*a+ln(d)*a*ln(x)-m*ln(x)*ln((f*x+e)/e)*a+ln(d)*l
n(x)*b*ln(c)-m*dilog((f*x+e)/e)*b*ln(c)-1/2*m*b*n*ln(x)^2*ln(1+f*x/e)-m*ln(
x)*ln((f*x+e)/e)*b*ln(x^n)+b*m*n*polylog(3,-f*x/e)-1/2*I*m*dilog((f*x+e)/e)
*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*m*dilog((f*x+e)/e)*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2+1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*ln(x)*b*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x
+e)^m)^2*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3
*ln(x)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f
*x+e)^m)^2*b/n*ln(x^n)^2+1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2
+1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*(f*x+e)^m
)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*ln(c)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*
csgn(I*d*(f*x+e)^m)*ln(x)*b*csgn(I*c*x^n)^3-1/2*I*Pi*csgn(I*d)*csgn(I*(f*x+
e)^m)*csgn(I*d*(f*x+e)^m)*a*ln(x)+1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*
ln(x)*b*ln(c)+1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*ln(x
)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*ln
(x)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x+e)
^m)*csgn(I*d*(f*x+e)^m)*b/n*ln(x^n)^2-1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+1/4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/n*
ln(x^n)^2+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(
f*x+e)^m)*ln(x)*b*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*m*ln(x)*ln((f*x+e)/e)*b*P
i*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2+1/2*I*m*dilog((f*x+e)/e)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)-1/2*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*ln(x)*b*ln(c)
-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*c)*csgn(I*c*x^n)^2
-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*ln(x)*b*csgn(I*c)*csgn(
I*c*x^n)^2+1/2*I*m*ln(x)*ln((f*x+e)/e)*b*Pi*csgn(I*c*x^n)^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x + e)^m) - integrate(-1/2*(b*f*m*n*x*log(x)^2 - 2*(b*f*m*log(c) + a*f*m)*x
*log(x) + 2*(b*f*log(c)*log(d) + a*f*log(d))*x + 2*(b*log(c)*log(d) + a*log
(d))*e - 2*(b*f*m*x*log(x) - b*f*x*log(d) - b*e*log(d))*log(x^n))/(f*x^2 +
x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x, x)
```

3.75 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx$

Optimal. Leaf size=164

$$\frac{bfmn\log(x)}{e}-\frac{bfmn\log^2(x)}{2e}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{bfmn\log(e+fx)}{e}+\frac{bfmn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{e}$$

```
[Out] b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-b*f*m*n*l
n(f*x+e)/e+b*f*m*n*ln(-f*x/e)*ln(f*x+e)/e-f*m*(a+b*ln(c*x^n))*ln(f*x+e)/e-b
*n*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x+b*f*m*n*polylog(2,1+
f*x/e)/e
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2442, 36, 29, 31, 2423, 2338, 2441, 2352}

$$\frac{bfmn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{e}-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx)^m\right)}{x}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{fm\log(e+fx)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{bn\log\left(d(e+fx)^m\right)}{x}-\frac{bfmn\log^2(x)}{2e}+\frac{bfmn\log(x)}{e}-\frac{bfmn\log(e+fx)}{e}+\frac{bfmn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (b*f*m*n*Log[x])/e - (b*f*m*n*Log[x]^2)/(2*e) + (f*m*Log[x]*(a + b*Log[c*x^
n]))/e - (b*f*m*n*Log[e + f*x])/e + (b*f*m*n*Log[-((f*x)/e)]*Log[e + f*x])/
e - (f*m*(a + b*Log[c*x^n])*Log[e + f*x])/e - (b*n*Log[d*(e + f*x)^m])/x -
((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x + (b*f*m*n*PolyLog[2, 1 + (f*x)/e
])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^2}\,dx = \frac{fm\log(x)(a+b\log(cx^n))}{e} - \frac{fm(a+b\log(cx^n))\log(e+fx)}{e} -$$

$$= \frac{fm\log(x)(a+b\log(cx^n))}{e} - \frac{fm(a+b\log(cx^n))\log(e+fx)}{e} -$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)(a+b\log(cx^n))}{e} + \frac{bfmn\log\left(-\frac{fx}{e}\right.}{e}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)(a+b\log(cx^n))}{e} + \frac{bfmn\log\left(-\frac{fx}{e}\right.}{e}$$

$$= \frac{bfmn\log(x)}{e} - \frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)(a+b\log(cx^n))}{e} - \frac{b}{}$$

Mathematica [A]
time = 0.08, size = 117, normalized size = 0.71

$$-\frac{bfmnx\log^2(x)+2(a+bn+b\log(cx^n))\left(fmx\log(e+fx)+e\log\left(d(e+fx)^m\right)\right)-2fmx\log(x)\left(a+bn+b\log(cx^n)+bn\log(e+fx)-bn\log\left(1+\frac{fx}{e}\right)\right)+2bfmnx\mathrm{Li}_2\left(-\frac{fx}{e}\right)}{2ex}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/2*(b*f*m*n*x*Log[x]^2 + 2*(a + b*n + b*Log[c*x^n])*(f*m*x*Log[e + f*x] +
 e*Log[d*(e + f*x)^m]) - 2*f*m*x*Log[x]*(a + b*n + b*Log[c*x^n] + b*n*Log[e
 + f*x] - b*n*Log[1 + (f*x)/e]) + 2*b*f*m*n*x*PolyLog[2, -((f*x)/e)])/(e*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 1892, normalized size = 11.54

method	result	size
risch	Expression too large to display	1892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*f*m*n*ln(-f*x/e)*ln(f*x+e)/e+b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e-b*f*m*
n*ln(f*x+e)/e-ln(d)*b/x*ln(x^n)-1/2*I/x*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^
2+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c)*csgn(I*c*x
^n)^2+1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn
(I*c*x^n)^2-1/2*I/x*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I/x*Pi*b*n*c
sgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/2*I/x*Pi*a*csgn(I*d)*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)-1/x*ln(d)*a+m*f*b*ln(x^n)*ln(x)/e+(-b/x*ln(x^n)-1/
2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^
n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*
b*n+2*a)/x)*ln((f*x+e)^m)-1/2*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*
b/x*ln(x^n)-1/2*I/x*Pi*ln(c)*b*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1/4*
Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c)*csgn(I*c*x^n)^2-1/x*ln(d
)*ln(c)*b-1/x*ln(d)*b*n-1/4*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x*
b*csgn(I*c*x^n)^3+1/2*I/x*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/2*I/x*Pi*ln(c)*b*csg
n(I*d*(f*x+e)^m)^3-f*m/e*ln(f*x+e)*b*ln(c)+f*m/e*ln(x)*b*ln(c)+f*m/e*ln(x)*
a-f*m/e*ln(f*x+e)*a-1/2*I/x*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/x*
Pi*ln(c)*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x
+e)^m)^2*b/x*ln(x^n)-m*f*b*ln(x^n)/e*ln(f*x+e)+1/2*I*Pi*csgn(I*d*(f*x+e)^m)
^3*b/x*ln(x^n)+1/2*I*f*m/e*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*f*m
/e*ln(x)*b*Pi*csgn(I*c*x^n)^3+1/2*I*f*m/e*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^3+1/
2*I*Pi*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/x*ln(x^n)+1/4*Pi^2
*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I/x*Pi
*b*n*csgn(I*d*(f*x+e)^m)^3-1/2*I/x*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/4
*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^
```

```
2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn(I*c*x^n)^3-1/2*I
*f*m/e*ln(x)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*csgn(I*d*(f*
x+e)^m)^3/x*b*csgn(I*c*x^n)^3+1/2*I/x*Pi*a*csgn(I*d*(f*x+e)^m)^3+1/2*I/x*Pi
*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I/x*Pi*ln(c)*b*csgn(I*d)*c
sgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/2*I/x*Pi*b*n*csgn(I*d)*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)+n*b*f*m/e*dilog(-f*x/e)+1/4*Pi^2*csgn(I*d)*csgn(I*
(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*
I*f*m/e*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*f*m/e*ln(x)*b*Pi*c
sgn(I*c)*csgn(I*c*x^n)^2+1/2*I*f*m/e*ln(f*x+e)*b*Pi*csgn(I*c)*csgn(I*x^n)*c
sgn(I*c*x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x*b*c
sgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(
I*d*(f*x+e)^m)/x*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*(f*x+e)^m)*csg
n(I*d*(f*x+e)^m)^2/x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*f*m/e*ln(f
*x+e)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I/x*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*
d*(f*x+e)^m)^2-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*c)*csgn(I*c*x^n)^2
-1/4*Pi^2*csgn(I*d*(f*x+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*cs
gn(I*d)*csgn(I*d*(f*x+e)^m)^2/x*b*csgn(I*c*x^n)^3
```

Maxima [A]
time = 0.39, size = 198, normalized size = 1.21

$$-\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bfmne^{(-1)}-\left(afm+\left(fmn+fm\log\left(c\right)\right)b\right)e^{(-1)}\log\left(fx+e\right)+\frac{\left(2\,bfmnx\log\left(fx+e\right)\log\left(x\right)-bfmnx\log\left(x\right)^2+2\left(afm+\left(fmn+fm\log\left(c\right)\right)b\right)x\log\left(x\right)-2\left(\left(n\log\left(d\right)+\log\left(c\right)\log\left(d\right)\right)b+a\log\left(d\right)\right)e-2\left(be\log\left(x^n\right)+\left(b(n+\log\left(c\right))+a\right)e\right)\log\left(\left(fx+e\right)^m\right)-2\left(bfmx\log\left(fx+e\right)-bfmx\log\left(x\right)+be\log\left(d\right)\right)\log\left(x^n\right)\right)e^{(-1)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*f*m*n*e^(-1) - (a*f*m
+ (f*m*n + f*m*log(c))*b)*e^(-1)*log(f*x + e) + 1/2*(2*b*f*m*n*x*log(f*x +
e)*log(x) - b*f*m*n*x*log(x)^2 + 2*(a*f*m + (f*m*n + f*m*log(c))*b)*x*log(x
) - 2*((n*log(d) + log(c)*log(d))*b + a*log(d))*e - 2*(b*e*log(x^n) + (b*(n
 + log(c)) + a)*e)*log((f*x + e)^m) - 2*(b*f*m*x*log(f*x + e) - b*f*m*x*log
(x) + b*e*log(d))*log(x^n))*e^(-1)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^2, x)
```

3.76 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx$

Optimal. Leaf size=234

$$-\frac{3bfmn}{4ex}-\frac{bf^2mn\log(x)}{4e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm(a+b\log(cx^n))}{2ex}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{bf^2mn\log(}{4e^2}$$

```
[Out] -3/4*b*f*m*n/e/x-1/4*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/2*f*m*
(a+b*ln(c*x^n))/e/x-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/4*b*f^2*m*n*ln(f*
x+e)/e^2-1/2*b*f^2*m*n*ln(-f*x/e)*ln(f*x+e)/e^2+1/2*f^2*m*(a+b*ln(c*x^n))*l
n(f*x+e)/e^2-1/4*b*n*ln(d*(f*x+e)^m)/x^2-1/2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m
)/x^2-1/2*b*f^2*m*n*polylog(2,1+f*x/e)/e^2
```

Rubi [A]
time = 0.11, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 46, 2423, 2338, 2441, 2352}

$$-\frac{bf^2mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{2e^2}-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}-\frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log(e+fx)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\log(cx^n))}{2ex}-\frac{bn\log(d(e+fx)^m)}{4x^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{bf^2mn\log(x)}{4e^2}+\frac{bf^2mn\log(e+fx)}{4e^2}-\frac{bf^2mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{2e^2}-\frac{3bfmn}{4ex}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-3*b*f*m*n)/(4*e*x) - (b*f^2*m*n*Log[x])/(4*e^2) + (b*f^2*m*n*Log[x]^2)/(4
*e^2) - (f*m*(a + b*Log[c*x^n]))/(2*e*x) - (f^2*m*Log[x]*(a + b*Log[c*x^n])
)/(2*e^2) + (b*f^2*m*n*Log[e + f*x])/(4*e^2) - (b*f^2*m*n*Log[-((f*x)/e)]*L
og[e + f*x])/(2*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x])/(2*e^2) - (b
*n*Log[d*(e + f*x)^m])/(4*x^2) - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(2
*x^2) - (b*f^2*m*n*PolyLog[2, 1 + (f*x)/e])/(2*e^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^3}\,dx = -\frac{fm(a+b\log(cx^n))}{2ex} - \frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2} + \frac{f^2m(a+b\ldots$$

$$= -\frac{bfmn}{2ex} - \frac{fm(a+b\log(cx^n))}{2ex} - \frac{f^2m\log(x)(a+b\log(cx^n))}{2e^2} + \ldots$$

$$= -\frac{bfmn}{2ex} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{2ex} - \frac{f^2m\log(x)(a\ldots}{2e\ldots}$$

$$= -\frac{bfmn}{2ex} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{2ex} - \frac{f^2m\log(x)(a\ldots}{2e\ldots}$$

$$= -\frac{3bfmn}{4ex} - \frac{bf^2mn\log(x)}{4e^2} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n)\ldots}{2ex}$$

Mathematica [A]
time = 0.11, size = 232, normalized size = 0.99

$$-\frac{2aefmx+3befmnx-bf^2mnx^2\log^2(x)+2befmx\log(cx^n)-2af^2mx^2\log(e+fx)-bf^2mnx^2\log(e+fx)-2bf^2mx^2\log(cx^n)\log(e+fx)+2ae^2\log(d(e+fx)^m)+be^2n\log(d(e+fx)^m)+2be^2\log(cx^n)\log(d(e+fx)^m)+f^2mx^2\log(x)\left(2a+bn+2b\log(cx^n)+2bn\log(e+fx)-2bn\log\left(1+\frac{fx}{e}\right)\right)-2bf^2mnx^2\text{Li}_2\left(-\frac{fx}{e}\right)}{4e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/4*(2*a*e*f*m*x + 3*b*e*f*m*n*x - b*f^2*m*n*x^2*Log[x]^2 + 2*b*e*f*m*x*Lo
g[c*x^n] - 2*a*f^2*m*x^2*Log[e + f*x] - b*f^2*m*n*x^2*Log[e + f*x] - 2*b*f^
2*m*x^2*Log[c*x^n]*Log[e + f*x] + 2*a*e^2*Log[d*(e + f*x)^m] + b*e^2*n*Log[
d*(e + f*x)^m] + 2*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + f^2*m*x^2*Log[x]*(
2*a + b*n + 2*b*Log[c*x^n] + 2*b*n*Log[e + f*x] - 2*b*n*Log[1 + (f*x)/e]) -
 2*b*f^2*m*n*x^2*PolyLog[2, -((f*x)/e)])/(e^2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 2100, normalized size = 8.97

method	result	size
risch	Expression too large to display	2100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*csgn(I*c*x^n)^3+1/4*I/x^2*Pi*ln(d)*b*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*b*f^2*m*n*ln(-f*x/e)*ln(f*x+e)/e^2-1/4*b*
f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2+1/4*b*f^2*m*n*ln(f*x+e)/e^2+1/8
*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*c*x^n)^3
+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*c*x^n)^2+1
/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1
/4*I/x^2*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/4*I/x^2*Pi*ln(c)*b*csgn(I*d*(f*x+e)^m
)^3+1/8*I/x^2*Pi*b*n*csgn(I*d*(f*x+e)^m)^3-1/4*I/x^2*Pi*a*csgn(I*d)*csgn(I*
d*(f*x+e)^m)^2-1/2/x^2*ln(d)*a+(-1/2*b/x^2*ln(x^n)-1/4*(-I*b*Pi*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n
)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+b*n+2*a)/x^2)*ln((f*x+e)
^m)-1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I/e^2*f^2*m*
ln(f*x+e)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I/x^2*Pi*ln(c)*b*csgn(I*(f*x+e
)^m)*csgn(I*d*(f*x+e)^m)^2-1/8*I/x^2*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2
-1/8*I/x^2*Pi*b*n*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/2*ln(d)*b/x^2*l
n(x^n)-1/2*m*f^2*b*ln(x^n)/e^2*ln(x)-1/4*I/x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*
c*x^n)^2-1/4*I/x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I/x^2*Pi*ln(c
)*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2*b*cs
gn(I*c*x^n)^3+1/4*I/x^2*Pi*a*csgn(I*d*(f*x+e)^m)^3-1/4*I/e^2*f^2*m*ln(x)*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)
^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2*b*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2/e*f*m/x*b*ln(c)-1/2/e^2*f^2*m*ln(x)*
b*ln(c)+1/2/e^2*f^2*m*ln(f*x+e)*b*ln(c)-1/2*n*f^2*b*m/e^2*dilog(-f*x/e)-3/4
*b*f*m*n/e/x-1/2/e*f*m/x*a+1/2/e^2*f^2*m*ln(f*x+e)*a-1/2/e^2*f^2*m*ln(x)*a+
1/4*I/x^2*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/2*m*f^2*b*
ln(x^n)/e^2*ln(f*x+e)-1/2*m*f*b*ln(x^n)/e/x-1/4*I/x^2*Pi*a*csgn(I*(f*x+e)^m
)*csgn(I*d*(f*x+e)^m)^2+1/4*I*Pi*csgn(I*d*(f*x+e)^m)^3*b/x^2*ln(x^n)-1/8*Pi
^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^3-1/8*Pi^2*csgn(I*(f
```

```
*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c*x^n)^3-1/8*Pi^2*csgn(I*d*(f*x
+e)^m)^3/x^2*b*csgn(I*c)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^2
*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)+1/4*I/e*f*m/x*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)-1/2/x^2*ln(d)*ln(c)*b-1/4/x^2*ln(d)*b*n+1/4*I/e^2*f^2*m*ln(f*x+e)*b*Pi*c
sgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f
*x+e)^m)/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I/x^2*Pi*b*n*csgn(I*d)*csgn(
I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I/e*f*m/x*b*Pi*csgn(I*c*x^n)^3+1/4*I/e
^2*f^2*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)-1/4*I/e*f*m/x*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I/
e*f*m/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^
m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I/x^2*
Pi*ln(c)*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/4*I*Pi*csgn(I*
d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/x^2*ln(x^n)-1/8*Pi^2*csgn(I*d)*c
sgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*Pi^2*csg
n(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x
^n)-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^2*b*csgn(I*c
)*csgn(I*c*x^n)^2-1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b/x^2*ln(x^n)-1/
4*I*Pi*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/x^2*ln(x^n)+1/8*Pi^2*csgn(
I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*c*x^n)^2
```

Maxima [A]
time = 0.40, size = 271, normalized size = 1.16

$$\frac{1}{2}\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bf^2mne^{(-2)}+\frac{1}{4}\left(2\,af^2m+\left(f^2mn+2\,f^2m\log\left(c\right)\right)b\right)e^{(-2)}\log\left(fx+e\right)-\frac{\left(2\,bf^2mnx^2\log\left(fx+e\right)\log\left(x\right)-bf^2mnx^2\log\left(x\right)^2+\left(2\,af^2m+\left(f^2mn+2\,f^2m\log\left(c\right)\right)b\right)x^2\log\left(x\right)+\left(2\,afm+\left(3\,fmn+2\,fm\log\left(c\right)\right)b\right)xe+\left(\left(n\log\left(d\right)+2\,\log\left(c\right)\log\left(d\right)\right)b+2\,a\log\left(d\right)\right)e^2+\left(2\,be^2\log\left(x^n\right)+\left(b(n+2\,\log\left(c\right))+2\,a\right)e^2\right)\log\left(\left(fx+e\right)^m\right)-2\left(bf^2mx^2\log\left(fx+e\right)-bf^2mx^2\log\left(x\right)-bfmxe-be^2\log\left(d\right)\right)\log\left(x^n\right)\right)e^{(-2)}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*f^2*m*n*e^(-2) + 1/
4*(2*a*f^2*m + (f^2*m*n + 2*f^2*m*log(c))*b)*e^(-2)*log(f*x + e) - 1/4*(2*b
*f^2*m*n*x^2*log(f*x + e)*log(x) - b*f^2*m*n*x^2*log(x)^2 + (2*a*f^2*m + (f
^2*m*n + 2*f^2*m*log(c))*b)*x^2*log(x) + (2*a*f*m + (3*f*m*n + 2*f*m*log(c)
)*b)*x*e + ((n*log(d) + 2*log(c)*log(d))*b + 2*a*log(d))*e^2 + (2*b*e^2*log
(x^n) + (b*(n + 2*log(c)) + 2*a)*e^2)*log((f*x + e)^m) - 2*(b*f^2*m*x^2*log
(f*x + e) - b*f^2*m*x^2*log(x) - b*f*m*x*e - b*e^2*log(d))*log(x^n))*e^(-2)
/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^3, x)
```

3.77 $\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx$

Optimal. Leaf size=274

$$-\frac{5bfmn}{36ex^2}+\frac{4bf^2mn}{9e^2x}+\frac{bf^3mn\log(x)}{9e^3}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}+\frac{f^3m\log(x$$

```
[Out] -5/36*b*f*m*n/e/x^2+4/9*b*f^2*m*n/e^2/x+1/9*b*f^3*m*n*ln(x)/e^3-1/6*b*f^3*m
*n*ln(x)^2/e^3-1/6*f*m*(a+b*ln(c*x^n))/e/x^2+1/3*f^2*m*(a+b*ln(c*x^n))/e^2/
x+1/3*f^3*m*ln(x)*(a+b*ln(c*x^n))/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3+1/3*b*f^3
*m*n*ln(-f*x/e)*ln(f*x+e)/e^3-1/3*f^3*m*(a+b*ln(c*x^n))*ln(f*x+e)/e^3-1/9*b
*n*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3+1/3*b*f^3*m*
n*polylog(2,1+f*x/e)/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 46, 2423, 2338, 2441, 2352}

$$\frac{bf^3mn\text{PolyLog}\left(2,\frac{fx}{e}+1\right)}{3e^3}-\frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{3x^3}+\frac{f^3m\log(x)(a+b\log(cx^n))}{3e^3}-\frac{f^3m\log(e+fx)(a+b\log(cx^n))}{3e^3}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}-\frac{fm(a+b\log(cx^n))}{6ex^2}-\frac{bn\log(d(e+fx)^m)}{9x^3}-\frac{bf^3mn\log^2(x)}{6e^3}+\frac{bf^3mn\log(x)}{9e^3}-\frac{bf^3mn\log(e+fx)}{9e^3}+\frac{bf^3mn\log\left(-\frac{fx}{e}\right)\log(e+fx)}{3e^3}+\frac{4bf^2mn}{9e^2x}-\frac{5bfmn}{36ex^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] (-5*b*f*m*n)/(36*e*x^2) + (4*b*f^2*m*n)/(9*e^2*x) + (b*f^3*m*n*Log[x])/(9*e
^3) - (b*f^3*m*n*Log[x]^2)/(6*e^3) - (f*m*(a + b*Log[c*x^n]))/(6*e*x^2) + (
f^2*m*(a + b*Log[c*x^n]))/(3*e^2*x) + (f^3*m*Log[x]*(a + b*Log[c*x^n]))/(3*
e^3) - (b*f^3*m*n*Log[e + f*x])/(9*e^3) + (b*f^3*m*n*Log[-((f*x)/e)]*Log[e
+ f*x])/(3*e^3) - (f^3*m*(a + b*Log[c*x^n])*Log[e + f*x])/(3*e^3) - (b*n*Lo
g[d*(e + f*x)^m])/(9*x^3) - ((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(3*x^3)
 + (b*f^3*m*n*PolyLog[2, 1 + (f*x)/e])/(3*e^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\log(cx^n))\log(d(e+fx)^m)}{x^4}\,dx &= -\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}+\frac{f^3m\log(x)\,(a+\\
&= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2m(a+b\log(cx^n))}{3e^2x}\\
&= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2}{}\\
&= -\frac{bfmn}{12ex^2}+\frac{bf^2mn}{3e^2x}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm(a+b\log(cx^n))}{6ex^2}+\frac{f^2}{}\\
&= -\frac{5bfmn}{36ex^2}+\frac{4bf^2mn}{9e^2x}+\frac{bf^3mn\log(x)}{9e^3}-\frac{bf^3mn\log^2(x)}{6e^3}-\frac{fm(a}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 280, normalized size = 1.02

$$-\frac{6ae^2fmx+5be^2fmnx-12aef^2mx^2-16bef^2mnx^2+6bf^3mnx^3\log^2(x)+6be^2fmx\log(cx^n)-12bef^2mx^2\log(cx^n)+12af^3mx^3\log(e+fx)+4bf^3mnx^3\log(e+fx)+12bf^3mx^3\log(cx^n)\log(e+fx)+12ae^3\log(d(e+fx)^m)+4be^3n\log(d(e+fx)^m)+12be^3\log(cx^n)\log(d(e+fx)^m)-4f^3mx^3\log(x)\left(3a+bn+3b\log(cx^n)+3bn\log(e+fx)-3bn\log\left(1+\frac{fx}{e}\right)\right)+12bf^3mnx^3\operatorname{Li}_2\left(-\frac{fx}{e}\right)}{36e^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] -1/36*(6*a*e^2*f*m*x + 5*b*e^2*f*m*n*x - 12*a*e*f^2*m*x^2 - 16*b*e*f^2*m*n*
x^2 + 6*b*f^3*m*n*x^3*Log[x]^2 + 6*b*e^2*f*m*x*Log[c*x^n] - 12*b*e*f^2*m*x^
2*Log[c*x^n] + 12*a*f^3*m*x^3*Log[e + f*x] + 4*b*f^3*m*n*x^3*Log[e + f*x] +
 12*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 12*a*e^3*Log[d*(e + f*x)^m] + 4*b
*e^3*n*Log[d*(e + f*x)^m] + 12*b*e^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 4*f^3*
m*x^3*Log[x]*(3*a + b*n + 3*b*Log[c*x^n] + 3*b*n*Log[e + f*x] - 3*b*n*Log[1
 + (f*x)/e]) + 12*b*f^3*m*n*x^3*PolyLog[2, -((f*x)/e)])/(e^3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 2282, normalized size = 8.33

method	result	size
risch	Expression too large to display	2282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*f^3*m*n*ln(-f*x/e)*ln(f*x+e)/e^3+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m
)*csgn(I*d*(f*x+e)^m)/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/9*b*f^3*m
*n*ln(x)/e^3-1/6*b*f^3*m*n*ln(x)^2/e^3-1/9*b*f^3*m*n*ln(f*x+e)/e^3-1/6*I/e^
3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*I/e*f*m/x^2*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2-1/12*I/e*f*m/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1
/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/6/e*f*m/x^2*b*ln(
c)+1/3/e^2*f^2*m/x*b*ln(c)+1/3/e^3*f^3*m*ln(x)*b*ln(c)-1/3/e^3*f^3*m*ln(f*x
+e)*b*ln(c)-1/3/x^3*ln(d)*a-1/6*I/x^3*Pi*a*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-
1/6*I/x^3*Pi*a*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/6*I/x^3*Pi*ln(c)*b
*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2-1/18*I/x^3*Pi*b*n*csgn(I*d)*csgn(I
*d*(f*x+e)^m)^2-1/18*I/x^3*Pi*b*n*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2+1
/3*n*f^3*b*m/e^3*dilog(-f*x/e)+(-1/3*b/x^3*ln(x^n)-1/18*(-3*I*b*Pi*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6*b*ln(c)+2*b*n+6*a)/x^3)
*ln((f*x+e)^m)-1/6*I/x^3*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I/x^3*P
i*ln(c)*b*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2-1/6/e*f*m/x^2*a+1/3/e^3*f^3*m*ln(
x)*a-1/3/e^3*f^3*m*ln(f*x+e)*a-1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)+1/6*I/e^3*f^3*m*ln(f*x+e)*b*Pi*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)-1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/12*
Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*csgn(I*c*x^n)^3+
1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*c*x^n)^2+1
/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+
1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*c*
x^n)^2-1/3*ln(d)*b/x^3*ln(x^n)+1/3/e^2*f^2*m/x*a-1/12*Pi^2*csgn(I*(f*x+e)^m
)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/6*I/x^3
```

```
*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/6*I*Pi*csgn(I*d)*csgn(I*(
f*x+e)^m)*csgn(I*d*(f*x+e)^m)*b/x^3*ln(x^n)-5/36*b*f*m*n/e/x^2+4/9*b*f^2*m*
n/e^2/x+1/12*Pi^2*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*x^n)
*csgn(I*c*x^n)^2+1/3*m*f^3*b*ln(x^n)/e^3*ln(x)+1/3*m*f^2*b*ln(x^n)/e^2/x-1/
3*m*f^3*b*ln(x^n)/e^3*ln(f*x+e)-1/6*m*f*b*ln(x^n)/e/x^2+1/6*I/x^3*Pi*a*csgn
(I*d*(f*x+e)^m)^3+1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I*c*x^n)^3+1/1
2*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/12
*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*csgn(I*c)*csgn(
I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)/x^3*b*
csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2/x^3*b
*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/9/x^3*ln(d)*b*n-1/3/x^3*ln(d)*ln(c)*
b-1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x+e)^m)^2*b/x^3*ln(x^n)-1/6*I*Pi*csgn(I*(f
*x+e)^m)*csgn(I*d*(f*x+e)^m)^2*b/x^3*ln(x^n)-1/12*Pi^2*csgn(I*(f*x+e)^m)*cs
gn(I*d*(f*x+e)^m)^2/x^3*b*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3/x
^3*b*csgn(I*c)*csgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d*(f*x+e)^m)^3/x^3*b*csgn(I
*x^n)*csgn(I*c*x^n)^2+1/6*I*Pi*csgn(I*d*(f*x+e)^m)^3*b/x^3*ln(x^n)+1/6*I/x^
3*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/6*I/x^3*Pi*ln(c)*b*csgn(I*d*(f*x+e)^m)^3+1/1
8*I/x^3*Pi*b*n*csgn(I*d*(f*x+e)^m)^3+1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*c)*c
sgn(I*c*x^n)^2+1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I/x^3*P
i*ln(c)*b*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/18*I/x^3*Pi*b*n
*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*x+e)^m)+1/6*I/e^2*f^2*m/x*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2+1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^
n)^2-1/6*I/e^3*f^3*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+1/6*I/e^3*f^3*m*ln(f*x+e)*b
*Pi*csgn(I*c*x^n)^3+1/6*I/x^3*Pi*a*csgn(I*d)*csgn(I*(f*x+e)^m)*csgn(I*d*(f*
x+e)^m)-1/6*I/x^3*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^2+1/12*I/e*f*m/x^2*b*P
i*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/6*I/e^2*f^2*m/x*b*Pi*csgn(I*c*x^n)^
3+1/12*I/e*f*m/x^2*b*Pi*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x+e
)^m)^2/x^3*b*csgn(I*c*x^n)^3
```

Maxima [A]
time = 0.40, size = 320, normalized size = 1.17

$$-\frac{1}{3}\left(\log\left(fxe^{(-1)}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-fxe^{(-1)}\right)\right)bf^3mne^{(-3)}-\frac{1}{9}\left(3\,af^3m+\left(f^3mn+3\,f^3m\log\left(c\right)\right)b\right)e^{(-3)}\log\left(fx+e\right)+\frac{\left(12\,bf^3mnx^3\log\left(fx+e\right)\log\left(x\right)-6\,bf^3mnx^3\log\left(x\right)^2+4\left(3\,af^3m+\left(f^3mn+3\,f^3m\log\left(c\right)\right)b\right)x^3\log\left(x\right)+4\left(3\,af^2m+\left(4\,f^2mn+3\,f^2m\log\left(c\right)\right)b\right)x^2e-\left(6\,afm+\left(5\,fmn+6\,fm\log\left(c\right)\right)b\right)xe^2-4\left(\left(n\log\left(d\right)+3\,\log\left(c\right)\log\left(d\right)\right)b+3\,a\log\left(d\right)\right)e^3-4\left(3\,be^3\log\left(x^n\right)+\left(b\left(n+3\,\log\left(c\right)\right)+3\,a\right)e^3\right)\log\left(\left(fx+e\right)^m\right)-6\left(2\,bf^3mx^3\log\left(fx+e\right)-2\,bf^3mx^3\log\left(x\right)-2\,bf^2mx^2e+bfmxe^2+2\,be^3\log\left(d\right)\right)\log\left(x^n\right)\right)e^{(-3)}}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(f*x*e^(-1) + 1)*log(x) + dilog(-f*x*e^(-1)))*b*f^3*m*n*e^(-3) - 1
/9*(3*a*f^3*m + (f^3*m*n + 3*f^3*m*log(c))*b)*e^(-3)*log(f*x + e) + 1/36*(1
2*b*f^3*m*n*x^3*log(f*x + e)*log(x) - 6*b*f^3*m*n*x^3*log(x)^2 + 4*(3*a*f^3
*m + (f^3*m*n + 3*f^3*m*log(c))*b)*x^3*log(x) + 4*(3*a*f^2*m + (4*f^2*m*n +
 3*f^2*m*log(c))*b)*x^2*e - (6*a*f*m + (5*f*m*n + 6*f*m*log(c))*b)*x*e^2 -
4*((n*log(d) + 3*log(c)*log(d))*b + 3*a*log(d))*e^3 - 4*(3*b*e^3*log(x^n) +
 (b*(n + 3*log(c)) + 3*a)*e^3)*log((f*x + e)^m) - 6*(2*b*f^3*m*x^3*log(f*x
+ e) - 2*b*f^3*m*x^3*log(x) - 2*b*f^2*m*x^2*e + b*f*m*x*e^2 + 2*b*e^3*log(d
))*log(x^n))*e^(-3)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x+e)**m)/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x + e)^m*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n)))/x^4, x)
```

3.78 $\int x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=452

$$\frac{8abe^2mnx}{9f^2}-\frac{26b^2e^2mn^2x}{27f^2}+\frac{19b^2emn^2x^2}{108f}-\frac{2}{27}b^2mn^2x^3+\frac{8b^2e^2mnx\log(cx^n)}{9f^2}-\frac{5bemnx^2(a+b\log(cx^n))}{18f}+\frac{4}{27}b$$

```
[Out] 8/9*a*b*e^2*m*n*x/f^2-26/27*b^2*e^2*m*n^2*x/f^2+19/108*b^2*e*m*n^2*x^2/f-2/
27*b^2*m*n^2*x^3+8/9*b^2*e^2*m*n*x*ln(c*x^n)/f^2-5/18*b*e*m*n*x^2*(a+b*ln(c
*x^n))/f+4/27*b*m*n*x^3*(a+b*ln(c*x^n))-1/3*e^2*m*x*(a+b*ln(c*x^n))^2/f^2+1
/6*e*m*x^2*(a+b*ln(c*x^n))^2/f-1/9*m*x^3*(a+b*ln(c*x^n))^2+2/27*b^2*e^3*m*n
^2*ln(f*x+e)/f^3+2/27*b^2*n^2*x^3*ln(d*(f*x+e)^m)-2/9*b*n*x^3*(a+b*ln(c*x^n
))*ln(d*(f*x+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)-2/9*b*e^3*m*n*
(a+b*ln(c*x^n))*ln(1+f*x/e)/f^3+1/3*e^3*m*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^3
-2/9*b^2*e^3*m*n^2*polylog(2,-f*x/e)/f^3+2/3*b*e^3*m*n*(a+b*ln(c*x^n))*poly
log(2,-f*x/e)/f^3-2/3*b^2*e^3*m*n^2*polylog(3,-f*x/e)/f^3
```

Rubi [A]
time = 0.45, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2342, 2341, 2425, 45, 2393, 2332, 2354, 2438, 2395, 2333, 2421, 6724}

$$\frac{2be^3mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{3f^3}-\frac{2b^2e^3mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{9f^3}-\frac{2b^2e^3mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{3f^3}+\frac{1}{3}x^3(a+b\log(cx^n))^2\log(d(e+fx)^m)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{e^3m\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{3f^3}-\frac{2be^3mn\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))}{9f^3}-\frac{e^2mx(a+b\log(cx^n))^2}{3f^2}+\frac{emx^2(a+b\log(cx^n))^2}{6f}-\frac{5bemnx^2(a+b\log(cx^n))}{18f}-\frac{1}{9}mx^3(a+b\log(cx^n))^2+\frac{4}{27}bmnx^3(a+b\log(cx^n))+\frac{8abe^2mnx}{9f^2}+\frac{8b^2e^2mnx\log(cx^n)}{9f^2}+\frac{2}{27}b^2n^2x^3\log(d(e+fx)^m)+\frac{2b^2e^3mn^2\log(e+fx)}{27f^3}-\frac{26b^2e^2mn^2x}{27f^2}+\frac{19b^2emn^2x^2}{108f}-\frac{2}{27}b^2mn^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (8*a*b*e^2*m*n*x)/(9*f^2) - (26*b^2*e^2*m*n^2*x)/(27*f^2) + (19*b^2*e*m*n^2
*x^2)/(108*f) - (2*b^2*m*n^2*x^3)/27 + (8*b^2*e^2*m*n*x*Log[c*x^n])/(9*f^2)
 - (5*b*e*m*n*x^2*(a + b*Log[c*x^n]))/(18*f) + (4*b*m*n*x^3*(a + b*Log[c*x^
n]))/27 - (e^2*m*x*(a + b*Log[c*x^n])^2)/(3*f^2) + (e*m*x^2*(a + b*Log[c*x^
n])^2)/(6*f) - (m*x^3*(a + b*Log[c*x^n])^2)/9 + (2*b^2*e^3*m*n^2*Log[e + f*
x])/(27*f^3) + (2*b^2*n^2*x^3*Log[d*(e + f*x)^m])/27 - (2*b*n*x^3*(a + b*Lo
g[c*x^n])*Log[d*(e + f*x)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)
^m])/3 - (2*b*e^3*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(9*f^3) + (e^3*m
*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(3*f^3) - (2*b^2*e^3*m*n^2*PolyLog[
2, -((f*x)/e)])/(9*f^3) + (2*b*e^3*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*x
)/e)])/(3*f^3) - (2*b^2*e^3*m*n^2*PolyLog[3, -((f*x)/e)])/(3*f^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx)^m\right)\,dx = \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d(e+\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d(e+\right.$$

$$= \frac{2}{27}b^2n^2x^3\log\left(d(e+fx)^m\right) - \frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d(e+\right.$$

$$= -\frac{2b^2e^2mn^2x}{27f^2} + \frac{b^2emn^2x^2}{27f} - \frac{2}{81}b^2mn^2x^3 + \frac{2b^2e^3mn^2\log(e+f}{27f^3}$$

$$= \frac{2abe^2mnx}{9f^2} - \frac{2b^2e^2mn^2x}{27f^2} + \frac{5b^2emn^2x^2}{54f} - \frac{4}{81}b^2mn^2x^3 - \frac{bemn}{}$$

$$= \frac{8abe^2mnx}{9f^2} - \frac{8b^2e^2mn^2x}{27f^2} + \frac{19b^2emn^2x^2}{108f} - \frac{2}{27}b^2mn^2x^3 + \frac{2b^2e}{}$$

$$= \frac{8abe^2mnx}{9f^2} - \frac{26b^2e^2mn^2x}{27f^2} + \frac{19b^2emn^2x^2}{108f} - \frac{2}{27}b^2mn^2x^3 + \frac{8b^2}{}$$

Mathematica [A]
time = 0.21, size = 788, normalized size = 1.74

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-36*a^2*e^2*f*m*x + 96*a*b*e^2*f*m*n*x - 104*b^2*e^2*f*m*n^2*x + 18*a^2*e*
f^2*m*x^2 - 30*a*b*e*f^2*m*n*x^2 + 19*b^2*e*f^2*m*n^2*x^2 - 12*a^2*f^3*m*x^
3 + 16*a*b*f^3*m*n*x^3 - 8*b^2*f^3*m*n^2*x^3 - 72*a*b*e^2*f*m*x*Log[c*x^n]
+ 96*b^2*e^2*f*m*n*x*Log[c*x^n] + 36*a*b*e*f^2*m*x^2*Log[c*x^n] - 30*b^2*e*
f^2*m*n*x^2*Log[c*x^n] - 24*a*b*f^3*m*x^3*Log[c*x^n] + 16*b^2*f^3*m*n*x^3*L
og[c*x^n] - 36*b^2*e^2*f*m*x*Log[c*x^n]^2 + 18*b^2*e*f^2*m*x^2*Log[c*x^n]^2
 - 12*b^2*f^3*m*x^3*Log[c*x^n]^2 + 36*a^2*e^3*m*Log[e + f*x] - 24*a*b*e^3*m
*n*Log[e + f*x] + 8*b^2*e^3*m*n^2*Log[e + f*x] - 72*a*b*e^3*m*n*Log[x]*Log[
e + f*x] + 24*b^2*e^3*m*n^2*Log[x]*Log[e + f*x] + 36*b^2*e^3*m*n^2*Log[x]^2
*Log[e + f*x] + 72*a*b*e^3*m*Log[c*x^n]*Log[e + f*x] - 24*b^2*e^3*m*n*Log[c
*x^n]*Log[e + f*x] - 72*b^2*e^3*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + 36*b^2
*e^3*m*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*f^3*x^3*Log[d*(e + f*x)^m] - 24*a
*b*f^3*n*x^3*Log[d*(e + f*x)^m] + 8*b^2*f^3*n^2*x^3*Log[d*(e + f*x)^m] + 72
*a*b*f^3*x^3*Log[c*x^n]*Log[d*(e + f*x)^m] - 24*b^2*f^3*n*x^3*Log[c*x^n]*Lo
```

```
g[d*(e + f*x)^m] + 36*b^2*f^3*x^3*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 72*a*b*
e^3*m*n*Log[x]*Log[1 + (f*x)/e] - 24*b^2*e^3*m*n^2*Log[x]*Log[1 + (f*x)/e]
- 36*b^2*e^3*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*e^3*m*n*Log[x]*Log[c*
x^n]*Log[1 + (f*x)/e] + 24*b*e^3*m*n*(3*a - b*n + 3*b*Log[c*x^n])*PolyLog[2
, -((f*x)/e)] - 72*b^2*e^3*m*n^2*PolyLog[3, -((f*x)/e)])/(108*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 12902, normalized size = 28.54

method	result	size
risch	Expression too large to display	12902

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/54*(3*(3*b^2*f^2*m*x^2*e - 2*(f^3*m - 3*f^3*log(d))*b^2*x^3 - 6*b^2*f*m*x
*e^2 + 6*b^2*m*e^3*log(f*x + e))*log(x^n)^2 + 2*(9*b^2*f^3*x^3*log(x^n)^2 +
 6*(3*a*b*f^3 - (f^3*n - 3*f^3*log(c))*b^2)*x^3*log(x^n) + (9*a^2*f^3 - 6*(
f^3*n - 3*f^3*log(c))*a*b + (2*f^3*n^2 - 6*f^3*n*log(c) + 9*f^3*log(c)^2)*b
^2)*x^3)*log((f*x + e)^m))/f^3 - integrate(1/27*((9*(f^4*m - 3*f^4*log(d))*
a^2 - 6*(f^4*m*n - 3*(f^4*m - 3*f^4*log(d))*log(c))*a*b + (2*f^4*m*n^2 - 6*
f^4*m*n*log(c) + 9*(f^4*m - 3*f^4*log(d))*log(c)^2)*b^2)*x^4 - 27*(b^2*f^3*
log(c)^2*log(d) + 2*a*b*f^3*log(c)*log(d) + a^2*f^3*log(d))*x^3*e - 3*(3*b^
2*f^2*m*n*x^2*e^2 + 6*b^2*f*m*n*x*e^3 - 2*(3*(f^4*m - 3*f^4*log(d))*a*b - (
2*f^4*m*n - 3*f^4*n*log(d) - 3*(f^4*m - 3*f^4*log(d))*log(c))*b^2)*x^4 + (1
8*a*b*f^3*log(d) - (f^3*m*n + 6*f^3*n*log(d) - 18*f^3*log(c)*log(d))*b^2)*x
^3*e - 6*(b^2*f*m*n*x*e^3 + b^2*m*n*e^4)*log(f*x + e))*log(x^n))/(f^4*x^2 +
 f^3*x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x +
 e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^2*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

3.79 $\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=373

$$-\frac{3abemnx}{2f}+\frac{7b^2emn^2x}{4f}-\frac{3}{8}b^2mn^2x^2-\frac{3b^2emnx\log(cx^n)}{2f}+\frac{1}{2}bmnx^2(a+b\log(cx^n))+\frac{emx(a+b\log(cx^n))^2}{2f}$$

```
[Out] -3/2*a*b*e*m*n*x/f+7/4*b^2*e*m*n^2*x/f-3/8*b^2*m*n^2*x^2-3/2*b^2*e*m*n*x*ln
(c*x^n)/f+1/2*b*m*n*x^2*(a+b*ln(c*x^n))+1/2*e*m*x*(a+b*ln(c*x^n))^2/f-1/4*m
*x^2*(a+b*ln(c*x^n))^2-1/4*b^2*e^2*m*n^2*ln(f*x+e)/f^2+1/4*b^2*n^2*x^2*ln(d
*(f*x+e)^m)-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln(c*x
^n))^2*ln(d*(f*x+e)^m)+1/2*b*e^2*m*n*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2-1/2*e^
2*m*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^2+1/2*b^2*e^2*m*n^2*polylog(2,-f*x/e)/f
^2-b*e^2*m*n*(a+b*ln(c*x^n))*polylog(2,-f*x/e)/f^2+b^2*e^2*m*n^2*polylog(3,
-f*x/e)/f^2
```

Rubi [A]
time = 0.35, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2342, 2341, 2425, 45, 2393, 2332, 2354, 2438, 2395, 2333, 2421, 6724}

$$-\frac{be^2mn\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f^2}+\frac{b^2e^2mn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{2f^2}+\frac{b^2e^2mn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{f^2}-\frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+fx)^m)+\frac{1}{2}x^2(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{be^2mn\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))}{2f^2}-\frac{e^2m\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{2f^2}+\frac{emx(a+b\log(cx^n))^2}{2f}+\frac{1}{2}bmnx^2(a+b\log(cx^n))-\frac{1}{4}mx^2(a+b\log(cx^n))^2-\frac{3abemnx}{2f}-\frac{3b^2emnx\log(cx^n)}{2f}+\frac{1}{4}b^2n^2x^2\log(d(e+fx)^m)-\frac{b^2e^2mn^2\log(e+fx)}{4f^2}+\frac{7b^2emn^2x}{4f}-\frac{3}{8}b^2mn^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-3*a*b*e*m*n*x)/(2*f) + (7*b^2*e*m*n^2*x)/(4*f) - (3*b^2*m*n^2*x^2)/8 - (3
*b^2*e*m*n*x*Log[c*x^n])/(2*f) + (b*m*n*x^2*(a + b*Log[c*x^n]))/2 + (e*m*x*
(a + b*Log[c*x^n])^2)/(2*f) - (m*x^2*(a + b*Log[c*x^n])^2)/4 - (b^2*e^2*m*n
^2*Log[e + f*x])/(4*f^2) + (b^2*n^2*x^2*Log[d*(e + f*x)^m])/4 - (b*n*x^2*(a
 + b*Log[c*x^n])*Log[d*(e + f*x)^m])/2 + (x^2*(a + b*Log[c*x^n])^2*Log[d*(e
 + f*x)^m])/2 + (b*e^2*m*n*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(2*f^2) - (
e^2*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(2*f^2) + (b^2*e^2*m*n^2*PolyL
og[2, -((f*x)/e)])/(2*f^2) - (b*e^2*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*
x)/e)])/f^2 + (b^2*e^2*m*n^2*PolyLog[3, -((f*x)/e)])/f^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^2\log(d(e+fx)^m)\,dx = \frac{1}{4}b^2n^2x^2\log(d(e+fx)^m) - \frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+$$

$$= \frac{1}{4}b^2n^2x^2\log(d(e+fx)^m) - \frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+$$

$$= \frac{1}{4}b^2n^2x^2\log(d(e+fx)^m) - \frac{1}{2}bnx^2(a+b\log(cx^n))\log(d(e+$$

$$= \frac{b^2emn^2x}{4f} - \frac{1}{8}b^2mn^2x^2 - \frac{b^2e^2mn^2\log(e+fx)}{4f^2} + \frac{1}{4}b^2n^2x^2\log($$

$$= -\frac{abemnx}{2f} + \frac{b^2emn^2x}{4f} - \frac{1}{4}b^2mn^2x^2 + \frac{1}{4}bmnx^2(a+b\log(cx^n)$$

$$= -\frac{3abemnx}{2f} + \frac{3b^2emn^2x}{4f} - \frac{3}{8}b^2mn^2x^2 - \frac{b^2emnx\log(cx^n)}{2f} +$$

$$= -\frac{3abemnx}{2f} + \frac{7b^2emn^2x}{4f} - \frac{3}{8}b^2mn^2x^2 - \frac{3b^2emnx\log(cx^n)}{2f} +$$

Mathematica [A]
time = 0.16, size = 674, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*a^2*e*f*m*x - 12*a*b*e*f*m*n*x + 14*b^2*e*f*m*n^2*x - 2*a^2*f^2*m*x^2 +
4*a*b*f^2*m*n*x^2 - 3*b^2*f^2*m*n^2*x^2 + 8*a*b*e*f*m*x*Log[c*x^n] - 12*b^2
*e*f*m*n*x*Log[c*x^n] - 4*a*b*f^2*m*x^2*Log[c*x^n] + 4*b^2*f^2*m*n*x^2*Log[
c*x^n] + 4*b^2*e*f*m*x*Log[c*x^n]^2 - 2*b^2*f^2*m*x^2*Log[c*x^n]^2 - 4*a^2*
e^2*m*Log[e + f*x] + 4*a*b*e^2*m*n*Log[e + f*x] - 2*b^2*e^2*m*n^2*Log[e + f
*x] + 8*a*b*e^2*m*n*Log[x]*Log[e + f*x] - 4*b^2*e^2*m*n^2*Log[x]*Log[e + f*
x] - 4*b^2*e^2*m*n^2*Log[x]^2*Log[e + f*x] - 8*a*b*e^2*m*Log[c*x^n]*Log[e +
 f*x] + 4*b^2*e^2*m*n*Log[c*x^n]*Log[e + f*x] + 8*b^2*e^2*m*n*Log[x]*Log[c*
x^n]*Log[e + f*x] - 4*b^2*e^2*m*Log[c*x^n]^2*Log[e + f*x] + 4*a^2*f^2*x^2*L
og[d*(e + f*x)^m] - 4*a*b*f^2*n*x^2*Log[d*(e + f*x)^m] + 2*b^2*f^2*n^2*x^2*
Log[d*(e + f*x)^m] + 8*a*b*f^2*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] - 4*b^2*f^
2*n*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 4*b^2*f^2*x^2*Log[c*x^n]^2*Log[d*(e
 + f*x)^m] - 8*a*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] + 4*b^2*e^2*m*n^2*Log[x]
*Log[1 + (f*x)/e] + 4*b^2*e^2*m*n^2*Log[x]^2*Log[1 + (f*x)/e] - 8*b^2*e^2*m
*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 4*b*e^2*m*n*(-2*a + b*n - 2*b*Log[c
*x^n])*PolyLog[2, -((f*x)/e)] + 8*b^2*e^2*m*n^2*PolyLog[3, -((f*x)/e)])/(8*
f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 11828, normalized size = 31.71

method	result	size
risch	Expression too large to display	11828

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*((2*b^2*f*m*x*e - (f^2*m - 2*f^2*log(d))*b^2*x^2 - 2*b^2*m*e^2*log(f*x
+ e))*log(x^n)^2 + (2*b^2*f^2*x^2*log(x^n)^2 + 2*(2*a*b*f^2 - (f^2*n - 2*f^
2*log(c))*b^2)*x^2*log(x^n) + (2*a^2*f^2 - 2*(f^2*n - 2*f^2*log(c))*a*b + (
f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^2)*x^2)*log((f*x + e)^m))/f^2
+ integrate(-1/4*((2*(f^3*m - 2*f^3*log(d))*a^2 - 2*(f^3*m*n - 2*(f^3*m - 2
*f^3*log(d))*log(c))*a*b + (f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3
*log(d))*log(c)^2)*b^2)*x^3 - 4*(b^2*f^2*log(c)^2*log(d) + 2*a*b*f^2*log(c)
*log(d) + a^2*f^2*log(d))*x^2*e + 2*(2*b^2*f*m*n*x*e^2 + 2*((f^3*m - 2*f^3*
log(d))*a*b - (f^3*m*n - f^3*n*log(d) - (f^3*m - 2*f^3*log(d))*log(c))*b^2)
*x^3 - (4*a*b*f^2*log(d) - (f^2*m*n + 2*f^2*n*log(d) - 4*f^2*log(c)*log(d))
*b^2)*x^2*e - 2*(b^2*f*m*n*x*e^2 + b^2*m*n*e^3)*log(f*x + e))*log(x^n))/(f^
3*x^2 + f^2*x*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x + e)^m*
d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

3.80 $\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx$

Optimal. Leaf size=288

$$2abmnx-4b^2mn^2x+2bmn(a-bn)x+4b^2mnx\log(cx^n)-mx(a+b\log(cx^n))^2-\frac{2bemn(a-bn)\log(e+fx)}{f}-$$

```
[Out] 2*a*b*m*n*x-4*b^2*m*n^2*x+2*b*m*n*(-b*n+a)*x+4*b^2*m*n*x*ln(c*x^n)-m*x*(a+b
*ln(c*x^n))^2-2*b*e*m*n*(-b*n+a)*ln(f*x+e)/f-2*a*b*n*x*ln(d*(f*x+e)^m)+2*b^
2*n^2*x*ln(d*(f*x+e)^m)-2*b^2*n*x*ln(c*x^n)*ln(d*(f*x+e)^m)+x*(a+b*ln(c*x^n
))^2*ln(d*(f*x+e)^m)-2*b^2*e*m*n*ln(c*x^n)*ln(1+f*x/e)/f+e*m*(a+b*ln(c*x^n)
)^2*ln(1+f*x/e)/f-2*b^2*e*m*n^2*polylog(2,-f*x/e)/f+2*b*e*m*n*(a+b*ln(c*x^n
))*polylog(2,-f*x/e)/f-2*b^2*e*m*n^2*polylog(3,-f*x/e)/f
```

Rubi [A]
time = 0.24, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2333, 2332, 2418, 6, 45, 2393, 2354, 2438, 2395, 2421, 6724}

$$\frac{2bemn\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f}-\frac{2b^2emn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{f}-\frac{2b^2emn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{f}+x(a+b\log(cx^n))^2\log(d(e+fx)^m)+\frac{em\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{f}-mx(a+b\log(cx^n))^2-2abnx\log(d(e+fx)^m)-\frac{2bemn(a-bn)\log(e+fx)}{f}+2abmnx+2bmnx(a-bn)-2b^2nx\log(cx^n)\log(d(e+fx)^m)-\frac{2b^2emn\log(cx^n)\log\left(\frac{fx}{e}+1\right)}{f}+4b^2mnx\log(cx^n)+2b^2n^2x\log(d(e+fx)^m)-4b^2mn^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] 2*a*b*m*n*x - 4*b^2*m*n^2*x + 2*b*m*n*(a - b*n)*x + 4*b^2*m*n*x*Log[c*x^n]
- m*x*(a + b*Log[c*x^n])^2 - (2*b*e*m*n*(a - b*n)*Log[e + f*x])/f - 2*a*b*n
*x*Log[d*(e + f*x)^m] + 2*b^2*n^2*x*Log[d*(e + f*x)^m] - 2*b^2*n*x*Log[c*x^
n]*Log[d*(e + f*x)^m] + x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m] - (2*b^2*
e*m*n*Log[c*x^n]*Log[1 + (f*x)/e])/f + (e*m*(a + b*Log[c*x^n])^2*Log[1 + (f
*x)/e])/f - (2*b^2*e*m*n^2*PolyLog[2, -((f*x)/e)])/f + (2*b*e*m*n*(a + b*Lo
g[c*x^n])*PolyLog[2, -((f*x)/e)])/f - (2*b^2*e*m*n^2*PolyLog[3, -((f*x)/e)]
)/f
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^2 \log(d(e+fx)^m)\,dx = -2abnx\log(d(e+fx)^m) + 2b^2n^2x\log(d(e+fx)^m) - 2b^2nx\log$$

$$= -2abnx\log(d(e+fx)^m) + 2b^2n^2x\log(d(e+fx)^m) - 2b^2nx\log$$

$$= -2abnx\log(d(e+fx)^m) + 2b^2n^2x\log(d(e+fx)^m) - 2b^2nx\log$$

$$= -2abnx\log(d(e+fx)^m) + 2b^2n^2x\log(d(e+fx)^m) - 2b^2nx\log$$

$$= 2bmn(a-bn)x - \frac{2bemn(a-bn)\log(e+fx)}{f} - 2abnx\log(d(e$$

$$= -2b^2mn^2x + 2bmn(a-bn)x + 2b^2mnx\log(cx^n) - mx(a+b\log$$

$$= 2abmnx - 2b^2mn^2x + 2bmn(a-bn)x + 2b^2mnx\log(cx^n) - m$$

$$= 2abmnx - 4b^2mn^2x + 2bmn(a-bn)x + 4b^2mnx\log(cx^n) - m$$

Mathematica [A]
time = 0.12, size = 507, normalized size = 1.76

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a^2*f*m*x) + 4*a*b*f*m*n*x - 6*b^2*f*m*n^2*x - 2*a*b*f*m*x*Log[c*x^n] +
4*b^2*f*m*n*x*Log[c*x^n] - b^2*f*m*x*Log[c*x^n]^2 + a^2*e*m*Log[e + f*x] -
2*a*b*e*m*n*Log[e + f*x] + 2*b^2*e*m*n^2*Log[e + f*x] - 2*a*b*e*m*n*Log[x]*
Log[e + f*x] + 2*b^2*e*m*n^2*Log[x]*Log[e + f*x] + b^2*e*m*n^2*Log[x]^2*Log
[e + f*x] + 2*a*b*e*m*Log[c*x^n]*Log[e + f*x] - 2*b^2*e*m*n*Log[c*x^n]*Log[
e + f*x] - 2*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + b^2*e*m*Log[c*x^n]^
2*Log[e + f*x] + a^2*f*x*Log[d*(e + f*x)^m] - 2*a*b*f*n*x*Log[d*(e + f*x)^m
] + 2*b^2*f*n^2*x*Log[d*(e + f*x)^m] + 2*a*b*f*x*Log[c*x^n]*Log[d*(e + f*x)
^m] - 2*b^2*f*n*x*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*f*x*Log[c*x^n]^2*Log[
d*(e + f*x)^m] + 2*a*b*e*m*n*Log[x]*Log[1 + (f*x)/e] - 2*b^2*e*m*n^2*Log[x]
*Log[1 + (f*x)/e] - b^2*e*m*n^2*Log[x]^2*Log[1 + (f*x)/e] + 2*b^2*e*m*n*Log
[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 2*b*e*m*n*(a - b*n + b*Log[c*x^n])*PolyLo
g[2, -((f*x)/e)] - 2*b^2*e*m*n^2*PolyLog[3, -((f*x)/e)])/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 10356, normalized size = 35.96

method	result	size
risch	Expression too large to display	10356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] ((b^2*m*e*log(f*x + e) - (f*m - f*log(d))*b^2*x)*log(x^n)^2 + (b^2*f*x*log(
x^n)^2 - 2*((f*n - f*log(c))*b^2 - a*b*f)*x*log(x^n) - (2*(f*n - f*log(c))*
a*b - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^2 - a^2*f)*x)*log((f*x + e)^m
))/f - integrate((((f^2*m - f^2*log(d))*a^2 - 2*(f^2*m*n - (f^2*m - f^2*log
(d))*log(c))*a*b + (2*f^2*m*n^2 - 2*f^2*m*n*log(c) + (f^2*m - f^2*log(d))*l
og(c)^2)*b^2)*x^2 - (b^2*f*log(c)^2*log(d) + 2*a*b*f*log(c)*log(d) + a^2*f*
log(d))*x*e + 2*(((f^2*m - f^2*log(d))*a*b - (2*f^2*m*n - f^2*n*log(d) - (f
^2*m - f^2*log(d))*log(c))*b^2)*x^2 - (a*b*f*log(d) + (f*m*n - f*n*log(d) +
 f*log(c)*log(d))*b^2)*x*e + (b^2*f*m*n*x*e + b^2*m*n*e^2)*log(f*x + e))*lo
g(x^n))/(f^2*x^2 + f*x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2, x)
```

3.81 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx$

Optimal. Leaf size=131

$$\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx}{e}\right)}{3bn}-m(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{fx}{e}\right)+2bmn$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)+2*b*m*n*(a+b*ln(c*x^n))*polylo
g(3,-f*x/e)-2*b^2*m*n^2*polylog(4,-f*x/e)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2422, 2354, 2421, 2430, 6724}

$$-m\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^2+2bmn\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))-2b^2mn^2\text{PolyLog}\left(4,-\frac{fx}{e}\right)+\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn}-\frac{m\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/(3*b*n) - (m*(a + b*Log[c*x^n])^3
*Log[1 + (f*x)/e])/(3*b*n) - m*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x)/e)]
+ 2*b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] - 2*b^2*m*n^2*PolyLog[4
, -((f*x)/e)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
```

```
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{(fm)\int\frac{(a+b\log(cx^n))^3}{e+fx}\,dx}{3bn}$$
$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log(1}{3bn}$$
$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log(1}{3bn}$$
$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log(1}{3bn}$$
$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{3bn} - \frac{m(a+b\log(cx^n))^3\log(1}{3bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(131) = 262.
time = 0.09, size = 329, normalized size = 2.51

$$a^2\log(x)\log(d(e+fx)^m) - abn\log^2(x)\log(d(e+fx)^m) + \frac{1}{3}b^2n^2\log^3(x)\log(d(e+fx)^m) + 2ab\log(x)\log(cx^n)\log(d(e+fx)^m) - b^2n\log^2(x)\log(cx^n)\log(d(e+fx)^m) + b^2\log(x)\log^2(cx^n)\log(d(e+fx)^m) - a^2m\log(x)\log\left(1+\frac{fx}{e}\right) + abmn\log^2(x)\log\left(1+\frac{fx}{e}\right) - \frac{1}{3}b^2mn^2\log^3(x)\log\left(1+\frac{fx}{e}\right) - 2abm\log(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) + b^2mn\log^2(x)\log(cx^n)\log\left(1+\frac{fx}{e}\right) - b^2m\log(x)\log^2(cx^n)\log\left(1+\frac{fx}{e}\right) - m(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{fx}{e}\right) + 2bmn(a+b\log(cx^n))\operatorname{Li}_3\left(-\frac{fx}{e}\right) - 2b^2mn^2\operatorname{Li}_4\left(-\frac{fx}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] a^2*Log[x]*Log[d*(e + f*x)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x)^m] + (b^2*n^
2*Log[x]^3*Log[d*(e + f*x)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^
```

```
m] - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x)^m] + b^2*Log[x]*Log[c*x^n]^2
*Log[d*(e + f*x)^m] - a^2*m*Log[x]*Log[1 + (f*x)/e] + a*b*m*n*Log[x]^2*Log[
1 + (f*x)/e] - (b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e])/3 - 2*a*b*m*Log[x]*Log
[c*x^n]*Log[1 + (f*x)/e] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - b
^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - m*(a + b*Log[c*x^n])^2*PolyLog[
2, -((f*x)/e)] + 2*b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] - 2*b^2*
m*n^2*PolyLog[4, -((f*x)/e)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 21792, normalized size = 166.35

method	result	size
risch	Expression too large to display	21792

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x + e)^m) - integrate(1/3*
(b^2*f*m*n^2*x*log(x)^3 - 3*(b^2*f*m*n*log(c) + a*b*f*m*n)*x*log(x)^2 + 3*(
b^2*f*m*log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x*log(x) + 3*(b^2*f*m*x*log(
x) - b^2*f*x*log(d) - b^2*e*log(d))*log(x^n)^2 - 3*(b^2*f*log(c)^2*log(d) +
 2*a*b*f*log(c)*log(d) + a^2*f*log(d))*x - 3*(b^2*log(c)^2*log(d) + 2*a*b*l
og(c)*log(d) + a^2*log(d))*e - 3*(b^2*f*m*n*x*log(x)^2 - 2*(b^2*f*m*log(c)
+ a*b*f*m)*x*log(x) + 2*(b^2*f*log(c)*log(d) + a*b*f*log(d))*x + 2*(b^2*log
(c)*log(d) + a*b*log(d))*e)*log(x^n))/(f*x^2 + x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x,
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x, x)
```

3.82 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx$

Optimal. Leaf size=248

$$\frac{2b^2fmn^2\log(x)}{e}-\frac{2bfmn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{e}-\frac{fm\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{e}-\frac{2b^2fmn^2\log(e}{e}$$

```
[Out] 2*b^2*f*m*n^2*ln(x)/e-2*b*f*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-f*m*ln(1+e/f/
x)*(a+b*ln(c*x^n))^2/e-2*b^2*f*m*n^2*ln(f*x+e)/e-2*b^2*n^2*ln(d*(f*x+e)^m)/
x-2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)
/x+2*b^2*f*m*n^2*polylog(2,-e/f/x)/e+2*b*f*m*n*(a+b*ln(c*x^n))*polylog(2,-e
/f/x)/e+2*b^2*f*m*n^2*polylog(3,-e/f/x)/e
```

Rubi [A]
time = 0.20, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2342, 2341, 2425, 36, 29, 31, 2379, 2438, 2421, 6724}

$$\frac{2bfmn\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}+\frac{2b^2fmn^2\text{PolyLog}\left(2,-\frac{e}{fx}\right)}{e}+\frac{2b^2fmn^2\text{PolyLog}\left(3,-\frac{e}{fx}\right)}{e}-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{2bfmn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{e}-\frac{fm\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^2}{e}-\frac{2b^2n^2\log(d(e+fx)^m)}{x}+\frac{2b^2fmn^2\log(x)}{e}-\frac{2b^2fmn^2\log(e+fx)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (2*b^2*f*m*n^2*Log[x])/e - (2*b*f*m*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/
e - (f*m*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/e - (2*b^2*f*m*n^2*Log[e +
f*x])/e - (2*b^2*n^2*Log[d*(e + f*x)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[
d*(e + f*x)^m])/x - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x + (2*b^2*f*
m*n^2*PolyLog[2, -(e/(f*x))])/e + (2*b*f*m*n*(a + b*Log[c*x^n])*PolyLog[2,
-(e/(f*x))])/e + (2*b^2*f*m*n^2*PolyLog[3, -(e/(f*x))])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^2}\,dx = -\frac{2b^2n^2\log(d(e+fx)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}$$

$$= -\frac{2b^2n^2\log(d(e+fx)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}$$

$$= -\frac{2b^2n^2\log(d(e+fx)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{x}$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm(a+b\log(cx^n))^2}{e} - \frac{2b^2fmn^2\log(e+fx)}{e}$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm(a+b\log(cx^n))^2}{e} + \frac{fm(a+b\log(cx^n))^3}{3ben}$$

$$= \frac{2b^2fmn^2\log(x)}{e} + \frac{fm(a+b\log(cx^n))^2}{e} + \frac{fm(a+b\log(cx^n))^3}{3ben}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 600 vs. $2(248) = 496$.

time = 0.21, size = 600, normalized size = 2.42

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/3*(-3*a^2*f*m*x*Log[x] - 6*a*b*f*m*n*x*Log[x] - 6*b^2*f*m*n^2*x*Log[x] +
 3*a*b*f*m*n*x*Log[x]^2 + 3*b^2*f*m*n^2*x*Log[x]^2 - b^2*f*m*n^2*x*Log[x]^3
 - 6*a*b*f*m*x*Log[x]*Log[c*x^n] - 6*b^2*f*m*n*x*Log[x]*Log[c*x^n] + 3*b^2*
f*m*n*x*Log[x]^2*Log[c*x^n] - 3*b^2*f*m*x*Log[x]*Log[c*x^n]^2 + 3*a^2*f*m*x
*Log[e + f*x] + 6*a*b*f*m*n*x*Log[e + f*x] + 6*b^2*f*m*n^2*x*Log[e + f*x] -
 6*a*b*f*m*n*x*Log[x]*Log[e + f*x] - 6*b^2*f*m*n^2*x*Log[x]*Log[e + f*x] +
3*b^2*f*m*n^2*x*Log[x]^2*Log[e + f*x] + 6*a*b*f*m*x*Log[c*x^n]*Log[e + f*x]
 + 6*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] - 6*b^2*f*m*n*x*Log[x]*Log[c*x^n]*
Log[e + f*x] + 3*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x] + 3*a^2*e*Log[d*(e + f
*x)^m] + 6*a*b*e*n*Log[d*(e + f*x)^m] + 6*b^2*e*n^2*Log[d*(e + f*x)^m] + 6*
a*b*e*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e*n*Log[c*x^n]*Log[d*(e + f*x)^
m] + 3*b^2*e*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 6*a*b*f*m*n*x*Log[x]*Log[1 +
 (f*x)/e] + 6*b^2*f*m*n^2*x*Log[x]*Log[1 + (f*x)/e] - 3*b^2*f*m*n^2*x*Log[x
]^2*Log[1 + (f*x)/e] + 6*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 6
*b*f*m*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)] - 6*b^2*f*m*n^2*
x*PolyLog[3, -((f*x)/e)])/(e*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 10967, normalized size = 44.22

method	result	size
risch	Expression too large to display	10967

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -((b^2*f*m*x*log(f*x + e) - b^2*f*m*x*log(x) + b^2*e*log(d))*log(x^n)^2 + (
b^2*e*log(x^n)^2 + 2*(b^2*(n + log(c)) + a*b)*e*log(x^n) + ((2*n^2 + 2*n*lo
g(c) + log(c)^2)*b^2 + 2*a*b*(n + log(c)) + a^2)*e)*log((f*x + e)^m))*e^(-1
)/x + integrate((((f*m + f*log(d))*a^2 + 2*(f*m*n + (f*m + f*log(d))*log(c)
)*a*b + (2*f*m*n^2 + 2*f*m*n*log(c) + (f*m + f*log(d))*log(c)^2)*b^2)*x*e +
 (b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e^2 + 2*(((f*m +
f*log(d))*a*b + (f*m*n + f*n*log(d) + (f*m + f*log(d))*log(c))*b^2)*x*e + (
(n*log(d) + log(c)*log(d))*b^2 + a*b*log(d))*e^2 + (b^2*f^2*m*n*x^2 + b^2*f
*m*n*x*e)*log(f*x + e) - (b^2*f^2*m*n*x^2 + b^2*f*m*n*x*e)*log(x))*log(x^n)
)/(f*x^3*e + x^2*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^2
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

3.83 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx$

Optimal. Leaf size=344

$$-\frac{7b^2fmn^2}{4ex}-\frac{b^2f^2mn^2\log(x)}{4e^2}-\frac{3bfmn(a+b\log(cx^n))}{2ex}+\frac{bf^2mn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\lo}{2ex}$$

```
[Out] -7/4*b^2*f*m*n^2/e/x-1/4*b^2*f^2*m*n^2*ln(x)/e^2-3/2*b*f*m*n*(a+b*ln(c*x^n)
)/e/x+1/2*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-1/2*f*m*(a+b*ln(c*x^n))
^2/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2+1/4*b^2*f^2*m*n^2*ln(f*x
+e)/e^2-1/4*b^2*n^2*ln(d*(f*x+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e
)^m)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2-1/2*b^2*f^2*m*n^2*polylo
g(2,-e/f/x)/e^2-b*f^2*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-b^2*f^2*m*n
^2*polylog(3,-e/f/x)/e^2
```

Rubi [A]
time = 0.35, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2342, 2341, 2425, 46, 2380, 2379, 2438, 2421, 6724}

$$-\frac{bf^2mn\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e^2}-\frac{b^2f^2mn^2\text{PolyLog}\left(2,-\frac{e}{fx}\right)}{2e^2}-\frac{b^2f^2mn^2\text{PolyLog}\left(3,-\frac{e}{fx}\right)}{e^2}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{2x^2}-\frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}+\frac{f^2m\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^2}{2e^2}+\frac{bf^2mn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{2e^2}-\frac{fm(a+b\log(cx^n))^2}{2ex}-\frac{3bfmn(a+b\log(cx^n))}{2ex}-\frac{b^2n^2\log(d(e+fx)^m)}{4x^2}-\frac{b^2f^2mn^2\log(x)}{4e^2}+\frac{b^2f^2mn^2\log(e+fx)}{4e^2}-\frac{7b^2fmn^2}{4ex}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-7*b^2*f*m*n^2)/(4*e*x) - (b^2*f^2*m*n^2*Log[x])/(4*e^2) - (3*b*f*m*n*(a +
 b*Log[c*x^n]))/(2*e*x) + (b*f^2*m*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n]))/(
2*e^2) - (f*m*(a + b*Log[c*x^n])^2)/(2*e*x) + (f^2*m*Log[1 + e/(f*x)]*(a +
b*Log[c*x^n])^2)/(2*e^2) + (b^2*f^2*m*n^2*Log[e + f*x])/(4*e^2) - (b^2*n^2*
Log[d*(e + f*x)^m])/(4*x^2) - (b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(
2*x^2) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(2*x^2) - (b^2*f^2*m*n^2
*PolyLog[2, -(e/(f*x))])/(2*e^2) - (b*f^2*m*n*(a + b*Log[c*x^n])*PolyLog[2,
 -(e/(f*x))])/e^2 - (b^2*f^2*m*n^2*PolyLog[3, -(e/(f*x))])/e^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^3}\,dx = -\frac{b^2n^2\log(d(e+fx)^m)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}$$

$$= -\frac{b^2n^2\log(d(e+fx)^m)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}$$

$$= -\frac{b^2n^2\log(d(e+fx)^m)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(d(e+fx)^m)}{2x^2}$$

$$= -\frac{b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} + \frac{b^2f^2mn^2\log(e+fx)}{4e^2} - \frac{b^2n^2\log}{}$$

$$= -\frac{3b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{bfmn(a+b\log(cx^n))}{2ex} - \frac{f^2m(}{}$$

$$= -\frac{7b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{3bfmn(a+b\log(cx^n))}{2ex} - \frac{f^2m}{}$$

$$= -\frac{7b^2fmn^2}{4ex} - \frac{b^2f^2mn^2\log(x)}{4e^2} - \frac{3bfmn(a+b\log(cx^n))}{2ex} - \frac{f^2m}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 796 vs. 2(344) = 688.
time = 0.23, size = 796, normalized size = 2.31

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/12*(6*a^2*e*f*m*x + 18*a*b*e*f*m*n*x + 21*b^2*e*f*m*n^2*x + 6*a^2*f^2*m*
x^2*Log[x] + 6*a*b*f^2*m*n*x^2*Log[x] + 3*b^2*f^2*m*n^2*x^2*Log[x] - 6*a*b*
f^2*m*n*x^2*Log[x]^2 - 3*b^2*f^2*m*n^2*x^2*Log[x]^2 + 2*b^2*f^2*m*n^2*x^2*L
og[x]^3 + 12*a*b*e*f*m*x*Log[c*x^n] + 18*b^2*e*f*m*n*x*Log[c*x^n] + 12*a*b*
f^2*m*x^2*Log[x]*Log[c*x^n] + 6*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] - 6*b^2*f
^2*m*n*x^2*Log[x]^2*Log[c*x^n] + 6*b^2*e*f*m*x*Log[c*x^n]^2 + 6*b^2*f^2*m*x
^2*Log[x]*Log[c*x^n]^2 - 6*a^2*f^2*m*x^2*Log[e + f*x] - 6*a*b*f^2*m*n*x^2*L
og[e + f*x] - 3*b^2*f^2*m*n^2*x^2*Log[e + f*x] + 12*a*b*f^2*m*n*x^2*Log[x]*
```

```
Log[e + f*x] + 6*b^2*f^2*m*n^2*x^2*Log[x]*Log[e + f*x] - 6*b^2*f^2*m*n^2*x^
2*Log[x]^2*Log[e + f*x] - 12*a*b*f^2*m*x^2*Log[c*x^n]*Log[e + f*x] - 6*b^2*
f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] + 12*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*
Log[e + f*x] - 6*b^2*f^2*m*x^2*Log[c*x^n]^2*Log[e + f*x] + 6*a^2*e^2*Log[d*
(e + f*x)^m] + 6*a*b*e^2*n*Log[d*(e + f*x)^m] + 3*b^2*e^2*n^2*Log[d*(e + f*
x)^m] + 12*a*b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^2*e^2*n*Log[c*x^n]*L
og[d*(e + f*x)^m] + 6*b^2*e^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 12*a*b*f^2*
m*n*x^2*Log[x]*Log[1 + (f*x)/e] - 6*b^2*f^2*m*n^2*x^2*Log[x]*Log[1 + (f*x)/
e] + 6*b^2*f^2*m*n^2*x^2*Log[x]^2*Log[1 + (f*x)/e] - 12*b^2*f^2*m*n*x^2*Log
[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 6*b*f^2*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^
n])*PolyLog[2, -((f*x)/e)] + 12*b^2*f^2*m*n^2*x^2*PolyLog[3, -((f*x)/e)])/(
e^2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 12159, normalized size = 35.35

method	result	size
risch	Expression too large to display	12159

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^2*f^2*m*x^2*log(f*x + e) - b^2*f^2*m*x^2*log(x) - b^2*f*m*x*e - b
^2*e^2*log(d))*log(x^n)^2 - (2*b^2*e^2*log(x^n)^2 + 2*(b^2*(n + 2*log(c)) +
 2*a*b)*e^2*log(x^n) + ((n^2 + 2*n*log(c) + 2*log(c)^2)*b^2 + 2*a*b*(n + 2*
log(c)) + 2*a^2)*e^2)*log((f*x + e)^m))*e^(-2)/x^2 - integrate(-1/4*((2*(f*
m + 2*f*log(d))*a^2 + 2*(f*m*n + 2*(f*m + 2*f*log(d))*log(c))*a*b + (f*m*n^
2 + 2*f*m*n*log(c) + 2*(f*m + 2*f*log(d))*log(c)^2)*b^2)*x*e^2 + 4*(b^2*log
(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e^3 + 2*(2*b^2*f^2*m*n*x^2
*e + (2*(f*m + 2*f*log(d))*a*b + (3*f*m*n + 2*f*n*log(d) + 2*(f*m + 2*f*log
(d))*log(c))*b^2)*x*e^2 + 2*((n*log(d) + 2*log(c)*log(d))*b^2 + 2*a*b*log(d
))*e^3 - 2*(b^2*f^3*m*n*x^3 + b^2*f^2*m*n*x^2*e)*log(f*x + e) + 2*(b^2*f^3*
m*n*x^3 + b^2*f^2*m*n*x^2*e)*log(x))*log(x^n))/(f*x^4*e^2 + x^3*e^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^3
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

3.84 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx$

Optimal. Leaf size=420

$$-\frac{19b^2fmn^2}{108ex^2}+\frac{26b^2f^2mn^2}{27e^2x}+\frac{2b^2f^3mn^2\log(x)}{27e^3}-\frac{5bfmn(a+b\log(cx^n))}{18ex^2}+\frac{8bf^2mn(a+b\log(cx^n))}{9e^2x}-\frac{2bf^3mn\,\mathrm{lo}}{}$$

```
[Out] -19/108*b^2*f*m*n^2/e/x^2+26/27*b^2*f^2*m*n^2/e^2/x+2/27*b^2*f^3*m*n^2*ln(x
)/e^3-5/18*b*f*m*n*(a+b*ln(c*x^n))/e/x^2+8/9*b*f^2*m*n*(a+b*ln(c*x^n))/e^2/
x-2/9*b*f^3*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^3-1/6*f*m*(a+b*ln(c*x^n))^2/e
/x^2+1/3*f^2*m*(a+b*ln(c*x^n))^2/e^2/x-1/3*f^3*m*ln(1+e/f/x)*(a+b*ln(c*x^n)
)^2/e^3-2/27*b^2*f^3*m*n^2*ln(f*x+e)/e^3-2/27*b^2*n^2*ln(d*(f*x+e)^m)/x^3-2
/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)/x^3-1/3*(a+b*ln(c*x^n))^2*ln(d*(f*x+
e)^m)/x^3+2/9*b^2*f^3*m*n^2*polylog(2,-e/f/x)/e^3+2/3*b*f^3*m*n*(a+b*ln(c*x
^n))*polylog(2,-e/f/x)/e^3+2/3*b^2*f^3*m*n^2*polylog(3,-e/f/x)/e^3
```

Rubi [A]
time = 0.50, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2342, 2341, 2425, 46, 2380, 2379, 2438, 2421, 6724}

$$\frac{2bf^3mn\,\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{3e^3}+\frac{2b^2f^3mn^2\,\text{PolyLog}\left(2,-\frac{e}{fx}\right)}{9e^3}+\frac{2b^2f^3mn^2\,\text{PolyLog}\left(3,-\frac{e}{fx}\right)}{3e^3}-\frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{3x^3}-\frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}-\frac{f^3m\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^2}{3e^3}-\frac{2bf^3mn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{9e^3}+\frac{f^2m(a+b\log(cx^n))^2}{3e^2x}+\frac{8bf^2mn(a+b\log(cx^n))}{9e^2x}-\frac{fm(a+b\log(cx^n))^2}{6ex^2}-\frac{5bfmn(a+b\log(cx^n))}{18ex^2}-\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3}+\frac{2b^2f^3mn^2\log(x)}{27e^3}-\frac{2b^2f^3mn^2\log(e+fx)}{27e^3}+\frac{26b^2f^2mn^2}{27e^2x}-\frac{19b^2fmn^2}{108ex^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] (-19*b^2*f*m*n^2)/(108*e*x^2) + (26*b^2*f^2*m*n^2)/(27*e^2*x) + (2*b^2*f^3*
m*n^2*Log[x])/(27*e^3) - (5*b*f*m*n*(a + b*Log[c*x^n]))/(18*e*x^2) + (8*b*f
^2*m*n*(a + b*Log[c*x^n]))/(9*e^2*x) - (2*b*f^3*m*n*Log[1 + e/(f*x)]*(a + b
*Log[c*x^n]))/(9*e^3) - (f*m*(a + b*Log[c*x^n])^2)/(6*e*x^2) + (f^2*m*(a +
b*Log[c*x^n])^2)/(3*e^2*x) - (f^3*m*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/
(3*e^3) - (2*b^2*f^3*m*n^2*Log[e + f*x])/(27*e^3) - (2*b^2*n^2*Log[d*(e + f
*x)^m])/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(9*x^3) -
((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(3*x^3) + (2*b^2*f^3*m*n^2*PolyLo
g[2, -(e/(f*x))]])/(9*e^3) + (2*b*f^3*m*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/
(f*x))])/(3*e^3) + (2*b^2*f^3*m*n^2*PolyLog[3, -(e/(f*x))])/(3*e^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x^4}\,dx = -\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}$$

$$= -\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}$$

$$= -\frac{2b^2n^2\log(d(e+fx)^m)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx)^m)}{9x^3}$$

$$= -\frac{b^2fmn^2}{27ex^2} + \frac{2b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{2b^2f^3mn^2\log(e+}{27e^3}$$

$$= -\frac{5b^2fmn^2}{54ex^2} + \frac{8b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{bfmn(a+b\log(c}{9ex^2}$$

$$= -\frac{19b^2fmn^2}{108ex^2} + \frac{26b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{5bfmn(a+b\log}{18ex^2}$$

$$= -\frac{19b^2fmn^2}{108ex^2} + \frac{26b^2f^2mn^2}{27e^2x} + \frac{2b^2f^3mn^2\log(x)}{27e^3} - \frac{5bfmn(a+b\log}{18ex^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 909 vs. 2(420) = 840.
time = 0.26, size = 909, normalized size = 2.16

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x^4,x]
```

```
[Out] -1/108*(18*a^2*e^2*f*m*x + 30*a*b*e^2*f*m*n*x + 19*b^2*e^2*f*m*n^2*x - 36*a
^2*e*f^2*m*x^2 - 96*a*b*e*f^2*m*n*x^2 - 104*b^2*e*f^2*m*n^2*x^2 - 36*a^2*f^
3*m*x^3*Log[x] - 24*a*b*f^3*m*n*x^3*Log[x] - 8*b^2*f^3*m*n^2*x^3*Log[x] + 3
6*a*b*f^3*m*n*x^3*Log[x]^2 + 12*b^2*f^3*m*n^2*x^3*Log[x]^2 - 12*b^2*f^3*m*n
^2*x^3*Log[x]^3 + 36*a*b*e^2*f*m*x*Log[c*x^n] + 30*b^2*e^2*f*m*n*x*Log[c*x^
n] - 72*a*b*e*f^2*m*x^2*Log[c*x^n] - 96*b^2*e*f^2*m*n*x^2*Log[c*x^n] - 72*a
*b*f^3*m*x^3*Log[x]*Log[c*x^n] - 24*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n] + 36*
```

```
b^2*f^3*m*n*x^3*Log[x]^2*Log[c*x^n] + 18*b^2*e^2*f*m*x*Log[c*x^n]^2 - 36*b^
2*e*f^2*m*x^2*Log[c*x^n]^2 - 36*b^2*f^3*m*x^3*Log[x]*Log[c*x^n]^2 + 36*a^2*
f^3*m*x^3*Log[e + f*x] + 24*a*b*f^3*m*n*x^3*Log[e + f*x] + 8*b^2*f^3*m*n^2*
x^3*Log[e + f*x] - 72*a*b*f^3*m*n*x^3*Log[x]*Log[e + f*x] - 24*b^2*f^3*m*n^
2*x^3*Log[x]*Log[e + f*x] + 36*b^2*f^3*m*n^2*x^3*Log[x]^2*Log[e + f*x] + 72
*a*b*f^3*m*x^3*Log[c*x^n]*Log[e + f*x] + 24*b^2*f^3*m*n*x^3*Log[c*x^n]*Log[
e + f*x] - 72*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n]*Log[e + f*x] + 36*b^2*f^3*m
*x^3*Log[c*x^n]^2*Log[e + f*x] + 36*a^2*e^3*Log[d*(e + f*x)^m] + 24*a*b*e^3
*n*Log[d*(e + f*x)^m] + 8*b^2*e^3*n^2*Log[d*(e + f*x)^m] + 72*a*b*e^3*Log[c
*x^n]*Log[d*(e + f*x)^m] + 24*b^2*e^3*n*Log[c*x^n]*Log[d*(e + f*x)^m] + 36*
b^2*e^3*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 72*a*b*f^3*m*n*x^3*Log[x]*Log[1 +
 (f*x)/e] + 24*b^2*f^3*m*n^2*x^3*Log[x]*Log[1 + (f*x)/e] - 36*b^2*f^3*m*n^2
*x^3*Log[x]^2*Log[1 + (f*x)/e] + 72*b^2*f^3*m*n*x^3*Log[x]*Log[c*x^n]*Log[1
 + (f*x)/e] + 24*b*f^3*m*n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, -((f
*x)/e)] - 72*b^2*f^3*m*n^2*x^3*PolyLog[3, -((f*x)/e)])/(e^3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 13227, normalized size = 31.49

method	result	size
risch	Expression too large to display	13227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/54*(9*(2*b^2*f^3*m*x^3*log(f*x + e) - 2*b^2*f^3*m*x^3*log(x) - 2*b^2*f^2
*m*x^2*e + b^2*f*m*x*e^2 + 2*b^2*e^3*log(d))*log(x^n)^2 + 2*(9*b^2*e^3*log(
x^n)^2 + 6*(b^2*(n + 3*log(c)) + 3*a*b)*e^3*log(x^n) + ((2*n^2 + 6*n*log(c)
 + 9*log(c)^2)*b^2 + 6*a*b*(n + 3*log(c)) + 9*a^2)*e^3)*log((f*x + e)^m))*e
^(-3)/x^3 + integrate(1/27*((9*(f*m + 3*f*log(d))*a^2 + 6*(f*m*n + 3*(f*m +
 3*f*log(d))*log(c))*a*b + (2*f*m*n^2 + 6*f*m*n*log(c) + 9*(f*m + 3*f*log(d
))*log(c)^2)*b^2)*x*e^3 + 27*(b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a
^2*log(d))*e^4 - 3*(6*b^2*f^3*m*n*x^3*e + 3*b^2*f^2*m*n*x^2*e^2 - (6*(f*m +
 3*f*log(d))*a*b + (5*f*m*n + 6*f*n*log(d) + 6*(f*m + 3*f*log(d))*log(c))*b
^2)*x*e^3 - 6*((n*log(d) + 3*log(c)*log(d))*b^2 + 3*a*b*log(d))*e^4 - 6*(b^
```

```
2*f^4*m*n*x^4 + b^2*f^3*m*n*x^3*e)*log(f*x + e) + 6*(b^2*f^4*m*n*x^4 + b^2*
f^3*m*n*x^3*e)*log(x))*log(x^n))/(f*x^5*e^3 + x^4*e^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x + e)^m*d)/x^4
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x+e)**m)/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x + e)^m*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

3.85 $\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx$

Optimal. Leaf size=603

$$\frac{21ab^2emn^2x}{4f}-\frac{45b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{21b^3emn^2x\log(cx^n)}{4f}-\frac{9}{8}b^2mn^2x^2(a+b\log(cx^n))-\frac{9bemnx(a+b\log}{4f}$$

[Out] 21/4*a*b^2*e*m*n^2*x/f-45/8*b^3*e*m*n^3*x/f+3/4*b^3*m*n^3*x^2+21/4*b^3*e*m*n^2*x*ln(c*x^n)/f-9/8*b^2*m*n^2*x^2*(a+b*ln(c*x^n))-9/4*b*e*m*n*x*(a+b*ln(c*x^n))^2/f+3/4*b*m*n*x^2*(a+b*ln(c*x^n))^2+1/2*e*m*x*(a+b*ln(c*x^n))^3/f-1/4*m*x^2*(a+b*ln(c*x^n))^3+3/8*b^3*e^2*m*n^3*ln(f*x+e)/f^2-3/8*b^3*n^3*x^2*ln(d*(f*x+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x+e)^m)-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)-3/4*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x/e)/f^2+3/4*b*e^2*m*n*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f^2-1/2*e^2*m*(a+b*ln(c*x^n))^3*ln(1+f*x/e)/f^2-3/4*b^3*e^2*m*n^3*polylog(2,-f*x/e)/f^2+3/2*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x/e)/f^2-3/2*b*e^2*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f^2-3/2*b^3*e^2*m*n^3*polylog(3,-f*x/e)/f^2+3*b^2*e^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x/e)/f^2-3*b^3*e^2*m*n^3*polylog(4,-f*x/e)/f^2

Rubi [A]
time = 0.63, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2342, 2341, 2425, 45, 2393, 2332, 2354, 2438, 2395, 2333, 2421, 6724, 2430}

Antiderivative was successfully verified.

[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]

[Out] (21*a*b^2*e*m*n^2*x)/(4*f) - (45*b^3*e*m*n^3*x)/(8*f) + (3*b^3*m*n^3*x^2)/4 + (21*b^3*e*m*n^2*x*Log[c*x^n])/(4*f) - (9*b^2*m*n^2*x^2*(a + b*Log[c*x^n]))/8 - (9*b*e*m*n*x*(a + b*Log[c*x^n])^2)/(4*f) + (3*b*m*n*x^2*(a + b*Log[c*x^n])^2)/4 + (e*m*x*(a + b*Log[c*x^n])^3)/(2*f) - (m*x^2*(a + b*Log[c*x^n])^3)/4 + (3*b^3*e^2*m*n^3*Log[e + f*x])/(8*f^2) - (3*b^3*n^3*x^2*Log[d*(e + f*x)^m])/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/4 - (3*b*n*x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/4 + (x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/2 - (3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*Log[1 + (f*x)/e])/(4*f^2) + (3*b*e^2*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/(4*f^2) - (e^2*m*(a + b*Log[c*x^n])^3*Log[1 + (f*x)/e])/(2*f^2) - (3*b^3*e^2*m*n^3*PolyLog[2, -((f*x)/e)])/(4*f^2) + (3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)])/(2*f^2) - (3*b*e^2*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x)/e)])/(2*f^2) - (3*b^3*e^2*m*n^3*PolyLog[3, -((f*x)/e)])/(2*f^2) + (3*b^2*e^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/f^2 - (3*b^3*e^2*m*n^3*PolyLog[4, -((f*x)/e)])/f^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx = -\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e-$$

$$= -\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e-$$

$$= -\frac{3}{8}b^3n^3x^2\log(d(e+fx)^m)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d(e-$$

$$= -\frac{3b^3emn^3x}{8f}+\frac{3}{16}b^3mn^3x^2+\frac{3b^3e^2mn^3\log(e+fx)}{8f^2}-\frac{3}{8}b^3n^3x^2\mathrm{l}$$

$$= \frac{3ab^2emn^2x}{4f}-\frac{3b^3emn^3x}{8f}+\frac{3}{8}b^3mn^3x^2-\frac{3}{8}b^2mn^2x^2(a+b\log(c$$

$$= \frac{9ab^2emn^2x}{4f}-\frac{9b^3emn^3x}{8f}+\frac{9}{16}b^3mn^3x^2+\frac{3b^3emn^2x\log(cx^n)}{4f}-$$

$$= \frac{21ab^2emn^2x}{4f}-\frac{21b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{9b^3emn^2x\log(cx^n)}{4f}$$

$$= \frac{21ab^2emn^2x}{4f}-\frac{45b^3emn^3x}{8f}+\frac{3}{4}b^3mn^3x^2+\frac{21b^3emn^2x\log(cx^n)}{4f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1431 vs. $2(603) = 1206$.
time = 0.34, size = 1431, normalized size = 2.37

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] (4*a^3*e*f*m*x - 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x - 45*b^3*e*f*m*n
^3*x - 2*a^3*f^2*m*x^2 + 6*a^2*b*f^2*m*n*x^2 - 9*a*b^2*f^2*m*n^2*x^2 + 6*b^
3*f^2*m*n^3*x^2 + 12*a^2*b*e*f*m*x*Log[c*x^n] - 36*a*b^2*e*f*m*n*x*Log[c*x^
n] + 42*b^3*e*f*m*n^2*x*Log[c*x^n] - 6*a^2*b*f^2*m*x^2*Log[c*x^n] + 12*a*b^
2*f^2*m*n*x^2*Log[c*x^n] - 9*b^3*f^2*m*n^2*x^2*Log[c*x^n] + 12*a*b^2*e*f*m*
x*Log[c*x^n]^2 - 18*b^3*e*f*m*n*x*Log[c*x^n]^2 - 6*a*b^2*f^2*m*x^2*Log[c*x^
n]^2 + 6*b^3*f^2*m*n*x^2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 - 2*b^3*
f^2*m*x^2*Log[c*x^n]^3 - 4*a^3*e^2*m*Log[e + f*x] + 6*a^2*b*e^2*m*n*Log[e +
 f*x] - 6*a*b^2*e^2*m*n^2*Log[e + f*x] + 3*b^3*e^2*m*n^3*Log[e + f*x] + 12*
a^2*b*e^2*m*n*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x]*Log[e + f*x]
+ 6*b^3*e^2*m*n^3*Log[x]*Log[e + f*x] - 12*a*b^2*e^2*m*n^2*Log[x]^2*Log[e +
```

```
 f*x] + 6*b^3*e^2*m*n^3*Log[x]^2*Log[e + f*x] + 4*b^3*e^2*m*n^3*Log[x]^3*Lo
g[e + f*x] - 12*a^2*b*e^2*m*Log[c*x^n]*Log[e + f*x] + 12*a*b^2*e^2*m*n*Log[
c*x^n]*Log[e + f*x] - 6*b^3*e^2*m*n^2*Log[c*x^n]*Log[e + f*x] + 24*a*b^2*e^
2*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*e^2*m*n^2*Log[x]*Log[c*x^n]*L
og[e + f*x] - 12*b^3*e^2*m*n^2*Log[x]^2*Log[c*x^n]*Log[e + f*x] - 12*a*b^2*
e^2*m*Log[c*x^n]^2*Log[e + f*x] + 6*b^3*e^2*m*n*Log[c*x^n]^2*Log[e + f*x] +
 12*b^3*e^2*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x] - 4*b^3*e^2*m*Log[c*x^n]^3
*Log[e + f*x] + 4*a^3*f^2*x^2*Log[d*(e + f*x)^m] - 6*a^2*b*f^2*n*x^2*Log[d*
(e + f*x)^m] + 6*a*b^2*f^2*n^2*x^2*Log[d*(e + f*x)^m] - 3*b^3*f^2*n^3*x^2*L
og[d*(e + f*x)^m] + 12*a^2*b*f^2*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] - 12*a*b
^2*f^2*n*x^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 6*b^3*f^2*n^2*x^2*Log[c*x^n]*L
og[d*(e + f*x)^m] + 12*a*b^2*f^2*x^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 6*b^
3*f^2*n*x^2*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 4*b^3*f^2*x^2*Log[c*x^n]^3*Lo
g[d*(e + f*x)^m] - 12*a^2*b*e^2*m*n*Log[x]*Log[1 + (f*x)/e] + 12*a*b^2*e^2*
m*n^2*Log[x]*Log[1 + (f*x)/e] - 6*b^3*e^2*m*n^3*Log[x]*Log[1 + (f*x)/e] + 1
2*a*b^2*e^2*m*n^2*Log[x]^2*Log[1 + (f*x)/e] - 6*b^3*e^2*m*n^3*Log[x]^2*Log[
1 + (f*x)/e] - 4*b^3*e^2*m*n^3*Log[x]^3*Log[1 + (f*x)/e] - 24*a*b^2*e^2*m*n
*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 12*b^3*e^2*m*n^2*Log[x]*Log[c*x^n]*Lo
g[1 + (f*x)/e] + 12*b^3*e^2*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - 12
*b^3*e^2*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - 6*b*e^2*m*n*(2*a^2 - 2*
a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog
[2, -((f*x)/e)] + 12*b^2*e^2*m*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[3,
-((f*x)/e)] - 24*b^3*e^2*m*n^3*PolyLog[4, -((f*x)/e)])/(8*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.25, size = 44991, normalized size = 74.61

method	result	size
risch	Expression too large to display	44991

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b^3*f*m*x*e - (f^2*m - 2*f^2*log(d))*b^3*x^2 - 2*b^3*m*e^2*log(f*
x + e))*log(x^n)^3 + (4*b^3*f^2*x^2*log(x^n)^3 + 6*(2*a*b^2*f^2 - (f^2*n -
```

```
2*f^2*log(c))*b^3)*x^2*log(x^n)^2 + 6*(2*a^2*b*f^2 - 2*(f^2*n - 2*f^2*log(c
))*a*b^2 + (f^2*n^2 - 2*f^2*n*log(c) + 2*f^2*log(c)^2)*b^3)*x^2*log(x^n) +
(4*a^3*f^2 - 6*(f^2*n - 2*f^2*log(c))*a^2*b + 6*(f^2*n^2 - 2*f^2*n*log(c) +
 2*f^2*log(c)^2)*a*b^2 - (3*f^2*n^3 - 6*f^2*n^2*log(c) + 6*f^2*n*log(c)^2 -
 4*f^2*log(c)^3)*b^3)*x^2)*log((f*x + e)^m))/f^2 + integrate(-1/8*((4*(f^3*
m - 2*f^3*log(d))*a^3 - 6*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c))*a^2*b
 + 6*(f^3*m*n^2 - 2*f^3*m*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(c)^2)*a*b
^2 - (3*f^3*m*n^3 - 6*f^3*m*n^2*log(c) + 6*f^3*m*n*log(c)^2 - 4*(f^3*m - 2*
f^3*log(d))*log(c)^3)*b^3)*x^3 - 8*(b^3*f^2*log(c)^3*log(d) + 3*a*b^2*f^2*l
og(c)^2*log(d) + 3*a^2*b*f^2*log(c)*log(d) + a^3*f^2*log(d))*x^2*e + 6*(2*b
^3*f*m*n*x*e^2 + 2*((f^3*m - 2*f^3*log(d))*a*b^2 - (f^3*m*n - f^3*n*log(d)
- (f^3*m - 2*f^3*log(d))*log(c))*b^3)*x^3 - (4*a*b^2*f^2*log(d) - (f^2*m*n
+ 2*f^2*n*log(d) - 4*f^2*log(c)*log(d))*b^3)*x^2*e - 2*(b^3*f*m*n*x*e^2 + b
^3*m*n*e^3)*log(f*x + e))*log(x^n)^2 + 6*((2*(f^3*m - 2*f^3*log(d))*a^2*b -
 2*(f^3*m*n - 2*(f^3*m - 2*f^3*log(d))*log(c))*a*b^2 + (f^3*m*n^2 - 2*f^3*m
*n*log(c) + 2*(f^3*m - 2*f^3*log(d))*log(c)^2)*b^3)*x^3 - 4*(b^3*f^2*log(c)
^2*log(d) + 2*a*b^2*f^2*log(c)*log(d) + a^2*b*f^2*log(d))*x^2*e)*log(x^n))/
(f^3*x^2 + f^2*x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log((f*x + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x + e)^m*d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln \left(d \left(e+f x\right)^m\right) \left(a+b \ln \left(c x^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

3.86 $\int (a+b\log(cx^n))^3 \log(d(e+fx)^m)\,dx$

Optimal. Leaf size=473

$$-12ab^2mn^2x+18b^3mn^3x-6b^2mn^2(a-bn)x-18b^3mn^2x\log(cx^n)+6bmnx(a+b\log(cx^n))^2-mx(a+b\log(cx$$

```
[Out] -12*a*b^2*m*n^2*x+18*b^3*m*n^3*x-6*b^2*m*n^2*(-b*n+a)*x-18*b^3*m*n^2*x*ln(c
*x^n)+6*b*m*n*x*(a+b*ln(c*x^n))^2-m*x*(a+b*ln(c*x^n))^3+6*b^2*e*m*n^2*(-b*n
+a)*ln(f*x+e)/f+6*a*b^2*n^2*x*ln(d*(f*x+e)^m)-6*b^3*n^3*x*ln(d*(f*x+e)^m)+6
*b^3*n^2*x*ln(c*x^n)*ln(d*(f*x+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)
^m)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)+6*b^3*e*m*n^2*ln(c*x^n)*ln(1+f*x/e)
/f-3*b*e*m*n*(a+b*ln(c*x^n))^2*ln(1+f*x/e)/f+e*m*(a+b*ln(c*x^n))^3*ln(1+f*x
/e)/f+6*b^3*e*m*n^3*polylog(2,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n))*polyl
og(2,-f*x/e)/f+3*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x/e)/f+6*b^3*e*m*n^
3*polylog(3,-f*x/e)/f-6*b^2*e*m*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x/e)/f+6*b
^3*e*m*n^3*polylog(4,-f*x/e)/f
```

Rubi [A]
time = 0.43, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {2333, 2332, 2418, 6, 45, 2393, 2354, 2438, 2395, 2421, 6724, 2430}

$$-\frac{6b^2emn^2\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f}-\frac{6b^2emn^2\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))}{f}+\frac{3bemn\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^2}{f}+\frac{6b^3emn^3\text{PolyLog}\left(2,-\frac{fx}{e}\right)}{f}+\frac{6b^3emn^3\text{PolyLog}\left(3,-\frac{fx}{e}\right)}{f}+\frac{6b^3emn^3\text{PolyLog}\left(4,-\frac{fx}{e}\right)}{f}+6ab^2n^2x\log(d(e+fx)^m)+\frac{6b^2emn^2(a-bn)\log(e+fx)}{f}-12ab^2mn^2x-6b^2mn^2x(a-bn)-3bnx(a+b\log(cx^n))^2\log(d(e+fx)^m)+x(a+b\log(cx^n))^3\log(d(e+fx)^m)-\frac{3bemn\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^2}{f}+\frac{em\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^3}{f}+6bmnx(a+b\log(cx^n))^2-mx(a+b\log(cx^n))^3+6b^3n^2x\log(cx^n)\log(d(e+fx)^m)+\frac{6b^3emn^2\log(cx^n)\log\left(\frac{fx}{e}+1\right)}{f}-18b^3mn^2x\log(cx^n)-6b^3n^3x\log(d(e+fx)^m)+18b^3mn^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] -12*a*b^2*m*n^2*x + 18*b^3*m*n^3*x - 6*b^2*m*n^2*(a - b*n)*x - 18*b^3*m*n^2
*x*Log[c*x^n] + 6*b*m*n*x*(a + b*Log[c*x^n])^2 - m*x*(a + b*Log[c*x^n])^3 +
 (6*b^2*e*m*n^2*(a - b*n)*Log[e + f*x])/f + 6*a*b^2*n^2*x*Log[d*(e + f*x)^m
] - 6*b^3*n^3*x*Log[d*(e + f*x)^m] + 6*b^3*n^2*x*Log[c*x^n]*Log[d*(e + f*x)
^m] - 3*b*n*x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m] + x*(a + b*Log[c*x^n]
)^3*Log[d*(e + f*x)^m] + (6*b^3*e*m*n^2*Log[c*x^n]*Log[1 + (f*x)/e])/f - (3
*b*e*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x)/e])/f + (e*m*(a + b*Log[c*x^n])
^3*Log[1 + (f*x)/e])/f + (6*b^3*e*m*n^3*PolyLog[2, -((f*x)/e)])/f - (6*b^2*
e*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x)/e)])/f + (3*b*e*m*n*(a + b*Lo
g[c*x^n])^2*PolyLog[2, -((f*x)/e)])/f + (6*b^3*e*m*n^3*PolyLog[3, -((f*x)/e
)])/f - (6*b^2*e*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)])/f + (6*b^
3*e*m*n^3*PolyLog[4, -((f*x)/e)])/f
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^3\log(d(e+fx)^m)\,dx = 6ab^2n^2x\log(d(e+fx)^m) - 6b^3n^3x\log(d(e+fx)^m) + 6b^3n^2x\,\mathrm{lo}$$

$$= 6ab^2n^2x\log(d(e+fx)^m) - 6b^3n^3x\log(d(e+fx)^m) + 6b^3n^2x\,\mathrm{lo}$$

$$= 6ab^2n^2x\log(d(e+fx)^m) - 6b^3n^3x\log(d(e+fx)^m) + 6b^3n^2x\,\mathrm{lo}$$

$$= 6ab^2n^2x\log(d(e+fx)^m) - 6b^3n^3x\log(d(e+fx)^m) + 6b^3n^2x\,\mathrm{lo}$$

$$= -6b^2mn^2(a-bn)x + \frac{6b^2emn^2(a-bn)\log(e+fx)}{f} + 6ab^2n^2x\,\mathrm{l}$$

$$= 6b^3mn^3x - 6b^2mn^2(a-bn)x - 6b^3mn^2x\log(cx^n) + 3bmnx(a+$$

$$= -6ab^2mn^2x + 6b^3mn^3x - 6b^2mn^2(a-bn)x - 6b^3mn^2x\log(cx^n$$

$$= -12ab^2mn^2x + 12b^3mn^3x - 6b^2mn^2(a-bn)x - 12b^3mn^2x\log($$

$$= -12ab^2mn^2x + 18b^3mn^3x - 6b^2mn^2(a-bn)x - 18b^3mn^2x\log($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1122 vs. $2(473) = 946$.
time = 0.26, size = 1122, normalized size = 2.37

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m],x]
```

```
[Out] (-(a^3*f*m*x) + 6*a^2*b*f*m*n*x - 18*a*b^2*f*m*n^2*x + 24*b^3*f*m*n^3*x - 3
*a^2*b*f*m*x*Log[c*x^n] + 12*a*b^2*f*m*n*x*Log[c*x^n] - 18*b^3*f*m*n^2*x*Lo
g[c*x^n] - 3*a*b^2*f*m*x*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[c*x^n]^2 - b^3*f*
m*x*Log[c*x^n]^3 + a^3*e*m*Log[e + f*x] - 3*a^2*b*e*m*n*Log[e + f*x] + 6*a*
b^2*e*m*n^2*Log[e + f*x] - 6*b^3*e*m*n^3*Log[e + f*x] - 3*a^2*b*e*m*n*Log[x
]*Log[e + f*x] + 6*a*b^2*e*m*n^2*Log[x]*Log[e + f*x] - 6*b^3*e*m*n^3*Log[x]
*Log[e + f*x] + 3*a*b^2*e*m*n^2*Log[x]^2*Log[e + f*x] - 3*b^3*e*m*n^3*Log[x
]^2*Log[e + f*x] - b^3*e*m*n^3*Log[x]^3*Log[e + f*x] + 3*a^2*b*e*m*Log[c*x^
```

```
n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[c*x^n]*Log[e + f*x] + 6*b^3*e*m*n^2*Log
[c*x^n]*Log[e + f*x] - 6*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x] + 6*b^3
*e*m*n^2*Log[x]*Log[c*x^n]*Log[e + f*x] + 3*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]
*Log[e + f*x] + 3*a*b^2*e*m*Log[c*x^n]^2*Log[e + f*x] - 3*b^3*e*m*n*Log[c*x
^n]^2*Log[e + f*x] - 3*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x] + b^3*e*m
*Log[c*x^n]^3*Log[e + f*x] + a^3*f*x*Log[d*(e + f*x)^m] - 3*a^2*b*f*n*x*Log
[d*(e + f*x)^m] + 6*a*b^2*f*n^2*x*Log[d*(e + f*x)^m] - 6*b^3*f*n^3*x*Log[d*
(e + f*x)^m] + 3*a^2*b*f*x*Log[c*x^n]*Log[d*(e + f*x)^m] - 6*a*b^2*f*n*x*Lo
g[c*x^n]*Log[d*(e + f*x)^m] + 6*b^3*f*n^2*x*Log[c*x^n]*Log[d*(e + f*x)^m] +
 3*a*b^2*f*x*Log[c*x^n]^2*Log[d*(e + f*x)^m] - 3*b^3*f*n*x*Log[c*x^n]^2*Log
[d*(e + f*x)^m] + b^3*f*x*Log[c*x^n]^3*Log[d*(e + f*x)^m] + 3*a^2*b*e*m*n*L
og[x]*Log[1 + (f*x)/e] - 6*a*b^2*e*m*n^2*Log[x]*Log[1 + (f*x)/e] + 6*b^3*e*
m*n^3*Log[x]*Log[1 + (f*x)/e] - 3*a*b^2*e*m*n^2*Log[x]^2*Log[1 + (f*x)/e] +
 3*b^3*e*m*n^3*Log[x]^2*Log[1 + (f*x)/e] + b^3*e*m*n^3*Log[x]^3*Log[1 + (f*
x)/e] + 6*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 6*b^3*e*m*n^2*Lo
g[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 3*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*Log[1
+ (f*x)/e] + 3*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] + 3*b*e*m*n*(
a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Po
lyLog[2, -((f*x)/e)] - 6*b^2*e*m*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -(
(f*x)/e)] + 6*b^3*e*m*n^3*PolyLog[4, -((f*x)/e)])/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 39644, normalized size = 83.81

method	result	size
risch	Expression too large to display	39644

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="maxima")
```

```
[Out] ((b^3*m*e*log(f*x + e) - (f*m - f*log(d))*b^3*x)*log(x^n)^3 + (b^3*f*x*log(
x^n)^3 - 3*((f*n - f*log(c))*b^3 - a*b^2*f)*x*log(x^n)^2 - 3*(2*(f*n - f*lo
g(c))*a*b^2 - (2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*b^3 - a^2*b*f)*x*log(x^
n) - (3*(f*n - f*log(c))*a^2*b - 3*(2*f*n^2 - 2*f*n*log(c) + f*log(c)^2)*a*
b^2 + (6*f*n^3 - 6*f*n^2*log(c) + 3*f*n*log(c)^2 - f*log(c)^3)*b^3 - a^3*f)
```

```
*x)*log((f*x + e)^m))/f - integrate((((f^2*m - f^2*log(d))*a^3 - 3*(f^2*m*n
 - (f^2*m - f^2*log(d))*log(c))*a^2*b + 3*(2*f^2*m*n^2 - 2*f^2*m*n*log(c) +
 (f^2*m - f^2*log(d))*log(c)^2)*a*b^2 - (6*f^2*m*n^3 - 6*f^2*m*n^2*log(c) +
 3*f^2*m*n*log(c)^2 - (f^2*m - f^2*log(d))*log(c)^3)*b^3)*x^2 - (b^3*f*log(
c)^3*log(d) + 3*a*b^2*f*log(c)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*f*l
og(d))*x*e + 3*(((f^2*m - f^2*log(d))*a*b^2 - (2*f^2*m*n - f^2*n*log(d) - (
f^2*m - f^2*log(d))*log(c))*b^3)*x^2 - (a*b^2*f*log(d) + (f*m*n - f*n*log(d
) + f*log(c)*log(d))*b^3)*x*e + (b^3*f*m*n*x*e + b^3*m*n*e^2)*log(f*x + e))
*log(x^n)^2 + 3*(((f^2*m - f^2*log(d))*a^2*b - 2*(f^2*m*n - (f^2*m - f^2*lo
g(d))*log(c))*a*b^2 + (2*f^2*m*n^2 - 2*f^2*m*n*log(c) + (f^2*m - f^2*log(d)
)*log(c)^2)*b^3)*x^2 - (b^3*f*log(c)^2*log(d) + 2*a*b^2*f*log(c)*log(d) + a
^2*b*f*log(d))*x*e)*log(x^n))/(f^2*x^2 + f*x*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3,x)
[Out] int(log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3, x)
```

3.87 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx$

Optimal. Leaf size=161

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{m(a+b\log(cx^n))^4\log\left(1+\frac{fx}{e}\right)}{4bn}-m(a+b\log(cx^n))^3\operatorname{Li}_2\left(-\frac{fx}{e}\right)+3bm$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f*x/
e)/b/n-m*(a+b*ln(c*x^n))^3*polylog(2,-f*x/e)+3*b*m*n*(a+b*ln(c*x^n))^2*poly
log(3,-f*x/e)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x/e)+6*b^3*m*n^3*pol
ylog(5,-f*x/e)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2422, 2354, 2421, 2430, 6724}

$$-6b^2mn^2\text{PolyLog}\left(4,-\frac{fx}{e}\right)(a+b\log(cx^n))-m\text{PolyLog}\left(2,-\frac{fx}{e}\right)(a+b\log(cx^n))^3+3bmn\text{PolyLog}\left(3,-\frac{fx}{e}\right)(a+b\log(cx^n))^2+6b^3mn^3\text{PolyLog}\left(5,-\frac{fx}{e}\right)+\frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn}-\frac{m\log\left(\frac{fx}{e}+1\right)(a+b\log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x)^m])/(4*b*n) - (m*(a + b*Log[c*x^n])^4
*Log[1 + (f*x)/e])/(4*b*n) - m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x)/e)]
+ 3*b*m*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x)/e)] - 6*b^2*m*n^2*(a + b*
Log[c*x^n])*PolyLog[4, -((f*x)/e)] + 6*b^3*m*n^3*PolyLog[5, -((f*x)/e)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
```

```
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}\,dx = \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{(fm)\int \frac{(a+b\log(cx^n))^4}{e+fx}\,dx}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{m(a+b\log(cx^n))^4\log(1+}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{m(a+b\log(cx^n))^4\log(1+}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{m(a+b\log(cx^n))^4\log(1+}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{m(a+b\log(cx^n))^4\log(1+}{4bn}$$
$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx)^m)}{4bn} - \frac{m(a+b\log(cx^n))^4\log(1+}{4bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 602 vs. 2(161) = 322.
time = 0.14, size = 602, normalized size = 3.74

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x,x]
```

```
[Out] a^3*Log[x]*Log[d*(e + f*x)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*x)^m])/2 +
 a*b^2*n^2*Log[x]^3*Log[d*(e + f*x)^m] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x)^
m])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x)^m] - 3*a*b^2*n*Log[x]^2*L
og[c*x^n]*Log[d*(e + f*x)^m] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*x)^
m] + 3*a*b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*x)^m] - (3*b^3*n*Log[x]^2*Log
[c*x^n]^2*Log[d*(e + f*x)^m])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e + f*x)^m
] - a^3*m*Log[x]*Log[1 + (f*x)/e] + (3*a^2*b*m*n*Log[x]^2*Log[1 + (f*x)/e])
/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (f*x)/e] + (b^3*m*n^3*Log[x]^4*Log[1 + (f
*x)/e])/4 - 3*a^2*b*m*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 3*a*b^2*m*n*Log[
x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Log[1 + (f
*x)/e] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] + (3*b^3*m*n*Log[x]
^2*Log[c*x^n]^2*Log[1 + (f*x)/e])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 + (f*
x)/e] - m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x)/e)] + 3*b*m*n*(a + b*Log[
c*x^n])^2*PolyLog[3, -((f*x)/e)] - 6*a*b^2*m*n^2*PolyLog[4, -((f*x)/e)] - 6
*b^3*m*n^2*Log[c*x^n]*PolyLog[4, -((f*x)/e)] + 6*b^3*m*n^3*PolyLog[5, -((f*
x)/e)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 60520, normalized size = 375.90

method	result	size
risch	Expression too large to display	60520

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log((f*x + e)^m) - integrate(-1/4*(b^3*f*m*n^3
*x*log(x)^4 - 4*(b^3*f*m*n^2*log(c) + a*b^2*f*m*n^2)*x*log(x)^3 + 6*(b^3*f*
m*n*log(c)^2 + 2*a*b^2*f*m*n*log(c) + a^2*b*f*m*n)*x*log(x)^2 - 4*(b^3*f*m*
x*log(x) - b^3*f*x*log(d) - b^3*e*log(d))*log(x^n)^3 - 4*(b^3*f*m*log(c)^3
```

```
+ 3*a*b^2*f*m*log(c)^2 + 3*a^2*b*f*m*log(c) + a^3*f*m)*x*log(x) + 6*(b^3*f*
m*n*x*log(x)^2 - 2*(b^3*f*m*log(c) + a*b^2*f*m)*x*log(x) + 2*(b^3*f*log(c)*
log(d) + a*b^2*f*log(d))*x + 2*(b^3*log(c)*log(d) + a*b^2*log(d))*e)*log(x^
n)^2 + 4*(b^3*f*log(c)^3*log(d) + 3*a*b^2*f*log(c)^2*log(d) + 3*a^2*b*f*log
(c)*log(d) + a^3*f*log(d))*x + 4*(b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*lo
g(d) + 3*a^2*b*log(c)*log(d) + a^3*log(d))*e - 4*(b^3*f*m*n^2*x*log(x)^3 -
3*(b^3*f*m*n*log(c) + a*b^2*f*m*n)*x*log(x)^2 + 3*(b^3*f*m*log(c)^2 + 2*a*b
^2*f*m*log(c) + a^2*b*f*m)*x*log(x) - 3*(b^3*f*log(c)^2*log(d) + 2*a*b^2*f*
log(c)*log(d) + a^2*b*f*log(d))*x - 3*(b^3*log(c)^2*log(d) + 2*a*b^2*log(c)
*log(d) + a^2*b*log(d))*e)*log(x^n))/(f*x^2 + x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x, x)
```

3.88 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx$

Optimal. Leaf size=411

$$\frac{6b^3fmn^3\log(x)}{e}-\frac{6b^2fmn^2\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{e}-\frac{3bfmn\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))^2}{e}-\frac{fm\log\left(\right.}{}$$

```
[Out] 6*b^3*f*m*n^3*ln(x)/e-6*b^2*f*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e-3*b*f*m*n
*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e-f*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e-6*b^3*
f*m*n^3*ln(f*x+e)/e-6*b^3*n^3*ln(d*(f*x+e)^m)/x-6*b^2*n^2*(a+b*ln(c*x^n))*l
n(d*(f*x+e)^m)/x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x-(a+b*ln(c*x^n))^
3*ln(d*(f*x+e)^m)/x+6*b^3*f*m*n^3*polylog(2,-e/f/x)/e+6*b^2*f*m*n^2*(a+b*ln
(c*x^n))*polylog(2,-e/f/x)/e+3*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x)/
e+6*b^3*f*m*n^3*polylog(3,-e/f/x)/e+6*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(3
,-e/f/x)/e+6*b^3*f*m*n^3*polylog(4,-e/f/x)/e
```

Rubi [A]
time = 0.34, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2342, 2341, 2425, 36, 29, 31, 2379, 2438, 2421, 6724, 2430}

$$\frac{6b^2fmn^2\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}+\frac{6b^2fmn^2\text{PolyLog}\left(3,-\frac{e}{fx}\right)(a+b\log(cx^n))}{e}+\frac{3bfmn\text{PolyLog}\left(2,-\frac{e}{fx}\right)(a+b\log(cx^n))^2}{e}+\frac{6b^3fmn^3\text{PolyLog}\left(2,-\frac{e}{fx}\right)}{e}+\frac{6b^3fmn^3\text{PolyLog}\left(3,-\frac{e}{fx}\right)}{e}+\frac{6b^3fmn^3\text{PolyLog}\left(4,-\frac{e}{fx}\right)}{e}-\frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)^m)}{x}-\frac{6b^2fmn^2\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))}{e}-\frac{3bn(a+b\log(cx^n))^2\log(d(e+fx)^m)}{x}-\frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x}-\frac{3bfmn\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^2}{e}-\frac{fm\log\left(\frac{e}{fx}+1\right)(a+b\log(cx^n))^3}{e}-\frac{6b^3n^3\log(d(e+fx)^m)}{x}+\frac{6b^3fmn^3\log(x)}{e}-\frac{6b^3fmn^3\log(e+fx)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] (6*b^3*f*m*n^3*Log[x])/e - (6*b^2*f*m*n^2*Log[1 + e/(f*x)]*(a + b*Log[c*x^n
]))/e - (3*b*f*m*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/e - (f*m*Log[1 +
e/(f*x)]*(a + b*Log[c*x^n])^3)/e - (6*b^3*f*m*n^3*Log[e + f*x])/e - (6*b^3*
n^3*Log[d*(e + f*x)^m])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m
])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/x - ((a + b*Log[c*x^
n])^3*Log[d*(e + f*x)^m])/x + (6*b^3*f*m*n^3*PolyLog[2, -(e/(f*x))])/e + (6
*b^2*f*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])/e + (3*b*f*m*n*(a +
 b*Log[c*x^n])^2*PolyLog[2, -(e/(f*x))])/e + (6*b^3*f*m*n^3*PolyLog[3, -(e/
(f*x))])/e + (6*b^2*f*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x))])/e +
(6*b^3*f*m*n^3*PolyLog[4, -(e/(f*x))])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
```

```
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^2}\,dx = -\frac{6b^3n^3\log(d(e+fx)^m)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)}{x}$$

$$= -\frac{6b^3n^3\log(d(e+fx)^m)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)}{x}$$

$$= -\frac{6b^3n^3\log(d(e+fx)^m)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log(d(e+fx)}{x}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn(a+b\log(cx^n))^2}{e} - \frac{6b^3fmn^3\log(e+}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn(a+b\log(cx^n))^2}{e} + \frac{fm(a+b\log(cx}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn(a+b\log(cx^n))^2}{e} + \frac{fm(a+b\log(cx}{e}$$

$$= \frac{6b^3fmn^3\log(x)}{e} + \frac{3bfmn(a+b\log(cx^n))^2}{e} + \frac{fm(a+b\log(cx}{e}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1347 vs. $2(411) = 822$.
time = 0.41, size = 1347, normalized size = 3.28

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^2,x]
```

```
[Out] -1/4*(-4*a^3*f*m*x*Log[x] - 12*a^2*b*f*m*n*x*Log[x] - 24*a*b^2*f*m*n^2*x*Lo
g[x] - 24*b^3*f*m*n^3*x*Log[x] + 6*a^2*b*f*m*n*x*Log[x]^2 + 12*a*b^2*f*m*n^
2*x*Log[x]^2 + 12*b^3*f*m*n^3*x*Log[x]^2 - 4*a*b^2*f*m*n^2*x*Log[x]^3 - 4*b
^3*f*m*n^3*x*Log[x]^3 + b^3*f*m*n^3*x*Log[x]^4 - 12*a^2*b*f*m*x*Log[x]*Log[
c*x^n] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n] - 24*b^3*f*m*n^2*x*Log[x]*Log[c
*x^n] + 12*a*b^2*f*m*n*x*Log[x]^2*Log[c*x^n] + 12*b^3*f*m*n^2*x*Log[x]^2*Lo
g[c*x^n] - 4*b^3*f*m*n^2*x*Log[x]^3*Log[c*x^n] - 12*a*b^2*f*m*x*Log[x]*Log[
c*x^n]^2 - 12*b^3*f*m*n*x*Log[x]*Log[c*x^n]^2 + 6*b^3*f*m*n*x*Log[x]^2*Log[
c*x^n]^2 - 4*b^3*f*m*x*Log[x]*Log[c*x^n]^3 + 4*a^3*f*m*x*Log[e + f*x] + 12*
a^2*b*f*m*n*x*Log[e + f*x] + 24*a*b^2*f*m*n^2*x*Log[e + f*x] + 24*b^3*f*m*n
^3*x*Log[e + f*x] - 12*a^2*b*f*m*n*x*Log[x]*Log[e + f*x] - 24*a*b^2*f*m*n^2
*x*Log[x]*Log[e + f*x] - 24*b^3*f*m*n^3*x*Log[x]*Log[e + f*x] + 12*a*b^2*f*
m*n^2*x*Log[x]^2*Log[e + f*x] + 12*b^3*f*m*n^3*x*Log[x]^2*Log[e + f*x] - 4*
b^3*f*m*n^3*x*Log[x]^3*Log[e + f*x] + 12*a^2*b*f*m*x*Log[c*x^n]*Log[e + f*x
] + 24*a*b^2*f*m*n*x*Log[c*x^n]*Log[e + f*x] + 24*b^3*f*m*n^2*x*Log[c*x^n]*
Log[e + f*x] - 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[e + f*x] - 24*b^3*f*m
*n^2*x*Log[x]*Log[c*x^n]*Log[e + f*x] + 12*b^3*f*m*n^2*x*Log[x]^2*Log[c*x^n
]*Log[e + f*x] + 12*a*b^2*f*m*x*Log[c*x^n]^2*Log[e + f*x] + 12*b^3*f*m*n*x*
Log[c*x^n]^2*Log[e + f*x] - 12*b^3*f*m*n*x*Log[x]*Log[c*x^n]^2*Log[e + f*x]
 + 4*b^3*f*m*x*Log[c*x^n]^3*Log[e + f*x] + 4*a^3*e*Log[d*(e + f*x)^m] + 12*
a^2*b*e*n*Log[d*(e + f*x)^m] + 24*a*b^2*e*n^2*Log[d*(e + f*x)^m] + 24*b^3*e
*n^3*Log[d*(e + f*x)^m] + 12*a^2*b*e*Log[c*x^n]*Log[d*(e + f*x)^m] + 24*a*b
^2*e*n*Log[c*x^n]*Log[d*(e + f*x)^m] + 24*b^3*e*n^2*Log[c*x^n]*Log[d*(e + f
*x)^m] + 12*a*b^2*e*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 12*b^3*e*n*Log[c*x^n]
^2*Log[d*(e + f*x)^m] + 4*b^3*e*Log[c*x^n]^3*Log[d*(e + f*x)^m] + 12*a^2*b*
f*m*n*x*Log[x]*Log[1 + (f*x)/e] + 24*a*b^2*f*m*n^2*x*Log[x]*Log[1 + (f*x)/e
] + 24*b^3*f*m*n^3*x*Log[x]*Log[1 + (f*x)/e] - 12*a*b^2*f*m*n^2*x*Log[x]^2*
Log[1 + (f*x)/e] - 12*b^3*f*m*n^3*x*Log[x]^2*Log[1 + (f*x)/e] + 4*b^3*f*m*n
^3*x*Log[x]^3*Log[1 + (f*x)/e] + 24*a*b^2*f*m*n*x*Log[x]*Log[c*x^n]*Log[1 +
 (f*x)/e] + 24*b^3*f*m*n^2*x*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] - 12*b^3*f*
m*n^2*x*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] + 12*b^3*f*m*n*x*Log[x]*Log[c*
x^n]^2*Log[1 + (f*x)/e] + 12*b*f*m*n*x*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a
+ b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -((f*x)/e)] - 24*b^2*f*m*n
^2*x*(a + b*n + b*Log[c*x^n])*PolyLog[3, -((f*x)/e)] + 24*b^3*f*m*n^3*x*Pol
yLog[4, -((f*x)/e)])/(e*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.22, size = 41923, normalized size = 102.00

method	result	size
risch	Expression too large to display	41923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -((b^3*f*m*x*log(f*x + e) - b^3*f*m*x*log(x) + b^3*e*log(d))*log(x^n)^3 + (
b^3*e*log(x^n)^3 + 3*(b^3*(n + log(c)) + a*b^2)*e*log(x^n)^2 + 3*((2*n^2 +
2*n*log(c) + log(c)^2)*b^3 + 2*a*b^2*(n + log(c)) + a^2*b)*e*log(x^n) + (3*
(2*n^2 + 2*n*log(c) + log(c)^2)*a*b^2 + (6*n^3 + 6*n^2*log(c) + 3*n*log(c)^
2 + log(c)^3)*b^3 + 3*a^2*b*(n + log(c)) + a^3)*e)*log((f*x + e)^m))*e^(-1)
/x + integrate((((f*m + f*log(d))*a^3 + 3*(f*m*n + (f*m + f*log(d))*log(c))
*a^2*b + 3*(2*f*m*n^2 + 2*f*m*n*log(c) + (f*m + f*log(d))*log(c)^2)*a*b^2 +
 (6*f*m*n^3 + 6*f*m*n^2*log(c) + 3*f*m*n*log(c)^2 + (f*m + f*log(d))*log(c)
^3)*b^3)*x*e + 3*(((f*m + f*log(d))*a*b^2 + (f*m*n + f*n*log(d) + (f*m + f*
log(d))*log(c))*b^3)*x*e + ((n*log(d) + log(c)*log(d))*b^3 + a*b^2*log(d))*
e^2 + (b^3*f^2*m*n*x^2 + b^3*f*m*n*x*e)*log(f*x + e) - (b^3*f^2*m*n*x^2 + b
^3*f*m*n*x*e)*log(x))*log(x^n)^2 + (b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*
log(d) + 3*a^2*b*log(c)*log(d) + a^3*log(d))*e^2 + 3*(((f*m + f*log(d))*a^2
*b + 2*(f*m*n + (f*m + f*log(d))*log(c))*a*b^2 + (2*f*m*n^2 + 2*f*m*n*log(c
) + (f*m + f*log(d))*log(c)^2)*b^3)*x*e + (b^3*log(c)^2*log(d) + 2*a*b^2*lo
g(c)*log(d) + a^2*b*log(d))*e^2)*log(x^n))/(f*x^3*e + x^2*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2,x)
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

3.89 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx$

Optimal. Leaf size=555

$$-\frac{45b^3fmn^3}{8ex}-\frac{3b^3f^2mn^3\log(x)}{8e^2}-\frac{21b^2fmn^2(a+b\log(cx^n))}{4ex}+\frac{3b^2f^2mn^2\log\left(1+\frac{e}{fx}\right)(a+b\log(cx^n))}{4e^2}-9bf$$

```
[Out] -45/8*b^3*f*m*n^3/e/x-3/8*b^3*f^2*m*n^3*ln(x)/e^2-21/4*b^2*f*m*n^2*(a+b*ln(
c*x^n))/e/x+3/4*b^2*f^2*m*n^2*ln(1+e/f/x)*(a+b*ln(c*x^n))/e^2-9/4*b*f*m*n*(
a+b*ln(c*x^n))^2/e/x+3/4*b*f^2*m*n*ln(1+e/f/x)*(a+b*ln(c*x^n))^2/e^2-1/2*f*
m*(a+b*ln(c*x^n))^3/e/x+1/2*f^2*m*ln(1+e/f/x)*(a+b*ln(c*x^n))^3/e^2+3/8*b^3
*f^2*m*n^3*ln(f*x+e)/e^2-3/8*b^3*n^3*ln(d*(f*x+e)^m)/x^2-3/4*b^2*n^2*(a+b*l
n(c*x^n))*ln(d*(f*x+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x+e)^m)/x^2
-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^2-3/4*b^3*f^2*m*n^3*polylog(2,-e/f
/x)/e^2-3/2*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-e/f/x)/e^2-3/2*b*f^2*m
*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x)/e^2-3/2*b^3*f^2*m*n^3*polylog(3,-e/f
/x)/e^2-3*b^2*f^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e/f/x)/e^2-3*b^3*f^2*m*n
^3*polylog(4,-e/f/x)/e^2
```

Rubi [A]
time = 0.59, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2342, 2341, 2425, 46, 2380, 2379, 2438, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] (-45*b^3*f*m*n^3)/(8*e*x) - (3*b^3*f^2*m*n^3*Log[x])/(8*e^2) - (21*b^2*f*m*
n^2*(a + b*Log[c*x^n]))/(4*e*x) + (3*b^2*f^2*m*n^2*Log[1 + e/(f*x)]*(a + b*
Log[c*x^n]))/(4*e^2) - (9*b*f*m*n*(a + b*Log[c*x^n])^2)/(4*e*x) + (3*b*f^2*
m*n*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^2)/(4*e^2) - (f*m*(a + b*Log[c*x^n]
)^3)/(2*e*x) + (f^2*m*Log[1 + e/(f*x)]*(a + b*Log[c*x^n])^3)/(2*e^2) + (3*b
^3*f^2*m*n^3*Log[e + f*x])/(8*e^2) - (3*b^3*n^3*Log[d*(e + f*x)^m])/(8*x^2)
 - (3*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e + f*x)^m])/(4*x^2) - (3*b*n*(a +
b*Log[c*x^n])^2*Log[d*(e + f*x)^m])/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[d*(
e + f*x)^m])/(2*x^2) - (3*b^3*f^2*m*n^3*PolyLog[2, -(e/(f*x))])/(4*e^2) - (
3*b^2*f^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x))])/(2*e^2) - (3*b*f
^2*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(e/(f*x))])/(2*e^2) - (3*b^3*f^2*m*
n^3*PolyLog[3, -(e/(f*x))])/(2*e^2) - (3*b^2*f^2*m*n^2*(a + b*Log[c*x^n])*P
olyLog[3, -(e/(f*x))])/e^2 - (3*b^3*f^2*m*n^3*PolyLog[4, -(e/(f*x))])/e^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
```

```
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx)^m)}{x^3}\,dx = -\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx$$

$$= -\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx$$

$$= -\frac{3b^3n^3\log(d(e+fx)^m)}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))\log(d(e+fx$$

$$= -\frac{3b^3fmn^3}{8ex} - \frac{3b^3f^2mn^3\log(x)}{8e^2} + \frac{3b^3f^2mn^3\log(e+fx)}{8e^2} - \frac{3b^3n$$

$$= -\frac{9b^3fmn^3}{8ex} - \frac{3b^3f^2mn^3\log(x)}{8e^2} - \frac{3b^2fmn^2(a+b\log(cx^n))}{4ex} - 3$$

$$= -\frac{21b^3fmn^3}{8ex} - \frac{3b^3f^2mn^3\log(x)}{8e^2} - \frac{9b^2fmn^2(a+b\log(cx^n))}{4ex} -$$

$$= -\frac{45b^3fmn^3}{8ex} - \frac{3b^3f^2mn^3\log(x)}{8e^2} - \frac{21b^2fmn^2(a+b\log(cx^n))}{4ex} -$$

$$= -\frac{45b^3fmn^3}{8ex} - \frac{3b^3f^2mn^3\log(x)}{8e^2} - \frac{21b^2fmn^2(a+b\log(cx^n))}{4ex} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1736 vs. $2(555) = 1110$.

time = 0.47, size = 1736, normalized size = 3.13

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x)^m])/x^3,x]
```

```
[Out] -1/8*(4*a^3*e*f*m*x + 18*a^2*b*e*f*m*n*x + 42*a*b^2*e*f*m*n^2*x + 45*b^3*e*
f*m*n^3*x + 4*a^3*f^2*m*x^2*Log[x] + 6*a^2*b*f^2*m*n*x^2*Log[x] + 6*a*b^2*f
^2*m*n^2*x^2*Log[x] + 3*b^3*f^2*m*n^3*x^2*Log[x] - 6*a^2*b*f^2*m*n*x^2*Log[
x]^2 - 6*a*b^2*f^2*m*n^2*x^2*Log[x]^2 - 3*b^3*f^2*m*n^3*x^2*Log[x]^2 + 4*a*
b^2*f^2*m*n^2*x^2*Log[x]^3 + 2*b^3*f^2*m*n^3*x^2*Log[x]^3 - b^3*f^2*m*n^3*x
^2*Log[x]^4 + 12*a^2*b*e*f*m*x*Log[c*x^n] + 36*a*b^2*e*f*m*n*x*Log[c*x^n] +
 42*b^3*e*f*m*n^2*x*Log[c*x^n] + 12*a^2*b*f^2*m*x^2*Log[x]*Log[c*x^n] + 12*
a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n] + 6*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]
 - 12*a*b^2*f^2*m*n*x^2*Log[x]^2*Log[c*x^n] - 6*b^3*f^2*m*n^2*x^2*Log[x]^2*
Log[c*x^n] + 4*b^3*f^2*m*n^2*x^2*Log[x]^3*Log[c*x^n] + 12*a*b^2*e*f*m*x*Log
[c*x^n]^2 + 18*b^3*e*f*m*n*x*Log[c*x^n]^2 + 12*a*b^2*f^2*m*x^2*Log[x]*Log[c
*x^n]^2 + 6*b^3*f^2*m*n*x^2*Log[x]*Log[c*x^n]^2 - 6*b^3*f^2*m*n*x^2*Log[x]^
2*Log[c*x^n]^2 + 4*b^3*e*f*m*x*Log[c*x^n]^3 + 4*b^3*f^2*m*x^2*Log[x]*Log[c*
x^n]^3 - 4*a^3*f^2*m*x^2*Log[e + f*x] - 6*a^2*b*f^2*m*n*x^2*Log[e + f*x] -
6*a*b^2*f^2*m*n^2*x^2*Log[e + f*x] - 3*b^3*f^2*m*n^3*x^2*Log[e + f*x] + 12*
a^2*b*f^2*m*n*x^2*Log[x]*Log[e + f*x] + 12*a*b^2*f^2*m*n^2*x^2*Log[x]*Log[e
 + f*x] + 6*b^3*f^2*m*n^3*x^2*Log[x]*Log[e + f*x] - 12*a*b^2*f^2*m*n^2*x^2*
Log[x]^2*Log[e + f*x] - 6*b^3*f^2*m*n^3*x^2*Log[x]^2*Log[e + f*x] + 4*b^3*f
^2*m*n^3*x^2*Log[x]^3*Log[e + f*x] - 12*a^2*b*f^2*m*x^2*Log[c*x^n]*Log[e +
f*x] - 12*a*b^2*f^2*m*n*x^2*Log[c*x^n]*Log[e + f*x] - 6*b^3*f^2*m*n^2*x^2*L
og[c*x^n]*Log[e + f*x] + 24*a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*Log[e + f*x
] + 12*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[e + f*x] - 12*b^3*f^2*m*n^2*
x^2*Log[x]^2*Log[c*x^n]*Log[e + f*x] - 12*a*b^2*f^2*m*x^2*Log[c*x^n]^2*Log[
e + f*x] - 6*b^3*f^2*m*n*x^2*Log[c*x^n]^2*Log[e + f*x] + 12*b^3*f^2*m*n*x^2
*Log[x]*Log[c*x^n]^2*Log[e + f*x] - 4*b^3*f^2*m*x^2*Log[c*x^n]^3*Log[e + f*
x] + 4*a^3*e^2*Log[d*(e + f*x)^m] + 6*a^2*b*e^2*n*Log[d*(e + f*x)^m] + 6*a*
b^2*e^2*n^2*Log[d*(e + f*x)^m] + 3*b^3*e^2*n^3*Log[d*(e + f*x)^m] + 12*a^2*
b*e^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 12*a*b^2*e^2*n*Log[c*x^n]*Log[d*(e +
f*x)^m] + 6*b^3*e^2*n^2*Log[c*x^n]*Log[d*(e + f*x)^m] + 12*a*b^2*e^2*Log[c*
x^n]^2*Log[d*(e + f*x)^m] + 6*b^3*e^2*n*Log[c*x^n]^2*Log[d*(e + f*x)^m] + 4
*b^3*e^2*Log[c*x^n]^3*Log[d*(e + f*x)^m] - 12*a^2*b*f^2*m*n*x^2*Log[x]*Log[
1 + (f*x)/e] - 12*a*b^2*f^2*m*n^2*x^2*Log[x]*Log[1 + (f*x)/e] - 6*b^3*f^2*m
*n^3*x^2*Log[x]*Log[1 + (f*x)/e] + 12*a*b^2*f^2*m*n^2*x^2*Log[x]^2*Log[1 +
(f*x)/e] + 6*b^3*f^2*m*n^3*x^2*Log[x]^2*Log[1 + (f*x)/e] - 4*b^3*f^2*m*n^3*
x^2*Log[x]^3*Log[1 + (f*x)/e] - 24*a*b^2*f^2*m*n*x^2*Log[x]*Log[c*x^n]*Log[
1 + (f*x)/e] - 12*b^3*f^2*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[1 + (f*x)/e] + 12
```

```
*b^3*f^2*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[1 + (f*x)/e] - 12*b^3*f^2*m*n*x^
2*Log[x]*Log[c*x^n]^2*Log[1 + (f*x)/e] - 6*b*f^2*m*n*x^2*(2*a^2 + 2*a*b*n +
 b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -((f
*x)/e)] + 12*b^2*f^2*m*n^2*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[3, -((f
*x)/e)] - 24*b^3*f^2*m*n^3*x^2*PolyLog[4, -((f*x)/e)])/(e^2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.27, size = 46399, normalized size = 83.60

method	result	size
risch	Expression too large to display	46399

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x+e)^m)/x^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="maxima")
[Out] 1/8*(4*(b^3*f^2*m*x^2*log(f*x + e) - b^3*f^2*m*x^2*log(x) - b^3*f*m*x*e - b
^3*e^2*log(d))*log(x^n)^3 - (4*b^3*e^2*log(x^n)^3 + 6*(b^3*(n + 2*log(c)) +
 2*a*b^2)*e^2*log(x^n)^2 + 6*((n^2 + 2*n*log(c) + 2*log(c)^2)*b^3 + 2*a*b^2
*(n + 2*log(c)) + 2*a^2*b)*e^2*log(x^n) + (6*(n^2 + 2*n*log(c) + 2*log(c)^2
)*a*b^2 + (3*n^3 + 6*n^2*log(c) + 6*n*log(c)^2 + 4*log(c)^3)*b^3 + 6*a^2*b*
(n + 2*log(c)) + 4*a^3)*e^2)*log((f*x + e)^m))*e^(-2)/x^2 - integrate(-1/8*
((4*(f*m + 2*f*log(d))*a^3 + 6*(f*m*n + 2*(f*m + 2*f*log(d))*log(c))*a^2*b
+ 6*(f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m + 2*f*log(d))*log(c)^2)*a*b^2 + (3*f
*m*n^3 + 6*f*m*n^2*log(c) + 6*f*m*n*log(c)^2 + 4*(f*m + 2*f*log(d))*log(c)^
3)*b^3)*x*e^2 + 6*(2*b^3*f^2*m*n*x^2*e + (2*(f*m + 2*f*log(d))*a*b^2 + (3*f
*m*n + 2*f*n*log(d) + 2*(f*m + 2*f*log(d))*log(c))*b^3)*x*e^2 + 2*((n*log(d
) + 2*log(c)*log(d))*b^3 + 2*a*b^2*log(d))*e^3 - 2*(b^3*f^3*m*n*x^3 + b^3*f
^2*m*n*x^2*e)*log(f*x + e) + 2*(b^3*f^3*m*n*x^3 + b^3*f^2*m*n*x^2*e)*log(x)
)*log(x^n)^2 + 8*(b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*l
og(c)*log(d) + a^3*log(d))*e^3 + 6*((2*(f*m + 2*f*log(d))*a^2*b + 2*(f*m*n
+ 2*(f*m + 2*f*log(d))*log(c))*a*b^2 + (f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m +
 2*f*log(d))*log(c)^2)*b^3)*x*e^2 + 4*(b^3*log(c)^2*log(d) + 2*a*b^2*log(c)
*log(d) + a^2*b*log(d))*e^3)*log(x^n))/(f*x^4*e^2 + x^3*e^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x + e)^m*d)/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x + e)^m*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(e + f*x)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

3.90 $\int x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=221

$$-\frac{3bemnx^2}{16f}+\frac{1}{16}bmnx^4+\frac{emx^2(a+b\log(cx^n))}{4f}-\frac{1}{8}mx^4(a+b\log(cx^n))+\frac{be^2mn\log(e+fx^2)}{16f^2}+\frac{be^2mn\log\left(\right.}{}$$

```
[Out] -3/16*b*e*m*n*x^2/f+1/16*b*m*n*x^4+1/4*e*m*x^2*(a+b*ln(c*x^n))/f-1/8*m*x^4*
(a+b*ln(c*x^n))+1/16*b*e^2*m*n*ln(f*x^2+e)/f^2+1/8*b*e^2*m*n*ln(-f*x^2/e)*l
n(f*x^2+e)/f^2-1/4*e^2*m*(a+b*ln(c*x^n))*ln(f*x^2+e)/f^2-1/16*b*n*x^4*ln(d*
(f*x^2+e)^m)+1/4*x^4*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/8*b*e^2*m*n*polylo
g(2,1+f*x^2/e)/f^2
```

Rubi [A]
time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2504, 2442, 45, 2423, 2441, 2352}

$$\frac{be^2mn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{8f^2}+\frac{1}{4}x^4(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)-\frac{e^2m\log(e+fx^2)(a+b\log(cx^n))}{4f^2}+\frac{emx^2(a+b\log(cx^n))}{4f}-\frac{1}{8}mx^4(a+b\log(cx^n))-\frac{1}{16}bnx^4\log\left(d\left(e+fx^2\right)^m\right)+\frac{be^2mn\log(e+fx^2)}{16f^2}+\frac{be^2mn\log\left(-\frac{fx^2}{e}\right)\log(e+fx^2)}{8f^2}-\frac{3bemnx^2}{16f}+\frac{1}{16}bmnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-3*b*e*m*n*x^2)/(16*f) + (b*m*n*x^4)/16 + (e*m*x^2*(a + b*Log[c*x^n]))/(4*
f) - (m*x^4*(a + b*Log[c*x^n]))/8 + (b*e^2*m*n*Log[e + f*x^2])/(16*f^2) + (
b*e^2*m*n*Log[-((f*x^2)/e)]*Log[e + f*x^2])/(8*f^2) - (e^2*m*(a + b*Log[c*x
^n])*Log[e + f*x^2])/(4*f^2) - (b*n*x^4*Log[d*(e + f*x^2)^m])/16 + (x^4*(a
+ b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/4 + (b*e^2*m*n*PolyLog[2, 1 + (f*x^2)
/e])/(8*f^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx = \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a+b\log(cx^n)) - \frac{e^2m(a+b$$

$$= -\frac{bemnx^2}{8f} + \frac{1}{32}bmnx^4 + \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a+$$

$$= -\frac{bemnx^2}{8f} + \frac{1}{32}bmnx^4 + \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a+$$

$$= -\frac{bemnx^2}{8f} + \frac{1}{32}bmnx^4 + \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a+$$

$$= -\frac{bemnx^2}{8f} + \frac{1}{32}bmnx^4 + \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a+$$

$$= -\frac{3bemnx^2}{16f} + \frac{1}{16}bmnx^4 + \frac{emx^2(a+b\log(cx^n))}{4f} - \frac{1}{8}mx^4(a$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 324, normalized size = 1.47

$$-\frac{-4aefmx^2+3befmnx^2+2af^2mx^4-bf^2mnx^4-4befmx^2\log(cx^n)+2bf^2mx^4\log(cx^n)+4be^2mn\log(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)+4be^2mn\log(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right)+4ae^2m\log(e+fx^2)-be^2mn\log(e+fx^2)-4be^2mn\log(x)\log(e+fx^2)+4be^2m\log(cx^n)\log(e+fx^2)-4af^2x^4\log\left(d(e+fx^2)^m\right)+bf^2nx^4\log\left(d(e+fx^2)^m\right)-4bf^2x^4\log(cx^n)\log\left(d(e+fx^2)^m\right)+4be^2mn\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)+4be^2mn\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{16f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] -1/16*(-4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 2*a*f^2*m*x^4 - b*f^2*m*n*x^4 - 4
*b*e*f*m*x^2*Log[c*x^n] + 2*b*f^2*m*x^4*Log[c*x^n] + 4*b*e^2*m*n*Log[x]*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*b*e^2*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt
[e]] + 4*a*e^2*m*Log[e + f*x^2] - b*e^2*m*n*Log[e + f*x^2] - 4*b*e^2*m*n*Lo
g[x]*Log[e + f*x^2] + 4*b*e^2*m*Log[c*x^n]*Log[e + f*x^2] - 4*a*f^2*x^4*Log
[d*(e + f*x^2)^m] + b*f^2*n*x^4*Log[d*(e + f*x^2)^m] - 4*b*f^2*x^4*Log[c*x^
n]*Log[d*(e + f*x^2)^m] + 4*b*e^2*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]]
+ 4*b*e^2*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 2270, normalized size = 10.27

method	result	size
risch	Expression too large to display	2270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)

[Out] -1/8*I/f*Pi*b*e*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x^2+1/8*I*e^2*m/f^2*l
n(f*x^2+e)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/16*b*m*n*x^4+1/4*b*e^
2*m*n/f^2*ln(x)*ln(f*x^2+e)-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4
*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4/f*x^2*a*e*m+1/4/f*ln(c)*x^2*b*e*m+1/8*b*
e^2*m*n/f^2-1/4*b*e^2*m*n/f^2*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b
*e^2*m*n/f^2*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b*e^2*m*n/f^2*ln(x)
*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b*e^2*m*n/f^2*ln(x)*ln((f*x+(-e*f
)^(1/2))/(-e*f)^(1/2))-1/4*m/f^2*b*ln(x^n)*e^2*ln(f*x^2+e)-1/16*Pi^2*csgn(I
*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c)*csgn(I*c*x^n)^2-1/16*
Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/8*x^4*a*m+1/16*b*e^2*m*n*ln(f*x^2+e)/f^2+1/8*I*Pi*x^4*a*csgn(I*d)
*csgn(I*d*(f*x^2+e)^m)^2-1/8*I*x^4*ln(d)*Pi*b*csgn(I*c*x^n)^3+1/8*I*Pi*x^4*
a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/4*x^4*ln(d)*a-1/4*e^2*m/f^2
*ln(f*x^2+e)*a-1/8*I*x^4*ln(d)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/1
6*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)+1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I
*d*(f*x^2+e)^m)*x^4*b*csgn(I*c)*csgn(I*c*x^n)^2-1/16*I*Pi*x^4*b*m*csgn(I*x^
n)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)*x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*Pi*csgn(I*d)*csgn(I*(
f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^4*ln(x^n)+1/32*I*Pi*csgn(I*d)*csgn(I*
(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^4*n-1/8*I*ln(c)*Pi*x^4*b*csgn(I*d*(f
*x^2+e)^m)^3-1/8*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*x^4*ln(x^n)+1/16*Pi^2*csgn(
I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*Pi^2*csgn(I*d*(f*x^
2+e)^m)^3*x^4*b*csgn(I*c)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csg
n(I*d*(f*x^2+e)^m)^2*x^4*b*csgn(I*c*x^n)^3+1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^
3*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m
)^2*b*x^4*ln(x^n)-1/8*I*Pi*x^4*a*csgn(I*d*(f*x^2+e)^m)^3-1/8*I/f*Pi*b*e*m*c
sgn(I*c*x^n)^3*x^2+1/8*I*e^2*m/f^2*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3+1/16*I*
Pi*x^4*b*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*ln(c)*Pi*x^4*b*csgn(I*
d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/16*Pi^2*csgn(I*d)*csgn(I*(f*
x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^4*b*csgn(I*c*x^n)^3+1/8*I*ln(c)*Pi*x^4*b*
csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/8*I*ln(c)*Pi*x^4*b*csgn(I*(f*x^2+e)^m)*
csgn(I*d*(f*x^2+e)^m)^2+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)*x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*I*Pi*x^4*a*csgn(I*d)*csgn(
I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/16*I*Pi*x^4*b*m*csgn(I*c)*csgn(I*c*x
^n)^2-1/32*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^4*n+1/4*x^4*ln(d)*ln(
c)*b-1/8*x^4*ln(c)*b*m-1/16*ln(d)*b*n*x^4+1/8*I*Pi*csgn(I*(f*x^2+e)^m)*csgn
(I*d*(f*x^2+e)^m)^2*b*x^4*ln(x^n)+1/8*I/f*Pi*b*e*m*csgn(I*c)*csgn(I*c*x^n)^
2*x^2+1/8*I/f*Pi*b*e*m*csgn(I*x^n)*csgn(I*c*x^n)^2*x^2-1/8*I*e^2*m/f^2*ln(f
*x^2+e)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/8*I*e^2*m/f^2*ln(f*x^2+e)*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)^2+1/16*I*Pi*x^4*b*m*csgn(I*c*x^n)^3+1/32*I*Pi*csgn(I
*d*(f*x^2+e)^m)^3*b*x^4*n-1/4*e^2*m/f^2*ln(f*x^2+e)*b*ln(c)-3/16*b*e*m*n*x^
```

```
2/f-1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+1/8*I*x^4*ln(d)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I*x^4*ln(d)*Pi*b*
csgn(I*c)*csgn(I*c*x^n)^2+(1/4*x^4*b*ln(x^n)+1/16*x^4*(-2*I*b*Pi*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)+2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+2*I*b*Pi*csgn(
I*x^n)*csgn(I*c*x^n)^2-2*I*b*Pi*csgn(I*c*x^n)^3+4*b*ln(c)-b*n+4*a))*ln((f*x
^2+e)^m)-1/8*m*b*ln(x^n)*x^4+1/4*ln(d)*b*x^4*ln(x^n)-1/16*Pi^2*csgn(I*d*(f*
x^2+e)^m)^3*x^4*b*csgn(I*c*x^n)^3-1/32*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)^2*b*x^4*n-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^4*b*csgn
(I*c)*csgn(I*c*x^n)^2+1/4*m/f*b*ln(x^n)*e*x^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*m*x^4*log(x^n) - ((m*n - 4*m*log(c))*b - 4*a*m)*x^4)*log(f*x^2 +
e) + integrate(-1/8*((4*(f*m - 2*f*log(d))*a - (f*m*n - 4*(f*m - 2*f*log(d)
)*log(c))*b)*x^5 - 8*(b*log(c)*log(d) + a*log(d))*x^3*e + 4*((f*m - 2*f*log
(d))*b*x^5 - 2*b*x^3*e*log(d))*log(x^n))/(f*x^2 + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*log(c*x^n) + a*x^3)*log((f*x^2 + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^3*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^3*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

3.91 $\int x(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=148

$$\frac{1}{2}bmnx^2-\frac{1}{2}mx^2(a+b\log(cx^n))-\frac{bn(e+fx^2)\log\left(d(e+fx^2)^m\right)}{4f}-\frac{ben\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right)}{4f}+\frac{(e+$$

```
[Out] 1/2*b*m*n*x^2-1/2*m*x^2*(a+b*ln(c*x^n))-1/4*b*n*(f*x^2+e)*ln(d*(f*x^2+e)^m)
/f-1/4*b*e*n*ln(-f*x^2/e)*ln(d*(f*x^2+e)^m)/f+1/2*(f*x^2+e)*(a+b*ln(c*x^n))
*ln(d*(f*x^2+e)^m)/f-1/4*b*e*m*n*polylog(2,1+f*x^2/e)/f
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2436, 2332, 2423, 2525, 2458, 45, 2393, 2354, 2438}

$$-\frac{bemn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4f}+\frac{(e+fx^2)(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2f}-\frac{1}{2}mx^2(a+b\log(cx^n))-\frac{bn(e+fx^2)\log\left(d(e+fx^2)^m\right)}{4f}-\frac{ben\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right)}{4f}+\frac{1}{2}bmnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (b*m*n*x^2)/2 - (m*x^2*(a + b*Log[c*x^n]))/2 - (b*n*(e + f*x^2)*Log[d*(e +
f*x^2)^m])/(4*f) - (b*e*n*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m])/(4*f) + (
(e + f*x^2)*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(2*f) - (b*e*m*n*PolyL
og[2, 1 + (f*x^2)/e])/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
```

```
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx = -\frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n))\log\left(d(e\right.}{2f}$$

$$= \frac{1}{4}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n)}{2f}$$

$$= \frac{1}{4}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n)}{2f}$$

$$= \frac{1}{4}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n)}{2f}$$

$$= \frac{1}{4}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n)}{2f}$$

$$= \frac{1}{4}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) + \frac{\left(e+fx^2\right)(a+b\log(cx^n)}{2f}$$

$$= \frac{1}{2}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) - \frac{bn(e+fx^2)\log\left(d(e+f\right.}{4f}$$

$$= \frac{1}{2}bmnx^2 - \frac{1}{2}mx^2(a+b\log(cx^n)) - \frac{bn(e+fx^2)\log\left(d(e+f\right.}{4f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 266, normalized size = 1.80

$$\frac{-2afmx^2+2bfmnx^2-2bfmx^2\log(cx^n)+2bemn\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+2bemn\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-bemn\log(e+fx^2)-2bemn\log(x)\log(e+fx^2)+2bem\log(cx^n)\log(e+fx^2)+2ae\log\left(d(e+fx^2)^m\right)+2afx^2\log\left(d(e+fx^2)^m\right)-bfnx^2\log\left(d(e+fx^2)^m\right)+2bfx^2\log(cx^n)\log\left(d(e+fx^2)^m\right)+2bemn\mathrm{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+2bemn\mathrm{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a*f*m*x^2 + 2*b*f*m*n*x^2 - 2*b*f*m*x^2*Log[c*x^n] + 2*b*e*m*n*Log[x]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt
[e]] - b*e*m*n*Log[e + f*x^2] - 2*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b*e*m*L
og[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f*x^2)^m] + 2*a*f*x^2*Log[d*(e
+ f*x^2)^m] - b*f*n*x^2*Log[d*(e + f*x^2)^m] + 2*b*f*x^2*Log[c*x^n]*Log[d*(
```

```
e + f*x^2)^m] + 2*b*e*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*e*m*n*
PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 2068, normalized size = 13.97

method	result	size
risch	Expression too large to display	2068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I*Pi*b*n*x^2*csgn(I
*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^2*
b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m
)*csgn(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d)*
csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2
+1/2*b*m*n*x^2+1/4*I*ln(c)*Pi*b*x^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I
*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*x^2*ln(x^n)+1/8*I*Pi*b*n*x^2*csgn(I*d*(f*x^2+
e)^m)^3+(1/2*x^2*b*ln(x^n)+1/4*x^2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*
b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)-b*n+2*a))*ln((f*x^2+e)^m)-1/2*x^2*a*m-1/4*b*
e*m*n/f*ln(f*x^2+e)+1/2*x^2*ln(d)*a+1/2*e*m/f*ln(f*x^2+e)*a+1/4*I*Pi*b*m*x^
2*csgn(I*c*x^n)^3+1/4*I*Pi*a*x^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I*Pi
*a*x^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*ln(d)*b*n*x^2+1/2*x^
2*ln(c)*ln(d)*b-1/2*x^2*ln(c)*b*m+1/2*e*m/f*ln(f*x^2+e)*b*ln(c)+1/4*I*e*m/f
*ln(f*x^2+e)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*Pi*b*m*x^2*csgn(I*c)*csgn
(I*c*x^n)^2+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^2*ln(x^n)+1/4*I*
Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*x^2*ln(x^n)+1/8*Pi^2*csgn(
I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c*x^n)^3-1/4*I*ln(c)*Pi
*b*x^2*csgn(I*d*(f*x^2+e)^m)^3-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x
^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x
^2+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csg
n(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*b*e*m*n/f*ln(x)*
ln(f*x^2+e)+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c*x^n)^
3+1/4*I*x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*x^2*Pi*ln(d)*b*csgn(
I*x^n)*csgn(I*c*x^n)^2-1/4*I*Pi*a*x^2*csgn(I*d*(f*x^2+e)^m)^3+1/4*I*e*m/f*l
n(f*x^2+e)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^
2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8
*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^2*b*csgn(I*c*x^
n)^3-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*c*x^
n)^2+1/2*b*e*m*n/f*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*I*e*m/f*ln
(f*x^2+e)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*m*b*ln(x^n)*x^2+1/2*
ln(d)*b*x^2*ln(x^n)+1/2*b*e*m*n/f*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2)
)-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^2*b*csgn(I*c*x^n)^3+1/8*Pi^2*csgn(I*d)
```

```
*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/8*Pi^2
*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^2*b*csgn(I*c)*csgn(I*x^n)*cs
gn(I*c*x^n)+1/8*I*Pi*b*n*x^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^2*ln
(x^n)-1/4*I*ln(c)*Pi*b*x^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)-1/4*I*e*m/f*ln(f*x^2+e)*Pi*b*csgn(I*c*x^n)^3+1/4*I*Pi*b*m*x^2*csgn(I*c)
*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I*ln(c)*Pi*b*x^2*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)^2+1/2*m/f*b*ln(x^n)*e*ln(f*x^2+e)-1/4*I*x^2*Pi*ln(d)*b*csgn(
I*c*x^n)^3-1/4*I*x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*P
i*b*m*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*Pi*a*x^2*csgn(I*d)*csgn(I*(f*x^
2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/2*b*e*m*n/f*dilog((-f*x+(-e*f)^(1/2))/(-e*f
)^(1/2))+1/2*b*e*m*n/f*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/8*Pi^2*csgn
(I*d*(f*x^2+e)^m)^3*x^2*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d*(f*x^
2+e)^m)^3*x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*m*x^2*log(x^n) - ((m*n - 2*m*log(c))*b - 2*a*m)*x^2)*log(f*x^2 + e
) + integrate(-1/2*((2*(f*m - f*log(d))*a - (f*m*n - 2*(f*m - f*log(d))*log
(c))*b)*x^3 - 2*(b*log(c)*log(d) + a*log(d))*x*e + 2*((f*m - f*log(d))*b*x^
3 - b*x*e*log(d))*log(x^n))/(f*x^2 + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

3.92 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx$

Optimal. Leaf size=113

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{m(a+b\log(cx^n))^2\log\left(1+\frac{fx^2}{e}\right)}{2bn}-\frac{1}{2}m(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{fx^2}{e}\right)+$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/b/n-1/2*m*(a+b*ln(c*x^n))^2*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))*polylog(2,-f*x^2/e)+1/4*b*m*n*polylog(3,-f
*x^2/e)
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2422, 2375, 2421, 6724}

$$-\frac{1}{2}m\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))+\frac{1}{4}bmn\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)+\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{m\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(2*b*n) - (m*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^2)/e])/(2*b*n) - (m*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)
/e)])/2 + (b*m*n*PolyLog[3, -((f*x^2)/e)])/4
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
```

```
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}\,dx &= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{(fm)\int\frac{x(a+b\log(cx^n))^2}{e+fx^2}\,dx}{bn}\\ &= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{m(a+b\log(cx^n))^2\log\left(1\right.}{2bn}\\ &= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{m(a+b\log(cx^n))^2\log\left(1\right.}{2bn}\\ &= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2bn}-\frac{m(a+b\log(cx^n))^2\log\left(1\right.}{2bn}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 297, normalized size = 2.63

$$\frac{1}{2}\left(bmn\log^2(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-2bm\log(x)\log(cx^n)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+bmn\log^2(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-2bm\log(x)\log(cx^n)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-bn\log^2(x)\log\left(d(e+fx^2)^m\right)+a\log\left(-\frac{fx^2}{e}\right)\log\left(d(e+fx^2)^m\right)+2b\log(x)\log(cx^n)\log\left(d(e+fx^2)^m\right)-2bm\log(cx^n)\,\mathrm{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-2bm\log(cx^n)\,\mathrm{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+am\mathrm{Li}_2\left(1+\frac{fx^2}{e}\right)+2bmn\mathrm{Li}_3\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+2bmn\mathrm{Li}_3\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] (b*m*n*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b*m*Log[x]*Log[c*x^n]*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] - 2*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b*n*Log[x]^2*L
og[d*(e + f*x^2)^m] + a*Log[-((f*x^2)/e)]*Log[d*(e + f*x^2)^m] + 2*b*Log[x]
*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b*m*Log[c*x^n]*PolyLog[2, ((-I)*Sqrt[f
]*x)/Sqrt[e]] - 2*b*m*Log[c*x^n]*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + a*m*Po
lyLog[2, 1 + (f*x^2)/e] + 2*b*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*
b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 2842, normalized size = 25.15

method	result	size
risch	Expression too large to display	2842

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*a*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))-ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^
(1/2))*b*m*(ln(x^n)-n*ln(x))+ln(x)*ln(f*x^2+e)*b*m*(ln(x^n)-n*ln(x))+1/2*I*
m*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c*x^n)^3+1/2*I*m*dilo
g((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c*x^n)^3-1/4*I*Pi*csgn(I*d*(
f*x^2+e)^m)^3*b/n*ln(x^n)^2+1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*a*ln
(x)-1/2*I*m*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*csgn(I*c)*csgn(I
*c*x^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*csgn(I*x^n)*cs
gn(I*c*x^n)^2+1/2*ln(d)*b/n*ln(x^n)^2-ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(
1/2))*b*m*(ln(x^n)-n*ln(x))-1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*c*x^n)^3+ln(d)*a*
ln(x)+ln(d)*ln(x)*b*ln(c)-1/2*I*m*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*
Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*m*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*m*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*
f)^(1/2))*b*Pi*csgn(I*c*x^n)^3+1/2*I*m*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(
1/2))*b*Pi*csgn(I*c*x^n)^3+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)
*b*csgn(I*c*x^n)^3+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*ln(
x)*b*csgn(I*c*x^n)^3+1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*ln(x)*b*csgn(I*c)*csg
n(I*c*x^n)^2+1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*ln(x)*b*csgn(I*x^n)*csgn(I*c*
x^n)^2-1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*ln(x)*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*ln(x)
*b*csgn(I*c*x^n)^3+1/2*I*ln(x)*Pi*ln(f*x^2+e)*b*m*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/2*I*m*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2-1/2*I*ln(x)*Pi*ln(f*x^2+e)*b*m*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)+1/2*I*m*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)-m*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*a-m*dilog((f*x
+(-e*f)^(1/2))/(-e*f)^(1/2))*a+1/4*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2
+e)^m)^2*b/n*ln(x^n)^2-1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x
^2+e)^m)*a*ln(x)-m*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*a-m*ln(x)*ln(
(f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*a+ln(x)*ln(f*x^2+e)*a*m-m*dilog((-f*x+(-e*
f)^(1/2))/(-e*f)^(1/2))*b*ln(c)-m*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*
ln(c)+1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*b*n*m*ln(x)^2*
ln(f*x^2+e)-1/2*b*n*m*ln(x)^2*ln(1+f*x^2/e)-1/2*b*n*m*ln(x)*polylog(2,-f*x^
2/e)-m*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*ln(c)-m*ln(x)*ln((f*x+(
-e*f)^(1/2))/(-e*f)^(1/2))*b*ln(c)+ln(x)*ln(c)*ln(f*x^2+e)*b*m-1/2*I*ln(x)*
Pi*b*csgn(I*c*x^n)^3*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))+1/2*I*m*ln(x)*ln((f*x+
(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*Pi
^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*ln(x)*b*csgn(I*c)*cs
```

```
gn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(d)*ln(x)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/
2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*ln(c)+1/2*I*ln(x)*Pi*b*csg
n(I*x^n)*csgn(I*c*x^n)^2*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))+1/2*I*ln(x)*Pi*b*c
sgn(I*c)*csgn(I*c*x^n)^2*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))-1/2*I*ln(x)*Pi*ln(
f*x^2+e)*b*m*csgn(I*c*x^n)^3+1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/n
*ln(x^n)^2+1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*ln(
c)+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)-1/2*I*ln(x)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)*(ln((f*x^2+e)^m)-m*ln(f*x^2+e))+1/4*b*m*n*polylog(3,-f*x^2/e)-1/2*I*ln(d
)*ln(x)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*m*ln(x)*ln((-f*x+(-e
*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*m*ln(x)*ln(
(f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/4*Pi^2*cs
gn(I*d*(f*x^2+e)^m)^3*ln(x)*b*csgn(I*c*x^n)^3-1/2*I*Pi*csgn(I*d*(f*x^2+e)^m
)^3*a*ln(x)+1/2*I*ln(x)*Pi*ln(f*x^2+e)*b*m*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*
Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*ln(x)*b*ln(c)+1/2*I*
Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*a*ln(x)-1/2*I*Pi*csgn(I*d*(f
*x^2+e)^m)^3*ln(x)*b*ln(c)-1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)^2*ln(x)*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)^2*ln(x)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*d)*csg
n(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*ln(x)*b*csgn(I*c)*csgn(I*c*x^n)^2+1/
4*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*ln(x)*b*csgn(I*x
^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*ln(x)*b*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn
(I*d*(f*x^2+e)^m)*b/n*ln(x^n)^2-1/2*I*m*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1
/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*m*ln(x)*ln((f*x+(-e*f)^(1/2))/(
-e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*m*dilog((-f*x+(-e*f)^(1
/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*m*dilog((
f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-di
log((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))*b*m*(ln(x^n)-n*ln(x))-dilog((f*x+(-e*
f)^(1/2))/(-e*f)^(1/2))*b*m*(ln(x^n)-n*ln(x))+1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*m*n*log(x)^2 - 2*b*m*log(x)*log(x^n) - 2*(b*m*log(c) + a*m)*log(x))
*log(f*x^2 + e) - integrate(-(b*f*m*n*x^2*log(x)^2 - 2*(b*f*m*log(c) + a*f*
m)*x^2*log(x) + (b*f*log(c)*log(d) + a*f*log(d))*x^2 + (b*log(c)*log(d) + a
*log(d))*e - (2*b*f*m*x^2*log(x) - b*f*x^2*log(d) - b*e*log(d))*log(x^n))/(
f*x^3 + x*e), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x, x)
```

3.93 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=195

$$\frac{bfmn\log(x)}{2e}-\frac{bfmn\log^2(x)}{2e}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{bfmn\log\left(e+fx^2\right)}{4e}+\frac{bfmn\log\left(-\frac{fx^2}{e}\right)\log\left(e+\right.}{4e}$$

```
[Out] 1/2*b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+f*m*ln(x)*(a+b*ln(c*x^n))/e-1/4*b
*f*m*n*ln(f*x^2+e)/e+1/4*b*f*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e-1/2*f*m*(a+b*ln
(c*x^n))*ln(f*x^2+e)/e-1/4*b*n*ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))*ln
(d*(f*x^2+e)^m)/x^2+1/4*b*f*m*n*polylog(2,1+f*x^2/e)/e
```

Rubi [A]
time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2504, 2442, 36, 29, 31, 2423, 2338, 2441, 2352}

$$\frac{bfmn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{4e}-\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^2\right)^m\right)}{2x^2}+\frac{fm\log(x)\left(a+b\log\left(cx^n\right)\right)}{e}-\frac{fm\log\left(e+fx^2\right)\left(a+b\log\left(cx^n\right)\right)}{2e}-\frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{4x^2}-\frac{bfmn\log\left(e+fx^2\right)}{4e}+\frac{bfmn\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{4e}-\frac{bfmn\log^2(x)}{2e}+\frac{bfmn\log(x)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (b*f*m*n*Log[x])/(2*e) - (b*f*m*n*Log[x]^2)/(2*e) + (f*m*Log[x]*(a + b*Log[
c*x^n]))/e - (b*f*m*n*Log[e + f*x^2])/(4*e) + (b*f*m*n*Log[-((f*x^2)/e)]*Lo
g[e + f*x^2])/(4*e) - (f*m*(a + b*Log[c*x^n])*Log[e + f*x^2])/(2*e) - (b*n*
Log[d*(e + f*x^2)^m])/(4*x^2) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(
2*x^2) + (b*f*m*n*PolyLog[2, 1 + (f*x^2)/e])/(4*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^3}\,dx = \frac{fm\log(x)\,(a+b\log(cx^n))}{e} - \frac{fm(a+b\log(cx^n))\log(e+fx^2)}{2e}$$

$$= \frac{fm\log(x)\,(a+b\log(cx^n))}{e} - \frac{fm(a+b\log(cx^n))\log(e+fx^2)}{2e}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\,(a+b\log(cx^n))}{e} - \frac{fm(a+b\log(c$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\,(a+b\log(cx^n))}{e} + \frac{bfmn\log\left(-\frac{fx^2}{e}\right.}{4}$$

$$= -\frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\,(a+b\log(cx^n))}{e} + \frac{bfmn\log\left(-\frac{fx^2}{e}\right.}{4}$$

$$= \frac{bfmn\log(x)}{2e} - \frac{bfmn\log^2(x)}{2e} + \frac{fm\log(x)\,(a+b\log(cx^n))}{e} - \frac{bf}{}$$

<u>**Mathematica**</u> [**C**] Result contains complex when optimal does not.
time = 0.09, size = 298, normalized size = 1.53

$$-\frac{-4afmx^2\log(x)-2bfmnx^2\log(x)+2bfmnx^2\log^2(x)-4bfmx^2\log(x)\log(cx^n)+2bfmnx^2\log(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)+2bfmnx^2\log(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right)+2afmx^2\log(e+fx^2)+bfmnx^2\log(e+fx^2)-2bfmnx^2\log(x)\log(e+fx^2)+2bfmx^2\log(cx^n)\log(e+fx^2)+2ae\log\left(d(e+fx^2)^m\right)+ben\log\left(d(e+fx^2)^m\right)+2be\log(cx^n)\log\left(d(e+fx^2)^m\right)+2bfmnx^2\text{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)+2bfmnx^2\text{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{4ex^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/4*(-4*a*f*m*x^2*Log[x] - 2*b*f*m*n*x^2*Log[x] + 2*b*f*m*n*x^2*Log[x]^2 -
 4*b*f*m*x^2*Log[x]*Log[c*x^n] + 2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)
/Sqrt[e]] + 2*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2*a*f*m*x
^2*Log[e + f*x^2] + b*f*m*n*x^2*Log[e + f*x^2] - 2*b*f*m*n*x^2*Log[x]*Log[e
 + f*x^2] + 2*b*f*m*x^2*Log[c*x^n]*Log[e + f*x^2] + 2*a*e*Log[d*(e + f*x^2)
^m] + b*e*n*Log[d*(e + f*x^2)^m] + 2*b*e*Log[c*x^n]*Log[d*(e + f*x^2)^m] +
2*b*f*m*n*x^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b*f*m*n*x^2*PolyLog[
2, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

<u>**Maple**</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 2101, normalized size = 10.77

method	result	size
risch	Expression too large to display	2101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3,x,method=_RETURNVERBOSE)

[Out] 1/2*b*f*m*n*ln(x)/e-1/2*b*f*m*n*ln(x)^2/e+(-1/2*b/x^2*ln(x^n)-1/4*(-I*b*Pi*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+b*n+2*a)/x^2)
*ln((f*x^2+e)^m)+1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b
*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*b*f*m*n/e*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e
*f)^(1/2))-1/2*b*f*m*n/e*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*b*f*
m*n*ln(x)/e*ln(f*x^2+e)+1/4*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^2*ln(x^n)+1/4*
I/x^2*Pi*ln(c)*b*csgn(I*d*(f*x^2+e)^m)^3-1/4*I/x^2*Pi*a*csgn(I*d)*csgn(I*d*
(f*x^2+e)^m)^2-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*c)*csgn(I*c*x^
n)^2-1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+m*f
*b*ln(x^n)*ln(x)/e-1/2/x^2*ln(d)*a+1/4*I/x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)-1/2*f*m/e*ln(f*x^2+e)*a-1/2*f*m/e*ln(f*x^2+e)*b*ln(c)-1/4*
b*f*m*n*ln(f*x^2+e)/e-1/2*b*f*m*n/e*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))
-1/2*b*f*m*n/e*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*I/x^2*Pi*a*csgn(I
*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/4*I/x^2*Pi*ln(c)*b*csgn(I*(
f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+f*m/e*ln(x)*b*ln(c)+1/8*Pi^2*csgn(I*d)*
csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-1/4*I*f*m/e*ln(f*x^2+e)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*f*m/e
*ln(f*x^2+e)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+f*m/e*ln(x)*a+1/4*I/x
^2*Pi*ln(d)*b*csgn(I*c*x^n)^3-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)^2/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I/x^2*Pi*ln(c)*b*csgn
(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/8*I/x^2*Pi*b*n*csgn(I*d)*
csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*I*f*m/e*ln(f*x^2+e)*b*Pi*csgn
(I*c*x^n)^3-1/4*I*f*m/e*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*
f*m/e*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*f*m/e*ln(x)*b*Pi*csgn(I*
c*x^n)^3-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^2*b
*csgn(I*c)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*
(f*x^2+e)^m)/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/8*Pi^2*csgn(I*d)*csgn(I*d*
(f*x^2+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I/x^2*Pi*b*n*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/4*I/x^2*Pi*ln(c)*b*csgn(I*d)*c
sgn(I*d*(f*x^2+e)^m)^2-1/4*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^2*ln(
x^n)+1/8*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^2*b*csg
n(I*c*x^n)^3+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*c)*csg
n(I*c*x^n)^2+1/4*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b
/x^2*ln(x^n)-1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*c*x^n)
^3-1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*c*x^n)
^3-1/4*I/x^2*Pi*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/8*I/x^2*Pi*
b*n*csgn(I*d*(f*x^2+e)^m)^3-1/2*m*f*b*ln(x^n)/e*ln(f*x^2+e)-1/2/x^2*ln(d)*l
n(c)*b-1/4/x^2*ln(d)*b*n-1/4*I/x^2*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^2-1/4
*I/x^2*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*f*m/e*ln(x)*b*Pi*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*ln(d)*b/x^2*ln(x^n)-1/4*I*Pi*csgn(I*(f*x^
2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^2*ln(x^n)-1/8*I/x^2*Pi*b*n*csgn(I*d)*cs
```

```
gn(I*d*(f*x^2+e)^m)^2+1/8*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^2*b*csgn(I*c*x^n)^
3+1/4*I/x^2*Pi*a*csgn(I*d*(f*x^2+e)^m)^3+1/8*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2
+e)^m)^2/x^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*csgn(I*(f*x^2+e)^m)*csg
n(I*d*(f*x^2+e)^m)^2/x^2*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*f*m/e*ln(x)*b*Pi
*csgn(I*c)*csgn(I*c*x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*m*log(x^n) + (m*n + 2*m*log(c))*b + 2*a*m)*log(f*x^2 + e)/x^2 + i
ntegrate(1/2*((2*(f*m + f*log(d))*a + (f*m*n + 2*(f*m + f*log(d))*log(c))*b
)*x^2 + 2*(b*log(c)*log(d) + a*log(d))*e + 2*((f*m + f*log(d))*b*x^2 + b*e*
log(d))*log(x^n))/(f*x^5 + x^3*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^3, x)
```

3.94 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$

Optimal. Leaf size=248

$$-\frac{3bfmn}{16ex^2}-\frac{bf^2mn\log(x)}{8e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{fm(a+b\log(cx^n))}{4ex^2}-\frac{f^2m\log(x)\,(a+b\log(cx^n))}{2e^2}+\frac{bf^2mn\log(e}{16e^2}$$

```
[Out] -3/16*b*f*m*n/e/x^2-1/8*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/4*f
*m*(a+b*ln(c*x^n))/e/x^2-1/2*f^2*m*ln(x)*(a+b*ln(c*x^n))/e^2+1/16*b*f^2*m*n
*ln(f*x^2+e)/e^2-1/8*b*f^2*m*n*ln(-f*x^2/e)*ln(f*x^2+e)/e^2+1/4*f^2*m*(a+b*
ln(c*x^n))*ln(f*x^2+e)/e^2-1/16*b*n*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c*x^n
))*ln(d*(f*x^2+e)^m)/x^4-1/8*b*f^2*m*n*polylog(2,1+f*x^2/e)/e^2
```

Rubi [A]

time = 0.15, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2504, 2442, 46, 2423, 2338, 2441, 2352}

$$-\frac{bf^2mn\text{PolyLog}\left(2,\frac{fx^2}{e}+1\right)}{8e^2}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{f^2m\log(x)\,(a+b\log(cx^n))}{2e^2}+\frac{f^2m\log\left(e+fx^2\right)(a+b\log(cx^n))}{4e^2}-\frac{fm(a+b\log(cx^n))}{4ex^2}-\frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{16x^4}+\frac{bf^2mn\log\left(e+fx^2\right)}{16e^2}-\frac{bf^2mn\log\left(-\frac{fx^2}{e}\right)\log\left(e+fx^2\right)}{8e^2}+\frac{bf^2mn\log^2(x)}{4e^2}-\frac{bf^2mn\log(x)}{8e^2}-\frac{3bfmn}{16ex^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] (-3*b*f*m*n)/(16*e*x^2) - (b*f^2*m*n*Log[x])/(8*e^2) + (b*f^2*m*n*Log[x]^2)
/(4*e^2) - (f*m*(a + b*Log[c*x^n]))/(4*e*x^2) - (f^2*m*Log[x]*(a + b*Log[c*
x^n]))/(2*e^2) + (b*f^2*m*n*Log[e + f*x^2])/(16*e^2) - (b*f^2*m*n*Log[-((f*
x^2)/e)]*Log[e + f*x^2])/(8*e^2) + (f^2*m*(a + b*Log[c*x^n])*Log[e + f*x^2]
)/(4*e^2) - (b*n*Log[d*(e + f*x^2)^m])/(16*x^4) - ((a + b*Log[c*x^n])*Log[d
*(e + f*x^2)^m])/(4*x^4) - (b*f^2*m*n*PolyLog[2, 1 + (f*x^2)/e])/(8*e^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = -\frac{fm(a+b\log(cx^n))}{4ex^2} - \frac{f^2m\log(x)\,(a+b\log(cx^n))}{2e^2} + \frac{f^2m(a+$$

$$= -\frac{bfmn}{8ex^2} - \frac{fm(a+b\log(cx^n))}{4ex^2} - \frac{f^2m\log(x)\,(a+b\log(cx^n))}{2e^2} +$$

$$= -\frac{bfmn}{8ex^2} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{4ex^2} - \frac{f^2m\log(x)\,($$

$$= -\frac{bfmn}{8ex^2} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{4ex^2} - \frac{f^2m\log(x)\,($$

$$= -\frac{bfmn}{8ex^2} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n))}{4ex^2} - \frac{f^2m\log(x)\,($$

$$= -\frac{3bfmn}{16ex^2} - \frac{bf^2mn\log(x)}{8e^2} + \frac{bf^2mn\log^2(x)}{4e^2} - \frac{fm(a+b\log(cx^n}{4ex^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 363, normalized size = 1.46

$$-\frac{4aefmx^2+3befmnx^2+8af^2mx^4\log(x)+2bf^2mnx^4\log(x)-4bf^2mnx^4\log^2(x)+4befmx^2\log(cx^n)+8bf^2mx^4\log(x)\log(cx^n)-4bf^2mnx^4\log(x)\log\left(1-\frac{i\sqrt{f}x}{\sqrt{e}}\right)-4bf^2mnx^4\log(x)\log\left(1+\frac{i\sqrt{f}x}{\sqrt{e}}\right)-4af^2mx^4\log(e+fx^2)-bf^2mnx^4\log(e+fx^2)+4bf^2mnx^4\log(x)\log(e+fx^2)-4bf^2mx^4\log(cx^n)\log(e+fx^2)+4ae^2\log\left(d(e+fx^2)^m\right)+be^2n\log\left(d(e+fx^2)^m\right)+4be^2\log(cx^n)\log\left(d(e+fx^2)^m\right)-4bf^2mnx^4\mathrm{Li}_2\left(-\frac{i\sqrt{f}x}{\sqrt{e}}\right)-4bf^2mnx^4\mathrm{Li}_2\left(\frac{i\sqrt{f}x}{\sqrt{e}}\right)}{16e^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] -1/16*(4*a*e*f*m*x^2 + 3*b*e*f*m*n*x^2 + 8*a*f^2*m*x^4*Log[x] + 2*b*f^2*m*n
*x^4*Log[x] - 4*b*f^2*m*n*x^4*Log[x]^2 + 4*b*e*f*m*x^2*Log[c*x^n] + 8*b*f^2
*m*x^4*Log[x]*Log[c*x^n] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqr
t[e]] - 4*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 4*a*f^2*m*x
^4*Log[e + f*x^2] - b*f^2*m*n*x^4*Log[e + f*x^2] + 4*b*f^2*m*n*x^4*Log[x]*L
og[e + f*x^2] - 4*b*f^2*m*x^4*Log[c*x^n]*Log[e + f*x^2] + 4*a*e^2*Log[d*(e
+ f*x^2)^m] + b*e^2*n*Log[d*(e + f*x^2)^m] + 4*b*e^2*Log[c*x^n]*Log[d*(e +
f*x^2)^m] - 4*b*f^2*m*n*x^4*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 4*b*f^2*
m*n*x^4*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e^2*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 2313, normalized size = 9.33

method	result	size
risch	Expression too large to display	2313

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^5,x,method=_RETURNVERBOSE)

[Out] (-1/4*b/x^4*ln(x^n)-1/16*(-2*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+2*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+2*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*b*Pi*csgn(I*c*x^n)^3+4*b*ln(c)+b*n+4*a)/x^4)*ln((f*x^2+e)^m)+1/4/e^2*f^2*m*ln(f*x^2+e)*b*ln(c)-1/8*b*f^2*m*n*ln(x)/e^2+1/4*b*f^2*m*n*ln(x)^2/e^2-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n)^3-1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c*x^n)^3-1/4/e*f*m/x^2*b*ln(c)+1/16*b*f^2*m*n*ln(f*x^2+e)/e^2-1/4/x^4*ln(d)*a+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c)*csgn(I*c*x^n)^2+1/4/e^2*f^2*m*ln(f*x^2+e)*a+1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c)*csgn(I*c*x^n)^2+1/8*I/x^4*ln(c)*Pi*b*csgn(I*d*(f*x^2+e)^m)^3-1/4*ln(d)*b/x^4*ln(x^n)-1/4/e*f*m/x^2*a+1/4*b*f^2*m*n/e^2*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*b*f^2*m*n/e^2*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2/e^2*f^2*m*ln(x)*b*ln(c)-1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*c)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/32*I/x^4*Pi*b*n*csgn(I*d*(f*x^2+e)^m)^3-1/8*I/x^4*Pi*a*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I/x^4*Pi*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/8*I/x^4*ln(d)*Pi*b*csgn(I*c*x^n)^3+1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4/x^4*ln(d)*ln(c)*b-1/16/x^4*ln(d)*b*n-3/16*b*f*m*n/e/x^2-1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/16*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*m*f*b*ln(x^n)/e/x^2-1/8*I/x^4*ln(d)*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-1/8*I/x^4*ln(d)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2/e^2*f^2*m*ln(x)*a+1/8*I/e*m*f/x^2*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^4*ln(x^n)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*c*x^n)^3-1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^4*b*csgn(I*c)*csgn(I*c*x^n)^2-1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*m*f^2*b*ln(x^n)/e^2*ln(x)-1/8*I/e*m*f/x^2*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-1/8*I/e*m*f/x^2*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I/e^2*f^2*m*ln(f*x^2+e)*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*m*f^2*b*ln(x^n)/e^2*ln(f*x^2+e)+1/8*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^4*ln(x^n)+1/4*b*f^2*m*n/e^2*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/4*b*f^2*m*n/e^2*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/4*b*f^2*m*n*ln(x)/e^2*ln(f*x^2+e)-1/8*I/x^4*ln(c)*Pi*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*c*x^n)^3+1/32*I/x^4*Pi*b*n*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/32*I/x^4*Pi*b*n*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-1/32*I/x^4*Pi*b*n*csgn(I

```
*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/8*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)
^m)^2*b/x^4*ln(x^n)-1/4*I/e^2*f^2*m*ln(x)*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+
1/8*I/x^4*ln(d)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/8*I*Pi*csgn(I*d)
*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b/x^4*ln(x^n)-1/16*Pi^2*csgn(I*d
)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^4*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/16*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^4*b*csgn(I*c)*csgn(I*x^n)*
csgn(I*c*x^n)+1/16*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
/x^4*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*I/x^4*ln(c)*Pi*b*csgn(I*(f*x
^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2+1/16*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^4*b*cs
gn(I*c*x^n)^3+1/8*I/x^4*Pi*a*csgn(I*d*(f*x^2+e)^m)^3+1/8*I/x^4*ln(c)*Pi*b*c
sgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/8*I/e^2*f^2*m*ln(f*x^2
+e)*b*Pi*csgn(I*c*x^n)^3+1/8*I/e*m*f/x^2*b*Pi*csgn(I*c*x^n)^3+1/8*I/x^4*Pi*
a*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*(4*b*m*log(x^n) + (m*n + 4*m*log(c))*b + 4*a*m)*log(f*x^2 + e)/x^4 +
integrate(1/8*((4*(f*m + 2*f*log(d))*a + (f*m*n + 4*(f*m + 2*f*log(d))*log(
c))*b)*x^2 + 8*(b*log(c)*log(d) + a*log(d))*e + 4*((f*m + 2*f*log(d))*b*x^2
 + 2*b*e*log(d))*log(x^n))/(f*x^7 + x^5*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^5, x)
```

3.95 $\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=251

$$-\frac{8bemnx}{9f}+\frac{4}{27}bmnx^3+\frac{2be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9f^{3/2}}+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+b\log(cx^n))-\frac{2e^{3/2}m\,\mathrm{ta}\cdots}{\cdots}$$

```
[Out] -8/9*b*e*m*n*x/f+4/27*b*m*n*x^3+2/9*b*e^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))
/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))/f-2/9*m*x^3*(a+b*ln(c*x^n))-2/3*e^(3/2)*
m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)-1/9*b*n*x^3*ln(d*(f*x^2
+e)^m)+1/3*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/3*I*b*e^(3/2)*m*n*polylo
g(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)-1/3*I*b*e^(3/2)*m*n*polylog(2,I*x*f^(1/2)
/e^(1/2))/f^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2505, 308, 211, 2423, 4940, 2438}

$$\frac{ibe^{3/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{3f^{3/2}}-\frac{ibe^{3/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{3f^{3/2}}-\frac{2e^{3/2}m\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3f^{3/2}}+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+b\log(cx^n))+\frac{2be^{3/2}mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9f^{3/2}}-\frac{1}{9}bnx^3\log\left(d(e+fx^2)^m\right)-\frac{8bemnx}{9f}+\frac{4}{27}bmnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-8*b*e*m*n*x)/(9*f) + (4*b*m*n*x^3)/27 + (2*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*
x)/Sqrt[e]])/(9*f^(3/2)) + (2*e*m*x*(a + b*Log[c*x^n]))/(3*f) - (2*m*x^3*(a
 + b*Log[c*x^n]))/9 - (2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c
*x^n]))/(3*f^(3/2)) - (b*n*x^3*Log[d*(e + f*x^2)^m])/9 + (x^3*(a + b*Log[c*
x^n])*Log[d*(e + f*x^2)^m])/3 + ((I/3)*b*e^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[
f]*x)/Sqrt[e]])/f^(3/2) - ((I/3)*b*e^(3/2)*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqr
t[e]])/f^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)\,dx=\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+b\log(cx^n))-\frac{2e^{3/2}m\tan^{-}}{}$$

$$=-\frac{2bemnx}{3f}+\frac{2}{27}bmnx^3+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+$$

$$=-\frac{2bemnx}{3f}+\frac{2}{27}bmnx^3+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+$$

$$=-\frac{2bemnx}{3f}+\frac{2}{27}bmnx^3+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+$$

$$=-\frac{8bemnx}{9f}+\frac{4}{27}bmnx^3+\frac{2emx(a+b\log(cx^n))}{3f}-\frac{2}{9}mx^3(a+$$

$$=-\frac{8bemnx}{9f}+\frac{4}{27}bmnx^3+\frac{2be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9f^{3/2}}+\frac{2emx(a}{}$$

Mathematica [A]
time = 0.10, size = 389, normalized size = 1.55

$$\frac{18ae\sqrt{f}\,mx-24be\sqrt{f}\,mnx-6af^{3/2}mx^3+4bf^{3/2}mnx^3-18ae^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)+6be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)+18be^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x)+18be\sqrt{f}\,mx\log(cx^n)-6bf^{3/2}mx^3\log(cx^n)-18be^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(cx^n)-9ibe^{3/2}mn\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+9ibe^{3/2}mn\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+9af^{3/2}x^3\log\left(d(e+fx^2)^m\right)-3bf^{3/2}nx^3\log\left(d(e+fx^2)^m\right)+9bf^{3/2}x^3\log(cx^n)\log\left(d(e+fx^2)^m\right)+9ibe^{3/2}mn\mathrm{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-9ibe^{3/2}mn\mathrm{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{27f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (18*a*e*Sqrt[f]*m*x - 24*b*e*Sqrt[f]*m*n*x - 6*a*f^(3/2)*m*x^3 + 4*b*f^(3/2
)*m*n*x^3 - 18*a*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*b*e^(3/2)*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]] + 18*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x] + 18*b*e*Sqrt[f]*m*x*Log[c*x^n] - 6*b*f^(3/2)*m*x^3*Log[c*x^n] - 18*b*
e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (9*I)*b*e^(3/2)*m*n*Log[
x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b*e^(3/2)*m*n*Log[x]*Log[1 + (I*S
qrt[f]*x)/Sqrt[e]] + 9*a*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 3*b*f^(3/2)*n*x
^3*Log[d*(e + f*x^2)^m] + 9*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] +
 (9*I)*b*e^(3/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b*e^(3/2)
*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 2321, normalized size = 9.25

method	result	size
risch	Expression too large to display	2321

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I*m*x^3*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I*Pi*csgn(I*d)*csgn(I*(f*
x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*a+1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)
^m)^2*x^3*b*ln(c)+1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4/27*b*m*n*x^3+1/3*x^3*ln(d)*a-1/6*I*Pi*c
sgn(I*d*(f*x^2+e)^m)^3*x^3*a-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csgn(I
*c*x^n)^3-2/3*m/f*b*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)-2/9*x^3
*a*m+2/3*a*e*m/f*x-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I
*x^n)*csgn(I*c*x^n)^2+1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^3*ln(x
^n)-1/18*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*x^3*n+(1/3*x^3*b*ln(x^n)+
1/18*x^3*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*
csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)
^3+6*b*ln(c)-2*b*n+6*a))*ln((f*x^2+e)^m)+1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2
+e)^m)^2*x^3*a+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*a-1
/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*ln(c)-1/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^
3*b*x^3*ln(x^n)-1/9*I*m*x^3*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2-2/9*m*b*ln(x^n)*
x^3+1/3*ln(d)*b*x^3*ln(x^n)+2/3*m/f*b*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2
))*n*ln(x)-1/3*m/f*b*n*e^2/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)
^(1/2))+1/3*m/f*b*n*e^2/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/
2))-1/6*I*x^3*Pi*ln(d)*b*csgn(I*c*x^n)^3-2/3*m/f*e^2/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*b*ln(c)+1/3*I*m/f*x*e*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/3*x^
3*ln(d)*ln(c)*b-2/9*x^3*ln(c)*b*m-1/9*ln(d)*b*n*x^3+1/12*Pi^2*csgn(I*d)*csg
n(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/12*Pi^2*cs
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*
b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(
I*d*(f*x^2+e)^m)*x^3*b*csgn(I*c)*csgn(I*c*x^n)^2-1/6*I*Pi*csgn(I*d)*csgn(I*
(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^3*ln(x^n)+1/18*I*Pi*csgn(I*d)*csgn(I
*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*x^3*n-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x
^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/
3*I*m/f*x*e*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2+2/3*m/f*b*ln(x^n)*x*e+1/18*I*Pi*
csgn(I*d*(f*x^2+e)^m)^3*b*x^3*n+1/9*I*m*x^3*Pi*b*csgn(I*c*x^n)^3+1/12*Pi^2*
csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*(f
*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d
*(f*x^2+e)^m)^3*x^3*b*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I*x^3*Pi*ln(d)*b*csgn(I
*c)*csgn(I*c*x^n)^2-8/9*b*e*m*n*x/f+2/9*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f
```

```
)^(1/2))*b*n-1/3*m/f*b*n*e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^
(1/2))+1/3*m/f*b*n*e^2/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-
1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c)*csgn(
I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csgn(I*c)*csgn
(I*c*x^n)^2-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*x^3*b*csg
n(I*x^n)*csgn(I*c*x^n)^2-1/3*I*m/f*x*e*Pi*b*csgn(I*c*x^n)^3+1/9*I*m*x^3*Pi*
b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m
)*csgn(I*d*(f*x^2+e)^m)*x^3*b*ln(c)+1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))*Pi*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csg
n(I*x^n)*csgn(I*c*x^n)^2+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
^2*x^3*b*ln(c)+1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*x^3*l
n(x^n)-1/18*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*x^3*n-1/12*P
i^2*csgn(I*d*(f*x^2+e)^m)^3*x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/12*
Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*x^3*b*csgn(I*c*x^n
)^3+2/3/f*ln(c)*b*e*m*x+1/6*I*x^3*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/
6*I*x^3*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2/3*m/f*e^2/(e*f)^(1
/2)*arctan(x*f/(e*f)^(1/2))*a-1/3*I*m/f*x*e*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)-1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*csgn(I*c)*
csgn(I*c*x^n)^2-1/3*I*m/f*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*Pi*b*csgn
(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
[Out] 1/9*(3*b*m*x^3*log(x^n) - ((m*n - 3*m*log(c))*b - 3*a*m)*x^3)*log(f*x^2 + e
) + integrate(-1/9*((3*(2*f*m - 3*f*log(d))*a - (2*f*m*n - 3*(2*f*m - 3*f*l
og(d))*log(c))*b)*x^4 - 9*(b*log(c)*log(d) + a*log(d))*x^2*e + 3*((2*f*m -
3*f*log(d))*b*x^4 - 3*b*x^2*e*log(d))*log(x^n))/(f*x^2 + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

3.96 $\int (a+b\log (cx^n)) \log \left(d(e+fx^2)^m\right) \, dx$

Optimal. Leaf size=194

$$4bmnx-\frac{2b\sqrt{e}\, mn\tan^{-1}\left(\frac{\sqrt{f}\, x}{\sqrt{e}}\right)}{\sqrt{f}}-2mx(a+b\log (cx^n))+\frac{2\sqrt{e}\, m\tan^{-1}\left(\frac{\sqrt{f}\, x}{\sqrt{e}}\right)(a+b\log (cx^n))}{\sqrt{f}}-bnx\log$$

```
[Out] 4*b*m*n*x-2*m*x*(a+b*ln(c*x^n))-b*n*x*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))*l
n(d*(f*x^2+e)^m)-2*b*m*n*arctan(x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+2*m*arct
an(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))*e^(1/2)/f^(1/2)-I*b*m*n*polylog(2,-I*
x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+I*b*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))*e
^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2498, 327, 211, 2417, 4940, 2438}

$$-\frac{ib\sqrt{e}\, mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\, x}{\sqrt{e}}\right)}{\sqrt{f}}+\frac{ib\sqrt{e}\, mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\, x}{\sqrt{e}}\right)}{\sqrt{f}}+\frac{2\sqrt{e}\, m\text{ArcTan}\left(\frac{\sqrt{f}\, x}{\sqrt{e}}\right)(a+b\log (cx^n))}{\sqrt{f}}+x(a+b\log (cx^n))\log \left(d(e+fx^2)^m\right)-2mx(a+b\log (cx^n))-\frac{2b\sqrt{e}\, mn\text{ArcTan}\left(\frac{\sqrt{f}\, x}{\sqrt{e}}\right)}{\sqrt{f}}-bnx\log \left(d(e+fx^2)^m\right)+4bmnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] 4*b*m*n*x - (2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - 2*m*x*(
a + b*Log[c*x^n]) + (2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x
^n]))/Sqrt[f] - b*n*x*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])*Log[d*(e
+ f*x^2)^m] - (I*b*Sqrt[e]*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f
] + (I*b*Sqrt[e]*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^2)^m], x]},
```

```
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int (a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)\,dx = -2mx(a+b\log(cx^n))+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx$$

$$= 2bmnx-2mx(a+b\log(cx^n))+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{f}}$$

$$= 2bmnx-2mx(a+b\log(cx^n))+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{f}}$$

$$= 4bmnx-2mx(a+b\log(cx^n))+\frac{2\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{f}}$$

$$= 4bmnx-\frac{2b\sqrt{e}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-2mx(a+b\log(cx^n))+$$

Mathematica [A]
time = 0.06, size = 332, normalized size = 1.71

$$\frac{-2a\sqrt{f}\,mx+4b\sqrt{f}\,mnx+2a\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-2b\sqrt{e}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-2b\sqrt{e}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x)-2b\sqrt{f}\,mx\log\left(cx^n\right)+2b\sqrt{e}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log\left(cx^n\right)+ib\sqrt{e}\,mn\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-ib\sqrt{e}\,mn\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+a\sqrt{f}\,x\log\left(d\left(e+fx^2\right)^m\right)-b\sqrt{f}\,nx\log\left(d\left(e+fx^2\right)^m\right)+b\sqrt{f}\,x\log\left(cx^n\right)\log\left(d\left(e+fx^2\right)^m\right)-ib\sqrt{e}\,mn\mathrm{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+ib\sqrt{e}\,mn\mathrm{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a*Sqrt[f]*m*x + 4*b*Sqrt[f]*m*n*x + 2*a*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sq
rt[e]] - 2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 2*b*Sqrt[e]*m*n*ArcT
an[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 2*b*Sqrt[f]*m*x*Log[c*x^n] + 2*b*Sqrt[e]*m
*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[e]*m*n*Log[x]*Log[1 - (I
*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
] + a*Sqrt[f]*x*Log[d*(e + f*x^2)^m] - b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] +
 b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[e]*m*n*PolyLog[2, (
(-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[e]*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]
])/Sqrt[f]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 2001, normalized size = 10.31

method	result	size
risch	Expression too large to display	2001

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x*a*m+1/4*Pi^2*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c)
*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(
I*d*(f*x^2+e)^m)*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(c)*Pi*x*b*csgn(I*d)*csg
n(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)-1/2*I*Pi*ln(x^n)*x*b*csgn(I*d)*csgn(
I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)+1/4*Pi^2*x*b*csgn(I*d*(f*x^2+e)^m)^3*c
sgn(I*x^n)*csgn(I*c*x^n)^2+4*b*m*n*x+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x^2
+e)^m)^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*m*x*b*Pi*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)-2*m*ln(c)*b*x+ln(d)*ln(c)*b*x-ln(d)*b*n*x-I*m*e/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c*x^n)^3+1/4*Pi^2*x*b*csgn(I*d)*csgn(
I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*csgn(I*x^n)*csgn(I*c*x^n)^2+ln(d)*a*x-
I*m*x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(d)*Pi*b*x*csgn(I*c)*csgn(I*
c*x^n)^2+1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c*x^n)^3+2*a
*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))-2*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*b*n+m*b*n*e/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*
b*n*e/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+2*m*e/(e*f)^(1/2)
*arctan(x*f/(e*f)^(1/2))*b*ln(c)-2*m*b*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*n*ln(x)+m*b*n*e/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m
*b*n*e/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-2*m*b*ln(x^n)
```

```
*x+ln(x^n)*ln(d)*x*b-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f
*x^2+e)^m)*csgn(I*c*x^n)^3-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*c
sgn(I*c)*csgn(I*c*x^n)^2-1/4*Pi^2*x*b*csgn(I*d*(f*x^2+e)^m)^3*csgn(I*c*x^n)
^3-1/2*I*Pi*x*a*csgn(I*d*(f*x^2+e)^m)^3+I*m*x*b*Pi*csgn(I*c*x^n)^3+(b*x*ln(
x^n)+1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn
(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*l
n(c)-2*b*n+2*a)*x)*ln((f*x^2+e)^m)+1/2*I*Pi*x*b*n*csgn(I*d)*csgn(I*(f*x^2+e
)^m)*csgn(I*d*(f*x^2+e)^m)+1/2*I*Pi*ln(x^n)*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e
)^m)^2-1/2*I*Pi*x*b*n*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2-I*m*x*b*Pi*csgn(I*c
)*csgn(I*c*x^n)^2-1/2*I*ln(d)*Pi*b*x*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/
4*Pi^2*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/4*Pi^2*x*b*csgn(I*d*(f*x^2+e)^m)^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/2*I*ln(c)*Pi*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1/2*I*
ln(c)*Pi*x*b*csgn(I*d*(f*x^2+e)^m)^3-1/2*I*Pi*ln(x^n)*x*b*csgn(I*d*(f*x^2+e
)^m)^3+1/2*I*Pi*x*b*n*csgn(I*d*(f*x^2+e)^m)^3+1/2*I*Pi*x*a*csgn(I*d)*csgn(I
*d*(f*x^2+e)^m)^2+1/2*I*Pi*x*a*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-
1/4*Pi^2*x*b*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)+I*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*cs
gn(I*c)*csgn(I*c*x^n)^2+2*m*b*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)
-1/4*Pi^2*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*x^n)*csgn(I*c*x^n)^2
-1/4*Pi^2*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*csgn(I*c)*csgn(I*
c*x^n)^2+1/2*I*Pi*ln(x^n)*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2-1
/2*I*ln(d)*Pi*b*x*csgn(I*c*x^n)^3+I*m*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*x*b*csgn(I*(f*x^2+e)^m)*csgn(I*d
*(f*x^2+e)^m)^2*csgn(I*c*x^n)^3-1/2*I*Pi*x*b*n*csgn(I*(f*x^2+e)^m)*csgn(I*d
*(f*x^2+e)^m)^2-1/2*I*Pi*x*a*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+
e)^m)+1/2*I*ln(c)*Pi*x*b*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2+1/4*Pi^2*x*b*csg
n(I*d*(f*x^2+e)^m)^3*csgn(I*c)*csgn(I*c*x^n)^2-I*m*e/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(d)*Pi*b*x*c
sgn(I*x^n)*csgn(I*c*x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] (b*m*x*log(x^n) - ((m*n - m*log(c))*b - a*m)*x)*log(f*x^2 + e) + integrate(
-(((2*f*m - f*log(d))*a - (2*f*m*n - (2*f*m - f*log(d))*log(c))*b)*x^2 - (b
*log(c)*log(d) + a*log(d))*e + ((2*f*m - f*log(d))*b*x^2 - b*e*log(d))*log(
x^n))/(f*x^2 + e), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)),x)
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)), x)
```

3.97 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=179

$$\frac{2b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{bn\log\left(d(e+fx^2)^m\right)}{x}-\frac{(a+b\log(c}{}$$

```
[Out] -b*n*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x+2*b*m*n*arctan
(x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+2*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c
*x^n))*f^(1/2)/e^(1/2)-I*b*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1
/2)+I*b*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2505, 211, 2423, 4940, 2438}

$$-\frac{ib\sqrt{f}\,mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{ib\sqrt{f}\,mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{2\sqrt{f}\,m\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x}+\frac{2b\sqrt{f}\,mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{bn\log\left(d(e+fx^2)^m\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*b*Sqrt[f]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (2*Sqrt[f]*m*ArcTan
[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] - (b*n*Log[d*(e + f*x^2)^
m])/x - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x - (I*b*Sqrt[f]*m*n*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (I*b*Sqrt[f]*m*n*PolyLog[2, (I*
Sqrt[f]*x)/Sqrt[e]])/Sqrt[e]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}} - \frac{(a+b\log(cx^n))\log}{x}$$

$$= \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}} - \frac{(a+b\log(cx^n))\log}{x}$$

$$= \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}} - \frac{bn\log\left(d(e+fx^2)^m\right)}{x}$$

$$= \frac{2b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{f}\,m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log}{\sqrt{e}}$$

Mathematica [A]
time = 0.07, size = 305, normalized size = 1.70

$$\frac{2a\sqrt{f}\,mx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)+2b\sqrt{f}\,mnx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-2b\sqrt{f}\,mnx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x)+2b\sqrt{f}\,mx\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(cx^n)+ib\sqrt{f}\,mnx\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-ib\sqrt{f}\,mnx\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-a\sqrt{e}\log\left(d(e+fx^2)^m\right)-b\sqrt{e}\,n\log\left(d(e+fx^2)^m\right)-b\sqrt{e}\log(cx^n)\log\left(d(e+fx^2)^m\right)-ib\sqrt{f}\,mnx\,\text{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+ib\sqrt{f}\,mnx\,\text{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 2*b*Sqrt[f]*m*n*x*ArcTan[(Sq
rt[f]*x)/Sqrt[e]] - 2*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] +
2*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + I*b*Sqrt[f]*m*n*x*
Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]] - a*Sqrt[e]*Log[d*(e + f*x^2)^m] - b*Sqrt[e]*n*Log[d*(
e + f*x^2)^m] - b*Sqrt[e]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - I*b*Sqrt[f]*m*n
*x*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + I*b*Sqrt[f]*m*n*x*PolyLog[2, (I*S
qrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 1972, normalized size = 11.02

method	result	size
risch	Expression too large to display	1972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(d)*b/x*ln(x^n)+1/2*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x*b*ln(c)+1/2*I*Pi*csgn
(I*d*(f*x^2+e)^m)^3*b/x*ln(x^n)-1/2*I/x*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)^
2+2*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*ln(c)-1/4*Pi^2*csgn(I*d)*csgn
(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*P
i*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*ln(c)-1/2*I*Pi*cs
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x*a+2*m*f*b/(e*f)^(1/2)*arctan(x*
f/(e*f)^(1/2))*ln(x^n)-1/x*ln(d)*a+(-b/x*ln(x^n)-1/2*(-I*b*Pi*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*
csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*n+2*a)/x)*ln((f*x^2+e)
^m)-1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*ln(c)+1/4*Pi^2*csgn(I*d*
(f*x^2+e)^m)^3/x*b*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x*a-1/4
*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)-1/4*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2
/x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*b*Pi*csgn(I*c*x^n)^3+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*
d*(f*x^2+e)^m)*b/x*ln(x^n)+2*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a-1/x*
ln(d)*ln(c)*b-1/x*ln(d)*b*n+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*
csgn(I*c)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*cs
gn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m
)^2/x*b*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I/x*Pi*ln(d)*b*csgn(I*c*x^n)^3+1/4*Pi
^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+I*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c
)*csgn(I*c*x^n)^2-1/4*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I*c)*csgn(I*c*x
^n)^2-1/4*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*c*x^n)^3-1/4*Pi
^2*csgn(I*d*(f*x^2+e)^m)^3/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I/x*Pi*ln(d)
*b*csgn(I*x^n)*csgn(I*c*x^n)^2+I*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*
```

```
Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(
I*d*(f*x^2+e)^m)*b*n/x-2*m*f*b/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*n*ln(x)-
1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x-1/2*I*Pi*csgn(I*(f*x^2+e)^
m)*csgn(I*d*(f*x^2+e)^m)^2/x*b*ln(c)+1/2*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*n/x
-1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x*a-1/4*Pi^2*csgn(I*(f*x^2+e)^m
)*csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*c*x^n)^3+1/4*Pi^2*csgn(I*(f*x^2+e)^m)*
csgn(I*d*(f*x^2+e)^m)^2/x*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*csgn(I*d*(
f*x^2+e)^m)^3/x*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*csgn(I*d)*cs
gn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x*b*csgn(I*c*x^n)^3+m*f*b*n/(-e*f)^
(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-m*f*b*n/(-e*f)^(1/2)*ln(x)
*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/2*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)
*csgn(I*d*(f*x^2+e)^m)/x*a-m*f*b*n/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-
e*f)^(1/2))+1/2*I/x*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-I*m*f/(e
*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+
2*m*f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*n+m*f*b*n/(-e*f)^(1/2)*dilog((-
f*x+(-e*f)^(1/2))/(-e*f)^(1/2))-1/2*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*
b/x*ln(x^n)-1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x*ln(x^n
)-1/2*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b*m*log(x^n) + (m*n + m*log(c))*b + a*m)*log(f*x^2 + e)/x + integrate((((
2*f*m + f*log(d))*a + (2*f*m*n + (2*f*m + f*log(d))*log(c))*b)*x^2 + (b*log
(c)*log(d) + a*log(d))*e + ((2*f*m + f*log(d))*b*x^2 + b*e*log(d))*log(x^n)
)/(f*x^4 + x^2*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^2, x)
```

3.98 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=227

$$-\frac{8bfmn}{9ex}-\frac{2bf^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}}-\frac{2fm(a+b\log(cx^n))}{3ex}-\frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}}-\frac{bn\,l}{}$$

```
[Out] -8/9*b*f*m*n/e/x-2/9*b*f^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)-2/3*f*
m*(a+b*ln(c*x^n))/e/x-2/3*f^(3/2)*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n
))/e^(3/2)-1/9*b*n*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)
^m)/x^3+1/3*I*b*f^(3/2)*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(3/2)-1/3*I*b
*f^(3/2)*m*n*polylog(2,I*x*f^(1/2)/e^(1/2))/e^(3/2)
```

Rubi [A]

time = 0.11, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2505, 331, 211, 2423, 4940, 2438}

$$\frac{ibf^{3/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{ibf^{3/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{2f^{3/2}m\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{3e^{3/2}}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{3x^3}-\frac{2fm(a+b\log(cx^n))}{3ex}-\frac{2bf^{3/2}mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}}-\frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{9x^3}-\frac{8bfmn}{9ex}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-8*b*f*m*n)/(9*e*x) - (2*b*f^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(9*e^(
3/2)) - (2*f*m*(a + b*Log[c*x^n]))/(3*e*x) - (2*f^(3/2)*m*ArcTan[(Sqrt[f]*x
)/Sqrt[e]]*(a + b*Log[c*x^n]))/(3*e^(3/2)) - (b*n*Log[d*(e + f*x^2)^m])/(9*
x^3) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(3*x^3) + ((I/3)*b*f^(3/2)
*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - ((I/3)*b*f^(3/2)*m*n*P
olyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^4}\,dx = -\frac{2fm(a+b\log(cx^n))}{3ex} - \frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n)}{3e^{3/2}}$$

$$= -\frac{2bfmn}{3ex} - \frac{2fm(a+b\log(cx^n))}{3ex} - \frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{3e^{3/2}}$$

$$= -\frac{2bfmn}{3ex} - \frac{2fm(a+b\log(cx^n))}{3ex} - \frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{3e^{3/2}}$$

$$= -\frac{8bfmn}{9ex} - \frac{2fm(a+b\log(cx^n))}{3ex} - \frac{2f^{3/2}m\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{3e^{3/2}}$$

$$= -\frac{8bfmn}{9ex} - \frac{2bf^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}} - \frac{2fm(a+b\log(cx^n))}{3ex} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 362, normalized size = 1.59

$$\frac{-8b\sqrt{e}\,fmnx^2-2bf^{3/2}mnx^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)-6a\sqrt{e}\,fmx^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{fx^2}{e}\right)+6bf^{3/2}mnx^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x)-6b\sqrt{e}\,fmx^2\log(cx^n)-6bf^{3/2}mx^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(cx^n)-3ibf^{3/2}mnx^3\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)+3ibf^{3/2}mnx^3\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-3ae^{3/2}\log\left(d(e+fx^2)^m\right)-be^{3/2}n\log\left(d(e+fx^2)^m\right)-3be^{3/2}\log(cx^n)\log\left(d(e+fx^2)^m\right)+3ibf^{3/2}mnx^3\text{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)-3ibf^{3/2}mnx^3\text{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-8*b*Sqrt[e]*f*m*n*x^2 - 2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] -
 6*a*Sqrt[e]*f*m*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((f*x^2)/e)] + 6*b*f^
(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 6*b*Sqrt[e]*f*m*x^2*Log[
c*x^n] - 6*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (3*I)*b
*f^(3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*f^(3/2)*m*
n*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*e^(3/2)*Log[d*(e + f*x^2)
^m] - b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 3*b*e^(3/2)*Log[c*x^n]*Log[d*(e +
f*x^2)^m] + (3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
(3*I)*b*f^(3/2)*m*n*x^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(9*e^(3/2)*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 2204, normalized size = 9.71

method	result	size
risch	Expression too large to display	2204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*b/x^3*ln(x^n)-1/18*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I
*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*
Pi*csgn(I*c*x^n)^3+6*b*ln(c)+2*b*n+6*a)/x^3)*ln((f*x^2+e)^m)+1/3*I*m*f^2/e/
(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)-1/3*m*f^2*b*n/e/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1
/3*m*f^2*b*n/e/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/12*
Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*csgn(I*c*x^n)^3+1/6*I*Pi*csgn(I*d*(f*x^2
+e)^m)^3/x^3*a-1/3/x^3*ln(d)*a-1/6*I/x^3*Pi*ln(d)*b*csgn(I*c)*csgn(I*c*x^n)
^2-2/3*m*f^2*b/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*ln(x^n)+1/12*Pi^2*csgn
(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*c*x^n)^2-
1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c*x^n)^3-2/3*m*f*b
*ln(x^n)/e/x-1/3*ln(d)*b/x^3*ln(x^n)+2/3*m*f^2*b/e/(e*f)^(1/2)*arctan(x*f/(
e*f)^(1/2))*n*ln(x)-1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*
csgn(I*x^n)*csgn(I*c*x^n)^2+1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2
))*b*Pi*csgn(I*c*x^n)^3-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*csgn(I*c)*c
sgn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/12*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*
c*x^n)^3-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*a+1/6*I*P
i*csgn(I*d*(f*x^2+e)^m)^3/x^3*b*ln(c)+1/6*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^
3*ln(x^n)+1/18*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b*n/x^3+1/6*I*Pi*csgn(I*d)*csgn
(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3*a-2/3*m*f^2/e/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*b*ln(c)-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
^2*b/x^3*ln(x^n)+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)/x^3*b*csgn(I*c*x^n)^3+1/12*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b
*csgn(I*c)*csgn(I*c*x^n)^2-2/3/e*f*m/x*b*ln(c)-1/3*I*m*f/e/x*b*Pi*csgn(I*c)
*csgn(I*c*x^n)^2-1/3*I*m*f/e/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-8/9*b*f*m*n
/e/x+1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3*b*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2/3/e*f*m/x*a+1/12*Pi^2*csgn(I*(f*x^2+e)^
m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/12*Pi^2*csgn
(I*d*(f*x^2+e)^m)^3/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/18*I*Pi*csg
n(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x^3-1/12*Pi^2*csgn(I*d)*csgn(I
*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/12*Pi^2*csgn(
I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)+1/3*I*m*f/e/x*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/18*I*Pi*csgn
(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x^3-1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)
^m)^2/x^3*b*ln(c)-1/6*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^3*
```

```
b*ln(c)-1/6*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^3*ln(x^n)-1/6*I*Pi*c
sgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^3*a-1/9/x^3*ln(d)*b*n-1/3/x^3*ln(d)*ln(c
)*b-1/6*I/x^3*Pi*ln(d)*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/3*I*m*f/e/x*b*Pi*csg
n(I*c*x^n)^3+1/6*I/x^3*Pi*ln(d)*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/12*
Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^3*b*csgn(I*c)*cs
gn(I*c*x^n)^2-1/12*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
/x^3*b*csgn(I*x^n)*csgn(I*c*x^n)^2+1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*c
sgn(I*d*(f*x^2+e)^m)/x^3*b*ln(c)+1/6*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csg
n(I*d*(f*x^2+e)^m)*b/x^3*ln(x^n)+1/18*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*cs
gn(I*d*(f*x^2+e)^m)*b*n/x^3-2/9*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*b*n-1/3*m*f^2*b*n/e/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1
/3*m*f^2*b*n/e/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/6*I/x^
3*Pi*ln(d)*b*csgn(I*c*x^n)^3-2/3*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*a-1/3*I*m*f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*(3*b*m*log(x^n) + (m*n + 3*m*log(c))*b + 3*a*m)*log(f*x^2 + e)/x^3 + i
ntegrate(1/9*((3*(2*f*m + 3*f*log(d))*a + (2*f*m*n + 3*(2*f*m + 3*f*log(d))
*log(c))*b)*x^2 + 9*(b*log(c)*log(d) + a*log(d))*e + 3*((2*f*m + 3*f*log(d)
)*b*x^2 + 3*b*e*log(d))*log(x^n))/(f*x^6 + x^4*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^4, x)
```

3.99 $\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x^6}\,dx$

Optimal. Leaf size=267

$$-\frac{16bfmn}{225ex^3}+\frac{12bf^2mn}{25e^2x}+\frac{2bf^{5/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{25e^{5/2}}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2f^2m(a+b\log(cx^n))}{5e^2x}+\frac{2f^{5/2}m\,\text{ta}}{}$$

```
[Out] -16/225*b*f*m*n/e/x^3+12/25*b*f^2*m*n/e^2/x+2/25*b*f^(5/2)*m*n*arctan(x*f^(
1/2)/e^(1/2))/e^(5/2)-2/15*f*m*(a+b*ln(c*x^n))/e/x^3+2/5*f^2*m*(a+b*ln(c*x^
n))/e^2/x+2/5*f^(5/2)*m*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/e^(5/2)-1
/25*b*n*ln(d*(f*x^2+e)^m)/x^5-1/5*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^5-1/5
*I*b*f^(5/2)*m*n*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(5/2)+1/5*I*b*f^(5/2)*m*
n*polylog(2,I*x*f^(1/2)/e^(1/2))/e^(5/2)
```

Rubi [A]
time = 0.13, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2505, 331, 211, 2423, 4940, 2438}

$$-\frac{ibf^{5/2}mn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{5e^{5/2}}+\frac{ibf^{5/2}mn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{5e^{5/2}}+\frac{2f^{5/2}m\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{5e^{5/2}}-\frac{(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{5x^5}+\frac{2f^2m(a+b\log(cx^n))}{5e^2x}-\frac{2fm(a+b\log(cx^n))}{15ex^3}+\frac{2bf^{5/2}mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{25e^{5/2}}-\frac{bn\log\left(d\left(e+fx^2\right)^m\right)}{25x^5}+\frac{12bf^2mn}{25e^2x}-\frac{16bfmn}{225ex^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

```
[Out] (-16*b*f*m*n)/(225*e*x^3) + (12*b*f^2*m*n)/(25*e^2*x) + (2*b*f^(5/2)*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]])/(25*e^(5/2)) - (2*f*m*(a + b*Log[c*x^n]))/(15*e*
x^3) + (2*f^2*m*(a + b*Log[c*x^n]))/(5*e^2*x) + (2*f^(5/2)*m*ArcTan[(Sqrt[f
]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(5*e^(5/2)) - (b*n*Log[d*(e + f*x^2)^m])/
(25*x^5) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(5*x^5) - ((I/5)*b*f^(
5/2)*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(5/2) + ((I/5)*b*f^(5/2)*m
*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{x^6}\,dx = -\frac{2fm(a+b\log(cx^n))}{15ex^3} + \frac{2f^2m(a+b\log(cx^n))}{5e^2x} + \frac{2f^{5/2}m\tan^{-1}}{}$$

$$= -\frac{2bfmn}{45ex^3} + \frac{2bf^2mn}{5e^2x} - \frac{2fm(a+b\log(cx^n))}{15ex^3} + \frac{2f^2m(a+b\log(}{5e^2x}$$

$$= -\frac{2bfmn}{45ex^3} + \frac{2bf^2mn}{5e^2x} - \frac{2fm(a+b\log(cx^n))}{15ex^3} + \frac{2f^2m(a+b\log(}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3} + \frac{2bf^2mn}{5e^2x} - \frac{2fm(a+b\log(cx^n))}{15ex^3} + \frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3} + \frac{12bf^2mn}{25e^2x} - \frac{2fm(a+b\log(cx^n))}{15ex^3} + \frac{2f^2m(a+b\log}{5e^2x}$$

$$= -\frac{16bfmn}{225ex^3} + \frac{12bf^2mn}{25e^2x} + \frac{2bf^{5/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{25e^{5/2}} - \frac{2fm(a+}{15}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 399, normalized size = 1.49

$$-\frac{16be^{3/2}fmnx^2 - 108b\sqrt{e}\,f^2mnx^4 - 18bf^{5/2}mnx^5\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right) + 30ae^{3/2}fmx^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{fx^2}{e}\right) + 90bf^{5/2}mnx^5\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(x) + 30be^{3/2}fmx^2\log(cx^n) - 90b\sqrt{e}\,f^2mx^4\log(cx^n) - 90bf^{5/2}mx^5\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(cx^n) - 45ibf^{5/2}mnx^5\log(x)\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right) + 45ibf^{5/2}mnx^5\log(x)\log\left(1+\frac{i\sqrt{f}\,x}{\sqrt{e}}\right) + 45ae^{5/2}\log\left(d(e+fx^2)^m\right) + 9be^{5/2}n\log\left(d(e+fx^2)^m\right) + 45be^{5/2}\log(cx^n)\log\left(d(e+fx^2)^m\right) + 45ibf^{5/2}mnx^5\operatorname{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right) - 45ibf^{5/2}mnx^5\operatorname{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{225e^{5/2}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x^6,x]
```

```
[Out] -1/225*(16*b*e^(3/2)*f*m*n*x^2 - 108*b*Sqrt[e]*f^2*m*n*x^4 - 18*b*f^(5/2)*m
*n*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 30*a*e^(3/2)*f*m*x^2*Hypergeometric2F1
[-3/2, 1, -1/2, -((f*x^2)/e)] + 90*b*f^(5/2)*m*n*x^5*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x] + 30*b*e^(3/2)*f*m*x^2*Log[c*x^n] - 90*b*Sqrt[e]*f^2*m*x^4*Log
[c*x^n] - 90*b*f^(5/2)*m*x^5*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - (45*I
)*b*f^(5/2)*m*n*x^5*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (45*I)*b*f^(5/2
)*m*n*x^5*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 45*a*e^(5/2)*Log[d*(e + f
*x^2)^m] + 9*b*e^(5/2)*n*Log[d*(e + f*x^2)^m] + 45*b*e^(5/2)*Log[c*x^n]*Log
[d*(e + f*x^2)^m] + (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, ((-I)*Sqrt[f]*x)/Sq
```

```
rt[e]] - (45*I)*b*f^(5/2)*m*n*x^5*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(e^(5/
2)*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 2385, normalized size = 8.93

method	result	size
risch	Expression too large to display	2385

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*m*f*b/e/x^3*ln(x^n)+1/20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^
m)^2/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2-2/5*m*f^3*b/e^2/(e*f)^(1/2)*arctan(x
*f/(e*f)^(1/2))*n*ln(x)+1/5*m*f^3*b*n/e^2/(-e*f)^(1/2)*ln(x)*ln((-f*x+(-e*f
)^(1/2))/(-e*f)^(1/2))-1/5*m*f^3*b*n/e^2/(-e*f)^(1/2)*ln(x)*ln((f*x+(-e*f)^
(1/2))/(-e*f)^(1/2))-2/15*m*f/e/x^3*a+1/5*I*m*f^3/e^2/(e*f)^(1/2)*arctan(x*
f/(e*f)^(1/2))*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/10*I*Pi*csgn(I*d*(f*x^2+e)^
m)^3/x^5*a+(-1/5*b/x^5*ln(x^n)-1/50*(-5*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)+5*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^
n)^2-5*I*b*Pi*csgn(I*c*x^n)^3+10*b*ln(c)+2*b*n+10*a)/x^5)*ln((f*x^2+e)^m)+1
/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c)*csgn(I*c*x^n)^2+
1/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)
^2-1/5*ln(d)*b/x^5*ln(x^n)+2/5/e^2*f^2*m/x*b*ln(c)+1/20*Pi^2*csgn(I*d*(f*x^
2+e)^m)^3/x^5*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/20*Pi^2*csgn(I*d)*csg
n(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*c*x^n)^3-1/10*I*Pi*csgn
(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b/x^5*ln(x^n)-1/50*I*Pi*csgn(I*(f*x
^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2*b*n/x^5-1/10*I*ln(d)/x^5*b*Pi*csgn(I*x^n)*
csgn(I*c*x^n)^2-1/5*m*f^3*b*n/e^2/(-e*f)^(1/2)*dilog((f*x+(-e*f)^(1/2))/(-e
*f)^(1/2))+2/25*m*f^3/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*n+2/5*m*f^3
/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*ln(c)+1/20*Pi^2*csgn(I*d*(f*x^2+
e)^m)^3/x^5*b*csgn(I*c*x^n)^3+2/5*m*f^2*b*ln(x^n)/e^2/x-1/5*ln(d)/x^5*b*ln(
c)-1/25*ln(d)*b*n/x^5-1/5*ln(d)/x^5*a+2/5*m*f^3*b/e^2/(e*f)^(1/2)*arctan(x*
f/(e*f)^(1/2))*ln(x^n)-1/5*I*m*f^3/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*
b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/20*Pi^2*csgn(I*d)*csgn(I*(f*x^2+
e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*x^n)*csgn(I*c*x^n)^2-1/20*Pi^2*csg
n(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/
20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)+1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/10*I*
Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b/x^5*ln(x^n)+1/50*I
*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)*b*n/x^5+1/15*I*m*f/
e/x^3*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+2/5/e^2*f^2*m/x*a+1/5*m*f^3*
b*n/e^2/(-e*f)^(1/2)*dilog((-f*x+(-e*f)^(1/2))/(-e*f)^(1/2))+1/10*I*ln(d)/x
^5*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/15*I*m*f/e/x^3*b*Pi*csgn(I*c*
x^n)^3-1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*c*x^n)^3-1/10*I*ln(d)/x^5*b*Pi*csgn(I*
```

```
c)*csgn(I*c*x^n)^2-1/15*I*m*f/e/x^3*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+2/5*m*f^
3/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a-2/15*m*f/e/x^3*b*ln(c)+12/25*b*
f^2*m*n/e^2/x+1/10*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)
/x^5*b*ln(c)+1/10*I*Pi*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/
x^5*a-1/15*I*m*f/e/x^3*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/5*I*m*f^3/e^2/(e*
f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*c*x^n)^3+1/20*Pi^2*csgn(I*d)*c
sgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)/x^5*b*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)+1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/5*I*m*f^3/e^2/
(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+1/20*P
i^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*csgn(I*c)*csgn(I*c*x^
n)^2-1/10*I*Pi*csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*b*ln(c)-1/10*I*Pi*csgn
(I*d)*csgn(I*d*(f*x^2+e)^m)^2*b/x^5*ln(x^n)-1/50*I*Pi*csgn(I*d)*csgn(I*d*(f
*x^2+e)^m)^2*b*n/x^5-1/10*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)^m)^2/
x^5*b*ln(c)-1/5*I*m*f^2/e^2/x*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-16/2
25*b*f*m*n/e/x^3-1/20*Pi^2*csgn(I*d)*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)/x^5*b*csgn(I*c)*csgn(I*c*x^n)^2-1/20*Pi^2*csgn(I*d)*csgn(I*d*(f*x^2+e)^
m)^2/x^5*b*csgn(I*c*x^n)^3-1/20*Pi^2*csgn(I*(f*x^2+e)^m)*csgn(I*d*(f*x^2+e)
^m)^2/x^5*b*csgn(I*c*x^n)^3-1/20*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*csgn(I*
c)*csgn(I*c*x^n)^2-1/20*Pi^2*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*csgn(I*x^n)*csgn
(I*c*x^n)^2+1/10*I*Pi*csgn(I*d*(f*x^2+e)^m)^3*b/x^5*ln(x^n)+1/50*I*Pi*csgn(
I*d*(f*x^2+e)^m)^3*b*n/x^5+1/10*I*ln(d)/x^5*b*Pi*csgn(I*c*x^n)^3-1/10*I*Pi*
csgn(I*d)*csgn(I*d*(f*x^2+e)^m)^2/x^5*a-1/10*I*Pi*csgn(I*(f*x^2+e)^m)*csgn(
I*d*(f*x^2+e)^m)^2/x^5*a+1/10*I*Pi*csgn(I*d*(f*x^2+e)^m)^3/x^5*b*ln(c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="maxima")
```

```
[Out] -1/25*(5*b*m*log(x^n) + (m*n + 5*m*log(c))*b + 5*a*m)*log(f*x^2 + e)/x^5 +
integrate(1/25*((5*(2*f*m + 5*f*log(d))*a + (2*f*m*n + 5*(2*f*m + 5*f*log(d
))*log(c))*b)*x^2 + 25*(b*log(c)*log(d) + a*log(d))*e + 5*((2*f*m + 5*f*log
(d))*b*x^2 + 5*b*e*log(d))*log(x^n))/(f*x^8 + x^6*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(f*x**2+e)**m)/x**6,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(f*x^2+e)^m)/x^6,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^2 + e)^m*d)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n)))/x^6, x)
```

3.100 $\int x(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=310

$$-\frac{3}{4}b^2mn^2x^2+bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx^n))^2+\frac{b^2emn^2\log(e+fx^2)}{4f}+\frac{1}{4}b^2n^2x^2\log\left(d(e+fx$$

```
[Out] -3/4*b^2*m*n^2*x^2+b*m*n*x^2*(a+b*ln(c*x^n))-1/2*m*x^2*(a+b*ln(c*x^n))^2+1/
4*b^2*e*m*n^2*ln(f*x^2+e)/f+1/4*b^2*n^2*x^2*ln(d*(f*x^2+e)^m)-1/2*b*n*x^2*(
a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m
)-1/2*b*e*m*n*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^2*ln(
1+f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(2,-f*x^2/e)/f+1/2*b*e*m*n*(a+b*ln(c*x^
n))*polylog(2,-f*x^2/e)/f-1/4*b^2*e*m*n^2*polylog(3,-f*x^2/e)/f
```

Rubi [A]
time = 0.35, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$,
Rules used = {2342, 2341, 2425, 272, 45, 2393, 2375, 2438, 2395, 2421, 6724}

$$\frac{bemn\,\mathrm{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))}{2f}-\frac{b^2emn^2\,\mathrm{PolyLog}\left(2,-\frac{fx^2}{e}\right)}{4f}-\frac{b^2emn^2\,\mathrm{PolyLog}\left(3,-\frac{fx^2}{e}\right)}{4f}-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)+\frac{1}{2}x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)-\frac{bemn\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))}{2f}+\frac{em\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^2}{2f}+bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx^n))^2+\frac{1}{4}b^2n^2x^2\log\left(d(e+fx^2)^m\right)+\frac{b^2emn^2\log(e+fx^2)}{4f}-\frac{3}{4}b^2mn^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-3*b^2*m*n^2*x^2)/4 + b*m*n*x^2*(a + b*Log[c*x^n]) - (m*x^2*(a + b*Log[c*x
^n])^2)/2 + (b^2*e*m*n^2*Log[e + f*x^2])/(4*f) + (b^2*n^2*x^2*Log[d*(e + f*
x^2)^m])/4 - (b*n*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/2 + (x^2*(a
+ b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/2 - (b*e*m*n*(a + b*Log[c*x^n])*Log
[1 + (f*x^2)/e])/(2*f) + (e*m*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)/e])/(2*f
) - (b^2*e*m*n^2*PolyLog[2, -((f*x^2)/e)])/(4*f) + (b*e*m*n*(a + b*Log[c*x^
n])*PolyLog[2, -((f*x^2)/e)])/(2*f) - (b^2*e*m*n^2*PolyLog[3, -((f*x^2)/e)]
)/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx=\frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e\right.$$

$$=\frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e\right.$$

$$=\frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e\right.$$

$$=\frac{1}{4}b^2n^2x^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{1}{2}bnx^2(a+b\log(cx^n))\log\left(d(e\right.$$

$$=-\frac{1}{2}b^2mn^2x^2+\frac{1}{2}bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx$$

$$=-\frac{3}{4}b^2mn^2x^2+bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx^n)$$

$$=-\frac{3}{4}b^2mn^2x^2+bmnx^2(a+b\log(cx^n))-\frac{1}{2}mx^2(a+b\log(cx^n)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 814, normalized size = 2.63

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^2*f*m*x^2 + 4*a*b*f*m*n*x^2 - 3*b^2*f*m*n^2*x^2 - 4*a*b*f*m*x^2*Log[c
*x^n] + 4*b^2*f*m*n*x^2*Log[c*x^n] - 2*b^2*f*m*x^2*Log[c*x^n]^2 + 4*a*b*e*m
*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1 - (I*
Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]
 + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 4*a*b*e*m
*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]*Log[1 + (I*
Sqrt[f]*x)/Sqrt[e]] - 2*b^2*e*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]]
 + 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2*a^2*e*m
*Log[e + f*x^2] - 2*a*b*e*m*n*Log[e + f*x^2] + b^2*e*m*n^2*Log[e + f*x^2] -
 4*a*b*e*m*n*Log[x]*Log[e + f*x^2] + 2*b^2*e*m*n^2*Log[x]*Log[e + f*x^2] +
2*b^2*e*m*n^2*Log[x]^2*Log[e + f*x^2] + 4*a*b*e*m*Log[c*x^n]*Log[e + f*x^2]
 - 2*b^2*e*m*n*Log[c*x^n]*Log[e + f*x^2] - 4*b^2*e*m*n*Log[x]*Log[c*x^n]*Lo
g[e + f*x^2] + 2*b^2*e*m*Log[c*x^n]^2*Log[e + f*x^2] + 2*a^2*f*x^2*Log[d*(e
 + f*x^2)^m] - 2*a*b*f*n*x^2*Log[d*(e + f*x^2)^m] + b^2*f*n^2*x^2*Log[d*(e
+ f*x^2)^m] + 4*a*b*f*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b^2*f*n*x^2*L
og[c*x^n]*Log[d*(e + f*x^2)^m] + 2*b^2*f*x^2*Log[c*x^n]^2*Log[d*(e + f*x^2)
^m] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sq
rt[e]] + 2*b*e*m*n*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sq
rt[e]] - 4*b^2*e*m*n^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 4*b^2*e*m*n^2
*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(4*f)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 12230, normalized size = 39.45

method	result	size
risch	Expression too large to display	12230

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*m*x^2*log(x^n)^2 - 2*((m*n - 2*m*log(c))*b^2 - 2*a*b*m)*x^2*log(
x^n) - (2*(m*n - 2*m*log(c))*a*b - (m*n^2 - 2*m*n*log(c) + 2*m*log(c)^2)*b^
```

```
2 - 2*a^2*m)*x^2)*log(f*x^2 + e) + integrate(-1/2*((2*(f*m - f*log(d))*a^2
- 2*(f*m*n - 2*(f*m - f*log(d))*log(c))*a*b + (f*m*n^2 - 2*f*m*n*log(c) + 2
*(f*m - f*log(d))*log(c)^2)*b^2)*x^3 - 2*(b^2*log(c)^2*log(d) + 2*a*b*log(c
)*log(d) + a^2*log(d))*x*e + 2*((f*m - f*log(d))*b^2*x^3 - b^2*x*e*log(d))*
log(x^n)^2 + 2*((2*(f*m - f*log(d))*a*b - (f*m*n - 2*(f*m - f*log(d))*log(c
))*b^2)*x^3 - 2*(b^2*log(c)*log(d) + a*b*log(d))*x*e)*log(x^n))/(f*x^2 + e)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log((f*x^2 + e)^
m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

3.101 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal. Leaf size=147

$$\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn}-\frac{m(a+b\log(cx^n))^3\log\left(1+\frac{fx^2}{e}\right)}{3bn}-\frac{1}{2}m(a+b\log(cx^n))^2\,\mathrm{Li}_2\left(-\frac{fx^2}{e}\right)$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/b/n-1/3*m*(a+b*ln(c*x^n))^3*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2/e)+1/2*b*m*n*(a+b*ln(c*
x^n))*polylog(3,-f*x^2/e)-1/4*b^2*m*n^2*polylog(4,-f*x^2/e)
```

Rubi [A]
time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$-\frac{1}{2}m\mathrm{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2+\frac{1}{2}bmn\mathrm{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{4}b^2mn^2\mathrm{PolyLog}\left(4,-\frac{fx^2}{e}\right)+\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn}-\frac{m\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/(3*b*n) - (m*(a + b*Log[c*x^n])
^3*Log[1 + (f*x^2)/e])/(3*b*n) - (m*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^
2)/e)])/2 + (b*m*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*x^2)/e)])/2 - (b^2*m*
n^2*PolyLog[4, -((f*x^2)/e)])/4
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
```

```
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{(2fm)\int\frac{x(a+b\log(cx^n))^3}{e+fx^2}\,dx}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(\right.}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(\right.}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(\right.}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{3bn} - \frac{m(a+b\log(cx^n))^3\log\left(\right.}{3bn}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 736, normalized size = 5.01

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] -(a^2*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + a*b*m*n*Log[x]^2*Log[1 - (
I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]])
/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + b^2*m*n*Log
[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^2*m*Log[x]*Log[c*x^n]^2
*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - a^2*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e
]] + a*b*m*n*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (b^2*m*n^2*Log[x]^3*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/3 - 2*a*b*m*Log[x]*Log[c*x^n]*Log[1 + (I*Sq
rt[f]*x)/Sqrt[e]] + b^2*m*n*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[
e]] - b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^2*Log[x]
*Log[d*(e + f*x^2)^m] - a*b*n*Log[x]^2*Log[d*(e + f*x^2)^m] + (b^2*n^2*Log[
x]^3*Log[d*(e + f*x^2)^m])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m]
 - b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Log[x]*Log[c*x^n]^2
*Log[d*(e + f*x^2)^m] - m*(a + b*Log[c*x^n])^2*PolyLog[2, ((-I)*Sqrt[f]*x)/
Sqrt[e]] - m*(a + b*Log[c*x^n])^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 2*a*b
*m*n*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*b^2*m*n*Log[c*x^n]*PolyLog[3,
 ((-I)*Sqrt[f]*x)/Sqrt[e]] + 2*a*b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
2*b^2*m*n*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*m*n^2*PolyLo
g[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 2*b^2*m*n^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[
e]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.25, size = 27214, normalized size = 185.13

method	result	size
risch	Expression too large to display	27214

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*m*n^2*log(x)^3 + 3*b^2*m*log(x)*log(x^n)^2 - 3*(b^2*m*n*log(c) + a
*b*m*n)*log(x)^2 - 3*(b^2*m*n*log(x)^2 - 2*(b^2*m*log(c) + a*b*m)*log(x))*l
og(x^n) + 3*(b^2*m*log(c)^2 + 2*a*b*m*log(c) + a^2*m)*log(x))*log(f*x^2 + e
) - integrate(1/3*(2*b^2*f*m*n^2*x^2*log(x)^3 - 6*(b^2*f*m*n*log(c) + a*b*f
*m*n)*x^2*log(x)^2 + 6*(b^2*f*m*log(c)^2 + 2*a*b*f*m*log(c) + a^2*f*m)*x^2*
log(x) - 3*(b^2*f*log(c)^2*log(d) + 2*a*b*f*log(c)*log(d) + a^2*f*log(d))*x
```

```
^2 + 3*(2*b^2*f*m*x^2*log(x) - b^2*f*x^2*log(d) - b^2*e*log(d))*log(x^n)^2
- 3*(b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e - 6*(b^2*f*m
*n*x^2*log(x)^2 - 2*(b^2*f*m*log(c) + a*b*f*m)*x^2*log(x) + (b^2*f*log(c)*l
og(d) + a*b*f*log(d))*x^2 + (b^2*log(c)*log(d) + a*b*log(d))*e)*log(x^n))/(
f*x^3 + x*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x, x)
```

3.102 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=276

$$\frac{b^2fmn^2\log(x)}{2e}-\frac{bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e}-\frac{fm\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{2e}-\frac{b^2fmn^2\log(e}{4e}$$

```
[Out] 1/2*b^2*f*m*n^2*ln(x)/e-1/2*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-1/2*f*m
*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/4*b^2*f*m*n^2*ln(f*x^2+e)/e-1/4*b^2*n^
2*ln(d*(f*x^2+e)^m)/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-1/2*(
a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2+1/4*b^2*f*m*n^2*polylog(2,-e/f/x^2)/
e+1/2*b*f*m*n*(a+b*ln(c*x^n))*polylog(2,-e/f/x^2)/e+1/4*b^2*f*m*n^2*polylog
(3,-e/f/x^2)/e
```

Rubi [A]
time = 0.21, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2342, 2341, 2425, 272, 36, 29, 31, 2379, 2438, 2421, 6724}

$$\frac{bfmn\,\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e}+\frac{b^2fmn^2\,\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{4e}+\frac{b^2fmn^2\,\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{4e}-\frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{2x^2}-\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{2x^2}-\frac{bfmn\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))}{2e}-\frac{fm\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))^2}{2e}-\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{4x^2}-\frac{b^2fmn^2\log(e+fx^2)}{4e}+\frac{b^2fmn^2\log(x)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (b^2*f*m*n^2*Log[x])/(2*e) - (b*f*m*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])
)/(2*e) - (f*m*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(2*e) - (b^2*f*m*n^
2*Log[e + f*x^2])/(4*e) - (b^2*n^2*Log[d*(e + f*x^2)^m])/(4*x^2) - (b*n*(a
+ b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(2*x^2) - ((a + b*Log[c*x^n])^2*Log[d
*(e + f*x^2)^m])/(2*x^2) + (b^2*f*m*n^2*PolyLog[2, -(e/(f*x^2))])/(4*e) + (
b*f*m*n*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(2*e) + (b^2*f*m*n^2*P
olyLog[3, -(e/(f*x^2))])/(4*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^3}\,dx = -\frac{b^2n^2\log(d(e+fx^2)^m)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(d(e+fx^2)}{2x^2}$$

$$= -\frac{b^2n^2\log(d(e+fx^2)^m)}{4x^2} - \frac{bn(a+b\log(cx^n))\log(d(e+fx^2)}{2x^2}$$

$$= -\frac{bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e} - \frac{fm\log\left(1+\frac{e}{fx^2}\right)(a}{2e}$$

$$= -\frac{bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e} - \frac{fm\log\left(1+\frac{e}{fx^2}\right)(a}{2e}$$

$$= \frac{b^2fmn^2\log(x)}{2e} - \frac{bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{2e} - \frac{fm\,l}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 946, normalized size = 3.43

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/12*(-12*a^2*f*m*x^2*Log[x] - 12*a*b*f*m*n*x^2*Log[x] - 6*b^2*f*m*n^2*x^2
*Log[x] + 12*a*b*f*m*n*x^2*Log[x]^2 + 6*b^2*f*m*n^2*x^2*Log[x]^2 - 4*b^2*f*
m*n^2*x^2*Log[x]^3 - 24*a*b*f*m*x^2*Log[x]*Log[c*x^n] - 12*b^2*f*m*n*x^2*Lo
g[x]*Log[c*x^n] + 12*b^2*f*m*n*x^2*Log[x]^2*Log[c*x^n] - 12*b^2*f*m*x^2*Log
[x]*Log[c*x^n]^2 + 12*a*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^2*f*m*n^2*x^
2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^2*f*m*n*x^2*Log[x]*Log[c*x
^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b*f*m*n*x^2*Log[x]*Log[1 + (I*Sqr
t[f]*x)/Sqrt[e]] + 6*b^2*f*m*n^2*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]]
```

```
- 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^2*f*m*n*
x^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*a^2*f*m*x^2*Log[e
+ f*x^2] + 6*a*b*f*m*n*x^2*Log[e + f*x^2] + 3*b^2*f*m*n^2*x^2*Log[e + f*x^2
] - 12*a*b*f*m*n*x^2*Log[x]*Log[e + f*x^2] - 6*b^2*f*m*n^2*x^2*Log[x]*Log[e
 + f*x^2] + 6*b^2*f*m*n^2*x^2*Log[x]^2*Log[e + f*x^2] + 12*a*b*f*m*x^2*Log[
c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*n*x^2*Log[c*x^n]*Log[e + f*x^2] - 12*b^2*
f*m*n*x^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 6*b^2*f*m*x^2*Log[c*x^n]^2*Log
[e + f*x^2] + 6*a^2*e*Log[d*(e + f*x^2)^m] + 6*a*b*e*n*Log[d*(e + f*x^2)^m]
 + 3*b^2*e*n^2*Log[d*(e + f*x^2)^m] + 12*a*b*e*Log[c*x^n]*Log[d*(e + f*x^2)
^m] + 6*b^2*e*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 6*b^2*e*Log[c*x^n]^2*Log[
d*(e + f*x^2)^m] + 6*b*f*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]] + 6*b*f*m*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLo
g[2, (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f*m*n^2*x^2*PolyLog[3, ((-I)*Sqrt[f]*x
)/Sqrt[e]] - 12*b^2*f*m*n^2*x^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 12568, normalized size = 45.54

method	result	size
risch	Expression too large to display	12568

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b^2*m*log(x^n)^2 + 2*(m*n + 2*m*log(c))*a*b + (m*n^2 + 2*m*n*log(c)
 + 2*m*log(c)^2)*b^2 + 2*a^2*m + 2*((m*n + 2*m*log(c))*b^2 + 2*a*b*m)*log(x
^n))*log(f*x^2 + e)/x^2 + integrate(1/2*((2*(f*m + f*log(d))*a^2 + 2*(f*m*n
 + 2*(f*m + f*log(d))*log(c))*a*b + (f*m*n^2 + 2*f*m*n*log(c) + 2*(f*m + f*
log(d))*log(c)^2)*b^2)*x^2 + 2*((f*m + f*log(d))*b^2*x^2 + b^2*e*log(d))*lo
g(x^n)^2 + 2*(b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e + 2
*((2*(f*m + f*log(d))*a*b + (f*m*n + 2*(f*m + f*log(d))*log(c))*b^2)*x^2 +
2*(b^2*log(c)*log(d) + a*b*log(d))*e)*log(x^n))/(f*x^5 + x^3*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^3, x)
```

3.103 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^5}\,dx$

Optimal. Leaf size=356

$$-\frac{7b^2fmn^2}{32ex^2}-\frac{b^2f^2mn^2\log(x)}{16e^2}-\frac{3bfmn(a+b\log(cx^n))}{8ex^2}+\frac{bf^2mn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{8e^2}-\frac{fm(a+b\log}{4ex^2}$$

```
[Out] -7/32*b^2*f*m*n^2/e/x^2-1/16*b^2*f^2*m*n^2*ln(x)/e^2-3/8*b*f*m*n*(a+b*ln(c*
x^n))/e/x^2+1/8*b*f^2*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e^2-1/4*f*m*(a+b*ln
(c*x^n))^2/e/x^2+1/4*f^2*m*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e^2+1/32*b^2*f^2
*m*n^2*ln(f*x^2+e)/e^2-1/32*b^2*n^2*ln(d*(f*x^2+e)^m)/x^4-1/8*b*n*(a+b*ln(c
*x^n))*ln(d*(f*x^2+e)^m)/x^4-1/4*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4-1/
16*b^2*f^2*m*n^2*polylog(2,-e/f/x^2)/e^2-1/4*b*f^2*m*n*(a+b*ln(c*x^n))*poly
log(2,-e/f/x^2)/e^2-1/8*b^2*f^2*m*n^2*polylog(3,-e/f/x^2)/e^2
```

Rubi [A]
time = 0.38, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2342, 2341, 2425, 272, 46, 2380, 2379, 2438, 2421, 6724}

$$-\frac{bf^2mn\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e^2}-\frac{b^2f^2mn^2\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{16e^2}-\frac{b^2f^2mn^2\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{8e^2}-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{4x^4}-\frac{bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{8x^4}+\frac{f^2m\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))^2}{4e^2}+\frac{bf^2mn\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))}{8e^2}-\frac{fm(a+b\log(cx^n))^2}{4ex^2}-\frac{3bfmn(a+b\log(cx^n))}{8ex^2}-\frac{b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{32x^4}+\frac{b^2f^2mn^2\log\left(e+fx^2\right)}{32e^2}-\frac{b^2f^2mn^2\log(x)}{16e^2}-\frac{7b^2fmn^2}{32ex^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] (-7*b^2*f*m*n^2)/(32*e*x^2) - (b^2*f^2*m*n^2*Log[x])/(16*e^2) - (3*b*f*m*n*
(a + b*Log[c*x^n]))/(8*e*x^2) + (b*f^2*m*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*
x^n]))/(8*e^2) - (f*m*(a + b*Log[c*x^n])^2)/(4*e*x^2) + (f^2*m*Log[1 + e/(f
*x^2)]*(a + b*Log[c*x^n])^2)/(4*e^2) + (b^2*f^2*m*n^2*Log[e + f*x^2])/(32*e
^2) - (b^2*n^2*Log[d*(e + f*x^2)^m])/(32*x^4) - (b*n*(a + b*Log[c*x^n])*Log
[d*(e + f*x^2)^m])/(8*x^4) - ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(4
*x^4) - (b^2*f^2*m*n^2*PolyLog[2, -(e/(f*x^2))])/(16*e^2) - (b*f^2*m*n*(a +
 b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(4*e^2) - (b^2*f^2*m*n^2*PolyLog[3
, -(e/(f*x^2))])/(8*e^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^5}\,dx = -\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4} - \frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4} - \frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4} - \frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right.}{8x^4}$$

$$= -\frac{b^2n^2\log\left(d(e+fx^2)^m\right)}{32x^4} - \frac{bn(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right.}{8x^4}$$

$$= -\frac{3b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{bfmn(a+b\log(cx^n))}{8ex^2} - \frac{f^2m(}{}$$

$$= -\frac{7b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{3bfmn(a+b\log(cx^n))}{8ex^2} - \frac{f^2m}{}$$

$$= -\frac{7b^2fmn^2}{32ex^2} - \frac{b^2f^2mn^2\log(x)}{16e^2} - \frac{3bfmn(a+b\log(cx^n))}{8ex^2} - \frac{f^2m}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 1111, normalized size = 3.12

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^5,x]
```

```
[Out] -1/96*(24*a^2*e*f*m*x^2 + 36*a*b*e*f*m*n*x^2 + 21*b^2*e*f*m*n^2*x^2 + 48*a^
2*f^2*m*x^4*Log[x] + 24*a*b*f^2*m*n*x^4*Log[x] + 6*b^2*f^2*m*n^2*x^4*Log[x]
 - 48*a*b*f^2*m*n*x^4*Log[x]^2 - 12*b^2*f^2*m*n^2*x^4*Log[x]^2 + 16*b^2*f^2
```

```
*m*n^2*x^4*Log[x]^3 + 48*a*b*e*f*m*x^2*Log[c*x^n] + 36*b^2*e*f*m*n*x^2*Log[
c*x^n] + 96*a*b*f^2*m*x^4*Log[x]*Log[c*x^n] + 24*b^2*f^2*m*n*x^4*Log[x]*Log
[c*x^n] - 48*b^2*f^2*m*n*x^4*Log[x]^2*Log[c*x^n] + 24*b^2*e*f*m*x^2*Log[c*x
^n]^2 + 48*b^2*f^2*m*x^4*Log[x]*Log[c*x^n]^2 - 48*a*b*f^2*m*n*x^4*Log[x]*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f^2*m*n^2*x^4*Log[x]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e
]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] -
48*a*b*f^2*m*n*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^2*f^2*m*n^2
*x^4*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 24*b^2*f^2*m*n^2*x^4*Log[x]^2*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 48*b^2*f^2*m*n*x^4*Log[x]*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] - 24*a^2*f^2*m*x^4*Log[e + f*x^2] - 12*a*b*f^2*m*
n*x^4*Log[e + f*x^2] - 3*b^2*f^2*m*n^2*x^4*Log[e + f*x^2] + 48*a*b*f^2*m*n*
x^4*Log[x]*Log[e + f*x^2] + 12*b^2*f^2*m*n^2*x^4*Log[x]*Log[e + f*x^2] - 24
*b^2*f^2*m*n^2*x^4*Log[x]^2*Log[e + f*x^2] - 48*a*b*f^2*m*x^4*Log[c*x^n]*Lo
g[e + f*x^2] - 12*b^2*f^2*m*n*x^4*Log[c*x^n]*Log[e + f*x^2] + 48*b^2*f^2*m*
n*x^4*Log[x]*Log[c*x^n]*Log[e + f*x^2] - 24*b^2*f^2*m*x^4*Log[c*x^n]^2*Log[
e + f*x^2] + 24*a^2*e^2*Log[d*(e + f*x^2)^m] + 12*a*b*e^2*n*Log[d*(e + f*x^
2)^m] + 3*b^2*e^2*n^2*Log[d*(e + f*x^2)^m] + 48*a*b*e^2*Log[c*x^n]*Log[d*(e
 + f*x^2)^m] + 12*b^2*e^2*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 24*b^2*e^2*Lo
g[c*x^n]^2*Log[d*(e + f*x^2)^m] - 12*b*f^2*m*n*x^4*(4*a + b*n + 4*b*Log[c*x
^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 12*b*f^2*m*n*x^4*(4*a + b*n + 4
*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 48*b^2*f^2*m*n^2*x^4*Pol
yLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 48*b^2*f^2*m*n^2*x^4*PolyLog[3, (I*Sqrt
[f]*x)/Sqrt[e]])/(e^2*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 13825, normalized size = 38.83

method	result	size
risch	Expression too large to display	13825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*b^2*m*log(x^n)^2 + 4*(m*n + 4*m*log(c))*a*b + (m*n^2 + 4*m*n*log(c
) + 8*m*log(c)^2)*b^2 + 8*a^2*m + 4*((m*n + 4*m*log(c))*b^2 + 4*a*b*m)*log(
```

```
x^n))*log(f*x^2 + e)/x^4 + integrate(1/16*((8*(f*m + 2*f*log(d))*a^2 + 4*(f
*m*n + 4*(f*m + 2*f*log(d))*log(c))*a*b + (f*m*n^2 + 4*f*m*n*log(c) + 8*(f*
m + 2*f*log(d))*log(c)^2)*b^2)*x^2 + 8*((f*m + 2*f*log(d))*b^2*x^2 + 2*b^2*
e*log(d))*log(x^n)^2 + 16*(b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*
log(d))*e + 4*((4*(f*m + 2*f*log(d))*a*b + (f*m*n + 4*(f*m + 2*f*log(d))*lo
g(c))*b^2)*x^2 + 8*(b^2*log(c)*log(d) + a*b*log(d))*e)*log(x^n))/(f*x^7 + x
^5*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**5,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^5,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^5, x)
```

3.104 $\int x^2(a+b\log(cx^n))^2 \log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=630

$$-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27f^{3/2}}-\frac{16b^2emnx\log(cx^n)}{9f}+\frac{8}{27}bmnx^3(a+$$

```
[Out] -16/9*a*b*e*m*n*x/f+52/27*b^2*e*m*n^2*x/f-4/27*b^2*m*n^2*x^3-4/27*b^2*e^(3/
2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))/f^(3/2)-16/9*b^2*e*m*n*x*ln(c*x^n)/f+8/2
7*b*m*n*x^3*(a+b*ln(c*x^n))+4/9*b*e^(3/2)*m*n*arctan(x*f^(1/2)/e^(1/2))*(a+
b*ln(c*x^n))/f^(3/2)+2/3*e*m*x*(a+b*ln(c*x^n))^2/f-2/9*m*x^3*(a+b*ln(c*x^n)
)^2+2/27*b^2*n^2*x^3*ln(d*(f*x^2+e)^m)-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*(f*
x^2+e)^m)+1/3*x^3*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-1/3*(-e)^(3/2)*m*(a+b
*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+1/3*(-e)^(3/2)*m*(a+b*ln(c
*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/3*b*(-e)^(3/2)*m*n*(a+b*ln(c*
x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b*(-e)^(3/2)*m*n*(a+b*ln
(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/9*I*b^2*e^(3/2)*m*n^2*po
lylog(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)+2/9*I*b^2*e^(3/2)*m*n^2*polylog(2,I*x
*f^(1/2)/e^(1/2))/f^(3/2)-2/3*b^2*(-e)^(3/2)*m*n^2*polylog(3,-x*f^(1/2)/(-e
)^(1/2))/f^(3/2)+2/3*b^2*(-e)^(3/2)*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))/f
^(3/2)
```

Rubi [A]
time = 0.69, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {2342, 2341, 2425, 308, 211, 2393, 2332, 2361, 12, 4940, 2438, 2395, 2333, 2367, 2354, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-16*a*b*e*m*n*x)/(9*f) + (52*b^2*e*m*n^2*x)/(27*f) - (4*b^2*m*n^2*x^3)/27
- (4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2)) - (16*b^2*
e*m*n*x*Log[c*x^n])/(9*f) + (8*b*m*n*x^3*(a + b*Log[c*x^n]))/27 + (4*b*e^(3
/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^(3/2)) + (2*e*
m*x*(a + b*Log[c*x^n])^2)/(3*f) - (2*m*x^3*(a + b*Log[c*x^n])^2)/9 - ((-e)^
(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) + (
(-e)^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)
) + (2*b^2*n^2*x^3*Log[d*(e + f*x^2)^m])/27 - (2*b*n*x^3*(a + b*Log[c*x^n])
*Log[d*(e + f*x^2)^m])/9 + (x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/
3 + (2*b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e
])])/(3*f^(3/2)) - (2*b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[
```

```
f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (((2*I)/9)*b^2*e^(3/2)*m*n^2*PolyLog[2, ((-I
)*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) + (((2*I)/9)*b^2*e^(3/2)*m*n^2*PolyLog[2, (I
*Sqrt[f]*x)/Sqrt[e]])/f^(3/2) - (2*b^2*(-e)^(3/2)*m*n^2*PolyLog[3, -((Sqrt[
f]*x)/Sqrt[-e])])/(3*f^(3/2)) + (2*b^2*(-e)^(3/2)*m*n^2*PolyLog[3, (Sqrt[f]
*x)/Sqrt[-e]])/(3*f^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
```

```
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)\,dx = \frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d\right.$$

$$=\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d\right.$$

$$=\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)-\frac{2}{9}bnx^3(a+b\log(cx^n))\log\left(d\right.$$

$$=\frac{4b^2emn^2x}{27f}-\frac{4}{81}b^2mn^2x^3+\frac{2}{27}b^2n^2x^3\log\left(d(e+fx^2)^m\right)-$$

$$=-\frac{4abemnx}{9f}+\frac{4b^2emn^2x}{27f}-\frac{8}{81}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan^{-1}}{27f^{3/2}}$$

$$=-\frac{16abemnx}{9f}+\frac{16b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan}{27f}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan}{27f}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan}{27f}$$

$$=-\frac{16abemnx}{9f}+\frac{52b^2emn^2x}{27f}-\frac{4}{27}b^2mn^2x^3-\frac{4b^2e^{3/2}mn^2\tan}{27f}$$

Mathematica [A]
time = 0.26, size = 1128, normalized size = 1.79

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (18*a^2*e*Sqrt[f]*m*x - 48*a*b*e*Sqrt[f]*m*n*x + 52*b^2*e*Sqrt[f]*m*n^2*x -
 6*a^2*f^(3/2)*m*x^3 + 8*a*b*f^(3/2)*m*n*x^3 - 4*b^2*f^(3/2)*m*n^2*x^3 - 18
*a^2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b*e^(3/2)*m*n*ArcTan[(Sqr
```

```
t[f]*x)/Sqrt[e]] - 4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 36*a*b
*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^2*e^(3/2)*m*n^2*ArcT
an[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sq
rt[e]]*Log[x]^2 + 36*a*b*e*Sqrt[f]*m*x*Log[c*x^n] - 48*b^2*e*Sqrt[f]*m*n*x*
Log[c*x^n] - 12*a*b*f^(3/2)*m*x^3*Log[c*x^n] + 8*b^2*f^(3/2)*m*n*x^3*Log[c*
x^n] - 36*a*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^2*e^(
3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*e^(3/2)*m*n*ArcTan
[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 18*b^2*e*Sqrt[f]*m*x*Log[c*x^n]^2
 - 6*b^2*f^(3/2)*m*x^3*Log[c*x^n]^2 - 18*b^2*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/S
qrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*e^(3/2)*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/
Sqrt[e]] + (6*I)*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
(9*I)*b^2*e^(3/2)*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^
2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b
*e^(3/2)*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^2*e^(3/2)*m*n^
2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^2*e^(3/2)*m*n^2*Log[x]^2*
Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 9*a^2*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 6*
a*b*f^(3/2)*n*x^3*Log[d*(e + f*x^2)^m] + 2*b^2*f^(3/2)*n^2*x^3*Log[d*(e + f
*x^2)^m] + 18*a*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^2*f^(3/
2)*n*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 9*b^2*f^(3/2)*x^3*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] + (6*I)*b*e^(3/2)*m*n*(3*a - b*n + 3*b*Log[c*x^n])*Poly
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b*e^(3/2)*m*n*(-3*a + b*n - 3*b*Lo
g[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^2*e^(3/2)*m*n^2*Poly
Log[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^2*e^(3/2)*m*n^2*PolyLog[3, (I*S
qrt[f]*x)/Sqrt[e]])/(27*f^(3/2))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}\left(a +b \ln \left(c \,x^{n}\right)\right)^{2} \ln \left(d \left(f \,x^{2}+e \right)^{m}\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^2*m*x^3*log(x^n)^2 - 6*((m*n - 3*m*log(c))*b^2 - 3*a*b*m)*x^3*log
(x^n) - (6*(m*n - 3*m*log(c))*a*b - (2*m*n^2 - 6*m*n*log(c) + 9*m*log(c)^2)
```

```
*b^2 - 9*a^2*m)*x^3)*log(f*x^2 + e) + integrate(-1/27*((9*(2*f*m - 3*f*log(
d))*a^2 - 6*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*a*b + (4*f*m*n^2 - 12
*f*m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log(c)^2)*b^2)*x^4 - 27*(b^2*log(c)^
2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*x^2*e + 9*((2*f*m - 3*f*log(d)
)*b^2*x^4 - 3*b^2*x^2*e*log(d))*log(x^n)^2 + 6*((3*(2*f*m - 3*f*log(d))*a*b
 - (2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*b^2)*x^4 - 9*(b^2*log(c)*log(d
) + a*b*log(d))*x^2*e)*log(x^n))/(f*x^2 + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log((f*x^2
 + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

3.105 $\int (a+b\log(cx^n))^2 \log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=546

$$4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+8b^2mnx\log(cx^n)-\frac{4b^2\sqrt{e}\,mn\tan^{-}}{}$$

```
[Out] 4*a*b*m*n*x-8*b^2*m*n^2*x+4*b*m*n*(-b*n+a)*x+8*b^2*m*n*x*ln(c*x^n)-2*m*x*(a
+b*ln(c*x^n))^2-2*a*b*n*x*ln(d*(f*x^2+e)^m)+2*b^2*n^2*x*ln(d*(f*x^2+e)^m)-2
*b^2*n*x*ln(c*x^n)*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-
m*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+m*(a+b*ln
(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+2*b*m*n*(a+b*ln(c*
x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-2*b*m*n*(a+b*ln(c
*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-2*b^2*m*n^2*polyl
og(3,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+2*b^2*m*n^2*polylog(3,x*f^(1
/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-4*b*m*n*(-b*n+a)*arctan(x*f^(1/2)/e^(1/2
))*e^(1/2)/f^(1/2)-4*b^2*m*n*arctan(x*f^(1/2)/e^(1/2))*ln(c*x^n)*e^(1/2)/f^
(1/2)-2*I*b^2*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+2*I*b^2*
m*n^2*polylog(2,-I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {2333, 2332, 2418, 6, 327, 211, 2393, 2361, 12, 4940, 2438, 2395, 2367, 2354, 2421, 6724}

$$\frac{2b\sqrt{-e}\,mn\text{PolyLog}\left(2,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{\sqrt{f}}-\frac{2b\sqrt{-e}\,mn\text{PolyLog}\left(2,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{\sqrt{f}}+\frac{2ib^2\sqrt{e}\,mn^2\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-\frac{2ib^2\sqrt{e}\,mn^2\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-\frac{2b^2\sqrt{-e}\,mn^2\text{PolyLog}\left(3,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{\sqrt{f}}+\frac{2b^2\sqrt{-e}\,mn^2\text{PolyLog}\left(3,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{\sqrt{f}}-\frac{4b\sqrt{e}\,mn(a-bn)\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}+x(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)-\frac{\sqrt{-e}\,m\log\left(1-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))^2}{\sqrt{f}}+\frac{\sqrt{-e}\,m\log\left(\frac{\sqrt{f}\,x}{\sqrt{-e}}+1\right)(a+b\log(cx^n))^2}{\sqrt{f}}-2mx(a+b\log(cx^n))^2-2abnx\log\left(d\left(e+fx^2\right)^m\right)+4abmnx+4bmnx(a-bn)-\frac{4b^2\sqrt{e}\,mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\log(cx^n)}{\sqrt{f}}-2b^2nx\log(cx^n)\log\left(d\left(e+fx^2\right)^m\right)+8b^2mnx\log(cx^n)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-8b^2mn^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] 4*a*b*m*n*x - 8*b^2*m*n^2*x + 4*b*m*n*(a - b*n)*x - (4*b*Sqrt[e]*m*n*(a - b
*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + 8*b^2*m*n*x*Log[c*x^n] - (4*b^2*
Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n])/Sqrt[f] - 2*m*x*(a + b*
Log[c*x^n])^2 - (Sqrt[-e]*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-
e]])/Sqrt[f] + (Sqrt[-e]*m*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e
]])/Sqrt[f] - 2*a*b*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*n^2*x*Log[d*(e + f*x^2
)^m] - 2*b^2*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^2*L
og[d*(e + f*x^2)^m] + (2*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])*PolyLog[2, -((Sq
rt[f]*x)/Sqrt[-e])])/Sqrt[f] - (2*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])*PolyLog
[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] + ((2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - ((2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[2, (I*S
qrt[f]*x)/Sqrt[e]])/Sqrt[f] - (2*b^2*Sqrt[-e]*m*n^2*PolyLog[3, -((Sqrt[f]*x
```

```
)/Sqrt[-e])])/Sqrt[f] + (2*b^2*Sqrt[-e]*m*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt[-
e]])/Sqrt[f]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
```

```
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)\,dx = -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= -2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+fx^2\right)^m\right)-2b^2nx$$

$$= 4bmn(a-bn)x-2abnx\log\left(d\left(e+fx^2\right)^m\right)+2b^2n^2x\log\left(d\left(e+\right.\right.$$

$$= 4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-2abnx\,\mathrm{l}$$

$$= -4b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan^{-1}\left(\frac{\sqrt{f}}{\sqrt{e}}\right.}{\sqrt{f}}$$

$$= 4abmnx-4b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan}{\sqrt{f}}$$

$$= 4abmnx-8b^2mn^2x+4bmn(a-bn)x-\frac{4b\sqrt{e}\,mn(a-bn)\tan}{\sqrt{f}}$$

Mathematica [A]
time = 0.20, size = 993, normalized size = 1.82

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^2*Sqrt[f]*m*x + 8*a*b*Sqrt[f]*m*n*x - 12*b^2*Sqrt[f]*m*n^2*x + 2*a^2*
Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x
)/Sqrt[e]] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*a*b*Sqrt[e
]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 4*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt
[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Lo
g[x]^2 - 4*a*b*Sqrt[f]*m*x*Log[c*x^n] + 8*b^2*Sqrt[f]*m*n*x*Log[c*x^n] + 4*
a*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]*m*n*Ar
cTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*b^2*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)
/Sqrt[e]]*Log[x]*Log[c*x^n] - 2*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 + 2*b^2*Sqrt[e
]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[e]*m*n*Log[x]
*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 - (I
*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[e]*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]] + (2*I)*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqr
t[e]] - (2*I)*a*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)
*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + I*b^2*Sqrt[e]*m*
n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n*Log[x]*
Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^2*Sqrt[f]*x*Log[d*(e + f*x^2)
^m] - 2*a*b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] + 2*b^2*Sqrt[f]*n^2*x*Log[d*(e
 + f*x^2)^m] + 2*a*b*Sqrt[f]*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 2*b^2*Sqrt
[f]*n*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^2*Sqrt[f]*x*Log[c*x^n]^2*Log[d*
(e + f*x^2)^m] - (2*I)*b*Sqrt[e]*m*n*(a - b*n + b*Log[c*x^n])*PolyLog[2, ((
-I)*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b*Sqrt[e]*m*n*(a - b*n + b*Log[c*x^n])*Poly
Log[2, (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[3, ((-I)*Sq
rt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[e]*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e
]])/Sqrt[f]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\,x^{n}\right)\right)^{2} \ln \left(d\left(f\,x^{2}+e\right)^{m}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] (b^2*m*x*log(x^n)^2 - 2*((m*n - m*log(c))*b^2 - a*b*m)*x*log(x^n) - (2*(m*n
 - m*log(c))*a*b - (2*m*n^2 - 2*m*n*log(c) + m*log(c)^2)*b^2 - a^2*m)*x)*lo
```

```
g(f*x^2 + e) + integrate(-((((2*f*m - f*log(d))*a^2 - 2*(2*f*m*n - (2*f*m -
f*log(d))*log(c))*a*b + (4*f*m*n^2 - 4*f*m*n*log(c) + (2*f*m - f*log(d))*lo
g(c)^2)*b^2)*x^2 + ((2*f*m - f*log(d))*b^2*x^2 - b^2*e*log(d))*log(x^n)^2 -
 (b^2*log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e + 2*(((2*f*m -
f*log(d))*a*b - (2*f*m*n - (2*f*m - f*log(d))*log(c))*b^2)*x^2 - (b^2*log(c
)*log(d) + a*b*log(d))*e)*log(x^n))/(f*x^2 + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2,x)
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2, x)
```

3.106 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=478

$$\frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}+\frac{\sqrt{f}\,m(a+b\log(cx^n))^2\log\left(1\right.}{\sqrt{-e}}$$

```
[Out] -2*b^2*n^2*ln(d*(f*x^2+e)^m)/x-2*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x-(a
+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x+m*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)
^(1/2))*f^(1/2)/(-e)^(1/2)-m*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*f
^(1/2)/(-e)^(1/2)-2*b*m*n*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*
f^(1/2)/(-e)^(1/2)+2*b*m*n*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*
f^(1/2)/(-e)^(1/2)+2*b^2*m*n^2*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e
)^(1/2)-2*b^2*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+4*b^
2*m*n^2*arctan(x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+4*b*m*n*arctan(x*f^(1/2)/
e^(1/2))*(a+b*ln(c*x^n))*f^(1/2)/e^(1/2)-2*I*b^2*m*n^2*polylog(2,-I*x*f^(1/
2)/e^(1/2))*f^(1/2)/e^(1/2)+2*I*b^2*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))*f^
(1/2)/e^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2342, 2341, 2425, 211, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724}

$$-\frac{2b\sqrt{f}\,mn\text{PolyLog}\left(2,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{\sqrt{-e}}+\frac{2b\sqrt{f}\,mn\text{PolyLog}\left(2,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{\sqrt{-e}}-\frac{2ib^2\sqrt{f}\,mn^2\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{2ib^2\sqrt{f}\,mn^2\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{2b^2\sqrt{f}\,mn^2\text{PolyLog}\left(3,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{\sqrt{-e}}-\frac{2b^2\sqrt{f}\,mn^2\text{PolyLog}\left(3,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{\sqrt{-e}}+\frac{4b\sqrt{f}\,mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}-\frac{2bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{x}-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x}+\frac{\sqrt{f}\,m\log\left(1-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))^2}{\sqrt{-e}}-\frac{\sqrt{f}\,m\log\left(\frac{\sqrt{f}\,x}{\sqrt{-e}}+1\right)(a+b\log(cx^n))^2}{\sqrt{-e}}+\frac{4b^2\sqrt{f}\,mn^2\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (4*b^2*Sqrt[f]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (4*b*Sqrt[f]*m*
n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] + (Sqrt[f]*m*(a +
 b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (Sqrt[f]*m*(a +
b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (2*b^2*n^2*Log[d*
(e + f*x^2)^m])/x - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/x - ((a
 + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x - (2*b*Sqrt[f]*m*n*(a + b*Log[c*
x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (2*b*Sqrt[f]*m*n*(a +
 b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - ((2*I)*b^2*Sqrt
[f]*m*n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + ((2*I)*b^2*Sqrt[f
]*m*n^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (2*b^2*Sqrt[f]*m*n^2*P
olyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] - (2*b^2*Sqrt[f]*m*n^2*PolyLog
[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^2)^m)}{x^2}\,dx = -\frac{2b^2n^2\log(d(e+fx^2)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx^2}{x}$$

$$= -\frac{2b^2n^2\log(d(e+fx^2)^m)}{x} - \frac{2bn(a+b\log(cx^n))\log(d(e+fx^2}{x}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{e}}$$

$$= \frac{4b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{4b\sqrt{f}\,mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{\sqrt{e}}$$

Mathematica [A]
time = 0.19, size = 917, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 4*a*b*Sqrt[f]*m*n*x*ArcTan
[(Sqrt[f]*x)/Sqrt[e]] + 4*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] -
 4*a*b*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 4*b^2*Sqrt[f]*m*n
^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 2*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqr
t[f]*x)/Sqrt[e]]*Log[x]^2 + 4*a*b*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*L
og[c*x^n] + 4*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 4*
b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 2*b^2*Sqr
t[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + (2*I)*a*b*Sqrt[f]*m*n*x
```

```
*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n^2*x*Log[x]*L
og[1 - (I*Sqrt[f]*x)/Sqrt[e]] - I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 - (I*S
qrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 - (I*S
qrt[f]*x)/Sqrt[e]] - (2*I)*a*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] - (2*I)*b^2*Sqrt[f]*m*n^2*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] +
 I*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*
Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - a^2*Sqrt[e
]*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*n*Log[d*(e + f*x^2)^m] - 2*b^2*Sqrt[
e]*n^2*Log[d*(e + f*x^2)^m] - 2*a*b*Sqrt[e]*Log[c*x^n]*Log[d*(e + f*x^2)^m]
 - 2*b^2*Sqrt[e]*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] - b^2*Sqrt[e]*Log[c*x^n]
^2*Log[d*(e + f*x^2)^m] - (2*I)*b*Sqrt[f]*m*n*x*(a + b*n + b*Log[c*x^n])*Po
lyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b*Sqrt[f]*m*n*x*(a + b*n + b*Log
[c*x^n])*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + (2*I)*b^2*Sqrt[f]*m*n^2*x*Poly
Log[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (2*I)*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, (I*
Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*m*log(x^n)^2 + 2*(m*n + m*log(c))*a*b + (2*m*n^2 + 2*m*n*log(c) + m*l
og(c)^2)*b^2 + a^2*m + 2*((m*n + m*log(c))*b^2 + a*b*m)*log(x^n))*log(f*x^2
 + e)/x + integrate((((2*f*m + f*log(d))*a^2 + 2*(2*f*m*n + (2*f*m + f*log(
d))*log(c))*a*b + (4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m + f*log(d))*log(c)^2
)*b^2)*x^2 + ((2*f*m + f*log(d))*b^2*x^2 + b^2*e*log(d))*log(x^n)^2 + (b^2*
log(c)^2*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e + 2*(((2*f*m + f*log(
d))*a*b + (2*f*m*n + (2*f*m + f*log(d))*log(c))*b^2)*x^2 + (b^2*log(c)*log(
d) + a*b*log(d))*e)*log(x^n))/(f*x^4 + x^2*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(f\,x^2+e\right)^m\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^2, x)
```

3.107 $\int \frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=571

$$-\frac{52b^2fmn^2}{27ex}-\frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}}-\frac{16bfmn(a+b\log(cx^n))}{9ex}-\frac{4bf^{3/2}mn\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log$$

```
[Out] -52/27*b^2*f*m*n^2/e/x-4/27*b^2*f^(3/2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))/e^(
3/2)-16/9*b*f*m*n*(a+b*ln(c*x^n))/e/x-4/9*b*f^(3/2)*m*n*arctan(x*f^(1/2)/e^
(1/2))*(a+b*ln(c*x^n))/e^(3/2)-2/3*f*m*(a+b*ln(c*x^n))^2/e/x-2/27*b^2*n^2*l
n(d*(f*x^2+e)^m)/x^3-2/9*b*n*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3-1/3*(a+b
*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^3+1/3*f^(3/2)*m*(a+b*ln(c*x^n))^2*ln(1-x*
f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-1/3*f^(3/2)*m*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/
2)/(-e)^(1/2))/(-e)^(3/2)-2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,-x*f^
(1/2)/(-e)^(1/2))/(-e)^(3/2)+2/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))*polylog(2,x*
f^(1/2)/(-e)^(1/2))/(-e)^(3/2)+2/9*I*b^2*f^(3/2)*m*n^2*polylog(2,-I*x*f^(1/
2)/e^(1/2))/e^(3/2)-2/9*I*b^2*f^(3/2)*m*n^2*polylog(2,I*x*f^(1/2)/e^(1/2))/
e^(3/2)+2/3*b^2*f^(3/2)*m*n^2*polylog(3,-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2
/3*b^2*f^(3/2)*m*n^2*polylog(3,x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)
```

Rubi [A]
time = 0.59, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2342, 2341, 2425, 331, 211, 2380, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724}

$$-\frac{2bf^{3/2}mn\text{PolyLog}\left(2,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{3(-e)^{3/2}}+\frac{2bf^{3/2}mn\text{PolyLog}\left(2,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))}{3(-e)^{3/2}}+\frac{2ib^2f^{3/2}mn^2\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}}-\frac{2ib^2f^{3/2}mn^2\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{9e^{3/2}}+\frac{2b^2f^{3/2}mn^2\text{PolyLog}\left(3,-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{3(-e)^{3/2}}-\frac{2b^2f^{3/2}mn^2\text{PolyLog}\left(3,\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)}{3(-e)^{3/2}}-\frac{4bf^{3/2}mn\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{9e^{3/2}}-\frac{2bn(a+b\log(cx^n))\log\left(d\left(e+fx^2\right)^m\right)}{9x^3}-\frac{(a+b\log(cx^n))^2\log\left(d\left(e+fx^2\right)^m\right)}{3x^3}+\frac{f^{3/2}m\log\left(1-\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)(a+b\log(cx^n))^2}{3(-e)^{3/2}}-\frac{f^{3/2}m\log\left(\frac{\sqrt{f}\,x}{\sqrt{-e}}+1\right)(a+b\log(cx^n))^2}{3(-e)^{3/2}}-\frac{16bfmn(a+b\log(cx^n))}{9ex}-\frac{2fm(a+b\log(cx^n))^2}{3ex}-\frac{4b^2f^{3/2}mn^2\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}}-\frac{2b^2n^2\log\left(d\left(e+fx^2\right)^m\right)}{27x^3}-\frac{52b^2fmn^2}{27ex}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-52*b^2*f*m*n^2)/(27*e*x) - (4*b^2*f^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(27*e^(3/2)) - (16*b*f*m*n*(a + b*Log[c*x^n]))/(9*e*x) - (4*b*f^(3/2)*m
*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) - (2*f*m*(a
+ b*Log[c*x^n])^2)/(3*e*x) + (f^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[
f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (f^(3/2)*m*(a + b*Log[c*x^n])^2*Log[1 + (
Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (2*b^2*n^2*Log[d*(e + f*x^2)^m])/(27
*x^3) - (2*b*n*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(9*x^3) - ((a + b*L
og[c*x^n])^2*Log[d*(e + f*x^2)^m])/(3*x^3) - (2*b*f^(3/2)*m*n*(a + b*Log[c*
x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) + (2*b*f^(3/2)*m*
n*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) + (((
2*I)/9)*b^2*f^(3/2)*m*n^2*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) - (
((2*I)/9)*b^2*f^(3/2)*m*n^2*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(3/2) + (2
```

```
*b^2*f^(3/2)*m*n^2*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (2
*b^2*f^(3/2)*m*n^2*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{x^4}\,dx = -\frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2\right.}{9x^3}$$

$$= -\frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{27x^3} - \frac{2bn(a+b\log(cx^n))\log\left(d(e+fx^2\right.}{9x^3}$$

$$= -\frac{4b^2fmn^2}{27ex} - \frac{2b^2n^2\log\left(d(e+fx^2)^m\right)}{27x^3} - \frac{2bn(a+b\log(cx^n))\,\mathrm{lo}}{9x^3}$$

$$= -\frac{4b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{2b^2n^2\log\left(d(e+f\right.}{27x^3}$$

$$= -\frac{16b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{4bfmn(a+b\log}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn(a+b\log}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn(a+b\log}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn(a+b\log}{9ex}$$

$$= -\frac{52b^2fmn^2}{27ex} - \frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16bfmn(a+b\log}{9ex}$$

Mathematica [A]
time = 0.25, size = 1083, normalized size = 1.90

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-18*a^2*Sqrt[e]*f*m*x^2 - 48*a*b*Sqrt[e]*f*m*n*x^2 - 52*b^2*Sqrt[e]*f*m*n^
2*x^2 - 18*a^2*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 12*a*b*f^(3/2)*m
```

```
*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f
]*x)/Sqrt[e]] + 36*a*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] +
 12*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 18*b^2*f^(3/
2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 36*a*b*Sqrt[e]*f*m*x^2*
Log[c*x^n] - 48*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 36*a*b*f^(3/2)*m*x^3*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f
]*x)/Sqrt[e]]*Log[c*x^n] + 36*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e
]]*Log[x]*Log[c*x^n] - 18*b^2*Sqrt[e]*f*m*x^2*Log[c*x^n]^2 - 18*b^2*f^(3/2)
*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - (18*I)*a*b*f^(3/2)*m*n*x^
3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x
]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]^2*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^n]*
Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b*f^(3/2)*m*n*x^3*Log[x]*Log[1 +
(I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]*Log[1 + (I*Sqrt
[f]*x)/Sqrt[e]] - (9*I)*b^2*f^(3/2)*m*n^2*x^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x
)/Sqrt[e]] + (18*I)*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f
]*x)/Sqrt[e]] - 9*a^2*e^(3/2)*Log[d*(e + f*x^2)^m] - 6*a*b*e^(3/2)*n*Log[d*
(e + f*x^2)^m] - 2*b^2*e^(3/2)*n^2*Log[d*(e + f*x^2)^m] - 18*a*b*e^(3/2)*Lo
g[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^2*e^(3/2)*n*Log[c*x^n]*Log[d*(e + f*x^2
)^m] - 9*b^2*e^(3/2)*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + (6*I)*b*f^(3/2)*m*
n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (
6*I)*b*f^(3/2)*m*n*x^3*(3*a + b*n + 3*b*Log[c*x^n])*PolyLog[2, (I*Sqrt[f]*x
)/Sqrt[e]] - (18*I)*b^2*f^(3/2)*m*n^2*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[
e]] + (18*I)*b^2*f^(3/2)*m*n^2*x^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]])/(27*e
^(3/2)*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4,x)
[Out] int((a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
[Out] -1/27*(9*b^2*m*log(x^n)^2 + 6*(m*n + 3*m*log(c))*a*b + (2*m*n^2 + 6*m*n*log
(c) + 9*m*log(c)^2)*b^2 + 9*a^2*m + 6*((m*n + 3*m*log(c))*b^2 + 3*a*b*m)*lo
```

```
g(x^n))*log(f*x^2 + e)/x^3 + integrate(1/27*((9*(2*f*m + 3*f*log(d))*a^2 +
6*(2*f*m*n + 3*(2*f*m + 3*f*log(d))*log(c))*a*b + (4*f*m*n^2 + 12*f*m*n*log
(c) + 9*(2*f*m + 3*f*log(d))*log(c)^2)*b^2)*x^2 + 9*((2*f*m + 3*f*log(d))*b
^2*x^2 + 3*b^2*e*log(d))*log(x^n)^2 + 27*(b^2*log(c)^2*log(d) + 2*a*b*log(c
)*log(d) + a^2*log(d))*e + 6*((3*(2*f*m + 3*f*log(d))*a*b + (2*f*m*n + 3*(2
*f*m + 3*f*log(d))*log(c))*b^2)*x^2 + 9*(b^2*log(c)*log(d) + a*b*log(d))*e)
*log(x^n))/(f*x^6 + x^4*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log((f*x^2 + e)^m*d)/x
^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(f*x**2+e)**m)/x**4,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^2)/x^4, x)
```

3.108 $\int x(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=514

$$\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{2}bmnx^2(a+b\log(cx^n))^2-\frac{1}{2}mx^2(a+b\log(cx^n))^3-\frac{3b^3emn^3\log(e}{8f}$$

```
[Out] 3/2*b^3*m*n^3*x^2-9/4*b^2*m*n^2*x^2*(a+b*ln(c*x^n))+3/2*b*m*n*x^2*(a+b*ln(c
*x^n))^2-1/2*m*x^2*(a+b*ln(c*x^n))^3-3/8*b^3*e*m*n^3*ln(f*x^2+e)/f-3/8*b^3*
n^3*x^2*ln(d*(f*x^2+e)^m)+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)
-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)+1/2*x^2*(a+b*ln(c*x^n))^3*
ln(d*(f*x^2+e)^m)+3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*ln(1+f*x^2/e)/f-3/4*b*e*m
*n*(a+b*ln(c*x^n))^2*ln(1+f*x^2/e)/f+1/2*e*m*(a+b*ln(c*x^n))^3*ln(1+f*x^2/e
)/f+3/8*b^3*e*m*n^3*polylog(2,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*p
olylog(2,-f*x^2/e)/f+3/4*b*e*m*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^2/e)/f+3/
8*b^3*e*m*n^3*polylog(3,-f*x^2/e)/f-3/4*b^2*e*m*n^2*(a+b*ln(c*x^n))*polylog
(3,-f*x^2/e)/f+3/8*b^3*e*m*n^3*polylog(4,-f*x^2/e)/f
```

Rubi [A]
time = 0.62, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.462$, Rules used = {2342, 2341, 2425, 272, 45, 2393, 2375, 2438, 2395, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (3*b^3*m*n^3*x^2)/2 - (9*b^2*m*n^2*x^2*(a + b*Log[c*x^n]))/4 + (3*b*m*n*x^2
*(a + b*Log[c*x^n])^2)/2 - (m*x^2*(a + b*Log[c*x^n])^3)/2 - (3*b^3*e*m*n^3*
Log[e + f*x^2])/(8*f) - (3*b^3*n^3*x^2*Log[d*(e + f*x^2)^m])/8 + (3*b^2*n^2
*x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/4 - (3*b*n*x^2*(a + b*Log[c*x
^n])^2*Log[d*(e + f*x^2)^m])/4 + (x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2
)^m])/2 + (3*b^2*e*m*n^2*(a + b*Log[c*x^n])*Log[1 + (f*x^2)/e])/(4*f) - (3*
b*e*m*n*(a + b*Log[c*x^n])^2*Log[1 + (f*x^2)/e])/(4*f) + (e*m*(a + b*Log[c*
x^n])^3*Log[1 + (f*x^2)/e])/(2*f) + (3*b^3*e*m*n^3*PolyLog[2, -((f*x^2)/e)]
)/(8*f) - (3*b^2*e*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^2)/e)])/(4*f)
 + (3*b*e*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^2)/e)])/(4*f) + (3*b^3
*e*m*n^3*PolyLog[3, -((f*x^2)/e)])/(8*f) - (3*b^2*e*m*n^2*(a + b*Log[c*x^n]
)*PolyLog[3, -((f*x^2)/e)])/(4*f) + (3*b^3*e*m*n^3*PolyLog[4, -((f*x^2)/e)]
)/(8*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d$$

$$=-\frac{3}{8}b^3n^3x^2\log\left(d(e+fx^2)^m\right)+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))\log(d$$

$$=\frac{3}{4}b^3mn^3x^2-\frac{3}{4}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{4}bmnx^2(a+b\log($$

$$=\frac{9}{8}b^3mn^3x^2-\frac{3}{2}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{2}bmnx^2(a+b\log($$

$$=\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{2}bmnx^2(a+b\log($$

$$=\frac{3}{2}b^3mn^3x^2-\frac{9}{4}b^2mn^2x^2(a+b\log(cx^n))+\frac{3}{2}bmnx^2(a+b\log($$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 1911, normalized size = 3.72

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-4*a^3*f*m*x^2 + 12*a^2*b*f*m*n*x^2 - 18*a*b^2*f*m*n^2*x^2 + 12*b^3*f*m*n^
3*x^2 - 12*a^2*b*f*m*x^2*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[c*x^n] - 18*b^
3*f*m*n^2*x^2*Log[c*x^n] - 12*a*b^2*f*m*x^2*Log[c*x^n]^2 + 12*b^3*f*m*n*x^2
*Log[c*x^n]^2 - 4*b^3*f*m*x^2*Log[c*x^n]^3 + 12*a^2*b*e*m*n*Log[x]*Log[1 -
(I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n
^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]^2*Log[1 -
 (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt
[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12
*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*e*m*
```

```
n^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n*Log[x
]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a^2*b*e*m*n*Log[x]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 6*b^3*e*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*e*
m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*e*m*n^3*Log[x]^2*Log[
1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*e*m*n^3*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/S
qrt[e]] + 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] -
 12*b^3*e*m*n^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*e
*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n*Lo
g[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*a^3*e*m*Log[e + f*x^2]
 - 6*a^2*b*e*m*n*Log[e + f*x^2] + 6*a*b^2*e*m*n^2*Log[e + f*x^2] - 3*b^3*e*
m*n^3*Log[e + f*x^2] - 12*a^2*b*e*m*n*Log[x]*Log[e + f*x^2] + 12*a*b^2*e*m*
n^2*Log[x]*Log[e + f*x^2] - 6*b^3*e*m*n^3*Log[x]*Log[e + f*x^2] + 12*a*b^2*
e*m*n^2*Log[x]^2*Log[e + f*x^2] - 6*b^3*e*m*n^3*Log[x]^2*Log[e + f*x^2] - 4
*b^3*e*m*n^3*Log[x]^3*Log[e + f*x^2] + 12*a^2*b*e*m*Log[c*x^n]*Log[e + f*x^
2] - 12*a*b^2*e*m*n*Log[c*x^n]*Log[e + f*x^2] + 6*b^3*e*m*n^2*Log[c*x^n]*Lo
g[e + f*x^2] - 24*a*b^2*e*m*n*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*b^3*e*m
*n^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*b^3*e*m*n^2*Log[x]^2*Log[c*x^n]*
Log[e + f*x^2] + 12*a*b^2*e*m*Log[c*x^n]^2*Log[e + f*x^2] - 6*b^3*e*m*n*Log
[c*x^n]^2*Log[e + f*x^2] - 12*b^3*e*m*n*Log[x]*Log[c*x^n]^2*Log[e + f*x^2]
+ 4*b^3*e*m*Log[c*x^n]^3*Log[e + f*x^2] + 4*a^3*f*x^2*Log[d*(e + f*x^2)^m]
- 6*a^2*b*f*n*x^2*Log[d*(e + f*x^2)^m] + 6*a*b^2*f*n^2*x^2*Log[d*(e + f*x^2
)^m] - 3*b^3*f*n^3*x^2*Log[d*(e + f*x^2)^m] + 12*a^2*b*f*x^2*Log[c*x^n]*Log
[d*(e + f*x^2)^m] - 12*a*b^2*f*n*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 6*b^
3*f*n^2*x^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*b^2*f*x^2*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] - 6*b^3*f*n*x^2*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + 4*b
^3*f*x^2*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + 6*b*e*m*n*(2*a^2 - 2*a*b*n + b
^2*n^2 - 2*b*(-2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)
*Sqrt[f]*x)/Sqrt[e]] + 6*b*e*m*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b
*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - 24
*a*b^2*e*m*n^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e*m*n^3*PolyLo
g[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*e*m*n^2*Log[c*x^n]*PolyLog[3, ((-I)
*Sqrt[f]*x)/Sqrt[e]] - 24*a*b^2*e*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
 12*b^3*e*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*e*m*n^2*Log[c*x^
n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 24*b^3*e*m*n^3*PolyLog[4, ((-I)*Sqrt
[f]*x)/Sqrt[e]] + 24*b^3*e*m*n^3*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(8*f)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.23, size = 47964, normalized size = 93.32

method	result	size
risch	Expression too large to display	47964

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^3*m*x^2*log(x^n)^3 - 6*((m*n - 2*m*log(c))*b^3 - 2*a*b^2*m)*x^2*lo
g(x^n)^2 - 6*(2*(m*n - 2*m*log(c))*a*b^2 - (m*n^2 - 2*m*n*log(c) + 2*m*log(
c)^2)*b^3 - 2*a^2*b*m)*x^2*log(x^n) - (6*(m*n - 2*m*log(c))*a^2*b - 6*(m*n^
2 - 2*m*n*log(c) + 2*m*log(c)^2)*a*b^2 + (3*m*n^3 - 6*m*n^2*log(c) + 6*m*n*
log(c)^2 - 4*m*log(c)^3)*b^3 - 4*a^3*m)*x^2)*log(f*x^2 + e) + integrate(-1/
4*((4*(f*m - f*log(d))*a^3 - 6*(f*m*n - 2*(f*m - f*log(d))*log(c))*a^2*b +
6*(f*m*n^2 - 2*f*m*n*log(c) + 2*(f*m - f*log(d))*log(c)^2)*a*b^2 - (3*f*m*n
^3 - 6*f*m*n^2*log(c) + 6*f*m*n*log(c)^2 - 4*(f*m - f*log(d))*log(c)^3)*b^3
)*x^3 + 4*((f*m - f*log(d))*b^3*x^3 - b^3*x*e*log(d))*log(x^n)^3 - 4*(b^3*l
og(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*log(c)*log(d) + a^3*log(
d))*x*e + 6*((2*(f*m - f*log(d))*a*b^2 - (f*m*n - 2*(f*m - f*log(d))*log(c)
)*b^3)*x^3 - 2*(b^3*log(c)*log(d) + a*b^2*log(d))*x*e)*log(x^n)^2 + 6*((2*(
f*m - f*log(d))*a^2*b - 2*(f*m*n - 2*(f*m - f*log(d))*log(c))*a*b^2 + (f*m*
n^2 - 2*f*m*n*log(c) + 2*(f*m - f*log(d))*log(c)^2)*b^3)*x^3 - 2*(b^3*log(c
)^2*log(d) + 2*a*b^2*log(c)*log(d) + a^2*b*log(d))*x*e)*log(x^n))/(f*x^2 +
e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
```

```
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log((f*x^2 + e)^m*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*x^2 + e)^m*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
[Out] int(x*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

3.109 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x}\,dx$

Optimal. Leaf size=181

$$\frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn}-\frac{m(a+b\log(cx^n))^4\log\left(1+\frac{fx^2}{e}\right)}{4bn}-\frac{1}{2}m(a+b\log(cx^n))^3\operatorname{Li}_2\left(-\frac{fx^2}{e}\right)+$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(f*x^2+e)^m)/b/n-1/4*m*(a+b*ln(c*x^n))^4*ln(1+f*
x^2/e)/b/n-1/2*m*(a+b*ln(c*x^n))^3*polylog(2,-f*x^2/e)+3/4*b*m*n*(a+b*ln(c*
x^n))^2*polylog(3,-f*x^2/e)-3/4*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x^2/
e)+3/8*b^3*m*n^3*polylog(5,-f*x^2/e)
```

Rubi [A]
time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$-\frac{3}{4}b^2mn^2\text{PolyLog}\left(4,-\frac{fx^2}{e}\right)(a+b\log(cx^n))-\frac{1}{2}m\text{PolyLog}\left(2,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^3+\frac{3}{4}bmn\text{PolyLog}\left(3,-\frac{fx^2}{e}\right)(a+b\log(cx^n))^2+\frac{3}{8}b^3mn^3\text{PolyLog}\left(5,-\frac{fx^2}{e}\right)+\frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn}-\frac{m\log\left(\frac{fx^2}{e}+1\right)(a+b\log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x^2)^m])/(4*b*n) - (m*(a + b*Log[c*x^n])
^4*Log[1 + (f*x^2)/e])/(4*b*n) - (m*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x^
2)/e)])/2 + (3*b*m*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x^2)/e)])/4 - (3*
b^2*m*n^2*(a + b*Log[c*x^n])*PolyLog[4, -((f*x^2)/e)])/4 + (3*b^3*m*n^3*Pol
yLog[5, -((f*x^2)/e)])/8
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x}\,dx = \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{(fm)\int \frac{x(a+b\log(cx^n))^4}{e+fx^2}\,dx}{2bn}$$

$$= \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log\left(d(e+fx^2)^m\right)}{4bn} - \frac{m(a+b\log(cx^n))^4\log}{4bn}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 1348, normalized size = 7.45

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x,x]
```

```
[Out] -(a^3*m*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]]) + (3*a^2*b*m*n*Log[x]^2*Log[
1 - (I*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/
Sqrt[e]] + (b^3*m*n^3*Log[x]^4*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a^2*b*
m*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[x]^2*L
og[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Lo
g[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - a^3
*m*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*a^2*b*m*n*Log[x]^2*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]])/2 - a*b^2*m*n^2*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e
]] + (b^3*m*n^3*Log[x]^4*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]])/4 - 3*a^2*b*m*Log[
x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 3*a*b^2*m*n*Log[x]^2*Log[c*x
^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - b^3*m*n^2*Log[x]^3*Log[c*x^n]*Log[1 +
(I*Sqrt[f]*x)/Sqrt[e]] - 3*a*b^2*m*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x
)/Sqrt[e]] + (3*b^3*m*n*Log[x]^2*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]
])/2 - b^3*m*Log[x]*Log[c*x^n]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^3*Log[x
]*Log[d*(e + f*x^2)^m] - (3*a^2*b*n*Log[x]^2*Log[d*(e + f*x^2)^m])/2 + a*b^
2*n^2*Log[x]^3*Log[d*(e + f*x^2)^m] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x^2)^m
])/4 + 3*a^2*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*a*b^2*n*Log[x]^2*
Log[c*x^n]*Log[d*(e + f*x^2)^m] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*
x^2)^m] + 3*a*b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - (3*b^3*n*Log[x
]^2*Log[c*x^n]^2*Log[d*(e + f*x^2)^m])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e
 + f*x^2)^m] - m*(a + b*Log[c*x^n])^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]]
- m*(a + b*Log[c*x^n])^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + 3*a^2*b*m*n*Po
lyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 6*a*b^2*m*n*Log[c*x^n]*PolyLog[3, ((-I
)*Sqrt[f]*x)/Sqrt[e]] + 3*b^3*m*n*Log[c*x^n]^2*PolyLog[3, ((-I)*Sqrt[f]*x)/
Sqrt[e]] + 3*a^2*b*m*n*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 6*a*b^2*m*n*Log[
c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + 3*b^3*m*n*Log[c*x^n]^2*PolyLog[3
, (I*Sqrt[f]*x)/Sqrt[e]] - 6*a*b^2*m*n^2*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e
]] - 6*b^3*m*n^2*Log[c*x^n]*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 6*a*b^2*
m*n^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^3*m*n^2*Log[c*x^n]*PolyLog[4,
 (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*m*n^3*PolyLog[5, ((-I)*Sqrt[f]*x)/Sqrt[e]]
+ 6*b^3*m*n^3*PolyLog[5, (I*Sqrt[f]*x)/Sqrt[e]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.45, size = 77072, normalized size = 425.81

method	result	size
risch	Expression too large to display	77072

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*m*n^3*log(x)^4 - 4*b^3*m*log(x)*log(x^n)^3 - 4*(b^3*m*n^2*log(c)
+ a*b^2*m*n^2)*log(x)^3 + 6*(b^3*m*n*log(c)^2 + 2*a*b^2*m*n*log(c) + a^2*b*
m*n)*log(x)^2 + 6*(b^3*m*n*log(x)^2 - 2*(b^3*m*log(c) + a*b^2*m)*log(x))*lo
g(x^n)^2 - 4*(b^3*m*n^2*log(x)^3 - 3*(b^3*m*n*log(c) + a*b^2*m*n)*log(x)^2
+ 3*(b^3*m*log(c)^2 + 2*a*b^2*m*log(c) + a^2*b*m)*log(x))*log(x^n) - 4*(b^3
*m*log(c)^3 + 3*a*b^2*m*log(c)^2 + 3*a^2*b*m*log(c) + a^3*m)*log(x))*log(f*
x^2 + e) - integrate(-1/2*(b^3*f*m*n^3*x^2*log(x)^4 - 4*(b^3*f*m*n^2*log(c)
 + a*b^2*f*m*n^2)*x^2*log(x)^3 + 6*(b^3*f*m*n*log(c)^2 + 2*a*b^2*f*m*n*log(
c) + a^2*b*f*m*n)*x^2*log(x)^2 - 4*(b^3*f*m*log(c)^3 + 3*a*b^2*f*m*log(c)^2
 + 3*a^2*b*f*m*log(c) + a^3*f*m)*x^2*log(x) - 2*(2*b^3*f*m*x^2*log(x) - b^3
*f*x^2*log(d) - b^3*e*log(d))*log(x^n)^3 + 2*(b^3*f*log(c)^3*log(d) + 3*a*b
^2*f*log(c)^2*log(d) + 3*a^2*b*f*log(c)*log(d) + a^3*f*log(d))*x^2 + 6*(b^3
*f*m*n*x^2*log(x)^2 - 2*(b^3*f*m*log(c) + a*b^2*f*m)*x^2*log(x) + (b^3*f*lo
g(c)*log(d) + a*b^2*f*log(d))*x^2 + (b^3*log(c)*log(d) + a*b^2*log(d))*e)*l
og(x^n)^2 + 2*(b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*log(
c)*log(d) + a^3*log(d))*e - 2*(2*b^3*f*m*n^2*x^2*log(x)^3 - 6*(b^3*f*m*n*lo
g(c) + a*b^2*f*m*n)*x^2*log(x)^2 + 6*(b^3*f*m*log(c)^2 + 2*a*b^2*f*m*log(c)
 + a^2*b*f*m)*x^2*log(x) - 3*(b^3*f*log(c)^2*log(d) + 2*a*b^2*f*log(c)*log(
d) + a^2*b*f*log(d))*x^2 - 3*(b^3*log(c)^2*log(d) + 2*a*b^2*log(c)*log(d) +
 a^2*b*log(d))*e)*log(x^n))/(f*x^3 + x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x, x)
```

3.110 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^3}\,dx$

Optimal. Leaf size=451

$$\frac{3b^3fmn^3\log(x)}{4e}-\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}-\frac{3bfmn\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{4e}-\frac{fm\log}{}$$

```
[Out] 3/4*b^3*f*m*n^3*ln(x)/e-3/4*b^2*f*m*n^2*ln(1+e/f/x^2)*(a+b*ln(c*x^n))/e-3/4
*b*f*m*n*ln(1+e/f/x^2)*(a+b*ln(c*x^n))^2/e-1/2*f*m*ln(1+e/f/x^2)*(a+b*ln(c*
x^n))^3/e-3/8*b^3*f*m*n^3*ln(f*x^2+e)/e-3/8*b^3*n^3*ln(d*(f*x^2+e)^m)/x^2-3
/4*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*
ln(d*(f*x^2+e)^m)/x^2-1/2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2+3/8*b^3*f
*m*n^3*polylog(2,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(2,-e/f
/x^2)/e+3/4*b*f*m*n*(a+b*ln(c*x^n))^2*polylog(2,-e/f/x^2)/e+3/8*b^3*f*m*n^3
*polylog(3,-e/f/x^2)/e+3/4*b^2*f*m*n^2*(a+b*ln(c*x^n))*polylog(3,-e/f/x^2)/
e+3/8*b^3*f*m*n^3*polylog(4,-e/f/x^2)/e
```

Rubi [A]
time = 0.37, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2342, 2341, 2425, 272, 36, 29, 31, 2379, 2438, 2421, 6724, 2430}

$$\frac{3b^2fmn^2\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}+\frac{3b^2fmn^2\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}+\frac{3bfmn\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)(a+b\log(cx^n))^2}{4e}+\frac{3b^3fmn^3\text{PolyLog}\left(2,-\frac{e}{fx^2}\right)}{8e}+\frac{3b^3fmn^3\text{PolyLog}\left(3,-\frac{e}{fx^2}\right)}{8e}+\frac{3b^3fmn^3\text{PolyLog}\left(4,-\frac{e}{fx^2}\right)}{8e}-\frac{3b^2n^2(a+b\log(cx^n))\log\left(d(e+fx^2)^m\right)}{4x^2}-\frac{3b^2fmn^2\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))}{4e}-\frac{3bn(a+b\log(cx^n))^2\log\left(d(e+fx^2)^m\right)}{4x^2}-\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{2x^2}-\frac{3bfmn\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))^2}{4e}-\frac{fm\log\left(\frac{e}{fx^2}+1\right)(a+b\log(cx^n))^3}{2e}-\frac{3b^3n^3\log\left(d(e+fx^2)^m\right)}{8x^2}-\frac{3b^3fmn^3\log\left(e+fx^2\right)}{8e}+\frac{3b^3fmn^3\log(x)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] (3*b^3*f*m*n^3*Log[x])/(4*e) - (3*b^2*f*m*n^2*Log[1 + e/(f*x^2)]*(a + b*Log
[c*x^n]))/(4*e) - (3*b*f*m*n*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^2)/(4*e)
 - (f*m*Log[1 + e/(f*x^2)]*(a + b*Log[c*x^n])^3)/(2*e) - (3*b^3*f*m*n^3*Log
[e + f*x^2])/(8*e) - (3*b^3*n^3*Log[d*(e + f*x^2)^m])/(8*x^2) - (3*b^2*n^2*
(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(4*x^2) - (3*b*n*(a + b*Log[c*x^n]
)^2*Log[d*(e + f*x^2)^m])/(4*x^2) - ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)
^m])/(2*x^2) + (3*b^3*f*m*n^3*PolyLog[2, -(e/(f*x^2))])/(8*e) + (3*b^2*f*m*
n^2*(a + b*Log[c*x^n])*PolyLog[2, -(e/(f*x^2))])/(4*e) + (3*b*f*m*n*(a + b*
Log[c*x^n])^2*PolyLog[2, -(e/(f*x^2))])/(4*e) + (3*b^3*f*m*n^3*PolyLog[3, -
(e/(f*x^2))])/(8*e) + (3*b^2*f*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -(e/(f*x
^2))])/(4*e) + (3*b^3*f*m*n^3*PolyLog[4, -(e/(f*x^2))])/(8*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
```

```
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^3}\,dx &= -\frac{3b^3n^3\log\left(d(e+fx^2)^m\right)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log\left(d(e+\right.}{4x^2}\\
&= -\frac{3b^3n^3\log\left(d(e+fx^2)^m\right)}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))\log\left(d(e+\right.}{4x^2}\\
&= -\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}-\frac{3bfmn\log\left(1+\right.}{}\\
&= -\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}-\frac{3bfmn\log\left(1+\right.}{}\\
&= \frac{3b^3fmn^3\log(x)}{4e}-\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}-\\
&= \frac{3b^3fmn^3\log(x)}{4e}-\frac{3b^2fmn^2\log\left(1+\frac{e}{fx^2}\right)(a+b\log(cx^n))}{4e}-
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 2248, normalized size = 4.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^3,x]
```

```
[Out] -1/8*(-8*a^3*f*m*x^2*Log[x] - 12*a^2*b*f*m*n*x^2*Log[x] - 12*a*b^2*f*m*n^2*
x^2*Log[x] - 6*b^3*f*m*n^3*x^2*Log[x] + 12*a^2*b*f*m*n*x^2*Log[x]^2 + 12*a*
b^2*f*m*n^2*x^2*Log[x]^2 + 6*b^3*f*m*n^3*x^2*Log[x]^2 - 8*a*b^2*f*m*n^2*x^2
*Log[x]^3 - 4*b^3*f*m*n^3*x^2*Log[x]^3 + 2*b^3*f*m*n^3*x^2*Log[x]^4 - 24*a^
2*b*f*m*x^2*Log[x]*Log[c*x^n] - 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n] - 12*b
^3*f*m*n^2*x^2*Log[x]*Log[c*x^n] + 24*a*b^2*f*m*n*x^2*Log[x]^2*Log[c*x^n] +
 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n] - 8*b^3*f*m*n^2*x^2*Log[x]^3*Log[c*
x^n] - 24*a*b^2*f*m*x^2*Log[x]*Log[c*x^n]^2 - 12*b^3*f*m*n*x^2*Log[x]*Log[c
*x^n]^2 + 12*b^3*f*m*n*x^2*Log[x]^2*Log[c*x^n]^2 - 8*b^3*f*m*x^2*Log[x]*Log
[c*x^n]^3 + 12*a^2*b*f*m*n*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a
*b^2*f*m*n^2*x^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f*m*n^3*x^2*
Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*a*b^2*f*m*n^2*x^2*Log[x]^2*Log[1
 - (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^3*f*m*n^3*x^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x
)/Sqrt[e]] + 4*b^3*f*m*n^3*x^2*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 24
*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*
f*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n
^2*x^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n*x^
2*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + 12*a^2*b*f*m*n*x^2*L
og[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*f*m*n^2*x^2*Log[x]*Log[1 +
(I*Sqrt[f]*x)/Sqrt[e]] + 6*b^3*f*m*n^3*x^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] - 12*a*b^2*f*m*n^2*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 6*b^
3*f*m*n^3*x^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 4*b^3*f*m*n^3*x^2*L
og[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n]*Lo
g[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] + 12*b^3*f*m*n*x^2*Log[x]*Log[c*x^n]^2*Log[1 + (I
*Sqrt[f]*x)/Sqrt[e]] + 4*a^3*f*m*x^2*Log[e + f*x^2] + 6*a^2*b*f*m*n*x^2*Log
[e + f*x^2] + 6*a*b^2*f*m*n^2*x^2*Log[e + f*x^2] + 3*b^3*f*m*n^3*x^2*Log[e
+ f*x^2] - 12*a^2*b*f*m*n*x^2*Log[x]*Log[e + f*x^2] - 12*a*b^2*f*m*n^2*x^2*
Log[x]*Log[e + f*x^2] - 6*b^3*f*m*n^3*x^2*Log[x]*Log[e + f*x^2] + 12*a*b^2*
f*m*n^2*x^2*Log[x]^2*Log[e + f*x^2] + 6*b^3*f*m*n^3*x^2*Log[x]^2*Log[e + f*
x^2] - 4*b^3*f*m*n^3*x^2*Log[x]^3*Log[e + f*x^2] + 12*a^2*b*f*m*x^2*Log[c*x
^n]*Log[e + f*x^2] + 12*a*b^2*f*m*n*x^2*Log[c*x^n]*Log[e + f*x^2] + 6*b^3*f
*m*n^2*x^2*Log[c*x^n]*Log[e + f*x^2] - 24*a*b^2*f*m*n*x^2*Log[x]*Log[c*x^n]
*Log[e + f*x^2] - 12*b^3*f*m*n^2*x^2*Log[x]*Log[c*x^n]*Log[e + f*x^2] + 12*
b^3*f*m*n^2*x^2*Log[x]^2*Log[c*x^n]*Log[e + f*x^2] + 12*a*b^2*f*m*x^2*Log[c
*x^n]^2*Log[e + f*x^2] + 6*b^3*f*m*n*x^2*Log[c*x^n]^2*Log[e + f*x^2] - 12*b
^3*f*m*n*x^2*Log[x]*Log[c*x^n]^2*Log[e + f*x^2] + 4*b^3*f*m*x^2*Log[c*x^n]^
3*Log[e + f*x^2] + 4*a^3*e*Log[d*(e + f*x^2)^m] + 6*a^2*b*e*n*Log[d*(e + f*
x^2)^m] + 6*a*b^2*e*n^2*Log[d*(e + f*x^2)^m] + 3*b^3*e*n^3*Log[d*(e + f*x^2
)^m] + 12*a^2*b*e*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*b^2*e*n*Log[c*x^n]
```

```
*Log[d*(e + f*x^2)^m] + 6*b^3*e*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 12*a*
b^2*e*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] + 6*b^3*e*n*Log[c*x^n]^2*Log[d*(e +
 f*x^2)^m] + 4*b^3*e*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + 6*b*f*m*n*x^2*(2*a
^2 + 2*a*b*n + b^2*n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*P
olyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 6*b*f*m*n*x^2*(2*a^2 + 2*a*b*n + b^2*
n^2 + 2*b*(2*a + b*n)*Log[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f
]*x)/Sqrt[e]] - 24*a*b^2*f*m*n^2*x^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
 12*b^3*f*m*n^3*x^2*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*b^3*f*m*n^2*x
^2*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - 24*a*b^2*f*m*n^2*x^2*P
olyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - 12*b^3*f*m*n^3*x^2*PolyLog[3, (I*Sqrt[f]
*x)/Sqrt[e]] - 24*b^3*f*m*n^2*x^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[
e]] + 24*b^3*f*m*n^3*x^2*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + 24*b^3*f*m*
n^3*x^2*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(e*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.27, size = 49628, normalized size = 110.04

method	result	size
risch	Expression too large to display	49628

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*m*log(x^n)^3 + 6*(m*n + 2*m*log(c))*a^2*b + 6*(m*n^2 + 2*m*n*lo
g(c) + 2*m*log(c)^2)*a*b^2 + (3*m*n^3 + 6*m*n^2*log(c) + 6*m*n*log(c)^2 + 4
*m*log(c)^3)*b^3 + 4*a^3*m + 6*((m*n + 2*m*log(c))*b^3 + 2*a*b^2*m)*log(x^n
)^2 + 6*(2*(m*n + 2*m*log(c))*a*b^2 + (m*n^2 + 2*m*n*log(c) + 2*m*log(c)^2)
*b^3 + 2*a^2*b*m)*log(x^n))*log(f*x^2 + e)/x^2 + integrate(1/4*(4*((f*m + f
*log(d))*b^3*x^2 + b^3*e*log(d))*log(x^n)^3 + (4*(f*m + f*log(d))*a^3 + 6*(
f*m*n + 2*(f*m + f*log(d))*log(c))*a^2*b + 6*(f*m*n^2 + 2*f*m*n*log(c) + 2*
(f*m + f*log(d))*log(c)^2)*a*b^2 + (3*f*m*n^3 + 6*f*m*n^2*log(c) + 6*f*m*n*
log(c)^2 + 4*(f*m + f*log(d))*log(c)^3)*b^3)*x^2 + 6*((2*(f*m + f*log(d))*a
*b^2 + (f*m*n + 2*(f*m + f*log(d))*log(c))*b^3)*x^2 + 2*(b^3*log(c)*log(d)
+ a*b^2*log(d))*e)*log(x^n)^2 + 4*(b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*l
og(d) + 3*a^2*b*log(c)*log(d) + a^3*log(d))*e + 6*((2*(f*m + f*log(d))*a^2*
```

```
b + 2*(f*m*n + 2*(f*m + f*log(d))*log(c))*a*b^2 + (f*m*n^2 + 2*f*m*n*log(c)
 + 2*(f*m + f*log(d))*log(c)^2)*b^3)*x^2 + 2*(b^3*log(c)^2*log(d) + 2*a*b^2
*log(c)*log(d) + a^2*b*log(d))*e)*log(x^n))/(f*x^5 + x^3*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^3, x)
```

3.111 $\int x^2(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)\,dx$

Optimal. Leaf size=1092

$$\frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27f^{3/2}}+\frac{52b^3emn^2x\log(cx^n)}{9f}-\frac{4}{9}b^2mn^2x^3(a$$

```
[Out] 16/81*b^3*m*n^3*x^3+2/3*e*m*x*(a+b*ln(c*x^n))^3/f-2/27*b^3*n^3*x^3*ln(d*(f*
x^2+e)^m)-160/27*b^3*e*m*n^3*x/f-2/9*m*x^3*(a+b*ln(c*x^n))^3+1/3*x^3*(a+b*l
n(c*x^n))^3*ln(d*(f*x^2+e)^m)+52/9*a*b^2*e*m*n^2*x/f+4/27*b^3*e^(3/2)*m*n^3
*arctan(x*f^(1/2)/e^(1/2))/f^(3/2)+2/3*b^3*(-e)^(3/2)*m*n^3*polylog(3,-x*f^
(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b^3*(-e)^(3/2)*m*n^3*polylog(3,x*f^(1/2)/(-e)
^(1/2))/f^(3/2)+2*b^3*(-e)^(3/2)*m*n^3*polylog(4,-x*f^(1/2)/(-e)^(1/2))/f^(
3/2)-2*b^3*(-e)^(3/2)*m*n^3*polylog(4,x*f^(1/2)/(-e)^(1/2))/f^(3/2)+1/3*(-e
)^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))/f^(3/2)-1/3*b*n*x^3*
(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-1/3*(-e)^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1
-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-4/9*b^2*m*n^2*x^3*(a+b*ln(c*x^n))+4/9*b*m*n*
x^3*(a+b*ln(c*x^n))^2+2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)+52/
9*b^3*e*m*n^2*x*ln(c*x^n)/f-8/3*b*e*m*n*x*(a+b*ln(c*x^n))^2/f+b*(-e)^(3/2)*
m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/3*b^2*(-e)
^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2*b^2*
(-e)^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2
*b^2*(-e)^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)/(-e)^(1/2))/f^(3/
2)-2/9*I*b^3*e^(3/2)*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))/f^(3/2)-4/9*b^2*e
^(3/2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/f^(3/2)+1/3*b*(-e)^(
3/2)*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-1/3*b*(-e)^(3
/2)*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/f^(3/2)-2/3*b^2*(-e)^(
3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/f^(3/2)-b*(-e)^
(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))/f^(3/2)+2/9*I*b
^3*e^(3/2)*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))/f^(3/2)
```

Rubi [A]
time = 1.23, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {2342, 2341, 2425, 308, 211, 2393, 2332, 2361, 12, 4940, 2438, 2395, 2333, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (52*a*b^2*e*m*n^2*x)/(9*f) - (160*b^3*e*m*n^3*x)/(27*f) + (16*b^3*m*n^3*x^3
)/81 + (4*b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(27*f^(3/2)) + (52
```

```
*b^3*e*m*n^2*x*Log[c*x^n])/(9*f) - (4*b^2*m*n^2*x^3*(a + b*Log[c*x^n]))/9 -
 (4*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*f^
(3/2)) - (8*b*e*m*n*x*(a + b*Log[c*x^n])^2)/(3*f) + (4*b*m*n*x^3*(a + b*Log
[c*x^n])^2)/9 + (2*e*m*x*(a + b*Log[c*x^n])^3)/(3*f) - (2*m*x^3*(a + b*Log[
c*x^n])^3)/9 + (b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/S
qrt[-e]])/(3*f^(3/2)) - ((-e)^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]
*x)/Sqrt[-e]])/(3*f^(3/2)) - (b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 +
 (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) + ((-e)^(3/2)*m*(a + b*Log[c*x^n])^3*Lo
g[1 + (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (2*b^3*n^3*x^3*Log[d*(e + f*x^2)
^m])/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/9 - (b*n*
x^3*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/3 + (x^3*(a + b*Log[c*x^n])^
3*Log[d*(e + f*x^2)^m])/3 - (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*Poly
Log[2, -((Sqrt[f]*x)/Sqrt[-e])]])/(3*f^(3/2)) + (b*(-e)^(3/2)*m*n*(a + b*Log
[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])]])/f^(3/2) + (2*b^2*(-e)^(3/2)
*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*f^(3/2)) - (
b*(-e)^(3/2)*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/f^(
3/2) + (((2*I)/9)*b^3*e^(3/2)*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/f
^(3/2) - (((2*I)/9)*b^3*e^(3/2)*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/f^
(3/2) + (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])]])/(3*f^(
3/2)) - (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)
/Sqrt[-e])]])/f^(3/2) - (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[
-e]])/(3*f^(3/2)) + (2*b^2*(-e)^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (
Sqrt[f]*x)/Sqrt[-e]])/f^(3/2) + (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[4, -((Sqrt[
f]*x)/Sqrt[-e])]])/f^(3/2) - (2*b^3*(-e)^(3/2)*m*n^3*PolyLog[4, (Sqrt[f]*x)/
Sqrt[-e]])/f^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)\,dx = -\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n))\,\mathrm{lo}$$

$$= -\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n))\,\mathrm{lo}$$

$$= -\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n))\,\mathrm{lo}$$

$$= -\frac{4b^3emn^3x}{27f}+\frac{4}{81}b^3mn^3x^3-\frac{2}{27}b^3n^3x^3\log\left(d\left(e+fx^2\right)^m\right)+$$

$$= \frac{4ab^2emn^2x}{9f}-\frac{4b^3emn^3x}{27f}+\frac{8}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan^{-1}}{27f^{3/2}}$$

$$= \frac{16ab^2emn^2x}{9f}-\frac{16b^3emn^3x}{27f}+\frac{4}{27}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan}{27f^3}$$

$$= \frac{52ab^2emn^2x}{9f}-\frac{52b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\tan}{27f^3}$$

$$= \frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\mathrm{ta}}{27}$$

$$= \frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\mathrm{ta}}{27}$$

$$= \frac{52ab^2emn^2x}{9f}-\frac{160b^3emn^3x}{27f}+\frac{16}{81}b^3mn^3x^3+\frac{4b^3e^{3/2}mn^3\mathrm{ta}}{27}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2544 vs. 2(1092) = 2184.
time = 0.57, size = 2544, normalized size = 2.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (54*a^3*e*Sqrt[f]*m*x - 216*a^2*b*e*Sqrt[f]*m*n*x + 468*a*b^2*e*Sqrt[f]*m*n
^2*x - 480*b^3*e*Sqrt[f]*m*n^3*x - 18*a^3*f^(3/2)*m*x^3 + 36*a^2*b*f^(3/2)*
m*n*x^3 - 36*a*b^2*f^(3/2)*m*n^2*x^3 + 16*b^3*f^(3/2)*m*n^3*x^3 - 54*a^3*e^
(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*e^(3/2)*m*n*ArcTan[(Sqrt[f]*
x)/Sqrt[e]] - 36*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*b^3*e
^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 162*a^2*b*e^(3/2)*m*n*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x] - 108*a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[
e]]*Log[x] + 36*b^3*e^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 162*
a*b^2*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*e^(3/2)*m
*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 54*b^3*e^(3/2)*m*n^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x]^3 + 162*a^2*b*e*Sqrt[f]*m*x*Log[c*x^n] - 432*a*b^2
*e*Sqrt[f]*m*n*x*Log[c*x^n] + 468*b^3*e*Sqrt[f]*m*n^2*x*Log[c*x^n] - 54*a^2
*b*f^(3/2)*m*x^3*Log[c*x^n] + 72*a*b^2*f^(3/2)*m*n*x^3*Log[c*x^n] - 36*b^3*
f^(3/2)*m*n^2*x^3*Log[c*x^n] - 162*a^2*b*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[
e]]*Log[c*x^n] + 108*a*b^2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^
n] - 36*b^3*e^(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 324*a*b^
2*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 108*b^3*e^(3/
2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 162*b^3*e^(3/2)*m*
n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] + 162*a*b^2*e*Sqrt[f]*m
*x*Log[c*x^n]^2 - 216*b^3*e*Sqrt[f]*m*n*x*Log[c*x^n]^2 - 54*a*b^2*f^(3/2)*m
*x^3*Log[c*x^n]^2 + 36*b^3*f^(3/2)*m*n*x^3*Log[c*x^n]^2 - 162*a*b^2*e^(3/2)
*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 54*b^3*e^(3/2)*m*n*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 162*b^3*e^(3/2)*m*n*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x]*Log[c*x^n]^2 + 54*b^3*e*Sqrt[f]*m*x*Log[c*x^n]^3 - 18*b^3*f^(3
/2)*m*x^3*Log[c*x^n]^3 - 54*b^3*e^(3/2)*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c
*x^n]^3 - (81*I)*a^2*b*e^(3/2)*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] +
(54*I)*a*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b
^3*e^(3/2)*m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*a*b^2*e^(3/
2)*m*n^2*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*e^(3/2)*m*n^3
*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*e^(3/2)*m*n^3*Log[x]^
3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (162*I)*a*b^2*e^(3/2)*m*n*Log[x]*Log[c*x
^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*e^(3/2)*m*n^2*Log[x]*Log[c*
x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*b^3*e^(3/2)*m*n^2*Log[x]^2*Log
[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (81*I)*b^3*e^(3/2)*m*n*Log[x]*Log[
c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (81*I)*a^2*b*e^(3/2)*m*n*Log[x]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*e^(3/2)*m*n^2*Log[x]*Log[1 + (
I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*e^(3/2)*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*
x)/Sqrt[e]] - (81*I)*a*b^2*e^(3/2)*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] + (27*I)*b^3*e^(3/2)*m*n^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] +
(27*I)*b^3*e^(3/2)*m*n^3*Log[x]^3*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (162*I)*
a*b^2*e^(3/2)*m*n*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)
```

```
*b^3*e^(3/2)*m*n^2*Log[x]*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (81*I
)*b^3*e^(3/2)*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (8
1*I)*b^3*e^(3/2)*m*n*Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + 2
7*a^3*f^(3/2)*x^3*Log[d*(e + f*x^2)^m] - 27*a^2*b*f^(3/2)*n*x^3*Log[d*(e +
f*x^2)^m] + 18*a*b^2*f^(3/2)*n^2*x^3*Log[d*(e + f*x^2)^m] - 6*b^3*f^(3/2)*n
^3*x^3*Log[d*(e + f*x^2)^m] + 81*a^2*b*f^(3/2)*x^3*Log[c*x^n]*Log[d*(e + f*
x^2)^m] - 54*a*b^2*f^(3/2)*n*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 18*b^3*f
^(3/2)*n^2*x^3*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 81*a*b^2*f^(3/2)*x^3*Log[c
*x^n]^2*Log[d*(e + f*x^2)^m] - 27*b^3*f^(3/2)*n*x^3*Log[c*x^n]^2*Log[d*(e +
 f*x^2)^m] + 27*b^3*f^(3/2)*x^3*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] + (9*I)*b
*e^(3/2)*m*n*(9*a^2 - 6*a*b*n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9
*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b*e^(3/2)*m
*n*(9*a^2 - 6*a*b*n + 2*b^2*n^2 - 6*b*(-3*a + b*n)*Log[c*x^n] + 9*b^2*Log[c
*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (162*I)*a*b^2*e^(3/2)*m*n^2*Po
lyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*e^(3/2)*m*n^3*PolyLog[3, ((
-I)*Sqrt[f]*x)/Sqrt[e]] - (162*I)*b^3*e^(3/2)*m*n^2*Log[c*x^n]*PolyLog[3, (
(-I)*Sqrt[f]*x)/Sqrt[e]] + (162*I)*a*b^2*e^(3/2)*m*n^2*PolyLog[3, (I*Sqrt[f
]*x)/Sqrt[e]] - (54*I)*b^3*e^(3/2)*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]]
+ (162*I)*b^3*e^(3/2)*m*n^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] +
(162*I)*b^3*e^(3/2)*m*n^3*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (162*I)*b^
3*e^(3/2)*m*n^3*PolyLog[4, (I*Sqrt[f]*x)/Sqrt[e]])/(81*f^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
```

```
[Out] 1/27*(9*b^3*m*x^3*log(x^n)^3 - 9*((m*n - 3*m*log(c))*b^3 - 3*a*b^2*m)*x^3*l
og(x^n)^2 - 3*(6*(m*n - 3*m*log(c))*a*b^2 - (2*m*n^2 - 6*m*n*log(c) + 9*m*l
og(c)^2)*b^3 - 9*a^2*b*m)*x^3*log(x^n) - (9*(m*n - 3*m*log(c))*a^2*b - 3*(2
*m*n^2 - 6*m*n*log(c) + 9*m*log(c)^2)*a*b^2 + (2*m*n^3 - 6*m*n^2*log(c) + 9
*m*n*log(c)^2 - 9*m*log(c)^3)*b^3 - 9*a^3*m)*x^3)*log(f*x^2 + e) + integrat
```

```
e(-1/27*((9*(2*f*m - 3*f*log(d))*a^3 - 9*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*
log(c))*a^2*b + 3*(4*f*m*n^2 - 12*f*m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log
(c)^2)*a*b^2 - (4*f*m*n^3 - 12*f*m*n^2*log(c) + 18*f*m*n*log(c)^2 - 9*(2*f*
m - 3*f*log(d))*log(c)^3)*b^3)*x^4 - 27*(b^3*log(c)^3*log(d) + 3*a*b^2*log(
c)^2*log(d) + 3*a^2*b*log(c)*log(d) + a^3*log(d))*x^2*e + 9*((2*f*m - 3*f*l
og(d))*b^3*x^4 - 3*b^3*x^2*e*log(d))*log(x^n)^3 + 9*((3*(2*f*m - 3*f*log(d)
)*a*b^2 - (2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*b^3)*x^4 - 9*(b^3*log(c
)*log(d) + a*b^2*log(d))*x^2*e)*log(x^n)^2 + 3*((9*(2*f*m - 3*f*log(d))*a^2
*b - 6*(2*f*m*n - 3*(2*f*m - 3*f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 - 12*f*
m*n*log(c) + 9*(2*f*m - 3*f*log(d))*log(c)^2)*b^3)*x^4 - 27*(b^3*log(c)^2*l
og(d) + 2*a*b^2*log(c)*log(d) + a^2*b*log(d))*x^2*e)*log(x^n))/(f*x^2 + e),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b^3*x^2*log(c*x^n)^3 + 3*a*b^2*x^2*log(c*x^n)^2 + 3*a^2*b*x^2*log
(c*x^n) + a^3*x^2)*log((f*x^2 + e)^m*d), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*x^2*log((f*x^2 + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
[Out] int(x^2*log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

3.112 $\int (a+b\log(cx^n))^3 \log\left(d(e+fx^2)^m\right) dx$

Optimal. Leaf size=977

$$-24ab^2mn^2x+36b^3mn^3x-12b^2mn^2(a-bn)x+\frac{12b^2\sqrt{e}\,mn^2(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}}-36b^3mn^2x\log(cx^n)+$$

[Out] -2*m*x*(a+b*ln(c*x^n))^3+36*b^3*m*n^3*x-36*b^3*m*n^2*x*ln(c*x^n)+12*b*m*n*x*(a+b*ln(c*x^n))^2-24*a*b^2*m*n^2*x-12*b^2*m*n^2*(-b*n+a)*x-6*b^3*n^3*x*ln(d*(f*x^2+e)^m)+x*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)+6*b^3*m*n^3*polylog(3,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-6*b^3*m*n^3*polylog(3,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*b^3*m*n^3*polylog(4,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-6*b^3*m*n^3*polylog(4,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*a*b^2*n^2*x*ln(d*(f*x^2+e)^m)+6*b^3*n^2*x*ln(c*x^n)*ln(d*(f*x^2+e)^m)-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)-m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+m*(a+b*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+12*b^2*m*n^2*(-b*n+a)*arctan(x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+12*b^3*m*n^2*arctan(x*f^(1/2)/e^(1/2))*ln(c*x^n)*e^(1/2)/f^(1/2)-6*I*b^3*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)+3*b*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-3*b*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+3*b*m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*b^2*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)-3*b*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))*(-e)^(1/2)/f^(1/2)+6*I*b^3*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))*e^(1/2)/f^(1/2)

Rubi [A]
time = 1.00, antiderivative size = 977, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {2333, 2332, 2418, 6, 327, 211, 2393, 2361, 12, 4940, 2438, 2395, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

[In] Int[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]

[Out] -24*a*b^2*m*n^2*x + 36*b^3*m*n^3*x - 12*b^2*m*n^2*(a - b*n)*x + (12*b^2*Sqrt[e]*m*n^2*(a - b*n)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] - 36*b^3*m*n^2*x*Log[c*x^n] + (12*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n])/Sqrt[f] + 12*b*m*n*x*(a + b*Log[c*x^n])^2 - 2*m*x*(a + b*Log[c*x^n])^3 + (3

```
*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f]
 - (Sqrt[-e]*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f]
- (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqr
t[f] + (Sqrt[-e]*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt
[f] + 6*a*b^2*n^2*x*Log[d*(e + f*x^2)^m] - 6*b^3*n^3*x*Log[d*(e + f*x^2)^m]
 + 6*b^3*n^2*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*b*n*x*(a + b*Log[c*x^n])
^2*Log[d*(e + f*x^2)^m] + x*(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m] - (6*
b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/
Sqrt[f] + (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/S
qrt[-e])])/Sqrt[f] + (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (S
qrt[f]*x)/Sqrt[-e]])/Sqrt[f] - (3*b*Sqrt[-e]*m*n*(a + b*Log[c*x^n])^2*PolyL
og[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] - ((6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[2,
((-I)*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + ((6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[2, (I
*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f] + (6*b^3*Sqrt[-e]*m*n^3*PolyLog[3, -((Sqrt[f]
*x)/Sqrt[-e])])/Sqrt[f] - (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*PolyLog[
3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[f] - (6*b^3*Sqrt[-e]*m*n^3*PolyLog[3, (Sq
rt[f]*x)/Sqrt[-e]])/Sqrt[f] + (6*b^2*Sqrt[-e]*m*n^2*(a + b*Log[c*x^n])*Poly
Log[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[f] + (6*b^3*Sqrt[-e]*m*n^3*PolyLog[4, -(
(Sqrt[f]*x)/Sqrt[-e])])/Sqrt[f] - (6*b^3*Sqrt[-e]*m*n^3*PolyLog[4, (Sqrt[f]
*x)/Sqrt[-e]])/Sqrt[f]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2418

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]},
Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, Int[Dist[x^(m - 1)/(e + f*x^
m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] &
& IntegerQ[m]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)\,dx = 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right) - 6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right) + 6b^3n^2$$

$$= 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right) - 6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right) + 6b^3n^2$$

$$= 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right) - 6b^3n^3x\log\left(d\left(e+fx^2\right)^m\right) + 6b^3n^2$$

$$= -12b^2mn^2(a-bn)x + 6ab^2n^2x\log\left(d\left(e+fx^2\right)^m\right) - 6b^3n^3x\log$$

$$= -12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2(a-bn)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{f}} +$$

$$= 12b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2(a-bn)\tan^{-1}\left(\right.}{\sqrt{f}}$$

$$= -12ab^2mn^2x + 12b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2}{}$$

$$= -24ab^2mn^2x + 24b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2}{}$$

$$= -24ab^2mn^2x + 36b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2}{}$$

$$= -24ab^2mn^2x + 36b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2}{}$$

$$= -24ab^2mn^2x + 36b^3mn^3x - 12b^2mn^2(a-bn)x + \frac{12b^2\sqrt{e}\,mn^2}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2302 vs. 2(977) = 1954.

time = 0.44, size = 2302, normalized size = 2.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m],x]
```

```
[Out] (-2*a^3*Sqrt[f]*m*x + 12*a^2*b*Sqrt[f]*m*n*x - 36*a*b^2*Sqrt[f]*m*n^2*x + 4
8*b^3*Sqrt[f]*m*n^3*x + 2*a^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a^2
*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[e]*m*n^2*ArcTan[
(Sqrt[f]*x)/Sqrt[e]] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6
*a^2*b*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] + 12*a*b^2*Sqrt[e]*m*
n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[
f]*x)/Sqrt[e]]*Log[x] + 6*a*b^2*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*L
og[x]^2 - 6*b^3*Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 2*b^3*
Sqrt[e]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 6*a^2*b*Sqrt[f]*m*x*Lo
g[c*x^n] + 24*a*b^2*Sqrt[f]*m*n*x*Log[c*x^n] - 36*b^3*Sqrt[f]*m*n^2*x*Log[c
*x^n] + 6*a^2*b*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2
*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sqrt[e]*m*n^2*
ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*a*b^2*Sqrt[e]*m*n*ArcTan[(Sqrt[
f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 12*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/
Sqrt[e]]*Log[x]*Log[c*x^n] + 6*b^3*Sqrt[e]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]
]*Log[x]^2*Log[c*x^n] - 6*a*b^2*Sqrt[f]*m*x*Log[c*x^n]^2 + 12*b^3*Sqrt[f]*m
*n*x*Log[c*x^n]^2 + 6*a*b^2*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n
]^2 - 6*b^3*Sqrt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 6*b^3*Sq
rt[e]*m*n*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 - 2*b^3*Sqrt[f]*m
*x*Log[c*x^n]^3 + 2*b^3*Sqrt[e]*m*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3
+ (3*I)*a^2*b*Sqrt[e]*m*n*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b
^2*Sqrt[e]*m*n^2*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*
m*n^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*a*b^2*Sqrt[e]*m*n^2*Log
[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n^3*Log[x]^2*Log
[1 - (I*Sqrt[f]*x)/Sqrt[e]] + I*b^3*Sqrt[e]*m*n^3*Log[x]^3*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*n^2*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[
f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m*n^2*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] - (3*I)*a^2*b*Sqrt[e]*m*n*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqr
t[e]] + (6*I)*a*b^2*Sqrt[e]*m*n^2*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (
6*I)*b^3*Sqrt[e]*m*n^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*a*b^2*
Sqrt[e]*m*n^2*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m
*n^3*Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - I*b^3*Sqrt[e]*m*n^3*Log[x]^3
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*Sqrt[e]*m*n*Log[x]*Log[c*x^n]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^2*Log[x]*Log[c*x^n]
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[e]*m*n^2*Log[x]^2*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[e]*m*n*Log[x]*Log[c*x^n]
```

```
^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + a^3*Sqrt[f]*x*Log[d*(e + f*x^2)^m] - 3*
a^2*b*Sqrt[f]*n*x*Log[d*(e + f*x^2)^m] + 6*a*b^2*Sqrt[f]*n^2*x*Log[d*(e + f
*x^2)^m] - 6*b^3*Sqrt[f]*n^3*x*Log[d*(e + f*x^2)^m] + 3*a^2*b*Sqrt[f]*x*Log
[c*x^n]*Log[d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[f]*n*x*Log[c*x^n]*Log[d*(e + f*
x^2)^m] + 6*b^3*Sqrt[f]*n^2*x*Log[c*x^n]*Log[d*(e + f*x^2)^m] + 3*a*b^2*Sqr
t[f]*x*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - 3*b^3*Sqrt[f]*n*x*Log[c*x^n]^2*L
og[d*(e + f*x^2)^m] + b^3*Sqrt[f]*x*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] - (3*
I)*b*Sqrt[e]*m*n*(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^
2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*Sqrt[e]*m*n*
(a^2 - 2*a*b*n + 2*b^2*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*P
olyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[e]*m*n^2*PolyLog[3, ((-
I)*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[3, ((-I)*Sqrt[f]*x
)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^2*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)
/Sqrt[e]] - (6*I)*a*b^2*Sqrt[e]*m*n^2*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (
6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e
]*m*n^2*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[e]*m*
n^3*PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[e]*m*n^3*PolyLog[
4, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[f]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(f\,x^{2}+e\right)^{m}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
[Out] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="maxima")
[Out] (b^3*m*x*log(x^n)^3 - 3*((m*n - m*log(c))*b^3 - a*b^2*m)*x*log(x^n)^2 - 3*(
2*(m*n - m*log(c))*a*b^2 - (2*m*n^2 - 2*m*n*log(c) + m*log(c)^2)*b^3 - a^2*
b*m)*x*log(x^n) - (3*(m*n - m*log(c))*a^2*b - 3*(2*m*n^2 - 2*m*n*log(c) + m
*log(c)^2)*a*b^2 + (6*m*n^3 - 6*m*n^2*log(c) + 3*m*n*log(c)^2 - m*log(c)^3)
*b^3 - a^3*m)*x)*log(f*x^2 + e) + integrate(-(((2*f*m - f*log(d))*b^3*x^2 -
 b^3*e*log(d))*log(x^n)^3 + ((2*f*m - f*log(d))*a^3 - 3*(2*f*m*n - (2*f*m -
 f*log(d))*log(c))*a^2*b + 3*(4*f*m*n^2 - 4*f*m*n*log(c) + (2*f*m - f*log(d
))*log(c)^2)*a*b^2 - (12*f*m*n^3 - 12*f*m*n^2*log(c) + 6*f*m*n*log(c)^2 - (
```

```
2*f*m - f*log(d))*log(c)^3)*b^3)*x^2 + 3*(((2*f*m - f*log(d))*a*b^2 - (2*f*
m*n - (2*f*m - f*log(d))*log(c))*b^3)*x^2 - (b^3*log(c)*log(d) + a*b^2*log(
d))*e)*log(x^n)^2 - (b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*
b*log(c)*log(d) + a^3*log(d))*e + 3*(((2*f*m - f*log(d))*a^2*b - 2*(2*f*m*n
 - (2*f*m - f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 - 4*f*m*n*log(c) + (2*f*m
- f*log(d))*log(c)^2)*b^3)*x^2 - (b^3*log(c)^2*log(d) + 2*a*b^2*log(c)*log(
d) + a^2*b*log(d))*e)*log(x^n))/(f*x^2 + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3,x)
[Out] int(log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3, x)
```

3.113 $\int \frac{(a+b\log(cx^n))^3 \log\left(d\left(e+fx^2\right)^m\right)}{x^2}\,dx$

Optimal. Leaf size=879

$$\frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log(cx^n))}{\sqrt{e}}+\frac{3b\sqrt{f}\,mn(a+b\log(cx^n))^2}{\sqrt{-e}}$$

```
[Out] -6*b^3*n^3*ln(d*(f*x^2+e)^m)/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/
x-3*b*n*(a+b*ln(c*x^n))^2*ln(d*(f*x^2+e)^m)/x-(a+b*ln(c*x^n))^3*ln(d*(f*x^2
+e)^m)/x+3*b*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^
(1/2)+m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-3*b
*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-m*(a+b
*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^2*m*n^2*(a+
b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-3*b*m*n*(a
+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^2*m
*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+3*b
*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6
*b^3*m*n^3*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^2*m*n^2*
(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^3*m
*n^3*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^2*m*n^2*(a+b*ln
(c*x^n))*polylog(3,x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)-6*b^3*m*n^3*pol
ylog(4,-x*f^(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+6*b^3*m*n^3*polylog(4,x*f^
(1/2)/(-e)^(1/2))*f^(1/2)/(-e)^(1/2)+12*b^3*m*n^3*arctan(x*f^(1/2)/e^(1/2))
*f^(1/2)/e^(1/2)+12*b^2*m*n^2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))*f^(
1/2)/e^(1/2)-6*I*b^3*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)+
6*I*b^3*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/2))*f^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 879, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2342, 2341, 2425, 211, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (12*b^3*Sqrt[f]*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/Sqrt[e] + (12*b^2*Sqrt[f
]*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/Sqrt[e] + (3*b*Sqrt
[f]*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] + (Sqr
t[f]*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (3*b*
Sqrt[f]*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] -
(Sqrt[f]*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e] - (
```

```
6*b^3*n^3*Log[d*(e + f*x^2)^m])/x - (6*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e
+ f*x^2)^m])/x - (3*b*n*(a + b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/x - ((a
+ b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x - (6*b^2*Sqrt[f]*m*n^2*(a + b*Log
[c*x^n])*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] - (3*b*Sqrt[f]*m*n*(
a + b*Log[c*x^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (6*b^2*
Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
 + (3*b*Sqrt[f]*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/
Sqrt[-e] - ((6*I)*b^3*Sqrt[f]*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/S
qrt[e] + ((6*I)*b^3*Sqrt[f]*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/Sqrt[e
] + (6*b^3*Sqrt[f]*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[-e] + (6
*b^2*Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/
Sqrt[-e] - (6*b^3*Sqrt[f]*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
- (6*b^2*Sqrt[f]*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]])
/Sqrt[-e] - (6*b^3*Sqrt[f]*m*n^3*PolyLog[4, -((Sqrt[f]*x)/Sqrt[-e])])/Sqrt[
-e] + (6*b^3*Sqrt[f]*m*n^3*PolyLog[4, (Sqrt[f]*x)/Sqrt[-e]])/Sqrt[-e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log\left(d(e+fx^2)^m\right)}{x^2}\,dx = -\frac{6b^3n^3\log\left(d(e+fx^2)^m\right)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log\left(d(e+\right.}{x}$$

$$= -\frac{6b^3n^3\log\left(d(e+fx^2)^m\right)}{x} - \frac{6b^2n^2(a+b\log(cx^n))\log\left(d(e+\right.}{x}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

$$= \frac{12b^3\sqrt{f}\,mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}} + \frac{12b^2\sqrt{f}\,mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right.}{\sqrt{e}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2166 vs. 2(879) = 1758.
time = 0.42, size = 2166, normalized size = 2.46

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^2,x]
```

```
[Out] (2*a^3*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 6*a^2*b*Sqrt[f]*m*n*x*ArcT
an[(Sqrt[f]*x)/Sqrt[e]] + 12*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[
e]] + 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 6*a^2*b*Sqrt[f]*
m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[
(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 12*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqr
t[e]]*Log[x] + 6*a*b^2*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2
 + 6*b^3*Sqrt[f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 2*b^3*Sqrt[
f]*m*n^3*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 + 6*a^2*b*Sqrt[f]*m*x*ArcTa
n[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*
x)/Sqrt[e]]*Log[c*x^n] + 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
*Log[c*x^n] - 12*a*b^2*Sqrt[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log
[c*x^n] - 12*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x
^n] + 6*b^3*Sqrt[f]*m*n^2*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n]
 + 6*a*b^2*Sqrt[f]*m*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 6*b^3*Sqr
t[f]*m*n*x*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 6*b^3*Sqrt[f]*m*n*x*A
rcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 + 2*b^3*Sqrt[f]*m*x*ArcTan[(
Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3 + (3*I)*a^2*b*Sqrt[f]*m*n*x*Log[x]*Log[1 -
 (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[f]*m*n^2*x*Log[x]*Log[1 - (I*Sqr
t[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]*Log[1 - (I*Sqrt[f]*x)/S
qrt[e]] - (3*I)*a*b^2*Sqrt[f]*m*n^2*x*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e
]] - (3*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + I*
b^3*Sqrt[f]*m*n^3*x*Log[x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*a*b^2*S
qrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*S
qrt[f]*m*n^2*x*Log[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3
*Sqrt[f]*m*n^2*x*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)
*b^3*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (3*
I)*a^2*b*Sqrt[f]*m*n*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*
Sqrt[f]*m*n^2*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m
*n^3*x*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*a*b^2*Sqrt[f]*m*n^2*x*
Log[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[f]*m*n^3*x*Log[x]^
2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - I*b^3*Sqrt[f]*m*n^3*x*Log[x]^3*Log[1 + (
I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*a*b^2*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]*Log[1 +
 (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^2*x*Log[x]*Log[c*x^n]*Log[1
 + (I*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b^3*Sqrt[f]*m*n^2*x*Log[x]^2*Log[c*x^n]*L
og[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (3*I)*b^3*Sqrt[f]*m*n*x*Log[x]*Log[c*x^n]^2
*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - a^3*Sqrt[e]*Log[d*(e + f*x^2)^m] - 3*a^2*
b*Sqrt[e]*n*Log[d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[e]*n^2*Log[d*(e + f*x^2)^m]
 - 6*b^3*Sqrt[e]*n^3*Log[d*(e + f*x^2)^m] - 3*a^2*b*Sqrt[e]*Log[c*x^n]*Log[
d*(e + f*x^2)^m] - 6*a*b^2*Sqrt[e]*n*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 6*b^
3*Sqrt[e]*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 3*a*b^2*Sqrt[e]*Log[c*x^n]^
2*Log[d*(e + f*x^2)^m] - 3*b^3*Sqrt[e]*n*Log[c*x^n]^2*Log[d*(e + f*x^2)^m]
- b^3*Sqrt[e]*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] - (3*I)*b*Sqrt[f]*m*n*x*(a^
2 + 2*a*b*n + 2*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*Poly
```

```
Log[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (3*I)*b*Sqrt[f]*m*n*x*(a^2 + 2*a*b*n + 2
*b^2*n^2 + 2*b*(a + b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[
f]*x)/Sqrt[e]] + (6*I)*a*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, ((-I)*Sqrt[f]*x)/Sq
rt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (
6*I)*b^3*Sqrt[f]*m*n^2*x*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] -
(6*I)*a*b^2*Sqrt[f]*m*n^2*x*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*S
qrt[f]*m*n^3*x*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^2*
x*Log[c*x^n]*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*Sqrt[f]*m*n^3*x*
PolyLog[4, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*Sqrt[f]*m*n^3*x*PolyLog[4,
 (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln \left(c\,x^{n}\right)\right)^{3} \ln \left(d\left(f\,x^{2}+e\right)^{m}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="maxima")
```

```
[Out] -(b^3*m*log(x^n)^3 + 3*(m*n + m*log(c))*a^2*b + 3*(2*m*n^2 + 2*m*n*log(c) +
 m*log(c)^2)*a*b^2 + (6*m*n^3 + 6*m*n^2*log(c) + 3*m*n*log(c)^2 + m*log(c)^
3)*b^3 + a^3*m + 3*((m*n + m*log(c))*b^3 + a*b^2*m)*log(x^n)^2 + 3*(2*(m*n
+ m*log(c))*a*b^2 + (2*m*n^2 + 2*m*n*log(c) + m*log(c)^2)*b^3 + a^2*b*m)*lo
g(x^n))*log(f*x^2 + e)/x + integrate((((2*f*m + f*log(d))*b^3*x^2 + b^3*e*l
og(d))*log(x^n)^3 + ((2*f*m + f*log(d))*a^3 + 3*(2*f*m*n + (2*f*m + f*log(d
))*log(c))*a^2*b + 3*(4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m + f*log(d))*log(c
)^2)*a*b^2 + (12*f*m*n^3 + 12*f*m*n^2*log(c) + 6*f*m*n*log(c)^2 + (2*f*m +
f*log(d))*log(c)^3)*b^3)*x^2 + 3*(((2*f*m + f*log(d))*a*b^2 + (2*f*m*n + (2
*f*m + f*log(d))*log(c))*b^3)*x^2 + (b^3*log(c)*log(d) + a*b^2*log(d))*e)*l
og(x^n)^2 + (b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*log(c)
*log(d) + a^3*log(d))*e + 3*(((2*f*m + f*log(d))*a^2*b + 2*(2*f*m*n + (2*f*
m + f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 + 4*f*m*n*log(c) + (2*f*m + f*log(
d))*log(c)^2)*b^3)*x^2 + (b^3*log(c)^2*log(d) + 2*a*b^2*log(c)*log(d) + a^2
*b*log(d))*e)*log(x^n))/(f*x^4 + x^2*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^2, x)
```

3.114 $\int \frac{(a+b\log(cx^n))^3\log\left(d\left(e+fx^2\right)^m\right)}{x^4}\,dx$

Optimal. Leaf size=1007

$$-\frac{160b^3fmn^3}{27ex}-\frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}}-\frac{52b^2fmn^2(a+b\log(cx^n))}{9ex}-\frac{4b^2f^{3/2}mn^2\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+}{9e^{3/2}}$$

```
[Out] -2/3*f*m*(a+b*ln(c*x^n))^3/e/x-2/27*b^3*n^3*ln(d*(f*x^2+e)^m)/x^3-160/27*b^
3*f*m*n^3/e/x-1/3*(a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^3-4/27*b^3*f^(3/2)*
m*n^3*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)+2/3*b^3*f^(3/2)*m*n^3*polylog(3,-x*
f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2/3*b^3*f^(3/2)*m*n^3*polylog(3,x*f^(1/2)/(-
e)^(1/2))/(-e)^(3/2)-2*b^3*f^(3/2)*m*n^3*polylog(4,-x*f^(1/2)/(-e)^(1/2))/(
-e)^(3/2)+2*b^3*f^(3/2)*m*n^3*polylog(4,x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2)-2/
9*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(f*x^2+e)^m)/x^3-1/3*b*n*(a+b*ln(c*x^n))^2*l
n(d*(f*x^2+e)^m)/x^3+1/3*f^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1-x*f^(1/2)/(-e)^(1
/2))/(-e)^(3/2)-1/3*f^(3/2)*m*(a+b*ln(c*x^n))^3*ln(1+x*f^(1/2)/(-e)^(1/2))/
(-e)^(3/2)-52/9*b^2*f*m*n^2*(a+b*ln(c*x^n))/e/x-8/3*b*f*m*n*(a+b*ln(c*x^n))
^2/e/x-4/9*b^2*f^(3/2)*m*n^2*arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*x^n))/e^(3
/2)+1/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*ln(1-x*f^(1/2)/(-e)^(1/2))/(-e)^(3/
2)-1/3*b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*ln(1+x*f^(1/2)/(-e)^(1/2))/(-e)^(3/2
)-2/3*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,-x*f^(1/2)/(-e)^(1/2))/(-
e)^(3/2)+2/3*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(2,x*f^(1/2)/(-e)^(1/
2))/(-e)^(3/2)+2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,-x*f^(1/2)/(-e
)^(1/2))/(-e)^(3/2)-2*b^2*f^(3/2)*m*n^2*(a+b*ln(c*x^n))*polylog(3,x*f^(1/2)
/(-e)^(1/2))/(-e)^(3/2)-2/9*I*b^3*f^(3/2)*m*n^3*polylog(2,I*x*f^(1/2)/e^(1/
2))/e^(3/2)-b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,-x*f^(1/2)/(-e)^(1/2)
)/(-e)^(3/2)+b*f^(3/2)*m*n*(a+b*ln(c*x^n))^2*polylog(2,x*f^(1/2)/(-e)^(1/2)
)/(-e)^(3/2)+2/9*I*b^3*f^(3/2)*m*n^3*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(3/2
)
```

Rubi [A]
time = 1.12, antiderivative size = 1007, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {2342, 2341, 2425, 331, 211, 2380, 2361, 12, 4940, 2438, 2367, 2354, 2421, 6724, 2430}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]
```

```
[Out] (-160*b^3*f*m*n^3)/(27*e*x) - (4*b^3*f^(3/2)*m*n^3*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(27*e^(3/2)) - (52*b^2*f*m*n^2*(a + b*Log[c*x^n]))/(9*e*x) - (4*b^2*f^
```

```
(3/2)*m*n^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*x^n]))/(9*e^(3/2)) - (
8*b*f*m*n*(a + b*Log[c*x^n])^2)/(3*e*x) - (2*f*m*(a + b*Log[c*x^n])^3)/(3*e
*x) + (b*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 - (Sqrt[f]*x)/Sqrt[-e]])/(3
*(-e)^(3/2)) + (f^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 - (Sqrt[f]*x)/Sqrt[-e]
])/(3*(-e)^(3/2)) - (b*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*Log[1 + (Sqrt[f]*x)
/Sqrt[-e]])/(3*(-e)^(3/2)) - (f^(3/2)*m*(a + b*Log[c*x^n])^3*Log[1 + (Sqrt[
f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) - (2*b^3*n^3*Log[d*(e + f*x^2)^m])/(27*x^3)
 - (2*b^2*n^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^2)^m])/(9*x^3) - (b*n*(a +
b*Log[c*x^n])^2*Log[d*(e + f*x^2)^m])/(3*x^3) - ((a + b*Log[c*x^n])^3*Log[d
*(e + f*x^2)^m])/(3*x^3) - (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[
2, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3/2)) - (b*f^(3/2)*m*n*(a + b*Log[c*x
^n])^2*PolyLog[2, -((Sqrt[f]*x)/Sqrt[-e])])/(-e)^(3/2) + (2*b^2*f^(3/2)*m*n
^2*(a + b*Log[c*x^n])*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(3*(-e)^(3/2)) + (b
*f^(3/2)*m*n*(a + b*Log[c*x^n])^2*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]])/(-e)^(3
/2) + (((2*I)/9)*b^3*f^(3/2)*m*n^3*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/e^
(3/2) - (((2*I)/9)*b^3*f^(3/2)*m*n^3*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/e^(
3/2) + (2*b^3*f^(3/2)*m*n^3*PolyLog[3, -((Sqrt[f]*x)/Sqrt[-e])])/(3*(-e)^(3
/2)) + (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((Sqrt[f]*x)/Sqr
t[-e])])/(-e)^(3/2) - (2*b^3*f^(3/2)*m*n^3*PolyLog[3, (Sqrt[f]*x)/Sqrt[-e]]
)/(3*(-e)^(3/2)) - (2*b^2*f^(3/2)*m*n^2*(a + b*Log[c*x^n])*PolyLog[3, (Sqrt
[f]*x)/Sqrt[-e]])/(-e)^(3/2) - (2*b^3*f^(3/2)*m*n^3*PolyLog[4, -((Sqrt[f]*x
)/Sqrt[-e])])/(-e)^(3/2) + (2*b^3*f^(3/2)*m*n^3*PolyLog[4, (Sqrt[f]*x)/Sqrt
[-e]])/(-e)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2425

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*
(a + b*Log[c*x^n])^p, x]}, Dist[Log[d*(e + f*x^m)^r], u, x] - Dist[f*m*r, I
nt[Dist[x^(m - 1)/(e + f*x^m), u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g
, r, m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^2)^m)}{x^4}\,dx = -\frac{2b^3n^3\log(d(e+fx^2)^m)}{27x^3} - \frac{2b^2n^2(a+b\log(cx^n))\log(d(e+\ldots}{9x^3}$$

$$= -\frac{2b^3n^3\log(d(e+fx^2)^m)}{27x^3} - \frac{2b^2n^2(a+b\log(cx^n))\log(d(e+\ldots}{9x^3}$$

$$= -\frac{4b^3fmn^3}{27ex} - \frac{2b^3n^3\log(d(e+fx^2)^m)}{27x^3} - \frac{2b^2n^2(a+b\log(cx^n)\ldots}{9x\ldots}$$

$$= -\frac{4b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{2b^3n^3\log(d(e+\ldots}{27x^3}$$

$$= -\frac{16b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{4b^2fmn^2(a+b\ldots}{9ex}$$

$$= -\frac{52b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{16b^2fmn^2(a+\ldots}{9ex}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2(a+\ldots}{9e\ldots}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2(a+\ldots}{9e\ldots}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2(a+\ldots}{9e\ldots}$$

$$= -\frac{160b^3fmn^3}{27ex} - \frac{4b^3f^{3/2}mn^3\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{27e^{3/2}} - \frac{52b^2fmn^2(a+\ldots}{9e\ldots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2488 vs. $2(1007) = 2014$.
time = 0.55, size = 2488, normalized size = 2.47

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^2)^m])/x^4,x]

[Out] (-18*a^3*Sqrt[e]*f*m*x^2 - 72*a^2*b*Sqrt[e]*f*m*n*x^2 - 156*a*b^2*Sqrt[e]*f
*m*n^2*x^2 - 160*b^3*Sqrt[e]*f*m*n^3*x^2 - 18*a^3*f^(3/2)*m*x^3*ArcTan[(Sqr
t[f]*x)/Sqrt[e]] - 18*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 1
2*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] - 4*b^3*f^(3/2)*m*n^3
*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]] + 54*a^2*b*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]
*x)/Sqrt[e]]*Log[x] + 36*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]
]*Log[x] + 12*b^3*f^(3/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x] - 54
*a*b^2*f^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 - 18*b^3*f^(3
/2)*m*n^3*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2 + 18*b^3*f^(3/2)*m*n^3*x
^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^3 - 54*a^2*b*Sqrt[e]*f*m*x^2*Log[c*x^
n] - 144*a*b^2*Sqrt[e]*f*m*n*x^2*Log[c*x^n] - 156*b^3*Sqrt[e]*f*m*n^2*x^2*L
og[c*x^n] - 54*a^2*b*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] -
 36*a*b^2*f^(3/2)*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] - 12*b^3*f
^(3/2)*m*n^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n] + 108*a*b^2*f^(3/2)
*m*n*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] + 36*b^3*f^(3/2)*m*n
^2*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n] - 54*b^3*f^(3/2)*m*n^2
*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[x]^2*Log[c*x^n] - 54*a*b^2*Sqrt[e]*f*m
*x^2*Log[c*x^n]^2 - 72*b^3*Sqrt[e]*f*m*n*x^2*Log[c*x^n]^2 - 54*a*b^2*f^(3/2
)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 - 18*b^3*f^(3/2)*m*n*x^3*A
rcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^2 + 54*b^3*f^(3/2)*m*n*x^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]]*Log[x]*Log[c*x^n]^2 - 18*b^3*Sqrt[e]*f*m*x^2*Log[c*x^n]^3
 - 18*b^3*f^(3/2)*m*x^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]]*Log[c*x^n]^3 - (27*I)*a
^2*b*f^(3/2)*m*n*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*a*b^2*f
^(3/2)*m*n^2*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (6*I)*b^3*f^(3/2)*
m*n^3*x^3*Log[x]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*a*b^2*f^(3/2)*m*n^
2*x^3*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^3*f^(3/2)*m*n^3*x^3
*Log[x]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[
x]^3*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*f^(3/2)*m*n*x^3*Log[x]*L
og[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^3*f^(3/2)*m*n^2*x^3*Log
[x]*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*b^3*f^(3/2)*m*n^2*x^
3*Log[x]^2*Log[c*x^n]*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*f^(3/2)*m
*n*x^3*Log[x]*Log[c*x^n]^2*Log[1 - (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*a^2*b*f^
(3/2)*m*n*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*a*b^2*f^(3/2)*
m*n^2*x^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (6*I)*b^3*f^(3/2)*m*n^3*x
^3*Log[x]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*a*b^2*f^(3/2)*m*n^2*x^3*L
og[x]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[x]
^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (9*I)*b^3*f^(3/2)*m*n^3*x^3*Log[x]^3*Lo
g[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*a*b^2*f^(3/2)*m*n*x^3*Log[x]*Log[c*x^
n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*f^(3/2)*m*n^2*x^3*Log[x]*Log
[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - (27*I)*b^3*f^(3/2)*m*n^2*x^3*Log[x
]^2*Log[c*x^n]*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] + (27*I)*b^3*f^(3/2)*m*n*x^3*
Log[x]*Log[c*x^n]^2*Log[1 + (I*Sqrt[f]*x)/Sqrt[e]] - 9*a^3*e^(3/2)*Log[d*(e
 + f*x^2)^m] - 9*a^2*b*e^(3/2)*n*Log[d*(e + f*x^2)^m] - 6*a*b^2*e^(3/2)*n^2

```
*Log[d*(e + f*x^2)^m] - 2*b^3*e^(3/2)*n^3*Log[d*(e + f*x^2)^m] - 27*a^2*b*e
^(3/2)*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 18*a*b^2*e^(3/2)*n*Log[c*x^n]*Log[
d*(e + f*x^2)^m] - 6*b^3*e^(3/2)*n^2*Log[c*x^n]*Log[d*(e + f*x^2)^m] - 27*a
*b^2*e^(3/2)*Log[c*x^n]^2*Log[d*(e + f*x^2)^m] - 9*b^3*e^(3/2)*n*Log[c*x^n]
^2*Log[d*(e + f*x^2)^m] - 9*b^3*e^(3/2)*Log[c*x^n]^3*Log[d*(e + f*x^2)^m] +
 (3*I)*b*f^(3/2)*m*n*x^3*(9*a^2 + 6*a*b*n + 2*b^2*n^2 + 6*b*(3*a + b*n)*Log
[c*x^n] + 9*b^2*Log[c*x^n]^2)*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (3*I)*
b*f^(3/2)*m*n*x^3*(9*a^2 + 6*a*b*n + 2*b^2*n^2 + 6*b*(3*a + b*n)*Log[c*x^n]
 + 9*b^2*Log[c*x^n]^2)*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]] - (54*I)*a*b^2*f^(
3/2)*m*n^2*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (18*I)*b^3*f^(3/2)*m*
n^3*x^3*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] - (54*I)*b^3*f^(3/2)*m*n^2*x^3
*Log[c*x^n]*PolyLog[3, ((-I)*Sqrt[f]*x)/Sqrt[e]] + (54*I)*a*b^2*f^(3/2)*m*n
^2*x^3*PolyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (18*I)*b^3*f^(3/2)*m*n^3*x^3*Pol
yLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*f^(3/2)*m*n^2*x^3*Log[c*x^n]*Po
lyLog[3, (I*Sqrt[f]*x)/Sqrt[e]] + (54*I)*b^3*f^(3/2)*m*n^3*x^3*PolyLog[4, (
(-I)*Sqrt[f]*x)/Sqrt[e]] - (54*I)*b^3*f^(3/2)*m*n^3*x^3*PolyLog[4, (I*Sqrt[
f]*x)/Sqrt[e]])/(27*e^(3/2)*x^3)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a +b \ln \left(c \,x^{n}\right)\right)^{3} \ln \left(d \left(f \,x^{2}+e \right)^{m}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(f*x^2+e)^m)/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^3*m*log(x^n)^3 + 9*(m*n + 3*m*log(c))*a^2*b + 3*(2*m*n^2 + 6*m*n
*log(c) + 9*m*log(c)^2)*a*b^2 + (2*m*n^3 + 6*m*n^2*log(c) + 9*m*n*log(c)^2
+ 9*m*log(c)^3)*b^3 + 9*a^3*m + 9*((m*n + 3*m*log(c))*b^3 + 3*a*b^2*m)*log(
x^n)^2 + 3*(6*(m*n + 3*m*log(c))*a*b^2 + (2*m*n^2 + 6*m*n*log(c) + 9*m*log(
c)^2)*b^3 + 9*a^2*b*m)*log(x^n))*log(f*x^2 + e)/x^3 + integrate(1/27*(9*((2
*f*m + 3*f*log(d))*b^3*x^2 + 3*b^3*e*log(d))*log(x^n)^3 + (9*(2*f*m + 3*f*l
og(d))*a^3 + 9*(2*f*m*n + 3*(2*f*m + 3*f*log(d))*log(c))*a^2*b + 3*(4*f*m*n
^2 + 12*f*m*n*log(c) + 9*(2*f*m + 3*f*log(d))*log(c)^2)*a*b^2 + (4*f*m*n^3
+ 12*f*m*n^2*log(c) + 18*f*m*n*log(c)^2 + 9*(2*f*m + 3*f*log(d))*log(c)^3)*
```

```
b^3)*x^2 + 9*((3*(2*f*m + 3*f*log(d))*a*b^2 + (2*f*m*n + 3*(2*f*m + 3*f*log
(d))*log(c))*b^3)*x^2 + 9*(b^3*log(c)*log(d) + a*b^2*log(d))*e)*log(x^n)^2
+ 27*(b^3*log(c)^3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*log(c)*log(d)
 + a^3*log(d))*e + 3*((9*(2*f*m + 3*f*log(d))*a^2*b + 6*(2*f*m*n + 3*(2*f*m
 + 3*f*log(d))*log(c))*a*b^2 + (4*f*m*n^2 + 12*f*m*n*log(c) + 9*(2*f*m + 3*
f*log(d))*log(c)^2)*b^3)*x^2 + 27*(b^3*log(c)^2*log(d) + 2*a*b^2*log(c)*log
(d) + a^2*b*log(d))*e)*log(x^n))/(f*x^6 + x^4*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="fricas")
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log((f*x^2 + e)^m*d)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(f*x**2+e)**m)/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(f*x^2+e)^m)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^2 + e)^m*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(f\,x^2+e\right)}^m\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4,x)
[Out] int((log(d*(e + f*x^2)^m)*(a + b*log(c*x^n))^3)/x^4, x)
```

3.115 $\int x^2 \log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal. Leaf size=403

$$-\frac{7be^5kn\sqrt{x}}{9f^5}+\frac{2be^4knx}{9f^4}-\frac{be^3knx^{3/2}}{9f^3}+\frac{5be^2knx^2}{72f^2}-\frac{11beknx^{5/2}}{225f}+\frac{1}{27}bknx^3+\frac{be^6kn\log\left(e+f\sqrt{x}\right)}{9f^6}-\frac{1}{9}bnx^3\log$$

```
[Out] 2/9*b*e^4*k*n*x/f^4-1/9*b*e^3*k*n*x^(3/2)/f^3+5/72*b*e^2*k*n*x^2/f^2-11/225
*b*e*k*n*x^(5/2)/f+1/27*b*k*n*x^3-1/6*e^4*k*x*(a+b*ln(c*x^n))/f^4+1/9*e^3*k
*x^(3/2)*(a+b*ln(c*x^n))/f^3-1/12*e^2*k*x^2*(a+b*ln(c*x^n))/f^2+1/15*e*k*x^
(5/2)*(a+b*ln(c*x^n))/f-1/18*k*x^3*(a+b*ln(c*x^n))+1/9*b*e^6*k*n*ln(e+f*x^(
1/2))/f^6-1/3*e^6*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^6+2/3*b*e^6*k*n*ln(-f
*x^(1/2)/e)*ln(e+f*x^(1/2))/f^6-1/9*b*n*x^3*ln(d*(e+f*x^(1/2))^k)+1/3*x^3*(
a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+2/3*b*e^6*k*n*polylog(2,1+f*x^(1/2)/e)
/f^6-7/9*b*e^5*k*n*x^(1/2)/f^5+1/3*e^5*k*(a+b*ln(c*x^n))*x^(1/2)/f^5
```

Rubi [A]
time = 0.23, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2442, 45, 2423, 2441, 2352}

$$\frac{2be^6kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3f^6}+\frac{1}{3}x^3(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^6}+\frac{e^5k\sqrt{x}(a+b\log(cx^n))}{3f^5}-\frac{e^4kx(a+b\log(cx^n))}{6f^4}+\frac{e^3kx^{3/2}(a+b\log(cx^n))}{9f^3}-\frac{e^2kx^2(a+b\log(cx^n))}{12f^2}+\frac{ekx^{5/2}(a+b\log(cx^n))}{15f}-\frac{1}{18}kx^3(a+b\log(cx^n))-\frac{1}{9}bnx^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{be^6kn\log\left(e+f\sqrt{x}\right)}{9f^6}+\frac{2be^6kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3f^6}-\frac{7be^5kn\sqrt{x}}{9f^5}+\frac{2be^4knx}{9f^4}-\frac{be^3knx^{3/2}}{9f^3}+\frac{5be^2knx^2}{72f^2}-\frac{11beknx^{5/2}}{225f}+\frac{1}{27}bknx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-7*b*e^5*k*n*Sqrt[x])/(9*f^5) + (2*b*e^4*k*n*x)/(9*f^4) - (b*e^3*k*n*x^(3/
2))/(9*f^3) + (5*b*e^2*k*n*x^2)/(72*f^2) - (11*b*e*k*n*x^(5/2))/(225*f) + (
b*k*n*x^3)/27 + (b*e^6*k*n*Log[e + f*Sqrt[x]])/(9*f^6) - (b*n*x^3*Log[d*(e
+ f*Sqrt[x])^k])/9 + (2*b*e^6*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])
/(3*f^6) + (e^5*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^5) - (e^4*k*x*(a + b*Log
[c*x^n]))/(6*f^4) + (e^3*k*x^(3/2)*(a + b*Log[c*x^n]))/(9*f^3) - (e^2*k*x^2
*(a + b*Log[c*x^n]))/(12*f^2) + (e*k*x^(5/2)*(a + b*Log[c*x^n]))/(15*f) - (
k*x^3*(a + b*Log[c*x^n]))/18 - (e^6*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n])
)/(3*f^6) + (x^3*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/3 + (2*b*e^6*
k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*f^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log (cx^n))\, dx = \frac{e^5 k\sqrt{x}\,(a+b\log (cx^n))}{3f^5} - \frac{e^4 kx(a+b\log (cx^n))}{6f^4} + \frac{e^3 kx^{3/}}{}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2bekn}{75}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2bekn}{75}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2bekn}{75}$$

$$= -\frac{2be^5kn\sqrt{x}}{3f^5} + \frac{be^4knx}{6f^4} - \frac{2be^3knx^{3/2}}{27f^3} + \frac{be^2knx^2}{24f^2} - \frac{2bekn}{75}$$

$$= -\frac{7be^5kn\sqrt{x}}{9f^5} + \frac{2be^4knx}{9f^4} - \frac{be^3knx^{3/2}}{9f^3} + \frac{5be^2knx^2}{72f^2} - \frac{11be}{2}$$

Mathematica [A]
time = 0.34, size = 434, normalized size = 1.08

$$-\frac{-1800ae^5fk\sqrt{x}+4200be^5fkn\sqrt{x}+900ae^4f^2kx-1200be^4f^2knx-600ae^3f^3kx^{3/2}+600be^3f^3knx^{3/2}+450ae^2f^4kx^2-375be^2f^4knx^2-360aef^5kx^{5/2}+264bef^5knx^{5/2}+300af^6kx^3-200bf^6knx^3-1800af^6x^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)+600bf^6nx^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)+1800be^6kn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-1800be^5fk\sqrt{x}\log(cx^n)+900be^4f^2kx\log(cx^n)-600be^3f^3kx^{3/2}\log(cx^n)+450be^2f^4kx^2\log(cx^n)-360bef^5kx^{5/2}\log(cx^n)+300bf^6kx^3\log(cx^n)-1800bf^6x^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+600e^6k\log\left(e+f\sqrt{x}\right)(3a-bn-3bn\log(x)+3b\log(cx^n))+3600be^6kn\operatorname{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{5400f^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/5400*(-1800*a*e^5*f*k*Sqrt[x] + 4200*b*e^5*f*k*n*Sqrt[x] + 900*a*e^4*f^2
*k*x - 1200*b*e^4*f^2*k*n*x - 600*a*e^3*f^3*k*x^(3/2) + 600*b*e^3*f^3*k*n*x
^(3/2) + 450*a*e^2*f^4*k*x^2 - 375*b*e^2*f^4*k*n*x^2 - 360*a*e*f^5*k*x^(5/2
) + 264*b*e*f^5*k*n*x^(5/2) + 300*a*f^6*k*x^3 - 200*b*f^6*k*n*x^3 - 1800*a*
f^6*x^3*Log[d*(e + f*Sqrt[x])^k] + 600*b*f^6*n*x^3*Log[d*(e + f*Sqrt[x])^k]
 + 1800*b*e^6*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 1800*b*e^5*f*k*Sqrt[x]*Lo
g[c*x^n] + 900*b*e^4*f^2*k*x*Log[c*x^n] - 600*b*e^3*f^3*k*x^(3/2)*Log[c*x^n
] + 450*b*e^2*f^4*k*x^2*Log[c*x^n] - 360*b*e*f^5*k*x^(5/2)*Log[c*x^n] + 300
*b*f^6*k*x^3*Log[c*x^n] - 1800*b*f^6*x^3*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n
] + 600*e^6*k*Log[e + f*Sqrt[x]]*(3*a - b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]
) + 3600*b*e^6*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/f^6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2 (a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima"
)
```

```
[Out] 1/441*(147*b*x^3*e*log(d)*log(x^n) - 49*((n*log(d) - 3*log(c)*log(d))*b - 3
*a*log(d))*x^3*e + 49*(3*b*x^3*e*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3*e)
*k*log(f*sqrt(x) + e) - (21*b*f*k*x^4*log(x^n) + (21*a*f*k - (13*f*k*n - 21
*f*k*log(c))*b)*x^4)/sqrt(x))*e^(-1) + integrate(1/18*(3*b*f^2*k*x^3*log(x^
n) + (3*a*f^2*k - (f^2*k*n - 3*f^2*k*log(c))*b)*x^3)/(f*e^(1/2*log(x) + 1)
+ e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x^2*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

3.116 $\int x \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx$

Optimal. Leaf size=313

$$-\frac{5be^3kn\sqrt{x}}{4f^3}+\frac{3be^2knx}{8f^2}-\frac{7beknx^{3/2}}{36f}+\frac{1}{8}bknx^2+\frac{be^4kn\log\left(e+f\sqrt{x}\right)}{4f^4}-\frac{1}{4}bnx^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{be^4kn\,\mathrm{lo}}{}$$

```
[Out] 3/8*b*e^2*k*n*x/f^2-7/36*b*e*k*n*x^(3/2)/f+1/8*b*k*n*x^2-1/4*e^2*k*x*(a+b*l
n(c*x^n))/f^2+1/6*e*k*x^(3/2)*(a+b*ln(c*x^n))/f-1/8*k*x^2*(a+b*ln(c*x^n))+1
/4*b*e^4*k*n*ln(e+f*x^(1/2))/f^4-1/2*e^4*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/
f^4+b*e^4*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^4-1/4*b*n*x^2*ln(d*(e+f*x^
(1/2))^k)+1/2*x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+b*e^4*k*n*polylog(2
,1+f*x^(1/2)/e)/f^4-5/4*b*e^3*k*n*x^(1/2)/f^3+1/2*e^3*k*(a+b*ln(c*x^n))*x^(
1/2)/f^3
```

Rubi [A]
time = 0.17, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2504, 2442, 45, 2423, 2441, 2352}

$$\frac{be^4kn\,\mathrm{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^4}+\frac{1}{2}x^2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2f^4}+\frac{e^3k\sqrt{x}(a+b\log(cx^n))}{2f^3}-\frac{e^2kx(a+b\log(cx^n))}{4f^2}+\frac{ekx^{3/2}(a+b\log(cx^n))}{6f}-\frac{1}{8}kx^2(a+b\log(cx^n))-\frac{1}{4}bnx^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{be^4kn\log\left(e+f\sqrt{x}\right)}{4f^4}+\frac{be^4kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^4}-\frac{5be^3kn\sqrt{x}}{4f^3}+\frac{3be^2knx}{8f^2}-\frac{7beknx^{3/2}}{36f}+\frac{1}{8}bknx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-5*b*e^3*k*n*Sqrt[x])/(4*f^3) + (3*b*e^2*k*n*x)/(8*f^2) - (7*b*e*k*n*x^(3/
2))/(36*f) + (b*k*n*x^2)/8 + (b*e^4*k*n*Log[e + f*Sqrt[x]])/(4*f^4) - (b*n*
x^2*Log[d*(e + f*Sqrt[x])^k])/4 + (b*e^4*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sq
rt[x])/e)])/f^4 + (e^3*k*Sqrt[x]*(a + b*Log[c*x^n]))/(2*f^3) - (e^2*k*x*(a
+ b*Log[c*x^n]))/(4*f^2) + (e*k*x^(3/2)*(a + b*Log[c*x^n]))/(6*f) - (k*x^2*
(a + b*Log[c*x^n]))/8 - (e^4*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*f^
4) + (x^2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/2 + (b*e^4*k*n*PolyL
og[2, 1 + (f*Sqrt[x])/e])/f^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \frac{e^3k\sqrt{x}\,(a+b\log(cx^n))}{2f^3} - \frac{e^2kx(a+b\log(cx^n))}{4f^2} + \frac{ekx^{3/2}(a$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 + \frac{e^3k\sqrt{x}\,(a$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 + \frac{e^3k\sqrt{x}\,(a$$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 - \frac{1}{4}bnx^2\log($$

$$= -\frac{be^3kn\sqrt{x}}{f^3} + \frac{be^2knx}{4f^2} - \frac{beknx^{3/2}}{9f} + \frac{1}{16}bknx^2 - \frac{1}{4}bnx^2\log($$

$$= -\frac{5be^3kn\sqrt{x}}{4f^3} + \frac{3be^2knx}{8f^2} - \frac{7beknx^{3/2}}{36f} + \frac{1}{8}bknx^2 + \frac{be^4kn\log$$

Mathematica [A]
time = 0.25, size = 336, normalized size = 1.07

$$-\frac{-36ae^3fk\sqrt{x}+90be^3fkn\sqrt{x}+18ae^2f^2kx-27be^2f^2knx-12aef^3kx^{3/2}+14bef^3knx^{3/2}+9af^4kx^2-9bf^4knx^2-36af^4x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+18bf^4nx^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+36be^4kn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-36be^3fk\sqrt{x}\log(cx^n)+18be^2f^2kx\log(cx^n)-12bef^3kx^{3/2}\log(cx^n)+9bf^4kx^2\log(cx^n)-36bf^4x^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+18e^4k\log\left(e+f\sqrt{x}\right)(2a-bn-2bn\log(x)+2b\log(cx^n))+72be^4kn\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{72f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/72*(-36*a*e^3*f*k*Sqrt[x] + 90*b*e^3*f*k*n*Sqrt[x] + 18*a*e^2*f^2*k*x -
27*b*e^2*f^2*k*n*x - 12*a*e*f^3*k*x^(3/2) + 14*b*e*f^3*k*n*x^(3/2) + 9*a*f^
4*k*x^2 - 9*b*f^4*k*n*x^2 - 36*a*f^4*x^2*Log[d*(e + f*Sqrt[x])^k] + 18*b*f^
4*n*x^2*Log[d*(e + f*Sqrt[x])^k] + 36*b*e^4*k*n*Log[1 + (f*Sqrt[x])/e]*Log[
x] - 36*b*e^3*f*k*Sqrt[x]*Log[c*x^n] + 18*b*e^2*f^2*k*x*Log[c*x^n] - 12*b*e
*f^3*k*x^(3/2)*Log[c*x^n] + 9*b*f^4*k*x^2*Log[c*x^n] - 36*b*f^4*x^2*Log[d*(
e + f*Sqrt[x])^k]*Log[c*x^n] + 18*e^4*k*Log[e + f*Sqrt[x]]*(2*a - b*n - 2*b
*n*Log[x] + 2*b*Log[c*x^n]) + 72*b*e^4*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/f^
4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima")
```

```
[Out] 1/100*(50*b*x^2*e*log(d)*log(x^n) - 25*((n*log(d) - 2*log(c)*log(d))*b - 2*
a*log(d))*x^2*e + 25*(2*b*x^2*e*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2*e)*
k*log(f*sqrt(x) + e) - (10*b*f*k*x^3*log(x^n) + (10*a*f*k - (9*f*k*n - 10*f
*k*log(c))*b)*x^3)/sqrt(x))*e^(-1) + integrate(1/8*(2*b*f^2*k*x^2*log(x^n)
+ (2*a*f^2*k - (f^2*k*n - 2*f^2*k*log(c))*b)*x^2)/(f*e^(1/2*log(x) + 1) + e
^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

3.117 $\int \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log (cx^n)) \, dx$

Optimal. Leaf size=209

$$-\frac{3bekn\sqrt{x}}{f}+bknx+\frac{be^2kn\log \left(e+f\sqrt{x}\right)}{f^2}-bnx\log \left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2be^2kn\log \left(e+f\sqrt{x}\right)\log \left(-\frac{f\sqrt{x}}{e}\right.}{f^2}$$

```
[Out] b*k*n*x-1/2*k*x*(a+b*ln(c*x^n))+b*e^2*k*n*ln(e+f*x^(1/2))/f^2-e^2*k*(a+b*ln
(c*x^n))*ln(e+f*x^(1/2))/f^2+2*b*e^2*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f
^2-b*n*x*ln(d*(e+f*x^(1/2))^k)+x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)+2*b*
e^2*k*n*polylog(2,1+f*x^(1/2)/e)/f^2-3*b*e*k*n*x^(1/2)/f+e*k*(a+b*ln(c*x^n)
)*x^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2498, 272, 45, 2417, 2504, 2441, 2352}

$$\frac{2be^2kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^2}+x(a+b\log (cx^n))\log \left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{e^2k\log \left(e+f\sqrt{x}\right)(a+b\log (cx^n))}{f^2}+\frac{ek\sqrt{x}(a+b\log (cx^n))}{f}-\frac{1}{2}kx(a+b\log (cx^n))-bnx\log \left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{be^2kn\log \left(e+f\sqrt{x}\right)}{f^2}+\frac{2be^2kn\log \left(e+f\sqrt{x}\right)\log \left(-\frac{f\sqrt{x}}{e}\right)}{f^2}-\frac{3bekn\sqrt{x}}{f}+bknx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-3*b*e*k*n*Sqrt[x])/f + b*k*n*x + (b*e^2*k*n*Log[e + f*Sqrt[x]])/f^2 - b*n
*x*Log[d*(e + f*Sqrt[x])^k] + (2*b*e^2*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt
[x])/e)])/f^2 + (e*k*Sqrt[x]*(a + b*Log[c*x^n]))/f - (k*x*(a + b*Log[c*x^n]
))/2 - (e^2*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + x*Log[d*(e + f*S
qrt[x])^k]*(a + b*Log[c*x^n]) + (2*b*e^2*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])
/f^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = \frac{ek\sqrt{x}\,(a+b\log(cx^n))}{f} - \frac{1}{2}kx(a+b\log(cx^n)) - \frac{e^2k\log(e+$$

$$= -\frac{2bekn\sqrt{x}}{f} + \frac{1}{2}bknx + \frac{ek\sqrt{x}\,(a+b\log(cx^n))}{f} - \frac{1}{2}kx(a+$$

$$= -\frac{2bekn\sqrt{x}}{f} + \frac{1}{2}bknx - bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right) + \frac{ek\sqrt{x}\,(}{}$$

$$= -\frac{2bekn\sqrt{x}}{f} + \frac{1}{2}bknx - bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right) + \frac{2be^2kn}{}$$

$$= -\frac{2bekn\sqrt{x}}{f} + \frac{1}{2}bknx - bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right) + \frac{2be^2kn}{}$$

$$= -\frac{3bekn\sqrt{x}}{f} + bknx + \frac{be^2kn\log\left(e+f\sqrt{x}\right)}{f^2} - bnx\log\left(d(e\right.$$

Mathematica [A]
time = 0.16, size = 218, normalized size = 1.04

$$\frac{aek\sqrt{x}}{f} - \frac{3bekn\sqrt{x}}{f} - \frac{akx}{2} + bknx + ax\log\left(d\left(e+f\sqrt{x}\right)^k\right) - bnx\log\left(d\left(e+f\sqrt{x}\right)^k\right) - \frac{be^2kn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)}{f^2} + \frac{bek\sqrt{x}\,\log(cx^n)}{f} - \frac{1}{2}bkx\log(cx^n) + bx\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n) - \frac{e^2k\log\left(e+f\sqrt{x}\right)(a-bn-bn\log(x)+b\log(cx^n))}{f^2} - \frac{2be^2kn\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (a*e*k*Sqrt[x])/f - (3*b*e*k*n*Sqrt[x])/f - (a*k*x)/2 + b*k*n*x + a*x*Log[d
*(e + f*Sqrt[x])^k] - b*n*x*Log[d*(e + f*Sqrt[x])^k] - (b*e^2*k*n*Log[1 + (
f*Sqrt[x])/e]*Log[x])/f^2 + (b*e*k*Sqrt[x]*Log[c*x^n])/f - (b*k*x*Log[c*x^n
])/2 + b*x*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] - (e^2*k*Log[e + f*Sqrt[x]]*
(a - b*n - b*n*Log[x] + b*Log[c*x^n]))/f^2 - (2*b*e^2*k*n*PolyLog[2, -((f*S
qrt[x])/e)])/f^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="maxima")
[Out] 1/9*(9*b*x*e*log(d)*log(x^n) - 9*((n*log(d) - log(c)*log(d))*b - a*log(d))*
x*e + 9*(b*x*e*log(x^n) - (b*(n - log(c)) - a)*x*e)*k*log(f*sqrt(x) + e) -
(3*b*f*k*x^2*log(x^n) + (3*a*f*k - (5*f*k*n - 3*f*k*log(c))*b)*x^2)/sqrt(x)
)*e^(-1) + integrate(1/2*(b*f^2*k*x*log(x^n) + (a*f^2*k - (f^2*k*n - f^2*k*
log(c))*b)*x)/(f*e^(1/2*log(x) + 1) + e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

3.118 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=117

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn}-\frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{2bn}-2k(a+b\log\left(cx^n\right))\operatorname{Li}_2\left(-\frac{f\sqrt{x}}{e}\right.$$

[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))^k)/b/n-1/2*k*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/b/n-2*k*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)+4*b*k*n*polylog(3,-f*x^(1/2)/e)

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2422, 2375, 2421, 6724}

$$-2k\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log\left(cx^n\right))+4bkn\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)+\frac{(a+b\log\left(cx^n\right))^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2bn}-\frac{k\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log\left(cx^n\right))^2}{2bn}$$

Antiderivative was successfully verified.

[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]

[Out] (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n])^2)/(2*b*n) - (k*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(2*b*n) - 2*k*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)] + 4*b*k*n*PolyLog[3, -((f*Sqrt[x])/e)]

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))^2}{2bn} - \frac{(fk)\int \frac{(a+b\log(cx^n))}{\left(e+f\sqrt{x}\right)\sqrt{}}}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)}{2bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)}{2bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))^2}{2bn} - \frac{k\log\left(1+\frac{f\sqrt{x}}{e}\right)}{2bn}$$

Mathematica [A]
time = 0.13, size = 186, normalized size = 1.59

$$\frac{1}{2}\left(4a\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log\left(-\frac{f\sqrt{x}}{e}\right)-bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log^2(x)+bkn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x)+2b\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(x)\log(cx^n)-2bk\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)\log(cx^n)+4ak\text{Li}_2\left(1+\frac{f\sqrt{x}}{e}\right)-4bk\log(cx^n)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)+8bkn\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x,x]
```

```
[Out] (4*a*Log[d*(e + f*Sqrt[x])^k]*Log[-((f*Sqrt[x])/e)] - b*n*Log[d*(e + f*Sqrt
[x])^k]*Log[x]^2 + b*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 2*b*Log[d*(e + f
*Sqrt[x])^k]*Log[x]*Log[c*x^n] - 2*b*k*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*
x^n] + 4*a*k*PolyLog[2, 1 + (f*Sqrt[x])/e] - 4*b*k*Log[c*x^n]*PolyLog[2, -(
(f*Sqrt[x])/e)] + 8*b*k*n*PolyLog[3, -((f*Sqrt[x])/e)])/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^{n}))\ln\left(d\left(e+f\sqrt{x}\,\right)^{k}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x,x)
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="maxima")
[Out] -1/2*(b*n*e*log(d)*log(x)^2 - 2*b*e*log(d)*log(x)*log(x^n) + (b*n*e*log(x)^
2 - 2*b*e*log(x)*log(x^n) - 2*(b*log(c) + a)*e*log(x))*k*log(f*sqrt(x) + e)
 - 2*(b*log(c)*log(d) + a*log(d))*e*log(x) - (b*f*k*n*x*log(x)^2 - 2*(b*f*k
*log(c) + a*f*k)*x*log(x) + 4*(a*f*k - (2*f*k*n - f*k*log(c))*b)*x - 2*(b*f
*k*x*log(x) - 2*b*f*k*x)*log(x^n))/sqrt(x))*e^(-1) + integrate(-1/4*(b*f^2*
k*n*log(x)^2 - 2*b*f^2*k*log(x)*log(x^n) - 2*(b*f^2*k*log(c) + a*f^2*k)*log
(x))/(f*e^(1/2*log(x) + 1) + e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x,x)
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x, x)
```

3.119 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx$

Optimal. Leaf size=248

$$-\frac{3bfkn}{e\sqrt{x}}+\frac{bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}-\frac{2bf^2kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{bf^2kn\,\mathrm{lo}\cdots}{2e^2\cdots}$$

```
[Out] -1/2*b*f^2*k*n*ln(x)/e^2+1/4*b*f^2*k*n*ln(x)^2/e^2-1/2*f^2*k*ln(x)*(a+b*ln(
c*x^n))/e^2+b*f^2*k*n*ln(e+f*x^(1/2))/e^2+f^2*k*(a+b*ln(c*x^n))*ln(e+f*x^(1
/2))/e^2-2*b*f^2*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^2-b*n*ln(d*(e+f*x^(
1/2))^k)/x-(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x-2*b*f^2*k*n*polylog(2,1+
f*x^(1/2)/e)/e^2-3*b*f*k*n/e/x^(1/2)-f*k*(a+b*ln(c*x^n))/e/x^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2442, 46, 2423, 2441, 2352, 2338}

$$-\frac{2bf^2kn\,\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^2}-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2}-\frac{f^2k\log(x)(a+b\log(cx^n))}{2e^2}-\frac{fk(a+b\log(cx^n))}{e\sqrt{x}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x}+\frac{bf^2kn\log^2(x)}{4e^2}+\frac{bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{2bf^2kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{bf^2kn\log(x)}{2e^2}-\frac{3bfkn}{e\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] (-3*b*f*k*n)/(e*Sqrt[x]) + (b*f^2*k*n*Log[e + f*Sqrt[x]])/e^2 - (b*n*Log[d*
(e + f*Sqrt[x])^k])/x - (2*b*f^2*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e
)])/e^2 - (b*f^2*k*n*Log[x])/(2*e^2) + (b*f^2*k*n*Log[x]^2)/(4*e^2) - (f*k*
(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (f^2*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^
n]))/e^2 - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x - (f^2*k*Log[x]*
(a + b*Log[c*x^n]))/(2*e^2) - (2*b*f^2*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/e
^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^2}\,dx = -\frac{fk(a+b\log(cx^n))}{e\sqrt{x}} + \frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2} -$$

$$= -\frac{2bfkn}{e\sqrt{x}} - \frac{fk(a+b\log(cx^n))}{e\sqrt{x}} + \frac{f^2k\log\left(e+f\sqrt{x}\right)(a+b\log$$

$$= -\frac{2bfkn}{e\sqrt{x}} + \frac{bf^2kn\log^2(x)}{4e^2} - \frac{fk(a+b\log(cx^n))}{e\sqrt{x}} + \frac{f^2k\log(e-$$

$$= -\frac{2bfkn}{e\sqrt{x}} - \frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x} - \frac{2bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}$$

$$= -\frac{2bfkn}{e\sqrt{x}} - \frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x} - \frac{2bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2}$$

$$= -\frac{3bfkn}{e\sqrt{x}} + \frac{bf^2kn\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{x} -$$

Mathematica [A]
time = 0.23, size = 250, normalized size = 1.01

$$-\frac{4aefk\sqrt{x}+12befkn\sqrt{x}+4ae^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)+4be^2n\log\left(d\left(e+f\sqrt{x}\right)^k\right)+2af^2kx\log(x)+2bf^2knx\log(x)-4bf^2knx\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-bf^2knx\log^2(x)+4befk\sqrt{x}\log(cx^n)+4be^2\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+2bf^2kx\log(x)\log(cx^n)-4f^2kx\log\left(e+f\sqrt{x}\right)(a+bn-bn\log(x)+b\log(cx^n))-8bf^2knx\,\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{4e^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^2,x]
```

```
[Out] -1/4*(4*a*e*f*k*Sqrt[x] + 12*b*e*f*k*n*Sqrt[x] + 4*a*e^2*Log[d*(e + f*Sqrt[
x])^k] + 4*b*e^2*n*Log[d*(e + f*Sqrt[x])^k] + 2*a*f^2*k*x*Log[x] + 2*b*f^2*
k*n*x*Log[x] - 4*b*f^2*k*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - b*f^2*k*n*x*Lo
g[x]^2 + 4*b*e*f*k*Sqrt[x]*Log[c*x^n] + 4*b*e^2*Log[d*(e + f*Sqrt[x])^k]*Lo
g[c*x^n] + 2*b*f^2*k*x*Log[x]*Log[c*x^n] - 4*f^2*k*x*Log[e + f*Sqrt[x]]*(a
+ b*n - b*n*Log[x] + b*Log[c*x^n]) - 8*b*f^2*k*n*x*PolyLog[2, -((f*Sqrt[x])
/e)])/(e^2*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="maxima"
)
```

```
[Out] -(b*e*log(d)*log(x^n) + (b*e*log(x^n) + (b*(n + log(c)) + a)*e)*k*log(f*sqr
t(x) + e) + ((n*log(d) + log(c)*log(d))*b + a*log(d))*e + (b*f*k*x*log(x^n)
 + (a*f*k + (3*f*k*n + f*k*log(c))*b)*x)/sqrt(x))*e^(-1)/x - integrate(1/2*
(b*f^2*k*log(x^n) + a*f^2*k + (f^2*k*n + f^2*k*log(c))*b)/(f*x*e^(1/2*log(x
) + 1) + x*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^2, x)
```

3.120 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx$

Optimal. Leaf size=346

$$-\frac{7bfkn}{36ex^{3/2}}+\frac{3bf^2kn}{8e^2x}-\frac{5bf^3kn}{4e^3\sqrt{x}}+\frac{bf^4kn\log\left(e+f\sqrt{x}\right)}{4e^4}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}-\frac{bf^4kn\log\left(e+f\sqrt{x}\right)\log}{e^4}$$

```
[Out] -7/36*b*f*k*n/e/x^(3/2)+3/8*b*f^2*k*n/e^2/x-1/8*b*f^4*k*n*ln(x)/e^4+1/8*b*f
^4*k*n*ln(x)^2/e^4-1/6*f*k*(a+b*ln(c*x^n))/e/x^(3/2)+1/4*f^2*k*(a+b*ln(c*x^
n))/e^2/x-1/4*f^4*k*ln(x)*(a+b*ln(c*x^n))/e^4+1/4*b*f^4*k*n*ln(e+f*x^(1/2))
/e^4+1/2*f^4*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^4-b*f^4*k*n*ln(-f*x^(1/2)/
e)*ln(e+f*x^(1/2))/e^4-1/4*b*n*ln(d*(e+f*x^(1/2))^k)/x^2-1/2*(a+b*ln(c*x^n)
)*ln(d*(e+f*x^(1/2))^k)/x^2-b*f^4*k*n*polylog(2,1+f*x^(1/2)/e)/e^4-5/4*b*f^
3*k*n/e^3/x^(1/2)-1/2*f^3*k*(a+b*ln(c*x^n))/e^3/x^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2442, 46, 2423, 2441, 2352, 2338}

$$-\frac{bf^4kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^4}-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{2x^2}+\frac{f^4k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{2e^4}-\frac{f^4k\log(x)(a+b\log(cx^n))}{4e^4}-\frac{f^3k(a+b\log(cx^n))}{2e^3\sqrt{x}}+\frac{f^2k(a+b\log(cx^n))}{4e^2x}-\frac{fk(a+b\log(cx^n))}{6ex^{3/2}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2}+\frac{bf^4kn\log^2(x)}{8e^4}+\frac{bf^4kn\log\left(e+f\sqrt{x}\right)}{4e^4}-\frac{bf^4kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^4}-\frac{bf^4kn\log(x)}{8e^4}-\frac{5bf^3kn}{4e^3\sqrt{x}}+\frac{3bf^2kn}{8e^2x}-\frac{7bfkn}{36ex^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] (-7*b*f*k*n)/(36*e*x^(3/2)) + (3*b*f^2*k*n)/(8*e^2*x) - (5*b*f^3*k*n)/(4*e^
3*Sqrt[x]) + (b*f^4*k*n*Log[e + f*Sqrt[x]])/(4*e^4) - (b*n*Log[d*(e + f*Sqr
t[x])^k])/(4*x^2) - (b*f^4*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e^
4 - (b*f^4*k*n*Log[x])/(8*e^4) + (b*f^4*k*n*Log[x]^2)/(8*e^4) - (f*k*(a + b
*Log[c*x^n]))/(6*e*x^(3/2)) + (f^2*k*(a + b*Log[c*x^n]))/(4*e^2*x) - (f^3*k
*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[x]) + (f^4*k*Log[e + f*Sqrt[x]]*(a + b*Log
[c*x^n]))/(2*e^4) - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(2*x^2) -
 (f^4*k*Log[x]*(a + b*Log[c*x^n]))/(4*e^4) - (b*f^4*k*n*PolyLog[2, 1 + (f*S
qrt[x])/e])/e^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^3}\,dx = -\frac{fk(a+b\log(cx^n))}{6ex^{3/2}} + \frac{f^2k(a+b\log(cx^n))}{4e^2x} - \frac{f^3k(a+b\log}{2e^3\sqrt{x}}$$

$$= -\frac{bfkn}{9ex^{3/2}} + \frac{bf^2kn}{4e^2x} - \frac{bf^3kn}{e^3\sqrt{x}} - \frac{fk(a+b\log(cx^n))}{6ex^{3/2}} + \frac{f^2k(a+}{}$$

$$= -\frac{bfkn}{9ex^{3/2}} + \frac{bf^2kn}{4e^2x} - \frac{bf^3kn}{e^3\sqrt{x}} + \frac{bf^4kn\log^2(x)}{8e^4} - \frac{fk(a+b\log}{6ex^{3/2}}$$

$$= -\frac{bfkn}{9ex^{3/2}} + \frac{bf^2kn}{4e^2x} - \frac{bf^3kn}{e^3\sqrt{x}} - \frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2} - \frac{bf^4}{}$$

$$= -\frac{bfkn}{9ex^{3/2}} + \frac{bf^2kn}{4e^2x} - \frac{bf^3kn}{e^3\sqrt{x}} - \frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{4x^2} - \frac{bf^4}{}$$

$$= -\frac{7bfkn}{36ex^{3/2}} + \frac{3bf^2kn}{8e^2x} - \frac{5bf^3kn}{4e^3\sqrt{x}} + \frac{bf^4kn\log\left(e+f\sqrt{x}\right)}{4e^4} - \frac{b}{}$$

Mathematica [A]
time = 0.28, size = 359, normalized size = 1.04

$$-\frac{12ae^3fk\sqrt{x} + 14be^3fkn\sqrt{x} - 18ae^2f^2kx - 27be^2f^2knx + 36aef^3kx^{3/2} + 90bef^3knx^{3/2} + 36ae^4\log\left(d\left(e+f\sqrt{x}\right)^k\right) + 18be^4n\log\left(d\left(e+f\sqrt{x}\right)^k\right) + 18af^4kx^2\log(x) + 9bf^4knx^2\log(x) - 36bf^4knx^2\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x) - 9bf^4knx^2\log^2(x) + 12be^3fk\sqrt{x}\log(cx^n) - 18be^2f^2kx\log(cx^n) + 36bef^3kx^{3/2}\log(cx^n) + 36be^4\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n) + 18bf^4kx^2\log(x)\log(cx^n) - 18f^4kx^2\log\left(e+f\sqrt{x}\right)(2a+bn-2bn\log(x)+2b\log(cx^n)) - 72bf^4knx^2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{72e^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^3,x]
```

```
[Out] -1/72*(12*a*e^3*f*k*Sqrt[x] + 14*b*e^3*f*k*n*Sqrt[x] - 18*a*e^2*f^2*k*x - 2
7*b*e^2*f^2*k*n*x + 36*a*e*f^3*k*x^(3/2) + 90*b*e*f^3*k*n*x^(3/2) + 36*a*e^
4*Log[d*(e + f*Sqrt[x])^k] + 18*b*e^4*n*Log[d*(e + f*Sqrt[x])^k] + 18*a*f^4
*k*x^2*Log[x] + 9*b*f^4*k*n*x^2*Log[x] - 36*b*f^4*k*n*x^2*Log[1 + (f*Sqrt[x
])/e]*Log[x] - 9*b*f^4*k*n*x^2*Log[x]^2 + 12*b*e^3*f*k*Sqrt[x]*Log[c*x^n] -
 18*b*e^2*f^2*k*x*Log[c*x^n] + 36*b*e*f^3*k*x^(3/2)*Log[c*x^n] + 36*b*e^4*L
og[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 18*b*f^4*k*x^2*Log[x]*Log[c*x^n] - 18*
f^4*k*x^2*Log[e + f*Sqrt[x]]*(2*a + b*n - 2*b*n*Log[x] + 2*b*Log[c*x^n]) -
72*b*f^4*k*n*x^2*PolyLog[2, -((f*Sqrt[x])/e)])/(e^4*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/36*(18*b*e*log(d)*log(x^n) + 9*(2*b*e*log(x^n) + (b*(n + 2*log(c)) + 2*a
)*e)*k*log(f*sqrt(x) + e) + 9*((n*log(d) + 2*log(c)*log(d))*b + 2*a*log(d))
*e + (6*b*f*k*x*log(x^n) + (6*a*f*k + (7*f*k*n + 6*f*k*log(c))*b)*x)/sqrt(x
))*e^(-1)/x^2 - integrate(1/8*(2*b*f^2*k*log(x^n) + 2*a*f^2*k + (f^2*k*n +
2*f^2*k*log(c))*b)/(f*x^2*e^(1/2*log(x) + 1) + x^2*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^3, x)
```

3.121 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^4}\,dx$

Optimal. Leaf size=434

$$-\frac{11bfkn}{225ex^{5/2}}+\frac{5bf^2kn}{72e^2x^2}-\frac{bf^3kn}{9e^3x^{3/2}}+\frac{2bf^4kn}{9e^4x}-\frac{7bf^5kn}{9e^5\sqrt{x}}+\frac{bf^6kn\log\left(e+f\sqrt{x}\right)}{9e^6}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^3}-\frac{2bf^6kn}{}$$

```
[Out] -11/225*b*f*k*n/e/x^(5/2)+5/72*b*f^2*k*n/e^2/x^2-1/9*b*f^3*k*n/e^3/x^(3/2)+
2/9*b*f^4*k*n/e^4/x-1/18*b*f^6*k*n*ln(x)/e^6+1/12*b*f^6*k*n*ln(x)^2/e^6-1/1
5*f*k*(a+b*ln(c*x^n))/e/x^(5/2)+1/12*f^2*k*(a+b*ln(c*x^n))/e^2/x^2-1/9*f^3*
k*(a+b*ln(c*x^n))/e^3/x^(3/2)+1/6*f^4*k*(a+b*ln(c*x^n))/e^4/x-1/6*f^6*k*ln(
x)*(a+b*ln(c*x^n))/e^6+1/9*b*f^6*k*n*ln(e+f*x^(1/2))/e^6+1/3*f^6*k*(a+b*ln(
c*x^n))*ln(e+f*x^(1/2))/e^6-2/3*b*f^6*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/
e^6-1/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^3-1/3*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)
)^k)/x^3-2/3*b*f^6*k*n*polylog(2,1+f*x^(1/2)/e)/e^6-7/9*b*f^5*k*n/e^5/x^(1/
2)-1/3*f^5*k*(a+b*ln(c*x^n))/e^5/x^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2442, 46, 2423, 2441, 2352, 2338}

$$-\frac{2bf^6kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3e^6}-\frac{(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^3}+\frac{f^6k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^6}-\frac{f^6k\log(x)(a+b\log(cx^n))}{6e^6}-\frac{f^5k(a+b\log(cx^n))}{3e^5\sqrt{x}}+\frac{f^4k(a+b\log(cx^n))}{6e^4x}-\frac{f^3k(a+b\log(cx^n))}{9e^3x^{3/2}}+\frac{f^2k(a+b\log(cx^n))}{12e^2x^2}-\frac{fk(a+b\log(cx^n))}{15ex^{5/2}}-\frac{bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^3}+\frac{bf^6kn\log^2(x)}{12e^6}+\frac{bf^6kn\log\left(e+f\sqrt{x}\right)}{9e^6}-\frac{2bf^6kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3e^6}-\frac{bf^6kn\log(x)}{18e^6}-\frac{7bf^5kn}{9e^5\sqrt{x}}+\frac{2bf^4kn}{9e^4x}-\frac{bf^3kn}{9e^3x^{3/2}}+\frac{5bf^2kn}{72e^2x^2}-\frac{11bfkn}{225ex^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] (-11*b*f*k*n)/(225*e*x^(5/2)) + (5*b*f^2*k*n)/(72*e^2*x^2) - (b*f^3*k*n)/(9
*e^3*x^(3/2)) + (2*b*f^4*k*n)/(9*e^4*x) - (7*b*f^5*k*n)/(9*e^5*Sqrt[x]) + (
b*f^6*k*n*Log[e + f*Sqrt[x]])/(9*e^6) - (b*n*Log[d*(e + f*Sqrt[x])^k])/(9*x
^3) - (2*b*f^6*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^6) - (b*f
^6*k*n*Log[x])/(18*e^6) + (b*f^6*k*n*Log[x]^2)/(12*e^6) - (f*k*(a + b*Log[c
*x^n]))/(15*e*x^(5/2)) + (f^2*k*(a + b*Log[c*x^n]))/(12*e^2*x^2) - (f^3*k*(
a + b*Log[c*x^n]))/(9*e^3*x^(3/2)) + (f^4*k*(a + b*Log[c*x^n]))/(6*e^4*x) -
 (f^5*k*(a + b*Log[c*x^n]))/(3*e^5*Sqrt[x]) + (f^6*k*Log[e + f*Sqrt[x]]*(a
+ b*Log[c*x^n]))/(3*e^6) - (Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/(3
*x^3) - (f^6*k*Log[x]*(a + b*Log[c*x^n]))/(6*e^6) - (2*b*f^6*k*n*PolyLog[2,
 1 + (f*Sqrt[x])/e])/(3*e^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^4}\,dx = -\frac{fk(a+b\log(cx^n))}{15ex^{5/2}} + \frac{f^2k(a+b\log(cx^n))}{12e^2x^2} - \frac{f^3k(a+b\log(\cdots}{9e^3x^{3/2}}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{fk(a+\cdots}{1\cdots}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} + \frac{bf^6kn\cdots}{12\cdots}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{bn\log(\cdots}{\cdots}$$

$$= -\frac{2bfkn}{75ex^{5/2}} + \frac{bf^2kn}{24e^2x^2} - \frac{2bf^3kn}{27e^3x^{3/2}} + \frac{bf^4kn}{6e^4x} - \frac{2bf^5kn}{3e^5\sqrt{x}} - \frac{bn\log(\cdots}{\cdots}$$

$$= -\frac{11bfkn}{225ex^{5/2}} + \frac{5bf^2kn}{72e^2x^2} - \frac{bf^3kn}{9e^3x^{3/2}} + \frac{2bf^4kn}{9e^4x} - \frac{7bf^5kn}{9e^5\sqrt{x}} + \frac{bf^6kn\cdots}{\cdots}$$

Mathematica [A]
time = 0.34, size = 457, normalized size = 1.05

$$-\frac{120ae^5fk\sqrt{x}+88be^5fkn\sqrt{x}-150ae^4f^2kx-125be^4f^2knx+200ae^3f^3kx^{3/2}+200be^3f^3knx^{3/2}-300ae^2f^4kx^2-400be^2f^4knx^2+600aef^5kx^{5/2}+1400bef^5knx^{5/2}+600ae^6\log\left(d\left(e+f\sqrt{x}\right)^k\right)+200be^6n\log\left(d\left(e+f\sqrt{x}\right)^k\right)+300af^6kx^3\log(x)+100bf^6knx^3\log(x)-600bf^6knx^3\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-150bf^6knx^3\log^2(x)+120be^5fk\sqrt{x}\log(cx^n)-150be^4f^2kx\log(cx^n)+200be^3f^3kx^{3/2}\log(cx^n)-300be^2f^4kx^2\log(cx^n)+600bef^5kx^{5/2}\log(cx^n)+600be^6\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+300bf^6kx^3\log(x)\log(cx^n)-200f^6kx^3\log\left(e+f\sqrt{x}\right)(3a+bn-3bn\log(x)+3b\log(cx^n))-1200bf^6knx^3\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{1800e^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^4,x]
```

```
[Out] -1/1800*(120*a*e^5*f*k*Sqrt[x] + 88*b*e^5*f*k*n*Sqrt[x] - 150*a*e^4*f^2*k*x
 - 125*b*e^4*f^2*k*n*x + 200*a*e^3*f^3*k*x^(3/2) + 200*b*e^3*f^3*k*n*x^(3/2
) - 300*a*e^2*f^4*k*x^2 - 400*b*e^2*f^4*k*n*x^2 + 600*a*e*f^5*k*x^(5/2) + 1
400*b*e*f^5*k*n*x^(5/2) + 600*a*e^6*Log[d*(e + f*Sqrt[x])^k] + 200*b*e^6*n*
Log[d*(e + f*Sqrt[x])^k] + 300*a*f^6*k*x^3*Log[x] + 100*b*f^6*k*n*x^3*Log[x
] - 600*b*f^6*k*n*x^3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 150*b*f^6*k*n*x^3*Log
[x]^2 + 120*b*e^5*f*k*Sqrt[x]*Log[c*x^n] - 150*b*e^4*f^2*k*x*Log[c*x^n] + 2
00*b*e^3*f^3*k*x^(3/2)*Log[c*x^n] - 300*b*e^2*f^4*k*x^2*Log[c*x^n] + 600*b*
e*f^5*k*x^(5/2)*Log[c*x^n] + 600*b*e^6*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n]
+ 300*b*f^6*k*x^3*Log[x]*Log[c*x^n] - 200*f^6*k*x^3*Log[e + f*Sqrt[x]]*(3*a
 + b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) - 1200*b*f^6*k*n*x^3*PolyLog[2, -((
f*Sqrt[x])/e)])/(e^6*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^{n}))\ln\left(d\left(e+f\sqrt{x}\,\right)^{k}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^4,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/225*(75*b*e*log(d)*log(x^n) + 25*(3*b*e*log(x^n) + (b*(n + 3*log(c)) + 3
*a)*e)*k*log(f*sqrt(x) + e) + 25*((n*log(d) + 3*log(c)*log(d))*b + 3*a*log(
d))*e + (15*b*f*k*x*log(x^n) + (15*a*f*k + (11*f*k*n + 15*f*k*log(c))*b)*x)
/sqrt(x))*e^(-1)/x^3 - integrate(1/18*(3*b*f^2*k*log(x^n) + 3*a*f^2*k + (f^
2*k*n + 3*f^2*k*log(c))*b)/(f*x^3*e^(1/2*log(x) + 1) + x^3*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,\sqrt{x}\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4,x)
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^4, x)
```

3.122 $\int x^2 \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=750

$$\frac{86b^2e^5n^2\sqrt{x}}{27f^5}+\frac{abe^4nx}{3f^4}-\frac{13b^2e^4n^2x}{27f^4}+\frac{14b^2e^3n^2x^{3/2}}{81f^3}-\frac{19b^2e^2n^2x^2}{216f^2}+\frac{182b^2en^2x^{5/2}}{3375f}-\frac{1}{27}b^2n^2x^3-\frac{2b^2e^6n^2\log(e}{27f^6}$$

```
[Out] 2/27*b*n*x^3*(a+b*ln(c*x^n))-1/6*e^4*x*(a+b*ln(c*x^n))^2/f^4+1/9*e^3*x^(3/2
)*(a+b*ln(c*x^n))^2/f^3-1/12*e^2*x^2*(a+b*ln(c*x^n))^2/f^2+1/15*e*x^(5/2)*(
a+b*ln(c*x^n))^2/f+2/27*b^2*n^2*x^3*ln(d*(e+f*x^(1/2)))-1/3*e^6*(a+b*ln(c*x
^n))^2*ln(1+f*x^(1/2)/e)/f^6+1/3*e^5*(a+b*ln(c*x^n))^2*x^(1/2)/f^5-13/27*b^
2*e^4*n^2*x/f^4+14/81*b^2*e^3*n^2*x^(3/2)/f^3-19/216*b^2*e^2*n^2*x^2/f^2+18
2/3375*b^2*e*n^2*x^(5/2)/f-1/18*x^3*(a+b*ln(c*x^n))^2-2/27*b^2*e^6*n^2*ln(e
+f*x^(1/2))/f^6-2/9*b*n*x^3*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))-4/9*b^2*e^6
*n^2*polylog(2,1+f*x^(1/2)/e)/f^6+8/3*b^2*e^6*n^2*polylog(3,-f*x^(1/2)/e)/f
^6+86/27*b^2*e^5*n^2*x^(1/2)/f^5-1/27*b^2*n^2*x^3+1/3*x^3*(a+b*ln(c*x^n))^2
*ln(d*(e+f*x^(1/2)))+1/3*b^2*e^4*n*x*ln(c*x^n)/f^4+1/9*b*e^4*n*x*(a+b*ln(c*
x^n))/f^4-2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*x^n))/f^3+5/36*b*e^2*n*x^2*(a+b*ln(
c*x^n))/f^2-22/225*b*e*n*x^(5/2)*(a+b*ln(c*x^n))/f+2/9*b*e^6*n*(a+b*ln(c*x^
n))*ln(e+f*x^(1/2))/f^6-4/9*b^2*e^6*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^
6-4/3*b*e^6*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^6-14/9*b*e^5*n*(a+b
*ln(c*x^n))*x^(1/2)/f^5+1/3*a*b*e^4*n*x/f^4
```

Rubi [A]
time = 0.55, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2504, 2442, 45, 2424, 2332, 2341, 2422, 2375, 2421, 6724, 2423, 2441, 2352}

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (86*b^2*e^5*n^2*Sqrt[x])/(27*f^5) + (a*b*e^4*n*x)/(3*f^4) - (13*b^2*e^4*n^2
*x)/(27*f^4) + (14*b^2*e^3*n^2*x^(3/2))/(81*f^3) - (19*b^2*e^2*n^2*x^2)/(21
6*f^2) + (182*b^2*e*n^2*x^(5/2))/(3375*f) - (b^2*n^2*x^3)/27 - (2*b^2*e^6*n
^2*Log[e + f*Sqrt[x]])/(27*f^6) + (2*b^2*n^2*x^3*Log[d*(e + f*Sqrt[x])])/27
 - (4*b^2*e^6*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(9*f^6) + (b^2*
e^4*n*x*Log[c*x^n])/(3*f^4) - (14*b*e^5*n*Sqrt[x]*(a + b*Log[c*x^n]))/(9*f^
5) + (b*e^4*n*x*(a + b*Log[c*x^n]))/(9*f^4) - (2*b*e^3*n*x^(3/2)*(a + b*Log
[c*x^n]))/(9*f^3) + (5*b*e^2*n*x^2*(a + b*Log[c*x^n]))/(36*f^2) - (22*b*e*n
*x^(5/2)*(a + b*Log[c*x^n]))/(225*f) + (2*b*n*x^3*(a + b*Log[c*x^n]))/27 +
(2*b*e^6*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(9*f^6) - (2*b*n*x^3*Log[
d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/9 + (e^5*Sqrt[x]*(a + b*Log[c*x^n])^
```

```
2)/(3*f^5) - (e^4*x*(a + b*Log[c*x^n])^2)/(6*f^4) + (e^3*x^(3/2)*(a + b*Log
[c*x^n])^2)/(9*f^3) - (e^2*x^2*(a + b*Log[c*x^n])^2)/(12*f^2) + (e*x^(5/2)*
(a + b*Log[c*x^n])^2)/(15*f) - (x^3*(a + b*Log[c*x^n])^2)/18 + (x^3*Log[d*(
e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/3 - (e^6*Log[1 + (f*Sqrt[x])/e]*(a +
b*Log[c*x^n])^2)/(3*f^6) - (4*b^2*e^6*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/(9
*f^6) - (4*b*e^6*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/(3*f^6)
 + (8*b^2*e^6*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(3*f^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{e^5\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{3f^5} - \frac{e^4x\left(a+b\log\left(cx^n\right)\right)^2}{6f^4} + \frac{e^3x^{3/2}(a \cdots}{\cdots}$$

$$= \frac{e^5\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{3f^5} - \frac{e^4x\left(a+b\log\left(cx^n\right)\right)^2}{6f^4} + \frac{e^3x^{3/2}(a \cdots}{\cdots}$$

$$= \frac{8b^2e^5n^2\sqrt{x}}{3f^5} + \frac{abe^4nx}{3f^4} + \frac{8b^2e^3n^2x^{3/2}}{81f^3} - \frac{b^2e^2n^2x^2}{24f^2} + \frac{8b^2en^2x \cdots}{375f \cdots}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n \cdots}{72f \cdots}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n \cdots}{72f \cdots}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n \cdots}{72f \cdots}$$

$$= \frac{28b^2e^5n^2\sqrt{x}}{9f^5} + \frac{abe^4nx}{3f^4} - \frac{4b^2e^4n^2x}{9f^4} + \frac{4b^2e^3n^2x^{3/2}}{27f^3} - \frac{5b^2e^2n \cdots}{72f \cdots}$$

$$= \frac{86b^2e^5n^2\sqrt{x}}{27f^5} + \frac{abe^4nx}{3f^4} - \frac{13b^2e^4n^2x}{27f^4} + \frac{14b^2e^3n^2x^{3/2}}{81f^3} - \frac{19b^2 \cdots}{2 \cdots}$$

Mathematica [A]
time = 0.55, size = 1319, normalized size = 1.76

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (a^2*e^5*Sqrt[x])/(3*f^5) - (14*a*b*e^5*n*Sqrt[x])/(9*f^5) + (86*b^2*e^5*n^
2*Sqrt[x])/(27*f^5) - (a^2*e^4*x)/(6*f^4) + (4*a*b*e^4*n*x)/(9*f^4) - (13*b
^2*e^4*n^2*x)/(27*f^4) + (a^2*e^3*x^(3/2))/(9*f^3) - (2*a*b*e^3*n*x^(3/2))/
(9*f^3) + (14*b^2*e^3*n^2*x^(3/2))/(81*f^3) - (a^2*e^2*x^2)/(12*f^2) + (5*a
*b*e^2*n*x^2)/(36*f^2) - (19*b^2*e^2*n^2*x^2)/(216*f^2) + (a^2*e*x^(5/2))/(
15*f) - (22*a*b*e*n*x^(5/2))/(225*f) + (182*b^2*e*n^2*x^(5/2))/(3375*f) - (
a^2*x^3)/18 + (2*a*b*n*x^3)/27 - (b^2*n^2*x^3)/27 - (a^2*e^6*Log[e + f*Sqrt
[x]])/(3*f^6) + (2*a*b*e^6*n*Log[e + f*Sqrt[x]])/(9*f^6) - (2*b^2*e^6*n^2*L
og[e + f*Sqrt[x]])/(27*f^6) + (a^2*x^3*Log[d*(e + f*Sqrt[x])])/3 - (2*a*b*n
*x^3*Log[d*(e + f*Sqrt[x])])/9 + (2*b^2*n^2*x^3*Log[d*(e + f*Sqrt[x])])/27
+ (2*a*b*e^6*n*Log[e + f*Sqrt[x]]*Log[x])/(3*f^6) - (2*b^2*e^6*n^2*Log[e +
f*Sqrt[x]]*Log[x])/(9*f^6) - (2*a*b*e^6*n*Log[1 + (f*Sqrt[x])/e]*Log[x])/(3
*f^6) + (2*b^2*e^6*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x])/(9*f^6) - (b^2*e^6*n^
2*Log[e + f*Sqrt[x]]*Log[x]^2)/(3*f^6) + (b^2*e^6*n^2*Log[1 + (f*Sqrt[x])/e
]*Log[x]^2)/(3*f^6) + (2*a*b*e^5*Sqrt[x]*Log[c*x^n])/(3*f^5) - (14*b^2*e^5*
n*Sqrt[x]*Log[c*x^n])/(9*f^5) - (a*b*e^4*x*Log[c*x^n])/(3*f^4) + (4*b^2*e^4
*n*x*Log[c*x^n])/(9*f^4) + (2*a*b*e^3*x^(3/2)*Log[c*x^n])/(9*f^3) - (2*b^2*
e^3*n*x^(3/2)*Log[c*x^n])/(9*f^3) - (a*b*e^2*x^2*Log[c*x^n])/(6*f^2) + (5*b
^2*e^2*n*x^2*Log[c*x^n])/(36*f^2) + (2*a*b*e*x^(5/2)*Log[c*x^n])/(15*f) - (
22*b^2*e*n*x^(5/2)*Log[c*x^n])/(225*f) - (a*b*x^3*Log[c*x^n])/9 + (2*b^2*n*
x^3*Log[c*x^n])/27 - (2*a*b*e^6*Log[e + f*Sqrt[x]]*Log[c*x^n])/(3*f^6) + (2
*b^2*e^6*n*Log[e + f*Sqrt[x]]*Log[c*x^n])/(9*f^6) + (2*a*b*x^3*Log[d*(e + f
*Sqrt[x])]*Log[c*x^n])/3 - (2*b^2*n*x^3*Log[d*(e + f*Sqrt[x])]*Log[c*x^n])/
9 + (2*b^2*e^6*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n])/(3*f^6) - (2*b^2*e^6
*n*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n])/(3*f^6) + (b^2*e^5*Sqrt[x]*Log
[c*x^n]^2)/(3*f^5) - (b^2*e^4*x*Log[c*x^n]^2)/(6*f^4) + (b^2*e^3*x^(3/2)*Lo
g[c*x^n]^2)/(9*f^3) - (b^2*e^2*x^2*Log[c*x^n]^2)/(12*f^2) + (b^2*e*x^(5/2)*
Log[c*x^n]^2)/(15*f) - (b^2*x^3*Log[c*x^n]^2)/18 - (b^2*e^6*Log[e + f*Sqrt[
x]]*Log[c*x^n]^2)/(3*f^6) + (b^2*x^3*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2)/3
 + (4*b*e^6*n*(-3*a + b*n - 3*b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/(
9*f^6) + (8*b^2*e^6*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(3*f^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(e+f\sqrt{x}\,\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima"
)
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas"
)
[Out] integral((b^2*x^2*log(c*x^n)^2 + 2*a*b*x^2*log(c*x^n) + a^2*x^2)*log(d*f*sq
rt(x) + d*e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^2*x^2*log((f*sqrt(x) + e)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
[Out] int(x^2*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

3.123 $\int x \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=598

$$\frac{21b^2e^3n^2\sqrt{x}}{4f^3}+\frac{abe^2nx}{2f^2}-\frac{7b^2e^2n^2x}{8f^2}+\frac{37b^2en^2x^{3/2}}{108f}-\frac{3}{16}b^2n^2x^2-\frac{b^2e^4n^2\log\left(e+f\sqrt{x}\right)}{4f^4}+\frac{1}{4}b^2n^2x^2\log\left(d\left(e+f\right.\right.$$

```
[Out] 1/2*a*b*e^2*n*x/f^2-7/8*b^2*e^2*n^2*x/f^2+37/108*b^2*e*n^2*x^(3/2)/f-3/16*b
^2*n^2*x^2+1/2*b^2*e^2*n*x*ln(c*x^n)/f^2+1/4*b*e^2*n*x*(a+b*ln(c*x^n))/f^2-
7/18*b*e*n*x^(3/2)*(a+b*ln(c*x^n))/f+1/4*b*n*x^2*(a+b*ln(c*x^n))-1/4*e^2*x*
(a+b*ln(c*x^n))^2/f^2+1/6*e*x^(3/2)*(a+b*ln(c*x^n))^2/f-1/8*x^2*(a+b*ln(c*x
^n))^2-1/4*b^2*e^4*n^2*ln(e+f*x^(1/2))/f^4+1/2*b*e^4*n*(a+b*ln(c*x^n))*ln(e
+f*x^(1/2))/f^4-b^2*e^4*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^4+1/4*b^2*n^
2*x^2*ln(d*(e+f*x^(1/2)))-1/2*b*n*x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))+1
/2*x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))-1/2*e^4*(a+b*ln(c*x^n))^2*ln(1
+f*x^(1/2)/e)/f^4-2*b*e^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^4-b^2
*e^4*n^2*polylog(2,1+f*x^(1/2)/e)/f^4+4*b^2*e^4*n^2*polylog(3,-f*x^(1/2)/e)
/f^4+21/4*b^2*e^3*n^2*x^(1/2)/f^3-5/2*b*e^3*n*(a+b*ln(c*x^n))*x^(1/2)/f^3+1
/2*e^3*(a+b*ln(c*x^n))^2*x^(1/2)/f^3
```

Rubi [A]
time = 0.44, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2504, 2442, 45, 2424, 2332, 2341, 2422, 2375, 2421, 6724, 2423, 2441, 2352}

$$-\frac{2be^4n\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^4}-\frac{b^2e^4n^2\,\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^4}+\frac{4b^2e^4n^2\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^4}+\frac{1}{2}x^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}bnx^2\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)-\frac{e^4\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{2f^4}+\frac{be^4n\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{2f^4}+\frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{2f^3}-\frac{5be^3n\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{2f^3}-\frac{e^2x\left(a+b\log\left(cx^n\right)\right)^2}{4f^2}+\frac{be^2nx\left(a+b\log\left(cx^n\right)\right)}{4f^2}+\frac{ex^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}{6f}-\frac{7benx^{3/2}\left(a+b\log\left(cx^n\right)\right)}{18f}-\frac{1}{8}x^2\left(a+b\log\left(cx^n\right)\right)^2+\frac{1}{4}bnx^2\left(a+b\log\left(cx^n\right)\right)+\frac{abe^2nx}{2f^2}+\frac{b^2e^2nx\log\left(cx^n\right)}{2f^2}+\frac{1}{4}b^2n^2x^2\log\left(d\left(e+f\sqrt{x}\right)\right)-\frac{b^2e^4n^2\log\left(e+f\sqrt{x}\right)}{4f^4}-\frac{b^2e^4n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^4}+\frac{21b^2e^3n^2\sqrt{x}}{4f^3}-\frac{7b^2e^2n^2x}{8f^2}+\frac{37b^2en^2x^{3/2}}{108f}-\frac{3}{16}b^2n^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (21*b^2*e^3*n^2*Sqrt[x])/(4*f^3) + (a*b*e^2*n*x)/(2*f^2) - (7*b^2*e^2*n^2*x
)/(8*f^2) + (37*b^2*e*n^2*x^(3/2))/(108*f) - (3*b^2*n^2*x^2)/16 - (b^2*e^4*
n^2*Log[e + f*Sqrt[x]])/(4*f^4) + (b^2*n^2*x^2*Log[d*(e + f*Sqrt[x])])/4 -
(b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^4 + (b^2*e^2*n*x*L
og[c*x^n])/(2*f^2) - (5*b*e^3*n*Sqrt[x]*(a + b*Log[c*x^n]))/(2*f^3) + (b*e^
2*n*x*(a + b*Log[c*x^n]))/(4*f^2) - (7*b*e*n*x^(3/2)*(a + b*Log[c*x^n]))/(1
8*f) + (b*n*x^2*(a + b*Log[c*x^n]))/4 + (b*e^4*n*Log[e + f*Sqrt[x]]*(a + b*
Log[c*x^n]))/(2*f^4) - (b*n*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/
2 + (e^3*Sqrt[x]*(a + b*Log[c*x^n])^2)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^
2)/(4*f^2) + (e*x^(3/2)*(a + b*Log[c*x^n])^2)/(6*f) - (x^2*(a + b*Log[c*x^n
])^2)/8 + (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/2 - (e^4*Log[1
+ (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(2*f^4) - (b^2*e^4*n^2*PolyLog[2, 1
+ (f*Sqrt[x])/e])/f^4 - (2*b*e^4*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[
x])/e)])/f^4 + (4*b^2*e^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \log \left(d\left(e+f\sqrt{x}\right)\right) \left(a+b\log \left(cx^n\right)\right)^2\, dx = \frac{e^3\sqrt{x}\left(a+b\log \left(cx^n\right)\right)^2}{2f^3} - \frac{e^2x\left(a+b\log \left(cx^n\right)\right)^2}{4f^2} + \frac{ex^{3/2}(a+}{6}$$

$$= \frac{e^3\sqrt{x}\left(a+b\log \left(cx^n\right)\right)^2}{2f^3} - \frac{e^2x\left(a+b\log \left(cx^n\right)\right)^2}{4f^2} + \frac{ex^{3/2}(a+}{6}$$

$$= \frac{4b^2e^3n^2\sqrt{x}}{f^3} + \frac{abe^2nx}{2f^2} + \frac{4b^2en^2x^{3/2}}{27f} - \frac{1}{16}b^2n^2x^2 - \frac{5be^3n\sqrt{x}}{}$$

$$= \frac{5b^2e^3n^2\sqrt{x}}{f^3} + \frac{abe^2nx}{2f^2} - \frac{3b^2e^2n^2x}{4f^2} + \frac{7b^2en^2x^{3/2}}{27f} - \frac{1}{8}b^2n^2x^2$$

$$= \frac{5b^2e^3n^2\sqrt{x}}{f^3} + \frac{abe^2nx}{2f^2} - \frac{3b^2e^2n^2x}{4f^2} + \frac{7b^2en^2x^{3/2}}{27f} - \frac{1}{8}b^2n^2x^2$$

$$= \frac{5b^2e^3n^2\sqrt{x}}{f^3} + \frac{abe^2nx}{2f^2} - \frac{3b^2e^2n^2x}{4f^2} + \frac{7b^2en^2x^{3/2}}{27f} - \frac{1}{8}b^2n^2x^2$$

$$= \frac{5b^2e^3n^2\sqrt{x}}{f^3} + \frac{abe^2nx}{2f^2} - \frac{3b^2e^2n^2x}{4f^2} + \frac{7b^2en^2x^{3/2}}{27f} - \frac{1}{8}b^2n^2x^2$$

$$= \frac{21b^2e^3n^2\sqrt{x}}{4f^3} + \frac{abe^2nx}{2f^2} - \frac{7b^2e^2n^2x}{8f^2} + \frac{37b^2en^2x^{3/2}}{108f} - \frac{3}{16}b^2n^2$$

Mathematica [A]
time = 0.29, size = 960, normalized size = 1.61

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (216*a^2*e^3*f*Sqrt[x] - 1080*a*b*e^3*f*n*Sqrt[x] + 2268*b^2*e^3*f*n^2*Sqrt
[x] - 108*a^2*e^2*f^2*x + 324*a*b*e^2*f^2*n*x - 378*b^2*e^2*f^2*n^2*x + 72*
a^2*e*f^3*x^(3/2) - 168*a*b*e*f^3*n*x^(3/2) + 148*b^2*e*f^3*n^2*x^(3/2) - 5
4*a^2*f^4*x^2 + 108*a*b*f^4*n*x^2 - 81*b^2*f^4*n^2*x^2 - 216*a^2*e^4*Log[e
+ f*Sqrt[x]] + 216*a*b*e^4*n*Log[e + f*Sqrt[x]] - 108*b^2*e^4*n^2*Log[e + f
```

```
*Sqrt[x]] + 216*a^2*f^4*x^2*Log[d*(e + f*Sqrt[x])] - 216*a*b*f^4*n*x^2*Log[
d*(e + f*Sqrt[x])] + 108*b^2*f^4*n^2*x^2*Log[d*(e + f*Sqrt[x])] + 432*a*b*e
^4*n*Log[e + f*Sqrt[x]]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]
- 432*a*b*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log[x] + 216*b^2*e^4*n^2*Log[1 + (f*
Sqrt[x])/e]*Log[x] - 216*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 + 216*b^2*
e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 432*a*b*e^3*f*Sqrt[x]*Log[c*x^n]
- 1080*b^2*e^3*f*n*Sqrt[x]*Log[c*x^n] - 216*a*b*e^2*f^2*x*Log[c*x^n] + 324*
b^2*e^2*f^2*n*x*Log[c*x^n] + 144*a*b*e*f^3*x^(3/2)*Log[c*x^n] - 168*b^2*e*f
^3*n*x^(3/2)*Log[c*x^n] - 108*a*b*f^4*x^2*Log[c*x^n] + 108*b^2*f^4*n*x^2*Lo
g[c*x^n] - 432*a*b*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n] + 216*b^2*e^4*n*Log[e
+ f*Sqrt[x]]*Log[c*x^n] + 432*a*b*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
 - 216*b^2*f^4*n*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 432*b^2*e^4*n*Log[
e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 432*b^2*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log
[x]*Log[c*x^n] + 216*b^2*e^3*f*Sqrt[x]*Log[c*x^n]^2 - 108*b^2*e^2*f^2*x*Log
[c*x^n]^2 + 72*b^2*e*f^3*x^(3/2)*Log[c*x^n]^2 - 54*b^2*f^4*x^2*Log[c*x^n]^2
 - 216*b^2*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 216*b^2*f^4*x^2*Log[d*(e +
 f*Sqrt[x])]*Log[c*x^n]^2 + 432*b*e^4*n*(-2*a + b*n - 2*b*Log[c*x^n])*PolyL
og[2, -((f*Sqrt[x])/e)] + 1728*b^2*e^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/(4
32*f^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^2 \ln\left(d\left(e+f\sqrt{x}\,\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
[Out] int(x*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x)*log(d*f*sqrt(x)
+ d*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*x*log((f*sqrt(x) + e)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\,\left(e+f\,\sqrt{x}\,\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

3.124 $\int \log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=405

$$\frac{14b^2en^2\sqrt{x}}{f}+abnx-3b^2n^2x-\frac{2b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2}+2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right)-\frac{4b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2}$$

```
[Out] a*b*n*x-3*b^2*n^2*x+b^2*n*x*ln(c*x^n)+b*n*x*(a+b*ln(c*x^n))-1/2*x*(a+b*ln(c
*x^n))^2-2*b^2*e^2*n^2*ln(e+f*x^(1/2))/f^2+2*b*e^2*n*(a+b*ln(c*x^n))*ln(e+f
*x^(1/2))/f^2-4*b^2*e^2*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^2+2*b^2*n^2*
x*ln(d*(e+f*x^(1/2)))-2*b*n*x*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))+x*(a+b*ln
(c*x^n))^2*ln(d*(e+f*x^(1/2)))-e^2*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/f^2-
4*b*e^2*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^2-4*b^2*e^2*n^2*polylog
(2,1+f*x^(1/2)/e)/f^2+8*b^2*e^2*n^2*polylog(3,-f*x^(1/2)/e)/f^2+14*b^2*e*n^
2*x^(1/2)/f-6*b*e*n*(a+b*ln(c*x^n))*x^(1/2)/f+e*(a+b*ln(c*x^n))^2*x^(1/2)/f
```

Rubi [A]
time = 0.30, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2498, 272, 45, 2417, 2332, 2341, 2422, 2375, 2421, 6724, 2504, 2441, 2352}

$$-\frac{4be^2n\,\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{4b^2e^2n^2\,\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^2}+\frac{8b^2e^2n^2\,\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2}-2bnx\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)+x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2+\frac{2be^2n\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{e^2\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{f^2}-\frac{6ben\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f}+\frac{e\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{f}+bnx\left(a+b\log\left(cx^n\right)\right)-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^2+abnx+b^2nx\log\left(cx^n\right)+2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right)-\frac{2b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2}-\frac{4b^2e^2n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^2}+\frac{14b^2en^2\sqrt{x}}{f}-3b^2n^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (14*b^2*e*n^2*Sqrt[x])/f + a*b*n*x - 3*b^2*n^2*x - (2*b^2*e^2*n^2*Log[e + f
*Sqrt[x]])/f^2 + 2*b^2*n^2*x*Log[d*(e + f*Sqrt[x])] - (4*b^2*e^2*n^2*Log[e
+ f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 + b^2*n*x*Log[c*x^n] - (6*b*e*n*Sqr
t[x]*(a + b*Log[c*x^n]))/f + b*n*x*(a + b*Log[c*x^n]) + (2*b*e^2*n*Log[e +
f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 - 2*b*n*x*Log[d*(e + f*Sqrt[x])]*(a + b*
Log[c*x^n]) + (e*Sqrt[x]*(a + b*Log[c*x^n])^2)/f - (x*(a + b*Log[c*x^n])^2)
/2 + x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2 - (e^2*Log[1 + (f*Sqrt[x
])/e]*(a + b*Log[c*x^n])^2)/f^2 - (4*b^2*e^2*n^2*PolyLog[2, 1 + (f*Sqrt[x])
/e])/f^2 - (4*b*e^2*n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/f^2
+ (8*b^2*e^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2\,dx = \frac{e\sqrt{x}\,(a+b\log(cx^n))^2}{f} - \frac{1}{2}x(a+b\log(cx^n))^2 - \frac{e^2\log\left(e+f\sqrt{\right.}}{}$$

$$= \frac{e\sqrt{x}\,(a+b\log(cx^n))^2}{f} - \frac{1}{2}x(a+b\log(cx^n))^2 - \frac{e^2\log\left(e+f\sqrt{\right.}}{}$$

$$= \frac{8b^2en^2\sqrt{x}}{f} + abnx - \frac{6ben\sqrt{x}\,(a+b\log(cx^n))}{f} + bnx(a+b\log$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + b^2nx\log(cx^n) - \frac{6ben\sqrt{x}\,(a+}{f}$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) + b$$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) - $$

$$= \frac{12b^2en^2\sqrt{x}}{f} + abnx - 2b^2n^2x + 2b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right) - $$

$$= \frac{14b^2en^2\sqrt{x}}{f} + abnx - 3b^2n^2x - \frac{2b^2e^2n^2\log\left(e+f\sqrt{x}\right)}{f^2} + 2b^2$$

Mathematica [A]
time = 0.23, size = 718, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] -1/2*(-2*a^2*e*f*Sqrt[x] + 12*a*b*e*f*n*Sqrt[x] - 28*b^2*e*f*n^2*Sqrt[x] +
a^2*f^2*x - 4*a*b*f^2*n*x + 6*b^2*f^2*n^2*x + 2*a^2*e^2*Log[e + f*Sqrt[x]]
- 4*a*b*e^2*n*Log[e + f*Sqrt[x]] + 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 2*a^2
*f^2*x*Log[d*(e + f*Sqrt[x])] + 4*a*b*f^2*n*x*Log[d*(e + f*Sqrt[x])] - 4*b^
2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] - 4*a*b*e^2*n*Log[e + f*Sqrt[x]]*Log[x]
+ 4*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] + 4*a*b*e^2*n*Log[1 + (f*Sqrt[x])
```

```
/e]*Log[x] - 4*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] + 2*b^2*e^2*n^2*Lo
g[e + f*Sqrt[x]]*Log[x]^2 - 2*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 -
 4*a*b*e*f*Sqrt[x]*Log[c*x^n] + 12*b^2*e*f*n*Sqrt[x]*Log[c*x^n] + 2*a*b*f^2
*x*Log[c*x^n] - 4*b^2*f^2*n*x*Log[c*x^n] + 4*a*b*e^2*Log[e + f*Sqrt[x]]*Log
[c*x^n] - 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] - 4*a*b*f^2*x*Log[d*(e
+ f*Sqrt[x])]*Log[c*x^n] + 4*b^2*f^2*n*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
- 4*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 4*b^2*e^2*n*Log[1 + (f
*Sqrt[x])/e]*Log[x]*Log[c*x^n] - 2*b^2*e*f*Sqrt[x]*Log[c*x^n]^2 + b^2*f^2*x
*Log[c*x^n]^2 + 2*b^2*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 - 2*b^2*f^2*x*Log
[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 8*b*e^2*n*(a - b*n + b*Log[c*x^n])*PolyL
og[2, -((f*Sqrt[x])/e)] - 16*b^2*e^2*n^2*PolyLog[3, -((f*Sqrt[x])/e)])/f^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\,x^{n}\right)\right)^{2} \ln \left(d\left(e+f\sqrt{x}\,\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/27*(27*b^2*x*e*log(d)*log(x^n)^2 - 54*((n*log(d) - log(c)*log(d))*b^2 - a
*b*log(d))*x*e*log(x^n) - 27*(2*(n*log(d) - log(c)*log(d))*a*b - (2*n^2*log
(d) - 2*n*log(c)*log(d) + log(c)^2*log(d))*b^2 - a^2*log(d))*x*e + 27*(b^2*
x*e*log(x^n)^2 - 2*(b^2*(n - log(c)) - a*b)*x*e*log(x^n) + ((2*n^2 - 2*n*lo
g(c) + log(c)^2)*b^2 - 2*a*b*(n - log(c)) + a^2)*x*e)*log(f*sqrt(x) + e) -
(9*b^2*f*x^2*log(x^n)^2 - 6*((5*f*n - 3*f*log(c))*b^2 - 3*a*b*f)*x^2*log(x^
n) - (6*(5*f*n - 3*f*log(c))*a*b - (38*f*n^2 - 30*f*n*log(c) + 9*f*log(c)^2
)*b^2 - 9*a^2*f)*x^2)/sqrt(x))*e^(-1) + integrate(1/2*(b^2*f^2*x*log(x^n)^2
 + 2*(a*b*f^2 - (f^2*n - f^2*log(c))*b^2)*x*log(x^n) + (a^2*f^2 - 2*(f^2*n
- f^2*log(c))*a*b + (2*f^2*n^2 - 2*f^2*n*log(c) + f^2*log(c)^2)*b^2)*x)/(f*
e^(1/2*log(x) + 1) + e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2,x)
```

```
[Out] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2, x)
```

3.125 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}\,dx$

Optimal. Leaf size=145

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}-\frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}-2\left(a+b\log\left(cx^n\right)\right)^2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/b/n-1/3*(a+b*ln(c*x^n))^3*ln(1+f*
x^(1/2)/e)/b/n-2*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)+8*b*n*(a+b*ln(c*
x^n))*polylog(3,-f*x^(1/2)/e)-16*b^2*n^2*polylog(4,-f*x^(1/2)/e)
```

Rubi [A]
time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+8bn\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-16b^2n^2\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}-\frac{\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/(3*b*n) - (Log[1 + (f*Sqrt[x]
)/e]*(a + b*Log[c*x^n])^3)/(3*b*n) - 2*(a + b*Log[c*x^n])^2*PolyLog[2, -((f
*Sqrt[x])/e)] + 8*b*n*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)] - 16*
b^2*n^2*PolyLog[4, -((f*Sqrt[x])/e)]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{f\int\frac{\left(a+b\log\left(cx^n\right)\right)^3}{\left(e+f\sqrt{x}\right)\sqrt{x}}\,dx}{6bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b \cdots}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b \cdots}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b \cdots}{3bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{3bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b \cdots}{3bn}$$

Mathematica [A]
time = 0.15, size = 263, normalized size = 1.81

$$\frac{1}{3}\left(\log\left(d\left(e+f\sqrt{x}\right)\right)\log(x)\left(b^2n^2\log^2(x)-3bn\log(x)\left(a+b\log\left(cx^n\right)\right)+3\left(a+b\log\left(cx^n\right)\right)^2\right)-3\left(a-bn\log(x)+b\log\left(cx^n\right)\right)^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)+2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\right)-3bn\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x)+4\log(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-8\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\right)-b^2n^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^3(x)+6\log^2(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-24\log(x)\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)+48\text{Li}_4\left(-\frac{f\sqrt{x}}{e}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*Log[x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[
c*x^n]) + 3*(a + b*Log[c*x^n])^2) - 3*(a - b*n*Log[x] + b*Log[c*x^n])^2*(Lo
g[1 + (f*Sqrt[x])/e]*Log[x] + 2*PolyLog[2, -((f*Sqrt[x])/e)]) - 3*b*n*(a -
b*n*Log[x] + b*Log[c*x^n])*(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*Poly
Log[2, -((f*Sqrt[x])/e)] - 8*PolyLog[3, -((f*Sqrt[x])/e)]) - b^2*n^2*(Log[1
 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[x])/e)] - 24*L
og[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt[x])/e)]))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^{n}))^{2}\ln\left(d\left(e+f\sqrt{x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x, x)
```

3.126 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^2}\,dx$

Optimal. Leaf size=441

$$-\frac{14b^2fn^2}{e\sqrt{x}}+\frac{2b^2f^2n^2\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x}-\frac{4b^2f^2n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-$$

```
[Out] -b^2*f^2*n^2*ln(x)/e^2+1/2*b^2*f^2*n^2*ln(x)^2/e^2-b*f^2*n*ln(x)*(a+b*ln(c*
x^n))/e^2-1/6*f^2*(a+b*ln(c*x^n))^3/b/e^2/n+2*b^2*f^2*n^2*ln(e+f*x^(1/2))/e
^2+2*b*f^2*n*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^2-4*b^2*f^2*n^2*ln(-f*x^(1/2
)/e)*ln(e+f*x^(1/2))/e^2-2*b^2*n^2*ln(d*(e+f*x^(1/2)))/x-2*b*n*(a+b*ln(c*x^
n))*ln(d*(e+f*x^(1/2)))/x-(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x+f^2*(a+b*
ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/e^2+4*b*f^2*n*(a+b*ln(c*x^n))*polylog(2,-f*x
^(1/2)/e)/e^2-4*b^2*f^2*n^2*polylog(2,1+f*x^(1/2)/e)/e^2-8*b^2*f^2*n^2*poly
log(3,-f*x^(1/2)/e)/e^2-14*b^2*f*n^2/e/x^(1/2)-6*b*f*n*(a+b*ln(c*x^n))/e/x^
(1/2)-f*(a+b*ln(c*x^n))^2/e/x^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {2504, 2442, 46, 2424, 2341, 2422, 2375, 2421, 6724, 2423, 2441, 2352, 2338, 2413, 12, 2339, 30}

$$\frac{4bf^2n\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)(a+b\log(cx^n))}{e^2}-\frac{4b^2f^2n^2\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e^2}-\frac{8b^2f^2n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x}-\frac{2bn\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))}{x}-\frac{f^2(a+b\log(cx^n))^3}{6be^2n}+\frac{f^2\log\left(\frac{f\sqrt{x}}{e}+1\right)(a+b\log(cx^n))^2}{e^2}+\frac{2bf^2n\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e^2}-\frac{bf^2n\log(x)(a+b\log(cx^n))}{e^2}-\frac{f(a+b\log(cx^n))^2}{e\sqrt{x}}-\frac{6bfn(a+b\log(cx^n))}{e\sqrt{x}}-\frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x}+\frac{b^2f^2n^2\log^2(x)}{2e^2}+\frac{2b^2f^2n^2\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{4b^2f^2n^2\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-\frac{b^2f^2n^2\log(x)}{e^2}-\frac{14b^2fn^2}{e\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] (-14*b^2*f*n^2)/(e*Sqrt[x]) + (2*b^2*f^2*n^2*Log[e + f*Sqrt[x]])/e^2 - (2*b
^2*n^2*Log[d*(e + f*Sqrt[x])])/x - (4*b^2*f^2*n^2*Log[e + f*Sqrt[x]]*Log[-(
(f*Sqrt[x])/e)])/e^2 - (b^2*f^2*n^2*Log[x])/e^2 + (b^2*f^2*n^2*Log[x]^2)/(2
*e^2) - (6*b*f*n*(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (2*b*f^2*n*Log[e + f*Sqr
t[x]]*(a + b*Log[c*x^n]))/e^2 - (2*b*n*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*
x^n]))/x - (b*f^2*n*Log[x]*(a + b*Log[c*x^n]))/e^2 - (f*(a + b*Log[c*x^n])^
2)/(e*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x + (f^2*Log
[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/e^2 - (f^2*(a + b*Log[c*x^n])^3)/
(6*b*e^2*n) - (4*b^2*f^2*n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2 + (4*b*f^2*
n*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 - (8*b^2*f^2*n^2*Pol
yLog[3, -((f*Sqrt[x])/e)])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
```

```
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^2}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2} -$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^2}{e^2} -$$

$$= -\frac{8b^2fn^2}{e\sqrt{x}} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n\log\left(e+f\sqrt{x}\right)(a}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n\log\left(e+f\sqrt{x}\right)(}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} + \frac{b^2f^2n^2\log^2(x)}{2e^2} - \frac{6bfn\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{2bf^2n}{}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{4b^2f^2n^2\log\left(e+f\sqrt{\right.}}{e^2}$$

$$= -\frac{12b^2fn^2}{e\sqrt{x}} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{4b^2f^2n^2\log\left(e+f\sqrt{\right.}}{e^2}$$

$$= -\frac{14b^2fn^2}{e\sqrt{x}} + \frac{2b^2f^2n^2\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{2b^2n^2\log\left(d\left(e+f\sqrt{\right.\right.}}{x}$$

Mathematica [A]
time = 0.31, size = 821, normalized size = 1.86

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^2,x]
```

```
[Out] -1/3*(3*a^2*e*f*Sqrt[x] + 18*a*b*e*f*n*Sqrt[x] + 42*b^2*e*f*n^2*Sqrt[x] - 3
*a^2*f^2*x*Log[e + f*Sqrt[x]] - 6*a*b*f^2*n*x*Log[e + f*Sqrt[x]] - 6*b^2*f^
2*n^2*x*Log[e + f*Sqrt[x]] + 3*a^2*e^2*Log[d*(e + f*Sqrt[x])] + 6*a*b*e^2*n
*Log[d*(e + f*Sqrt[x])] + 6*b^2*e^2*n^2*Log[d*(e + f*Sqrt[x])] + (3*a^2*f^2
*x*Log[x])/2 + 3*a*b*f^2*n*x*Log[x] + 3*b^2*f^2*n^2*x*Log[x] + 6*a*b*f^2*n*
x*Log[e + f*Sqrt[x]]*Log[x] + 6*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[x] - 6
```

```
*a*b*f^2*n*x*Log[1 + (f*Sqrt[x])/e]*Log[x] - 6*b^2*f^2*n^2*x*Log[1 + (f*Sqr
t[x])/e]*Log[x] - (3*a*b*f^2*n*x*Log[x]^2)/2 - (3*b^2*f^2*n^2*x*Log[x]^2)/2
 - 3*b^2*f^2*n^2*x*Log[e + f*Sqrt[x]]*Log[x]^2 + 3*b^2*f^2*n^2*x*Log[1 + (f
*Sqrt[x])/e]*Log[x]^2 + (b^2*f^2*n^2*x*Log[x]^3)/2 + 6*a*b*e*f*Sqrt[x]*Log[
c*x^n] + 18*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 6*a*b*f^2*x*Log[e + f*Sqrt[x]]*L
og[c*x^n] - 6*b^2*f^2*n*x*Log[e + f*Sqrt[x]]*Log[c*x^n] + 6*a*b*e^2*Log[d*(
e + f*Sqrt[x])]*Log[c*x^n] + 6*b^2*e^2*n*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
+ 3*a*b*f^2*x*Log[x]*Log[c*x^n] + 3*b^2*f^2*n*x*Log[x]*Log[c*x^n] + 6*b^2*f
^2*n*x*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 6*b^2*f^2*n*x*Log[1 + (f*Sqrt
[x])/e]*Log[x]*Log[c*x^n] - (3*b^2*f^2*n*x*Log[x]^2*Log[c*x^n])/2 + 3*b^2*e
*f*Sqrt[x]*Log[c*x^n]^2 - 3*b^2*f^2*x*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 3*b
^2*e^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + (3*b^2*f^2*x*Log[x]*Log[c*x^n]
^2)/2 - 12*b*f^2*n*x*(a + b*n + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)]
+ 24*b^2*f^2*n^2*x*PolyLog[3, -((f*Sqrt[x])/e)])/(e^2*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln \left(c\,x^{n}\right)\right)^{2} \ln \left(d\left(e+f\sqrt{x}\,\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\ln\left(c\,x^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^2, x)
```

3.127 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal. Leaf size=608

$$-\frac{37b^2fn^2}{108ex^{3/2}}+\frac{7b^2f^2n^2}{8e^2x}-\frac{21b^2f^3n^2}{4e^3\sqrt{x}}+\frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)}{4e^4}-\frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2}-\frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)\,\mathrm{l}}{e^4}$$

```
[Out] -37/108*b^2*f*n^2/e/x^(3/2)+7/8*b^2*f^2*n^2/e^2/x-1/8*b^2*f^4*n^2*ln(x)/e^4
+1/8*b^2*f^4*n^2*ln(x)^2/e^4-7/18*b*f*n*(a+b*ln(c*x^n))/e/x^(3/2)+3/4*b*f^2
*n*(a+b*ln(c*x^n))/e^2/x-1/4*b*f^4*n*ln(x)*(a+b*ln(c*x^n))/e^4-1/6*f*(a+b*l
n(c*x^n))^2/e/x^(3/2)+1/4*f^2*(a+b*ln(c*x^n))^2/e^2/x-1/12*f^4*(a+b*ln(c*x^
n))^3/b/e^4/n+1/4*b^2*f^4*n^2*ln(e+f*x^(1/2))/e^4+1/2*b*f^4*n*(a+b*ln(c*x^n
))*ln(e+f*x^(1/2))/e^4-b^2*f^4*n^2*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^4-1/4
*b^2*n^2*ln(d*(e+f*x^(1/2)))/x^2-1/2*b*n*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))
)/x^2-1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^2+1/2*f^4*(a+b*ln(c*x^n))
^2*ln(1+f*x^(1/2)/e)/e^4+2*b*f^4*n*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/
e^4-b^2*f^4*n^2*polylog(2,1+f*x^(1/2)/e)/e^4-4*b^2*f^4*n^2*polylog(3,-f*x^(
1/2)/e)/e^4-21/4*b^2*f^3*n^2/e^3/x^(1/2)-5/2*b*f^3*n*(a+b*ln(c*x^n))/e^3/x^
(1/2)-1/2*f^3*(a+b*ln(c*x^n))^2/e^3/x^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {2504, 2442, 46, 2424, 2341, 2422, 2375, 2421, 6724, 2423, 2441, 2352, 2338, 2413, 12, 2339, 30}

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] (-37*b^2*f*n^2)/(108*e*x^(3/2)) + (7*b^2*f^2*n^2)/(8*e^2*x) - (21*b^2*f^3*n
^2)/(4*e^3*Sqrt[x]) + (b^2*f^4*n^2*Log[e + f*Sqrt[x]])/(4*e^4) - (b^2*n^2*L
og[d*(e + f*Sqrt[x])])/(4*x^2) - (b^2*f^4*n^2*Log[e + f*Sqrt[x]]*Log[-((f*S
qrt[x])/e)])/e^4 - (b^2*f^4*n^2*Log[x])/(8*e^4) + (b^2*f^4*n^2*Log[x]^2)/(8
*e^4) - (7*b*f*n*(a + b*Log[c*x^n]))/(18*e*x^(3/2)) + (3*b*f^2*n*(a + b*Log
[c*x^n]))/(4*e^2*x) - (5*b*f^3*n*(a + b*Log[c*x^n]))/(2*e^3*Sqrt[x]) + (b*f
^4*n*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(2*e^4) - (b*n*Log[d*(e + f*Sqr
t[x])]*(a + b*Log[c*x^n]))/(2*x^2) - (b*f^4*n*Log[x]*(a + b*Log[c*x^n]))/(4
*e^4) - (f*(a + b*Log[c*x^n])^2)/(6*e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^2)
/(4*e^2*x) - (f^3*(a + b*Log[c*x^n])^2)/(2*e^3*Sqrt[x]) - (Log[d*(e + f*Sqr
t[x])]*(a + b*Log[c*x^n])^2)/(2*x^2) + (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*L
og[c*x^n])^2)/(2*e^4) - (f^4*(a + b*Log[c*x^n])^3)/(12*b*e^4*n) - (b^2*f^4*
```

```
n^2*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^4 + (2*b*f^4*n*(a + b*Log[c*x^n])*Poly
Log[2, -((f*Sqrt[x])/e)])/e^4 - (4*b^2*f^4*n^2*PolyLog[3, -((f*Sqrt[x])/e)]
)/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
```

```
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2}{x^3}\,dx = -\frac{f(a+b\log\left(cx^n\right))^2}{6ex^{3/2}} + \frac{f^2(a+b\log\left(cx^n\right))^2}{4e^2x} - \frac{f^3(a+b\log\left(cx^{\cdots}\right.}{2e^3\sqrt{x}}$$

$$= -\frac{f(a+b\log\left(cx^n\right))^2}{6ex^{3/2}} + \frac{f^2(a+b\log\left(cx^n\right))^2}{4e^2x} - \frac{f^3(a+b\log\left(cx^{\cdots}\right.}{2e^3\sqrt{x}}$$

$$= -\frac{4b^2fn^2}{27ex^{3/2}} + \frac{b^2f^2n^2}{2e^2x} - \frac{4b^2f^3n^2}{e^3\sqrt{x}} - \frac{7bfn(a+b\log\left(cx^n\right))}{18ex^{3/2}} + \frac{3bf^{\cdots}}{\cdots}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{7bfn(a+b\log\left(cx^n\right))}{18ex^{3/2}} + \frac{3b\cdots}{\cdots}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} + \frac{b^2f^4n^2\log^2(x)}{8e^4} - \frac{7bfn(a+\cdots}{18e\cdots}$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2} - \cdots$$

$$= -\frac{7b^2fn^2}{27ex^{3/2}} + \frac{3b^2f^2n^2}{4e^2x} - \frac{5b^2f^3n^2}{e^3\sqrt{x}} - \frac{b^2n^2\log\left(d\left(e+f\sqrt{x}\right)\right)}{4x^2} - \cdots$$

$$= -\frac{37b^2fn^2}{108ex^{3/2}} + \frac{7b^2f^2n^2}{8e^2x} - \frac{21b^2f^3n^2}{4e^3\sqrt{x}} + \frac{b^2f^4n^2\log\left(e+f\sqrt{x}\right)}{4e^4} - \cdots$$

Mathematica [A]
time = 0.36, size = 1078, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)/x^3,x]
```

```
[Out] -1/216*(36*a^2*e^3*f*Sqrt[x] + 84*a*b*e^3*f*n*Sqrt[x] + 74*b^2*e^3*f*n^2*Sq
rt[x] - 54*a^2*e^2*f^2*x - 162*a*b*e^2*f^2*n*x - 189*b^2*e^2*f^2*n^2*x + 10
8*a^2*e*f^3*x^(3/2) + 540*a*b*e*f^3*n*x^(3/2) + 1134*b^2*e*f^3*n^2*x^(3/2)
- 108*a^2*f^4*x^2*Log[e + f*Sqrt[x]] - 108*a*b*f^4*n*x^2*Log[e + f*Sqrt[x]]
 - 54*b^2*f^4*n^2*x^2*Log[e + f*Sqrt[x]] + 108*a^2*e^4*Log[d*(e + f*Sqrt[x]
)] + 108*a*b*e^4*n*Log[d*(e + f*Sqrt[x])] + 54*b^2*e^4*n^2*Log[d*(e + f*Sqr
```

```
t[x])] + 54*a^2*f^4*x^2*Log[x] + 54*a*b*f^4*n*x^2*Log[x] + 27*b^2*f^4*n^2*x
^2*Log[x] + 216*a*b*f^4*n*x^2*Log[e + f*Sqrt[x]]*Log[x] + 108*b^2*f^4*n^2*x
^2*Log[e + f*Sqrt[x]]*Log[x] - 216*a*b*f^4*n*x^2*Log[1 + (f*Sqrt[x])/e]*Log
[x] - 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 54*a*b*f^4*n*x^2*
Log[x]^2 - 27*b^2*f^4*n^2*x^2*Log[x]^2 - 108*b^2*f^4*n^2*x^2*Log[e + f*Sqrt
[x]]*Log[x]^2 + 108*b^2*f^4*n^2*x^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 18*b^
2*f^4*n^2*x^2*Log[x]^3 + 72*a*b*e^3*f*Sqrt[x]*Log[c*x^n] + 84*b^2*e^3*f*n*S
qrt[x]*Log[c*x^n] - 108*a*b*e^2*f^2*x*Log[c*x^n] - 162*b^2*e^2*f^2*n*x*Log[
c*x^n] + 216*a*b*e*f^3*x^(3/2)*Log[c*x^n] + 540*b^2*e*f^3*n*x^(3/2)*Log[c*x
^n] - 216*a*b*f^4*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n] - 108*b^2*f^4*n*x^2*Log
[e + f*Sqrt[x]]*Log[c*x^n] + 216*a*b*e^4*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
+ 108*b^2*e^4*n*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 108*a*b*f^4*x^2*Log[x]*
Log[c*x^n] + 54*b^2*f^4*n*x^2*Log[x]*Log[c*x^n] + 216*b^2*f^4*n*x^2*Log[e +
 f*Sqrt[x]]*Log[x]*Log[c*x^n] - 216*b^2*f^4*n*x^2*Log[1 + (f*Sqrt[x])/e]*Lo
g[x]*Log[c*x^n] - 54*b^2*f^4*n*x^2*Log[x]^2*Log[c*x^n] + 36*b^2*e^3*f*Sqrt[
x]*Log[c*x^n]^2 - 54*b^2*e^2*f^2*x*Log[c*x^n]^2 + 108*b^2*e*f^3*x^(3/2)*Log
[c*x^n]^2 - 108*b^2*f^4*x^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 + 108*b^2*e^4*L
og[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 54*b^2*f^4*x^2*Log[x]*Log[c*x^n]^2 - 2
16*b*f^4*n*x^2*(2*a + b*n + 2*b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)] +
864*b^2*f^4*n^2*x^2*PolyLog[3, -((f*Sqrt[x])/e)])/(e^4*x^2)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2} \ln\left(d\left(e+f\sqrt{x}\,\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*log(d*f*sqrt(x) + d*e)
/x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**(1/2)))/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*sqrt(x) + e)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^2)/x^3, x)
```

3.128 $\int x \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx$

Optimal. Leaf size=907

$$-\frac{255b^3e^3n^3\sqrt{x}}{8f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{45b^3e^2n^3x}{16f^2}-\frac{175b^3en^3x^{3/2}}{216f}+\frac{3}{8}b^3n^3x^2+\frac{3b^3e^4n^3\log\left(e+f\sqrt{x}\right)}{8f^4}-\frac{3}{8}b^3n^3x^2\log\left(\right.$$

```
[Out] 45/16*b^3*e^2*n^3*x/f^2-175/216*b^3*e*n^3*x^(3/2)/f-9/16*b^2*n^2*x^2*(a+b*l
n(c*x^n))+3/8*b*n*x^2*(a+b*ln(c*x^n))^2-1/4*e^2*x*(a+b*ln(c*x^n))^3/f^2+1/6
*e*x^(3/2)*(a+b*ln(c*x^n))^3/f-3/8*b^3*n^3*x^2*ln(d*(e+f*x^(1/2)))-1/2*e^4*
(a+b*ln(c*x^n))^3*ln(1+f*x^(1/2)/e)/f^4+1/2*e^3*(a+b*ln(c*x^n))^3*x^(1/2)/f
^3+3/8*b^3*n^3*x^2-1/8*x^2*(a+b*ln(c*x^n))^3-9/4*a*b^2*e^2*n^2*x/f^2+3/8*b^
3*e^4*n^3*ln(e+f*x^(1/2))/f^4+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(
1/2)))-3/4*b*n*x^2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))+3/2*b^3*e^4*n^3*po
lylog(2,1+f*x^(1/2)/e)/f^4-6*b^3*e^4*n^3*polylog(3,-f*x^(1/2)/e)/f^4-24*b^3
*e^4*n^3*polylog(4,-f*x^(1/2)/e)/f^4-255/8*b^3*e^3*n^3*x^(1/2)/f^3+1/2*x^2*
(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))-9/4*b^3*e^2*n^2*x*ln(c*x^n)/f^2-3/8*b
^2*e^2*n^2*x*(a+b*ln(c*x^n))/f^2+37/36*b^2*e*n^2*x^(3/2)*(a+b*ln(c*x^n))/f+
9/8*b*e^2*n*x*(a+b*ln(c*x^n))^2/f^2-7/12*b*e*n*x^(3/2)*(a+b*ln(c*x^n))^2/f-
3/4*b^2*e^4*n^2*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/f^4+3/2*b^3*e^4*n^3*ln(-f*x
^(1/2)/e)*ln(e+f*x^(1/2))/f^4+3/4*b*e^4*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/
e)/f^4+3*b^2*e^4*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/f^4-3*b*e^4*n*
(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/f^4+12*b^2*e^4*n^2*(a+b*ln(c*x^n)
)*polylog(3,-f*x^(1/2)/e)/f^4+63/4*b^2*e^3*n^2*(a+b*ln(c*x^n))*x^(1/2)/f^3-
15/4*b*e^3*n*(a+b*ln(c*x^n))^2*x^(1/2)/f^3
```

Rubi [A]
time = 0.88, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2504, 2442, 45, 2424, 2333, 2332, 2342, 2341, 2422, 2375, 2421, 2430, 6724, 2423, 2441, 2352}

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-255*b^3*e^3*n^3*Sqrt[x])/(8*f^3) - (9*a*b^2*e^2*n^2*x)/(4*f^2) + (45*b^3*
e^2*n^3*x)/(16*f^2) - (175*b^3*e*n^3*x^(3/2))/(216*f) + (3*b^3*n^3*x^2)/8 +
 (3*b^3*e^4*n^3*Log[e + f*Sqrt[x]])/(8*f^4) - (3*b^3*n^3*x^2*Log[d*(e + f*S
qrt[x])])/8 + (3*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(2*f
^4) - (9*b^3*e^2*n^2*x*Log[c*x^n])/(4*f^2) + (63*b^2*e^3*n^2*Sqrt[x]*(a + b
*Log[c*x^n]))/(4*f^3) - (3*b^2*e^2*n^2*x*(a + b*Log[c*x^n]))/(8*f^2) + (37*
b^2*e*n^2*x^(3/2)*(a + b*Log[c*x^n]))/(36*f) - (9*b^2*n^2*x^2*(a + b*Log[c*
```

```
x^n]))/16 - (3*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(4*f^4) +
 (3*b^2*n^2*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/4 - (15*b*e^3*n*
Sqrt[x]*(a + b*Log[c*x^n])^2)/(4*f^3) + (9*b*e^2*n*x*(a + b*Log[c*x^n])^2)/
(8*f^2) - (7*b*e*n*x^(3/2)*(a + b*Log[c*x^n])^2)/(12*f) + (3*b*n*x^2*(a + b
*Log[c*x^n])^2)/8 - (3*b*n*x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^2)
/4 + (3*b*e^4*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^2)/(4*f^4) + (e^3
*Sqrt[x]*(a + b*Log[c*x^n])^3)/(2*f^3) - (e^2*x*(a + b*Log[c*x^n])^3)/(4*f^
2) + (e*x^(3/2)*(a + b*Log[c*x^n])^3)/(6*f) - (x^2*(a + b*Log[c*x^n])^3)/8
+ (x^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/2 - (e^4*Log[1 + (f*Sqr
t[x])/e]*(a + b*Log[c*x^n])^3)/(2*f^4) + (3*b^3*e^4*n^3*PolyLog[2, 1 + (f*S
qrt[x])/e])/(2*f^4) + (3*b^2*e^4*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqr
t[x])/e)])/f^4 - (3*b*e^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e
)])/f^4 - (6*b^3*e^4*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/f^4 + (12*b^2*e^4*n^
2*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/f^4 - (24*b^3*e^4*n^3*Po
lyLog[4, -((f*Sqrt[x])/e)])/f^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3\,dx=\frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2f^3}-\frac{e^2x\left(a+b\log\left(cx^n\right)\right)^3}{4f^2}+\frac{ex^{3/2}(a+}{}$$

$$=\frac{e^3\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{2f^3}-\frac{e^2x\left(a+b\log\left(cx^n\right)\right)^3}{4f^2}+\frac{ex^{3/2}(a+}{}$$

$$=-\frac{15be^3n\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{4f^3}+\frac{9be^2nx\left(a+b\log\left(cx^n\right)\right)^2}{8f^2}-$$

$$=-\frac{24b^3e^3n^3\sqrt{x}}{f^3}-\frac{3ab^2e^2n^2x}{2f^2}-\frac{8b^3en^3x^{3/2}}{27f}+\frac{3}{32}b^3n^3x^2+$$

$$=-\frac{30b^3e^3n^3\sqrt{x}}{f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{3b^3e^2n^3x}{2f^2}-\frac{14b^3en^3x^{3/2}}{27f}+$$

$$=-\frac{63b^3e^3n^3\sqrt{x}}{2f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{21b^3e^2n^3x}{8f^2}-\frac{37b^3en^3x^{3/2}}{54f}+$$

$$=-\frac{63b^3e^3n^3\sqrt{x}}{2f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{21b^3e^2n^3x}{8f^2}-\frac{37b^3en^3x^{3/2}}{54f}+$$

$$=-\frac{63b^3e^3n^3\sqrt{x}}{2f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{21b^3e^2n^3x}{8f^2}-\frac{37b^3en^3x^{3/2}}{54f}+$$

$$=-\frac{63b^3e^3n^3\sqrt{x}}{2f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{21b^3e^2n^3x}{8f^2}-\frac{37b^3en^3x^{3/2}}{54f}+$$

$$=-\frac{255b^3e^3n^3\sqrt{x}}{8f^3}-\frac{9ab^2e^2n^2x}{4f^2}+\frac{45b^3e^2n^3x}{16f^2}-\frac{175b^3en^3x^{3/2}}{216f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1968 vs. $2(907) = 1814$.
time = 0.47, size = 1968, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (216*a^3*e^3*f*Sqrt[x] - 1620*a^2*b*e^3*f*n*Sqrt[x] + 6804*a*b^2*e^3*f*n^2*
Sqrt[x] - 13770*b^3*e^3*f*n^3*Sqrt[x] - 108*a^3*e^2*f^2*x + 486*a^2*b*e^2*f
^2*n*x - 1134*a*b^2*e^2*f^2*n^2*x + 1215*b^3*e^2*f^2*n^3*x + 72*a^3*e*f^3*x
^(3/2) - 252*a^2*b*e*f^3*n*x^(3/2) + 444*a*b^2*e*f^3*n^2*x^(3/2) - 350*b^3*
e*f^3*n^3*x^(3/2) - 54*a^3*f^4*x^2 + 162*a^2*b*f^4*n*x^2 - 243*a*b^2*f^4*n^
2*x^2 + 162*b^3*f^4*n^3*x^2 - 216*a^3*e^4*Log[e + f*Sqrt[x]] + 324*a^2*b*e^
4*n*Log[e + f*Sqrt[x]] - 324*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]] + 162*b^3*e^4
*n^3*Log[e + f*Sqrt[x]] + 216*a^3*f^4*x^2*Log[d*(e + f*Sqrt[x])] - 324*a^2*
b*f^4*n*x^2*Log[d*(e + f*Sqrt[x])] + 324*a*b^2*f^4*n^2*x^2*Log[d*(e + f*Sqr
t[x])] - 162*b^3*f^4*n^3*x^2*Log[d*(e + f*Sqrt[x])] + 648*a^2*b*e^4*n*Log[e
 + f*Sqrt[x]]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x] + 324*b^
3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x] - 648*a^2*b*e^4*n*Log[1 + (f*Sqrt[x])/e
]*Log[x] + 648*a*b^2*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x] - 324*b^3*e^4*n^
3*Log[1 + (f*Sqrt[x])/e]*Log[x] - 648*a*b^2*e^4*n^2*Log[e + f*Sqrt[x]]*Log[
x]^2 + 324*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^2 + 648*a*b^2*e^4*n^2*Log[
1 + (f*Sqrt[x])/e]*Log[x]^2 - 324*b^3*e^4*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]
^2 + 216*b^3*e^4*n^3*Log[e + f*Sqrt[x]]*Log[x]^3 - 216*b^3*e^4*n^3*Log[1 +
(f*Sqrt[x])/e]*Log[x]^3 + 648*a^2*b*e^3*f*Sqrt[x]*Log[c*x^n] - 3240*a*b^2*e
^3*f*n*Sqrt[x]*Log[c*x^n] + 6804*b^3*e^3*f*n^2*Sqrt[x]*Log[c*x^n] - 324*a^2
*b*e^2*f^2*x*Log[c*x^n] + 972*a*b^2*e^2*f^2*n*x*Log[c*x^n] - 1134*b^3*e^2*f
^2*n^2*x*Log[c*x^n] + 216*a^2*b*e*f^3*x^(3/2)*Log[c*x^n] - 504*a*b^2*e*f^3*
n*x^(3/2)*Log[c*x^n] + 444*b^3*e*f^3*n^2*x^(3/2)*Log[c*x^n] - 162*a^2*b*f^4
*x^2*Log[c*x^n] + 324*a*b^2*f^4*n*x^2*Log[c*x^n] - 243*b^3*f^4*n^2*x^2*Log[
c*x^n] - 648*a^2*b*e^4*Log[e + f*Sqrt[x]]*Log[c*x^n] + 648*a*b^2*e^4*n*Log[
e + f*Sqrt[x]]*Log[c*x^n] - 324*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[c*x^n] +
 648*a^2*b*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] - 648*a*b^2*f^4*n*x^2*
Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 324*b^3*f^4*n^2*x^2*Log[d*(e + f*Sqrt[x
])]*Log[c*x^n] + 1296*a*b^2*e^4*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 64
8*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] - 1296*a*b^2*e^4*n*Log[1
 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n] + 648*b^3*e^4*n^2*Log[1 + (f*Sqrt[x])/e
]*Log[x]*Log[c*x^n] - 648*b^3*e^4*n^2*Log[e + f*Sqrt[x]]*Log[x]^2*Log[c*x^n
] + 648*b^3*e^4*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2*Log[c*x^n] + 648*a*b^2*
e^3*f*Sqrt[x]*Log[c*x^n]^2 - 1620*b^3*e^3*f*n*Sqrt[x]*Log[c*x^n]^2 - 324*a*
b^2*e^2*f^2*x*Log[c*x^n]^2 + 486*b^3*e^2*f^2*n*x*Log[c*x^n]^2 + 216*a*b^2*e
*f^3*x^(3/2)*Log[c*x^n]^2 - 252*b^3*e*f^3*n*x^(3/2)*Log[c*x^n]^2 - 162*a*b^
2*f^4*x^2*Log[c*x^n]^2 + 162*b^3*f^4*n*x^2*Log[c*x^n]^2 - 648*a*b^2*e^4*Log
[e + f*Sqrt[x]]*Log[c*x^n]^2 + 324*b^3*e^4*n*Log[e + f*Sqrt[x]]*Log[c*x^n]^
2 + 648*a*b^2*f^4*x^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 - 324*b^3*f^4*n*x
^2*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^2 + 648*b^3*e^4*n*Log[e + f*Sqrt[x]]*L
og[x]*Log[c*x^n]^2 - 648*b^3*e^4*n*Log[1 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n]
^2 + 216*b^3*e^3*f*Sqrt[x]*Log[c*x^n]^3 - 108*b^3*e^2*f^2*x*Log[c*x^n]^3 +
72*b^3*e*f^3*x^(3/2)*Log[c*x^n]^3 - 54*b^3*f^4*x^2*Log[c*x^n]^3 - 216*b^3*e
^4*Log[e + f*Sqrt[x]]*Log[c*x^n]^3 + 216*b^3*f^4*x^2*Log[d*(e + f*Sqrt[x])]
```

```
*Log[c*x^n]^3 - 648*b*e^4*n*(2*a^2 - 2*a*b*n + b^2*n^2 - 2*b*(-2*a + b*n)*L
og[c*x^n] + 2*b^2*Log[c*x^n]^2)*PolyLog[2, -((f*Sqrt[x])/e)] + 2592*b^2*e^4
*n^2*(2*a - b*n + 2*b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)] - 10368*b^3*
e^4*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/(432*f^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^3\ln\left(d\left(e+f\sqrt{x}\,\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2))),x)
[Out] int(x*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
[Out] integral((b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3*a^2*b*x*log(c*x^n
) + a^3*x)*log(d*f*sqrt(x) + d*e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2))),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*x*log((f*sqrt(x) + e)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(d\left(e+f\,\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(x*log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

3.129 $\int \log \left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log \left(cx^n\right)\right)^3\,dx$

Optimal. Leaf size=639

$$-\frac{90b^3en^3\sqrt{x}}{f}-6ab^2n^2x+12b^3n^3x+\frac{6b^3e^2n^3\log\left(e+f\sqrt{x}\right)}{f^2}-6b^3n^3x\log\left(d\left(e+f\sqrt{x}\right)\right)+\frac{12b^3e^2n^3\log\left(e+\right.}{}$$

[Out] -6*a*b^2*n^2*x-6*b^3*n^2*x*ln(c*x^n)-3*b^2*n^2*x*(a+b*ln(c*x^n))+3*b*n*x*(a
+b*ln(c*x^n))^2+e*(a+b*ln(c*x^n))^3*x^(1/2)/f-6*b^3*n^3*x*ln(d*(e+f*x^(1/2)
))-e^2*(a+b*ln(c*x^n))^3*ln(1+f*x^(1/2)/e)/f^2+12*b^3*n^3*x-1/2*x*(a+b*ln(c
*x^n))^3+6*b^3*e^2*n^3*ln(e+f*x^(1/2))/f^2+6*b^2*n^2*x*(a+b*ln(c*x^n))*ln(d
*(e+f*x^(1/2)))-3*b*n*x*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))+12*b^3*e^2*n^
3*polylog(2,1+f*x^(1/2)/e)/f^2-24*b^3*e^2*n^3*polylog(3,-f*x^(1/2)/e)/f^2-4
8*b^3*e^2*n^3*polylog(4,-f*x^(1/2)/e)/f^2-90*b^3*e*n^3*x^(1/2)/f+x*(a+b*ln(
c*x^n))^3*ln(d*(e+f*x^(1/2)))+42*b^2*e*n^2*(a+b*ln(c*x^n))*x^(1/2)/f-9*b*e*
n*(a+b*ln(c*x^n))^2*x^(1/2)/f-6*b^2*e^2*n^2*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))
/f^2+12*b^3*e^2*n^3*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^2+3*b*e^2*n*(a+b*ln(
c*x^n))^2*ln(1+f*x^(1/2)/e)/f^2+12*b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(2,-f
*x^(1/2)/e)/f^2-6*b*e^2*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/f^2+24*
b^2*e^2*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/f^2

Rubi [A]
time = 0.58, antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {2498, 272, 45, 2417, 2333, 2332, 2342, 2341, 2422, 2375, 2421, 2430, 6724, 2504, 2441, 2352}

$$\frac{12b^2e^2n^2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{24b^2e^2n^2\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}-\frac{6be^2n\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2}{f^2}+\frac{12b^3e^2n^3\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{f^2}-\frac{24b^3e^2n^3\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)}{f^2}-\frac{48b^3e^2n^3\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)}{f^2}+6b^2n^2x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)-\frac{6b^2e^2n^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{f^2}+\frac{42b^2en^2\sqrt{x}\left(a+b\log\left(cx^n\right)\right)}{f}-3b^2n^2x\left(a+b\log\left(cx^n\right)\right)-6ab^2n^2x-3bnx\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^2+x\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3+\frac{3be^2n\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^2}{f^2}-\frac{e^2\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^3}{f^2}-\frac{9ben\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^2}{f}+\frac{e\sqrt{x}\left(a+b\log\left(cx^n\right)\right)^3}{f}+3bnx\left(a+b\log\left(cx^n\right)\right)^2-\frac{1}{2}x\left(a+b\log\left(cx^n\right)\right)^3-6b^3n^2x\log\left(cx^n\right)-6b^3n^3x\log\left(d\left(e+f\sqrt{x}\right)\right)+\frac{6b^3e^2n^3\log\left(e+f\sqrt{x}\right)}{f^2}+\frac{12b^3e^2n^3\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{f^2}-\frac{90b^3en^3\sqrt{x}}{f}+12b^3n^3x$$

Antiderivative was successfully verified.

[In] Int[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]

[Out] (-90*b^3*e*n^3*Sqrt[x])/f - 6*a*b^2*n^2*x + 12*b^3*n^3*x + (6*b^3*e^2*n^3*L
og[e + f*Sqrt[x]])/f^2 - 6*b^3*n^3*x*Log[d*(e + f*Sqrt[x])] + (12*b^3*e^2*n
^3*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/f^2 - 6*b^3*n^2*x*Log[c*x^n] +
 (42*b^2*e*n^2*Sqrt[x]*(a + b*Log[c*x^n]))/f - 3*b^2*n^2*x*(a + b*Log[c*x^n
]) - (6*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/f^2 + 6*b^2*n^2*
x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]) - (9*b*e*n*Sqrt[x]*(a + b*Log[c
*x^n])^2)/f + 3*b*n*x*(a + b*Log[c*x^n])^2 - 3*b*n*x*Log[d*(e + f*Sqrt[x])]
*(a + b*Log[c*x^n])^2 + (3*b*e^2*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n]
)^2)/f^2 + (e*Sqrt[x]*(a + b*Log[c*x^n])^3)/f - (x*(a + b*Log[c*x^n])^3)/2
+ x*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3 - (e^2*Log[1 + (f*Sqrt[x])/
e]*(a + b*Log[c*x^n])^3)/f^2 + (12*b^3*e^2*n^3*PolyLog[2, 1 + (f*Sqrt[x])/e
])/f^2 + (12*b^2*e^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/f

```
^2 - (6*b*e^2*n*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/f^2 - (2
4*b^3*e^2*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 + (24*b^2*e^2*n^2*(a + b*Lo
g[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/f^2 - (48*b^3*e^2*n^3*PolyLog[4, -(
(f*Sqrt[x])/e)])/f^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3\,dx = \frac{e\sqrt{x}\,(a+b\log(cx^n))^3}{f} - \frac{1}{2}x(a+b\log(cx^n))^3 - \frac{e^2\log\left(e+f\right.}{}$$

$$= \frac{e\sqrt{x}\,(a+b\log(cx^n))^3}{f} - \frac{1}{2}x(a+b\log(cx^n))^3 - \frac{e^2\log\left(e+f\right.}{}$$

$$= -\frac{9ben\sqrt{x}\,(a+b\log(cx^n))^2}{f} + 3bnx(a+b\log(cx^n))^2 + \frac{3be^2r}{}$$

$$= -\frac{48b^3en^3\sqrt{x}}{f} - 3ab^2n^2x + \frac{24b^2en^2\sqrt{x}\,(a+b\log(cx^n))}{f} - \frac{9}{}$$

$$= -\frac{72b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 3b^3n^3x - 3b^3n^2x\log(cx^n) + \frac{42b^2}{}$$

$$= -\frac{84b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 9b^3n^3x - 6b^3n^2x\log(cx^n) + \frac{42b^2}{}$$

$$= -\frac{84b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 9b^3n^3x - 6b^3n^3x\log\left(d\left(e+f\sqrt{x}\right.\right.$$

$$= -\frac{84b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 9b^3n^3x - 6b^3n^3x\log\left(d\left(e+f\sqrt{x}\right.\right.$$

$$= -\frac{84b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 9b^3n^3x - 6b^3n^3x\log\left(d\left(e+f\sqrt{x}\right.\right.$$

$$= -\frac{90b^3en^3\sqrt{x}}{f} - 6ab^2n^2x + 12b^3n^3x + \frac{6b^3e^2n^3\log\left(e+f\sqrt{x}\right.}{f^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1522 vs. $2(639) = 1278$.
time = 0.40, size = 1522, normalized size = 2.38

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3,x]
```

```
[Out] -1/2*(-2*a^3*e*f*Sqrt[x] + 18*a^2*b*e*f*n*Sqrt[x] - 84*a*b^2*e*f*n^2*Sqrt[x
] + 180*b^3*e*f*n^3*Sqrt[x] + a^3*f^2*x - 6*a^2*b*f^2*n*x + 18*a*b^2*f^2*n^
2*x - 24*b^3*f^2*n^3*x + 2*a^3*e^2*Log[e + f*Sqrt[x]] - 6*a^2*b*e^2*n*Log[e
 + f*Sqrt[x]] + 12*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]] - 12*b^3*e^2*n^3*Log[e
+ f*Sqrt[x]] - 2*a^3*f^2*x*Log[d*(e + f*Sqrt[x])] + 6*a^2*b*f^2*n*x*Log[d*(
e + f*Sqrt[x])] - 12*a*b^2*f^2*n^2*x*Log[d*(e + f*Sqrt[x])] + 12*b^3*f^2*n^
3*x*Log[d*(e + f*Sqrt[x])] - 6*a^2*b*e^2*n*Log[e + f*Sqrt[x]]*Log[x] + 12*a
*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x] - 12*b^3*e^2*n^3*Log[e + f*Sqrt[x]]*
Log[x] + 6*a^2*b*e^2*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 12*a*b^2*e^2*n^2*Log
[1 + (f*Sqrt[x])/e]*Log[x] + 12*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x] +
 6*a*b^2*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]^2 - 6*b^3*e^2*n^3*Log[e + f*Sqrt
[x]]*Log[x]^2 - 6*a*b^2*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 6*b^3*e^2
*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 - 2*b^3*e^2*n^3*Log[e + f*Sqrt[x]]*Log
[x]^3 + 2*b^3*e^2*n^3*Log[1 + (f*Sqrt[x])/e]*Log[x]^3 - 6*a^2*b*e*f*Sqrt[x]
*Log[c*x^n] + 36*a*b^2*e*f*n*Sqrt[x]*Log[c*x^n] - 84*b^3*e*f*n^2*Sqrt[x]*Lo
g[c*x^n] + 3*a^2*b*f^2*x*Log[c*x^n] - 12*a*b^2*f^2*n*x*Log[c*x^n] + 18*b^3*
f^2*n^2*x*Log[c*x^n] + 6*a^2*b*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n] - 12*a*b^2
*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n] + 12*b^3*e^2*n^2*Log[e + f*Sqrt[x]]*Lo
g[c*x^n] - 6*a^2*b*f^2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] + 12*a*b^2*f^2*n
*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n] - 12*b^3*f^2*n^2*x*Log[d*(e + f*Sqrt[x
])]*Log[c*x^n] - 12*a*b^2*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 12*b
^3*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n] + 12*a*b^2*e^2*n*Log[1 + (f
*Sqrt[x])/e]*Log[x]*Log[c*x^n] - 12*b^3*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[
x]*Log[c*x^n] + 6*b^3*e^2*n^2*Log[e + f*Sqrt[x]]*Log[x]^2*Log[c*x^n] - 6*b^
3*e^2*n^2*Log[1 + (f*Sqrt[x])/e]*Log[x]^2*Log[c*x^n] - 6*a*b^2*e*f*Sqrt[x]*
Log[c*x^n]^2 + 18*b^3*e*f*n*Sqrt[x]*Log[c*x^n]^2 + 3*a*b^2*f^2*x*Log[c*x^n]
^2 - 6*b^3*f^2*n*x*Log[c*x^n]^2 + 6*a*b^2*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]
^2 - 6*b^3*e^2*n*Log[e + f*Sqrt[x]]*Log[c*x^n]^2 - 6*a*b^2*f^2*x*Log[d*(e +
 f*Sqrt[x])]*Log[c*x^n]^2 + 6*b^3*f^2*n*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]
^2 - 6*b^3*e^2*n*Log[e + f*Sqrt[x]]*Log[x]*Log[c*x^n]^2 + 6*b^3*e^2*n*Log[1
 + (f*Sqrt[x])/e]*Log[x]*Log[c*x^n]^2 - 2*b^3*e*f*Sqrt[x]*Log[c*x^n]^3 + b^
3*f^2*x*Log[c*x^n]^3 + 2*b^3*e^2*Log[e + f*Sqrt[x]]*Log[c*x^n]^3 - 2*b^3*f^
2*x*Log[d*(e + f*Sqrt[x])]*Log[c*x^n]^3 + 12*b*e^2*n*(a^2 - 2*a*b*n + 2*b^2
*n^2 + 2*b*(a - b*n)*Log[c*x^n] + b^2*Log[c*x^n]^2)*PolyLog[2, -((f*Sqrt[x]
)/e)] - 48*b^2*e^2*n^2*(a - b*n + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)
] + 96*b^3*e^2*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/f^2
```

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a + b \ln \left(c\, x^{n}\right)\right)^{3} \ln \left(d \left(e + f \sqrt{x}\,\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2))),x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/27*(27*b^3*x*e*log(d)*log(x^n)^3 - 81*((n*log(d) - log(c)*log(d))*b^3 - a
*b^2*log(d))*x*e*log(x^n)^2 - 81*(2*(n*log(d) - log(c)*log(d))*a*b^2 - (2*n
^2*log(d) - 2*n*log(c)*log(d) + log(c)^2*log(d))*b^3 - a^2*b*log(d))*x*e*lo
g(x^n) - 27*(3*(n*log(d) - log(c)*log(d))*a^2*b - 3*(2*n^2*log(d) - 2*n*log
(c)*log(d) + log(c)^2*log(d))*a*b^2 + (6*n^3*log(d) - 6*n^2*log(c)*log(d) +
 3*n*log(c)^2*log(d) - log(c)^3*log(d))*b^3 - a^3*log(d))*x*e + 27*(b^3*x*e
*log(x^n)^3 - 3*(b^3*(n - log(c)) - a*b^2)*x*e*log(x^n)^2 + 3*((2*n^2 - 2*n
*log(c) + log(c)^2)*b^3 - 2*a*b^2*(n - log(c)) + a^2*b)*x*e*log(x^n) + (3*(
2*n^2 - 2*n*log(c) + log(c)^2)*a*b^2 - (6*n^3 - 6*n^2*log(c) + 3*n*log(c)^2
 - log(c)^3)*b^3 - 3*a^2*b*(n - log(c)) + a^3)*x*e)*log(f*sqrt(x) + e) - (9
*b^3*f*x^2*log(x^n)^3 - 9*((5*f*n - 3*f*log(c))*b^3 - 3*a*b^2*f)*x^2*log(x^
n)^2 - 3*(6*(5*f*n - 3*f*log(c))*a*b^2 - (38*f*n^2 - 30*f*n*log(c) + 9*f*lo
g(c)^2)*b^3 - 9*a^2*b*f)*x^2*log(x^n) - (9*(5*f*n - 3*f*log(c))*a^2*b - 3*(
38*f*n^2 - 30*f*n*log(c) + 9*f*log(c)^2)*a*b^2 + (130*f*n^3 - 114*f*n^2*log
(c) + 45*f*n*log(c)^2 - 9*f*log(c)^3)*b^3 - 9*a^3*f)*x^2)/sqrt(x))*e^(-1) +
 integrate(1/2*(b^3*f^2*x*log(x^n)^3 + 3*(a*b^2*f^2 - (f^2*n - f^2*log(c))*
b^3)*x*log(x^n)^2 + 3*(a^2*b*f^2 - 2*(f^2*n - f^2*log(c))*a*b^2 + (2*f^2*n^
2 - 2*f^2*n*log(c) + f^2*log(c)^2)*b^3)*x*log(x^n) + (a^3*f^2 - 3*(f^2*n -
f^2*log(c))*a^2*b + 3*(2*f^2*n^2 - 2*f^2*n*log(c) + f^2*log(c)^2)*a*b^2 - (
6*f^2*n^3 - 6*f^2*n^2*log(c) + 3*f^2*n*log(c)^2 - f^2*log(c)^3)*b^3)*x)/(f*
e^(1/2*log(x) + 1) + e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3,x)
```

```
[Out] int(log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3, x)
```

3.130 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x}\,dx$

Optimal. Leaf size=178

$$\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn}-\frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn}-2\left(a+b\log\left(cx^n\right)\right)^3\operatorname{Li}_2\left(-\frac{f\sqrt{x}}{e}\right.$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(e+f*x^(1/2)))/b/n-1/4*(a+b*ln(c*x^n))^4*ln(1+f*
x^(1/2)/e)/b/n-2*(a+b*ln(c*x^n))^3*polylog(2,-f*x^(1/2)/e)+12*b*n*(a+b*ln(c
*x^n))^2*polylog(3,-f*x^(1/2)/e)-48*b^2*n^2*(a+b*ln(c*x^n))*polylog(4,-f*x^
(1/2)/e)+96*b^3*n^3*polylog(5,-f*x^(1/2)/e)
```

Rubi [A]
time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$-48b^2n^2\text{PolyLog}\left(4,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)-2\text{PolyLog}\left(2,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^3+12bn\text{PolyLog}\left(3,-\frac{f\sqrt{x}}{e}\right)\left(a+b\log\left(cx^n\right)\right)^2+96b^3n^3\text{PolyLog}\left(5,-\frac{f\sqrt{x}}{e}\right)+\frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn}-\frac{\log\left(\frac{f\sqrt{x}}{e}+1\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^4)/(4*b*n) - (Log[1 + (f*Sqrt[x]
)/e]*(a + b*Log[c*x^n])^4)/(4*b*n) - 2*(a + b*Log[c*x^n])^3*PolyLog[2, -((f
*Sqrt[x])/e)] + 12*b*n*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*Sqrt[x])/e)] -
48*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[4, -((f*Sqrt[x])/e)] + 96*b^3*n^3*Pol
yLog[5, -((f*Sqrt[x])/e)]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{f\int\frac{(a+b\log(cx^n))^4}{\left(e+f\sqrt{x}\right)\sqrt{x}}\,dx}{8bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b}{4bn}$$

$$= \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^4}{4bn} - \frac{\log\left(1+\frac{f\sqrt{x}}{e}\right)(a+b}{4bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 403 vs. 2(178) = 356.
time = 0.24, size = 403, normalized size = 2.26

$$\frac{1}{8}\left(-2\log\left(d\left(e+f\sqrt{x}\right)\right)\log(x)\left(b^3n^3\log^3(x)-4b^2n^2\log^2(x)\left(a+b\log\left(cx^n\right)\right)+6bn\log(x)\left(a+b\log\left(cx^n\right)\right)^2-4\left(a+b\log\left(cx^n\right)\right)^3\right)-8\left(a-bn\log(x)+b\log\left(cx^n\right)\right)^3\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)+2\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)\right)-12bn\left(a-bn\log(x)+b\log\left(cx^n\right)\right)^2\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^2(x)+4\log(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-8\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)\right)-8b^2n^2\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^3(x)+6\log^2(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-24\log(x)\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)+48\text{Li}_4\left(-\frac{f\sqrt{x}}{e}\right)\right)-2b^3n^3\left(\log\left(1+\frac{f\sqrt{x}}{e}\right)\log^4(x)+8\log^3(x)\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)-48\log^2(x)\text{Li}_3\left(-\frac{f\sqrt{x}}{e}\right)+192\log(x)\text{Li}_4\left(-\frac{f\sqrt{x}}{e}\right)-384\text{Li}_5\left(-\frac{f\sqrt{x}}{e}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x,x]
```

```
[Out] (-2*Log[d*(e + f*Sqrt[x])]*Log[x]*(b^3*n^3*Log[x]^3 - 4*b^2*n^2*Log[x]^2*(a
 + b*Log[c*x^n]) + 6*b*n*Log[x]*(a + b*Log[c*x^n])^2 - 4*(a + b*Log[c*x^n])
^3) - 8*(a - b*n*Log[x] + b*Log[c*x^n])^3*(Log[1 + (f*Sqrt[x])/e]*Log[x] +
2*PolyLog[2, -((f*Sqrt[x])/e)]) - 12*b*n*(a - b*n*Log[x] + b*Log[c*x^n])^2*
(Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + 4*Log[x]*PolyLog[2, -((f*Sqrt[x])/e)] -
8*PolyLog[3, -((f*Sqrt[x])/e)]) - 8*b^2*n^2*(a - b*n*Log[x] + b*Log[c*x^n])
*(Log[1 + (f*Sqrt[x])/e]*Log[x]^3 + 6*Log[x]^2*PolyLog[2, -((f*Sqrt[x])/e)]
 - 24*Log[x]*PolyLog[3, -((f*Sqrt[x])/e)] + 48*PolyLog[4, -((f*Sqrt[x])/e)]
) - 2*b^3*n^3*(Log[1 + (f*Sqrt[x])/e]*Log[x]^4 + 8*Log[x]^3*PolyLog[2, -((f
*Sqrt[x])/e)] - 48*Log[x]^2*PolyLog[3, -((f*Sqrt[x])/e)] + 192*Log[x]*PolyL
og[4, -((f*Sqrt[x])/e)] - 384*PolyLog[5, -((f*Sqrt[x])/e)]))/8
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(e+f\sqrt{x}\,\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x, x)
```

3.131 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^2}\,dx$

Optimal. Leaf size=673

$$-\frac{90b^3fn^3}{e\sqrt{x}}+\frac{6b^3f^2n^3\log\left(e+f\sqrt{x}\right)}{e^2}-\frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x}-\frac{12b^3f^2n^3\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e^2}-$$

```
[Out] -3*b^3*f^2*n^3*ln(x)/e^2+3/2*b^3*f^2*n^3*ln(x)^2/e^2-3*b^2*f^2*n^2*ln(x)*(a
+b*ln(c*x^n))/e^2-1/2*f^2*(a+b*ln(c*x^n))^3/e^2-1/8*f^2*(a+b*ln(c*x^n))^4/b
/e^2/n+6*b^3*f^2*n^3*ln(e+f*x^(1/2))/e^2+6*b^2*f^2*n^2*(a+b*ln(c*x^n))*ln(e
+f*x^(1/2))/e^2-12*b^3*f^2*n^3*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^2-6*b^3*n
^3*ln(d*(e+f*x^(1/2)))/x-6*b^2*n^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))/x-3*
b*n*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^(1/2)))/x-(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(
1/2)))/x+3*b*f^2*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1/2)/e)/e^2+f^2*(a+b*ln(c*x^
n))^3*ln(1+f*x^(1/2)/e)/e^2+12*b^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x^(
1/2)/e)/e^2+6*b*f^2*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/e^2-12*b^3*
f^2*n^3*polylog(2,1+f*x^(1/2)/e)/e^2-24*b^3*f^2*n^3*polylog(3,-f*x^(1/2)/e)
/e^2-24*b^2*f^2*n^2*(a+b*ln(c*x^n))*polylog(3,-f*x^(1/2)/e)/e^2+48*b^3*f^2*
n^3*polylog(4,-f*x^(1/2)/e)/e^2-90*b^3*f*n^3/e/x^(1/2)-42*b^2*f*n^2*(a+b*ln
(c*x^n))/e/x^(1/2)-9*b*f*n*(a+b*ln(c*x^n))^2/e/x^(1/2)-f*(a+b*ln(c*x^n))^3/
e/x^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 19, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {2504, 2442, 46, 2424, 2342, 2341, 2422, 2375, 2421, 2430, 6724, 2423, 2441, 2352, 2338, 2413, 12, 2339, 30}

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] (-90*b^3*f*n^3)/(e*Sqrt[x]) + (6*b^3*f^2*n^3*Log[e + f*Sqrt[x]])/e^2 - (6*b
^3*n^3*Log[d*(e + f*Sqrt[x])])/x - (12*b^3*f^2*n^3*Log[e + f*Sqrt[x]]*Log[-
((f*Sqrt[x])/e)])/e^2 - (3*b^3*f^2*n^3*Log[x])/e^2 + (3*b^3*f^2*n^3*Log[x]^
2)/(2*e^2) - (42*b^2*f*n^2*(a + b*Log[c*x^n]))/(e*Sqrt[x]) + (6*b^2*f^2*n^2
*Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/e^2 - (6*b^2*n^2*Log[d*(e + f*Sqrt[
x])]*(a + b*Log[c*x^n]))/x - (3*b^2*f^2*n^2*Log[x]*(a + b*Log[c*x^n]))/e^2
- (9*b*f*n*(a + b*Log[c*x^n])^2)/(e*Sqrt[x]) - (3*b*n*Log[d*(e + f*Sqrt[x])
]*(a + b*Log[c*x^n])^2)/x + (3*b*f^2*n*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*
x^n])^2)/e^2 - (f^2*(a + b*Log[c*x^n])^3)/(2*e^2) - (f*(a + b*Log[c*x^n])^3
)/(e*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x + (f^2*Log[
```

```
1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^3)/e^2 - (f^2*(a + b*Log[c*x^n])^4)/(
8*b*e^2*n) - (12*b^3*f^2*n^3*PolyLog[2, 1 + (f*Sqrt[x])/e])/e^2 + (12*b^2*f
^2*n^2*(a + b*Log[c*x^n])*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 + (6*b*f^2*n*(a
 + b*Log[c*x^n])^2*PolyLog[2, -((f*Sqrt[x])/e)])/e^2 - (24*b^3*f^2*n^3*Poly
Log[3, -((f*Sqrt[x])/e)])/e^2 - (24*b^2*f^2*n^2*(a + b*Log[c*x^n])*PolyLog[
3, -((f*Sqrt[x])/e)])/e^2 + (48*b^3*f^2*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/e
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)\left(a+b\log\left(cx^n\right)\right)^3}{x^2}\,dx = -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2} -$$

$$= -\frac{f\left(a+b\log\left(cx^n\right)\right)^3}{e\sqrt{x}} + \frac{f^2\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)^3}{e^2} -$$

$$= -\frac{9bfn\left(a+b\log\left(cx^n\right)\right)^2}{e\sqrt{x}} + \frac{3bf^2n\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(c\right.\right.}{e^2}$$

$$= -\frac{48b^3fn^3}{e\sqrt{x}} - \frac{24b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} - \frac{9bfn\left(a+b\log\left(cx^n\right)\right.}{e\sqrt{x}}$$

$$= -\frac{72b^3fn^3}{e\sqrt{x}} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6b^2f^2n^2\log\left(e+f\sqrt{\right.}{e}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6b^2f^2n^2\log\left(e+f\sqrt{\right.}{e}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} + \frac{3b^3f^2n^3\log^2(x)}{2e^2} - \frac{42b^2fn^2\left(a+b\log\left(cx^n\right)\right)}{e\sqrt{x}} + \frac{6}{}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{12b^3f^2n^3\log\left(e+f\right.}{e}$$

$$= -\frac{84b^3fn^3}{e\sqrt{x}} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{x} - \frac{12b^3f^2n^3\log\left(e+f\right.}{e}$$

$$= -\frac{90b^3fn^3}{e\sqrt{x}} + \frac{6b^3f^2n^3\log\left(e+f\sqrt{x}\right)}{e^2} - \frac{6b^3n^3\log\left(d\left(e+f\sqrt{\right.\right.}{x}$$

Mathematica [A]
time = 0.74, size = 976, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^2,x]
```

```
[Out] -((e^2*Log[d*(e + f*Sqrt[x])]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 +
3*b*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[c*x^n] + 3*b^2*(a + b*n)*Log[c*x^n]^2 +
 b^3*Log[c*x^n]^3) + e*f*Sqrt[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3
 + 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]
) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n
])^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n]
)^3) - f^2*x*Log[e + f*Sqrt[x]]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3
+ 3*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n])
 + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n]
)^2 + 3*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])
^3) + (f^2*x*Log[x]*(a^3 + 3*a^2*b*n + 6*a*b^2*n^2 + 6*b^3*n^3 + 3*a^2*b*(-
(n*Log[x]) + Log[c*x^n]) + 6*a*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2
*(-(n*Log[x]) + Log[c*x^n]) + 3*a*b^2*(-(n*Log[x]) + Log[c*x^n])^2 + 3*b^3*
n*(-(n*Log[x]) + Log[c*x^n])^2 + b^3*(-(n*Log[x]) + Log[c*x^n])^3))/2 + 3*b
*f*n*Sqrt[x]*(a^2 + 2*a*b*n + 2*b^2*n^2 + 2*a*b*(-(n*Log[x]) + Log[c*x^n])
+ 2*b^2*n*(-(n*Log[x]) + Log[c*x^n]) + b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(2
*e + (e - f*Sqrt[x]*Log[1 + (f*Sqrt[x])/e])*Log[x] + (f*Sqrt[x]*Log[x]^2)/4
 - 2*f*Sqrt[x]*PolyLog[2, -((f*Sqrt[x])/e)]) + b^2*f*n^2*Sqrt[x]*(a + b*n -
 b*n*Log[x] + b*Log[c*x^n])*(24*e + 12*e*Log[x] + 3*e*Log[x]^2 - 3*f*Sqrt[x
]*Log[1 + (f*Sqrt[x])/e]*Log[x]^2 + (f*Sqrt[x]*Log[x]^3)/2 - 12*f*Sqrt[x]*L
og[x]*PolyLog[2, -((f*Sqrt[x])/e)] + 24*f*Sqrt[x]*PolyLog[3, -((f*Sqrt[x])/
e)]) + b^3*n^3*(6*f^2*x*Log[x]^2*PolyLog[2, -(e/(f*Sqrt[x]))] + f*Sqrt[x]*(
48*e + 24*e*Log[x] + 6*e*Log[x]^2 + e*Log[x]^3 - f*Sqrt[x]*Log[1 + e/(f*Sqr
t[x])]*Log[x]^3 + 24*f*Sqrt[x]*Log[x]*PolyLog[3, -(e/(f*Sqrt[x]))] + 48*f*S
qrt[x]*PolyLog[4, -(e/(f*Sqrt[x]))])))/(e^2*x))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(e+f\sqrt{x}\,\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2,x)
```

```
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^2, x)
```

3.132 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx$

Optimal. Leaf size=914

$$-\frac{175b^3fn^3}{216ex^{3/2}}+\frac{45b^3f^2n^3}{16e^2x}-\frac{255b^3f^3n^3}{8e^3\sqrt{x}}+\frac{3b^3f^4n^3\log\left(e+f\sqrt{x}\right)}{8e^4}-\frac{3b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right)}{8x^2}-\frac{3b^3f^4n^3\log\left(e+f\right.}{}$$

```
[Out] -1/6*f*(a+b*ln(c*x^n))^3/e/x^(3/2)+1/4*f^2*(a+b*ln(c*x^n))^3/e^2/x-3/8*b^3*
n^3*ln(d*(e+f*x^(1/2)))/x^2+1/2*f^4*(a+b*ln(c*x^n))^3*ln(1+f*x^(1/2)/e)/e^4
-1/2*f^3*(a+b*ln(c*x^n))^3/e^3/x^(1/2)-175/216*b^3*f*n^3/e/x^(3/2)+45/16*b^
3*f^2*n^3/e^2/x-3/16*b^3*f^4*n^3*ln(x)/e^4+3/16*b^3*f^4*n^3*ln(x)^2/e^4-1/1
6*f^4*(a+b*ln(c*x^n))^4/b/e^4/n+3/8*b^3*f^4*n^3*ln(e+f*x^(1/2))/e^4-3/4*b^2
*n^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2)))/x^2-3/4*b*n*(a+b*ln(c*x^n))^2*ln(d
*(e+f*x^(1/2)))/x^2-3/2*b^3*f^4*n^3*polylog(2,1+f*x^(1/2)/e)/e^4-6*b^3*f^4*
n^3*polylog(3,-f*x^(1/2)/e)/e^4+24*b^3*f^4*n^3*polylog(4,-f*x^(1/2)/e)/e^4-
255/8*b^3*f^3*n^3/e^3/x^(1/2)-1/2*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^2
-1/8*f^4*(a+b*ln(c*x^n))^3/e^4-37/36*b^2*f*n^2*(a+b*ln(c*x^n))/e/x^(3/2)+21
/8*b^2*f^2*n^2*(a+b*ln(c*x^n))/e^2/x-3/8*b^2*f^4*n^2*ln(x)*(a+b*ln(c*x^n))/
e^4-7/12*b*f*n*(a+b*ln(c*x^n))^2/e/x^(3/2)+9/8*b*f^2*n*(a+b*ln(c*x^n))^2/e^
2/x+3/4*b^2*f^4*n^2*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^4-3/2*b^3*f^4*n^3*ln(
-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^4+3/4*b*f^4*n*(a+b*ln(c*x^n))^2*ln(1+f*x^(1
/2)/e)/e^4+3*b^2*f^4*n^2*(a+b*ln(c*x^n))*polylog(2,-f*x^(1/2)/e)/e^4+3*b*f^
4*n*(a+b*ln(c*x^n))^2*polylog(2,-f*x^(1/2)/e)/e^4-12*b^2*f^4*n^2*(a+b*ln(c*
x^n))*polylog(3,-f*x^(1/2)/e)/e^4-63/4*b^2*f^3*n^2*(a+b*ln(c*x^n))/e^3/x^(1
/2)-15/4*b*f^3*n*(a+b*ln(c*x^n))^2/e^3/x^(1/2)
```

Rubi [A]
time = 1.01, antiderivative size = 914, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 19, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {2504, 2442, 46, 2424, 2342, 2341, 2422, 2375, 2421, 2430, 6724, 2423, 2441, 2352, 2338, 2413, 12, 2339, 30}

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] (-175*b^3*f*n^3)/(216*e*x^(3/2)) + (45*b^3*f^2*n^3)/(16*e^2*x) - (255*b^3*f
^3*n^3)/(8*e^3*Sqrt[x]) + (3*b^3*f^4*n^3*Log[e + f*Sqrt[x]])/(8*e^4) - (3*b
^3*n^3*Log[d*(e + f*Sqrt[x])])/(8*x^2) - (3*b^3*f^4*n^3*Log[e + f*Sqrt[x]]*
Log[-((f*Sqrt[x])/e)])/(2*e^4) - (3*b^3*f^4*n^3*Log[x])/(16*e^4) + (3*b^3*f
^4*n^3*Log[x]^2)/(16*e^4) - (37*b^2*f*n^2*(a + b*Log[c*x^n]))/(36*e*x^(3/2)
) + (21*b^2*f^2*n^2*(a + b*Log[c*x^n]))/(8*e^2*x) - (63*b^2*f^3*n^2*(a + b*
```

```
Log[c*x^n]))/(4*e^3*Sqrt[x]) + (3*b^2*f^4*n^2*Log[e + f*Sqrt[x]]*(a + b*Log
[c*x^n]))/(4*e^4) - (3*b^2*n^2*Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n]))/(
4*x^2) - (3*b^2*f^4*n^2*Log[x]*(a + b*Log[c*x^n]))/(8*e^4) - (7*b*f*n*(a +
b*Log[c*x^n])^2)/(12*e*x^(3/2)) + (9*b*f^2*n*(a + b*Log[c*x^n])^2)/(8*e^2*x
) - (15*b*f^3*n*(a + b*Log[c*x^n])^2)/(4*e^3*Sqrt[x]) - (3*b*n*Log[d*(e + f
*Sqrt[x])]*(a + b*Log[c*x^n])^2)/(4*x^2) + (3*b*f^4*n*Log[1 + (f*Sqrt[x])/e
]*(a + b*Log[c*x^n])^2)/(4*e^4) - (f^4*(a + b*Log[c*x^n])^3)/(8*e^4) - (f*(
a + b*Log[c*x^n])^3)/(6*e*x^(3/2)) + (f^2*(a + b*Log[c*x^n])^3)/(4*e^2*x) -
 (f^3*(a + b*Log[c*x^n])^3)/(2*e^3*Sqrt[x]) - (Log[d*(e + f*Sqrt[x])]*(a +
b*Log[c*x^n])^3)/(2*x^2) + (f^4*Log[1 + (f*Sqrt[x])/e]*(a + b*Log[c*x^n])^3
)/(2*e^4) - (f^4*(a + b*Log[c*x^n])^4)/(16*b*e^4*n) - (3*b^3*f^4*n^3*PolyLo
g[2, 1 + (f*Sqrt[x])/e])/(2*e^4) + (3*b^2*f^4*n^2*(a + b*Log[c*x^n])*PolyLo
g[2, -((f*Sqrt[x])/e)])/e^4 + (3*b*f^4*n*(a + b*Log[c*x^n])^2*PolyLog[2, -(
(f*Sqrt[x])/e)])/e^4 - (6*b^3*f^4*n^3*PolyLog[3, -((f*Sqrt[x])/e)])/e^4 - (
12*b^2*f^4*n^2*(a + b*Log[c*x^n])*PolyLog[3, -((f*Sqrt[x])/e)])/e^4 + (24*b
^3*f^4*n^3*PolyLog[4, -((f*Sqrt[x])/e)])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2424

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_
.))^(p_.)*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)], x]}, Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[
(a + b*Log[c*x^n])^(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, q}, x] && IGtQ[p, 0] && RationalQ[m] && RationalQ[q] && NeQ[q, -1] &
& (EqQ[p, 1] || (FractionQ[m] && IntegerQ[(q + 1)/m]) || (IGtQ[q, 0] && Int
egerQ[(q + 1)/m] && EqQ[d*e, 1]))
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)\right)(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{f(a+b\log(cx^n))^3}{6ex^{3/2}} + \frac{f^2(a+b\log(cx^n))^3}{4e^2x} - \frac{f^3(a+b\log(c}{2e^3\sqrt{x}}$$

$$= -\frac{f(a+b\log(cx^n))^3}{6ex^{3/2}} + \frac{f^2(a+b\log(cx^n))^3}{4e^2x} - \frac{f^3(a+b\log(c}{2e^3\sqrt{x}}$$

$$= -\frac{7bfn(a+b\log(cx^n))^2}{12ex^{3/2}} + \frac{9bf^2n(a+b\log(cx^n))^2}{8e^2x} - \frac{15bf^3n}{}$$

$$= -\frac{8b^3fn^3}{27ex^{3/2}} + \frac{3b^3f^2n^3}{2e^2x} - \frac{24b^3f^3n^3}{e^3\sqrt{x}} - \frac{4b^2fn^2(a+b\log(cx^n))}{9ex^{3/2}} +$$

$$= -\frac{14b^3fn^3}{27ex^{3/2}} + \frac{9b^3f^2n^3}{4e^2x} - \frac{30b^3f^3n^3}{e^3\sqrt{x}} - \frac{37b^2fn^2(a+b\log(cx^n))}{36ex^{3/2}}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}} + \frac{21b^3f^2n^3}{8e^2x} - \frac{63b^3f^3n^3}{2e^3\sqrt{x}} - \frac{37b^2fn^2(a+b\log(cx^n)}{36ex^{3/2}}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}} + \frac{21b^3f^2n^3}{8e^2x} - \frac{63b^3f^3n^3}{2e^3\sqrt{x}} + \frac{3b^3f^4n^3\log^2(x)}{16e^4} - \frac{37b^2}{}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}} + \frac{21b^3f^2n^3}{8e^2x} - \frac{63b^3f^3n^3}{2e^3\sqrt{x}} - \frac{3b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right.}{8x^2}$$

$$= -\frac{37b^3fn^3}{54ex^{3/2}} + \frac{21b^3f^2n^3}{8e^2x} - \frac{63b^3f^3n^3}{2e^3\sqrt{x}} - \frac{3b^3n^3\log\left(d\left(e+f\sqrt{x}\right)\right.}{8x^2}$$

$$= -\frac{175b^3fn^3}{216ex^{3/2}} + \frac{45b^3f^2n^3}{16e^2x} - \frac{255b^3f^3n^3}{8e^3\sqrt{x}} + \frac{3b^3f^4n^3\log\left(e+f\sqrt{\right.}}{8e^4}$$

Mathematica [A]
time = 1.10, size = 1549, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])]*(a + b*Log[c*x^n])^3)/x^3,x]
```

```
[Out] -1/432*(54*e^4*Log[d*(e + f*Sqrt[x])]*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^2 + 3*
b^3*n^3 + 6*b*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[c*x^n] + 6*b^2*(2*a + b*n)*Lo
g[c*x^n]^2 + 4*b^3*Log[c*x^n]^3) + 18*e^3*f*Sqrt[x]*(4*a^3 + 6*a^2*b*n + 6*
a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-
(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*
(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3
*(-(n*Log[x]) + Log[c*x^n])^3) - 27*e^2*f^2*x*(4*a^3 + 6*a^2*b*n + 6*a*b^2*
n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log
[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*L
og[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*
Log[x]) + Log[c*x^n])^3) + 54*e*f^3*x^(3/2)*(4*a^3 + 6*a^2*b*n + 6*a*b^2*n^
2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*(-(n*Log[x
]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*(-(n*Log
[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b^3*(-(n*Lo
g[x]) + Log[c*x^n])^3) - 54*f^4*x^2*Log[e + f*Sqrt[x]]*(4*a^3 + 6*a^2*b*n +
 6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n
*(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b
^2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*
b^3*(-(n*Log[x]) + Log[c*x^n])^3) + 27*f^4*x^2*Log[x]*(4*a^3 + 6*a^2*b*n +
6*a*b^2*n^2 + 3*b^3*n^3 + 12*a^2*b*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^2*n*
(-(n*Log[x]) + Log[c*x^n]) + 6*b^3*n^2*(-(n*Log[x]) + Log[c*x^n]) + 12*a*b^
2*(-(n*Log[x]) + Log[c*x^n])^2 + 6*b^3*n*(-(n*Log[x]) + Log[c*x^n])^2 + 4*b
^3*(-(n*Log[x]) + Log[c*x^n])^3) + 18*b*f*n*Sqrt[x]*(2*a^2 + 2*a*b*n + b^2*
n^2 + 4*a*b*(-(n*Log[x]) + Log[c*x^n]) + 2*b^2*n*(-(n*Log[x]) + Log[c*x^n])
 + 2*b^2*(-(n*Log[x]) + Log[c*x^n])^2)*(e*(4*e^2 - 9*e*f*Sqrt[x] + 36*f^2*x
) + 3*(2*e^3 - 3*e^2*f*Sqrt[x] + 6*e*f^2*x - 6*f^3*x^(3/2)*Log[1 + (f*Sqrt[
x])/e])*Log[x] + (9*f^3*x^(3/2)*Log[x]^2)/2 - 36*f^3*x^(3/2)*PolyLog[2, -((
f*Sqrt[x])/e)]) - 6*b^2*f*n^2*Sqrt[x]*(-2*a - b*n + 2*b*n*Log[x] - 2*b*Log[
c*x^n])*(16*e^3 - 54*e^2*f*Sqrt[x] + 432*e*f^2*x + 24*e^3*Log[x] - 54*e^2*f
*Sqrt[x]*Log[x] + 216*e*f^2*x*Log[x] + 18*e^3*Log[x]^2 - 27*e^2*f*Sqrt[x]*L
og[x]^2 + 54*e*f^2*x*Log[x]^2 - 54*f^3*x^(3/2)*Log[1 + (f*Sqrt[x])/e]*Log[x
]^2 + 9*f^3*x^(3/2)*Log[x]^3 - 216*f^3*x^(3/2)*Log[x]*PolyLog[2, -((f*Sqrt[
x])/e)] + 432*f^3*x^(3/2)*PolyLog[3, -((f*Sqrt[x])/e)]) + 4*b^3*n^3*(2*e*f*
Sqrt[x]*(16*e^2 - 81*e*f*Sqrt[x] + 1296*f^2*x) + 9*(e*f*Sqrt[x]*(2*e^2 - 3*
e*f*Sqrt[x] + 6*f^2*x) - 6*f^4*x^2*Log[1 + e/(f*Sqrt[x])])*Log[x]^3 + 9*f*S
qrt[x]*Log[x]^2*(e*(4*e^2 - 9*e*f*Sqrt[x] + 36*f^2*x) + 36*f^3*x^(3/2)*Poly
Log[2, -(e/(f*Sqrt[x]))]) + 6*f*Sqrt[x]*Log[x]*(e*(8*e^2 - 27*e*f*Sqrt[x] +
 216*f^2*x) + 216*f^3*x^(3/2)*PolyLog[3, -(e/(f*Sqrt[x]))]) + 2592*f^4*x^2*
PolyLog[4, -(e/(f*Sqrt[x]))]))/(e^4*x^2)
```

Maple [**F**]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\ln\left(d\left(e+f\sqrt{x}\,\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^(1/2)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*log(d*f*sqrt(x) + d*e)/x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**(1/2)))/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*sqrt(x) + e)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)\right)\,\left(a+b\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3,x)
[Out] int((log(d*(e + f*x^(1/2)))*(a + b*log(c*x^n))^3)/x^3, x)
```

3.133 $\int x^{3/2} \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log(cx^n))\,dx$

Optimal. Leaf size=367

$$\frac{24be^4kn\sqrt{x}}{25f^4}-\frac{7be^3knx}{25f^3}+\frac{32be^2knx^{3/2}}{225f^2}-\frac{9beknx^2}{100f}+\frac{8}{125}bknx^{5/2}-\frac{4be^5kn\log\left(e+f\sqrt{x}\right)}{25f^5}-\frac{4}{25}bnx^{5/2}\log\left(d(e$$

```
[Out] -7/25*b*e^3*k*n*x/f^3+32/225*b*e^2*k*n*x^(3/2)/f^2-9/100*b*e*k*n*x^2/f+8/12
5*b*k*n*x^(5/2)+1/5*e^3*k*x*(a+b*ln(c*x^n))/f^3-2/15*e^2*k*x^(3/2)*(a+b*ln(
c*x^n))/f^2+1/10*e*k*x^2*(a+b*ln(c*x^n))/f-2/25*k*x^(5/2)*(a+b*ln(c*x^n))-4
/25*b*e^5*k*n*ln(e+f*x^(1/2))/f^5+2/5*e^5*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))
/f^5-4/5*b*e^5*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^5-4/25*b*n*x^(5/2)*ln
(d*(e+f*x^(1/2))^k)+2/5*x^(5/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)-4/5*b
*e^5*k*n*polylog(2,1+f*x^(1/2)/e)/f^5+24/25*b*e^4*k*n*x^(1/2)/f^4-2/5*e^4*k
*(a+b*ln(c*x^n))*x^(1/2)/f^4
```

Rubi [A]
time = 0.20, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2504, 2442, 45, 2423, 2441, 2352}

$$-\frac{4be^5kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{5f^5}+\frac{2}{5}x^{5/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5f^5}-\frac{2e^4k\sqrt{x}(a+b\log(cx^n))}{5f^4}+\frac{e^3kx(a+b\log(cx^n))}{5f^3}-\frac{2e^2kx^{3/2}(a+b\log(cx^n))}{15f^2}+\frac{ekx^2(a+b\log(cx^n))}{10f}-\frac{2}{25}kx^{5/2}(a+b\log(cx^n))-\frac{4}{25}bnx^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{4be^5kn\log\left(e+f\sqrt{x}\right)}{25f^5}-\frac{4be^5kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{5f^5}+\frac{24be^4kn\sqrt{x}}{25f^4}-\frac{7be^3knx}{25f^3}+\frac{32be^2knx^{3/2}}{225f^2}-\frac{9beknx^2}{100f}+\frac{8}{125}bknx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (24*b*e^4*k*n*Sqrt[x])/(25*f^4) - (7*b*e^3*k*n*x)/(25*f^3) + (32*b*e^2*k*n*
x^(3/2))/(225*f^2) - (9*b*e*k*n*x^2)/(100*f) + (8*b*k*n*x^(5/2))/125 - (4*b
*e^5*k*n*Log[e + f*Sqrt[x]])/(25*f^5) - (4*b*n*x^(5/2)*Log[d*(e + f*Sqrt[x]
)^k])/25 - (4*b*e^5*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(5*f^5) -
 (2*e^4*k*Sqrt[x]*(a + b*Log[c*x^n]))/(5*f^4) + (e^3*k*x*(a + b*Log[c*x^n])
)/(5*f^3) - (2*e^2*k*x^(3/2)*(a + b*Log[c*x^n]))/(15*f^2) + (e*k*x^2*(a + b
*Log[c*x^n]))/(10*f) - (2*k*x^(5/2)*(a + b*Log[c*x^n]))/25 + (2*e^5*k*Log[e
 + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(5*f^5) + (2*x^(5/2)*Log[d*(e + f*Sqrt[x]
)^k]*(a + b*Log[c*x^n]))/5 - (4*b*e^5*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(5
*f^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = -\frac{2e^4k\sqrt{x}\,(a+b\log(cx^n))}{5f^4}+\frac{e^3kx(a+b\log(cx^n))}{5f^3}-\frac{2e}{}$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bk$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bk$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bk$$

$$=\frac{4be^4kn\sqrt{x}}{5f^4}-\frac{be^3knx}{5f^3}+\frac{4be^2knx^{3/2}}{45f^2}-\frac{beknx^2}{20f}+\frac{4}{125}bk$$

$$=\frac{24be^4kn\sqrt{x}}{25f^4}-\frac{7be^3knx}{25f^3}+\frac{32be^2knx^{3/2}}{225f^2}-\frac{9beknx^2}{100f}+\frac{8}{12}$$

Mathematica [A]
time = 0.26, size = 394, normalized size = 1.07

$$\frac{-1800ae^4fk\sqrt{x}+4320be^4fkn\sqrt{x}+900ae^3f^2kx-1260be^3f^2knx-600ae^2f^3kx^{3/2}+640be^2f^3knx^{3/2}+450aef^4kx^2-405bef^4knx^2-360af^5kx^{5/2}+288bf^5knx^{5/2}+1800af^5x^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-720bf^5nx^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)+1800be^5kn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-1800be^4fk\sqrt{x}\log(cx^n)+900be^3f^2kx\log(cx^n)-600be^2f^3kx^{3/2}\log(cx^n)+450bef^4kx^2\log(cx^n)-360bf^5kx^{5/2}\log(cx^n)+1800bf^5x^{5/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+360e^5k\log\left(e+f\sqrt{x}\right)(5a-2bn-5bn\log(x)+5b\log(cx^n))+3600be^5kn\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{4500f^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-1800*a*e^4*f*k*Sqrt[x] + 4320*b*e^4*f*k*n*Sqrt[x] + 900*a*e^3*f^2*k*x - 1
260*b*e^3*f^2*k*n*x - 600*a*e^2*f^3*k*x^(3/2) + 640*b*e^2*f^3*k*n*x^(3/2) +
 450*a*e*f^4*k*x^2 - 405*b*e*f^4*k*n*x^2 - 360*a*f^5*k*x^(5/2) + 288*b*f^5*
k*n*x^(5/2) + 1800*a*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k] - 720*b*f^5*n*x^(
5/2)*Log[d*(e + f*Sqrt[x])^k] + 1800*b*e^5*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x
] - 1800*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 900*b*e^3*f^2*k*x*Log[c*x^n] - 600*
b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 450*b*e*f^4*k*x^2*Log[c*x^n] - 360*b*f^5*k
*x^(5/2)*Log[c*x^n] + 1800*b*f^5*x^(5/2)*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n
] + 360*e^5*k*Log[e + f*Sqrt[x]]*(5*a - 2*b*n - 5*b*n*Log[x] + 5*b*Log[c*x^
n]) + 3600*b*e^5*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/(4500*f^5)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int(x^(3/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="max
ima")
```

```
[Out] 1/500*(50*b*k*x^2*e*log(x^n) + 40*(5*b*f*x*log(x^n) - ((2*f*n - 5*f*log(c))
*b - 5*a*f)*x)*k*x^(3/2)*log(f*sqrt(x) + e) - 5*((9*k*n - 10*k*log(c))*b -
10*a*k)*x^2*e + 40*(5*b*f*x*log(d)*log(x^n) + (5*a*f*log(d) - (2*f*n*log(d)
 - 5*f*log(c)*log(d))*b)*x)*x^(3/2) - 8*(5*b*f*k*x^2*log(x^n) + (5*a*f*k -
(4*f*k*n - 5*f*k*log(c))*b)*x^2)*sqrt(x))/f - integrate(1/25*(5*b*k*x*e^2*l
og(x^n) - ((2*k*n - 5*k*log(c))*b - 5*a*k)*x*e^2)/(f^2*sqrt(x) + f*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fri
cas")
```

```
[Out] integral((b*x^(3/2)*log(c*x^n) + a*x^(3/2))*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^(3/2)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\ln\left(d\left(e+f\,\sqrt{x}\,\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^(3/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

3.134 $\int \sqrt{x} \log \left(d\left(e+f\sqrt{x}\right)^k\right) (a+b\log(cx^n)) \, dx$

Optimal. Leaf size=283

$$\frac{16be^2kn\sqrt{x}}{9f^2}-\frac{5beknx}{9f}+\frac{8}{27}bknx^{3/2}-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)}{9f^3}-\frac{4}{9}bnx^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{4be^3kn\log\left(e+\right.}{}$$

```
[Out] -5/9*b*e*k*n*x/f+8/27*b*k*n*x^(3/2)+1/3*e*k*x*(a+b*ln(c*x^n))/f-2/9*k*x^(3/
2)*(a+b*ln(c*x^n))-4/9*b*e^3*k*n*ln(e+f*x^(1/2))/f^3+2/3*e^3*k*(a+b*ln(c*x^
n))*ln(e+f*x^(1/2))/f^3-4/3*b*e^3*k*n*ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/f^3-
4/9*b*n*x^(3/2)*ln(d*(e+f*x^(1/2))^k)+2/3*x^(3/2)*(a+b*ln(c*x^n))*ln(d*(e+f
*x^(1/2))^k)-4/3*b*e^3*k*n*polylog(2,1+f*x^(1/2)/e)/f^3+16/9*b*e^2*k*n*x^(1
/2)/f^2-2/3*e^2*k*(a+b*ln(c*x^n))*x^(1/2)/f^2
```

Rubi [A]
time = 0.15, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2504, 2442, 45, 2423, 2441, 2352}

$$-\frac{4be^3kn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3f^3}+\frac{2}{3}x^{3/2}(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)+\frac{2e^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3f^3}-\frac{2e^2k\sqrt{x}(a+b\log(cx^n))}{3f^2}+\frac{ekx(a+b\log(cx^n))}{3f}-\frac{2}{9}kx^{3/2}(a+b\log(cx^n))-\frac{4}{9}bnx^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)}{9f^3}-\frac{4be^3kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3f^3}+\frac{16be^2kn\sqrt{x}}{9f^2}-\frac{5beknx}{9f}+\frac{8}{27}bknx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (16*b*e^2*k*n*Sqrt[x])/(9*f^2) - (5*b*e*k*n*x)/(9*f) + (8*b*k*n*x^(3/2))/27
 - (4*b*e^3*k*n*Log[e + f*Sqrt[x]])/(9*f^3) - (4*b*n*x^(3/2)*Log[d*(e + f*S
qrt[x])^k])/9 - (4*b*e^3*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*f
^3) - (2*e^2*k*Sqrt[x]*(a + b*Log[c*x^n]))/(3*f^2) + (e*k*x*(a + b*Log[c*x^
n]))/(3*f) - (2*k*x^(3/2)*(a + b*Log[c*x^n]))/9 + (2*e^3*k*Log[e + f*Sqrt[x
]]*(a + b*Log[c*x^n]))/(3*f^3) + (2*x^(3/2)*Log[d*(e + f*Sqrt[x])^k]*(a + b
*Log[c*x^n]))/3 - (4*b*e^3*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*f^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \sqrt{x}\,\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))\,dx = -\frac{2e^2k\sqrt{x}\,(a+b\log(cx^n))}{3f^2}+\frac{ekx(a+b\log(cx^n))}{3f}-\frac{2}{9}kx$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{2e^2k\sqrt{x}\,(a+b\log($$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{2e^2k\sqrt{x}\,(a+b\log($$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{4}{9}bnx^{3/2}\log\left(d(e+\right.$$

$$=\frac{4be^2kn\sqrt{x}}{3f^2}-\frac{beknx}{3f}+\frac{4}{27}bknx^{3/2}-\frac{4}{9}bnx^{3/2}\log\left(d(e+\right.$$

$$=\frac{16be^2kn\sqrt{x}}{9f^2}-\frac{5beknx}{9f}+\frac{8}{27}bknx^{3/2}-\frac{4be^3kn\log(e+f}{9f^3}$$

Mathematica [A]
time = 0.19, size = 296, normalized size = 1.05

$$\frac{-18ae^2fk\sqrt{x}+48be^2fkn\sqrt{x}+9aef^2kx-15bef^2knx-6af^3kx^{3/2}+8bf^3knx^{3/2}+18af^3x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)-12bf^3nx^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)+18be^3kn\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-18be^2fk\sqrt{x}\,\log(cx^n)+9bef^2kx\log(cx^n)-6bf^3kx^{3/2}\log(cx^n)+18bf^3x^{3/2}\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+6e^3k\log\left(e+f\sqrt{x}\right)(3a-2bn-3bn\log(x)+3b\log(cx^n))+36be^3kn\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]),x]
```

```
[Out] (-18*a*e^2*f*k*Sqrt[x] + 48*b*e^2*f*k*n*Sqrt[x] + 9*a*e*f^2*k*x - 15*b*e*f^
2*k*n*x - 6*a*f^3*k*x^(3/2) + 8*b*f^3*k*n*x^(3/2) + 18*a*f^3*x^(3/2)*Log[d*
(e + f*Sqrt[x])^k] - 12*b*f^3*n*x^(3/2)*Log[d*(e + f*Sqrt[x])^k] + 18*b*e^3
*k*n*Log[1 + (f*Sqrt[x])/e]*Log[x] - 18*b*e^2*f*k*Sqrt[x]*Log[c*x^n] + 9*b*
e*f^2*k*x*Log[c*x^n] - 6*b*f^3*k*x^(3/2)*Log[c*x^n] + 18*b*f^3*x^(3/2)*Log[
d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 6*e^3*k*Log[e + f*Sqrt[x]]*(3*a - 2*b*n -
 3*b*n*Log[x] + 3*b*Log[c*x^n]) + 36*b*e^3*k*n*PolyLog[2, -((f*Sqrt[x])/e)]
)/(27*f^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

```
[Out] int(x^(1/2)*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="max
ima")
```

```
[Out] 2/9*(3*b*x*log(x^n) - (b*(2*n - 3*log(c)) - 3*a)*x)*k*sqrt(x)*log(f*sqrt(x)
 + e) + 2/9*(3*b*x*log(d)*log(x^n) - ((2*n*log(d) - 3*log(c)*log(d))*b - 3*
a*log(d))*x)*sqrt(x) - integrate(1/9*(3*b*f*k*x*log(x^n) + (3*a*f*k - (2*f*
k*n - 3*f*k*log(c))*b)*x)/(f*sqrt(x) + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*sqrt(x)*log((f*sqrt(x) + e)^k*d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\, \ln\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)),x)
[Out] int(x^(1/2)*log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)), x)
```

3.135 $\int \frac{\log \left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{3/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{4bfkn\log\left(e+f\sqrt{x}\right)}{e}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}+\frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e}+\frac{2bfkn\log(x)}{e}-$$

```
[Out] 2*b*f*k*n*ln(x)/e-1/2*b*f*k*n*ln(x)^2/e+f*k*ln(x)*(a+b*ln(c*x^n))/e-4*b*f*k
*n*ln(e+f*x^(1/2))/e-2*f*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e+4*b*f*k*n*ln(-
f*x^(1/2)/e)*ln(e+f*x^(1/2))/e+4*b*f*k*n*polylog(2,1+f*x^(1/2)/e)/e-4*b*n*l
n(d*(e+f*x^(1/2))^k)/x^(1/2)-2*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(1/2
)
```

Rubi [A]
time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2504, 2442, 36, 29, 31, 2423, 2441, 2352, 2338}

$$\frac{4bfkn\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{e}-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}+\frac{fk\log(x)(a+b\log(cx^n))}{e}-\frac{2fk\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{e}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}}-\frac{bfkn\log^2(x)}{2e}+\frac{2bfkn\log(x)}{e}-\frac{4bfkn\log\left(e+f\sqrt{x}\right)}{e}+\frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

```
[Out] (-4*b*f*k*n*Log[e + f*Sqrt[x]])/e - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/Sqrt[x
] + (4*b*f*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/e + (2*b*f*k*n*Log
[x])/e - (b*f*k*n*Log[x]^2)/(2*e) - (2*f*k*Log[e + f*Sqrt[x]]*(a + b*Log[c*
x^n]))/e - (2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/Sqrt[x] + (f*k*L
og[x]*(a + b*Log[c*x^n]))/e + (4*b*f*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/e
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+b\log\left(cx^n\right)\right)}{x^{3/2}}\,dx = -\frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\log\left(d\left(e+f\sqrt{x}\right.\right.}{\sqrt{}}$$

$$= -\frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\log\left(d\left(e+f\sqrt{x}\right.\right.}{\sqrt{}}$$

$$= -\frac{bfkn\log^2(x)}{2e} - \frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+b\log\left(cx^n\right)\right)}{e} - \frac{2\,\mathrm{lo}}{}$$

$$= -\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f}{}\right.}{e}$$

$$= -\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfkn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f}{}\right.}{e}$$

$$= -\frac{4bfkn\log\left(e+f\sqrt{x}\right)}{e} - \frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{\sqrt{x}} + \frac{4bfk}{}$$

Mathematica [A]
time = 0.27, size = 145, normalized size = 0.73

$$-\frac{2\log\left(d\left(e+f\sqrt{x}\right)^k\right)\left(a+2bn+b\log\left(cx^n\right)\right)}{\sqrt{x}} - \frac{2fk\log\left(e+f\sqrt{x}\right)\left(a+2bn-bn\log(x)+b\log\left(cx^n\right)\right)}{e} - \frac{fk\log(x)\left(4bn\log\left(1+\frac{f\sqrt{x}}{e}\right)+bn\log(x)-2\left(a+2bn+b\log\left(cx^n\right)\right)\right)}{2e} - \frac{4bfkn\,\mathrm{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(3/2),x]
```

```
[Out] (-2*Log[d*(e + f*Sqrt[x])^k]*(a + 2*b*n + b*Log[c*x^n]))/Sqrt[x] - (2*f*k*L
og[e + f*Sqrt[x]]*(a + 2*b*n - b*n*Log[x] + b*Log[c*x^n]))/e - (f*k*Log[x]*
(4*b*n*Log[1 + (f*Sqrt[x])/e] + b*n*Log[x] - 2*(a + 2*b*n + b*Log[c*x^n])))
/(2*e) - (4*b*f*k*n*PolyLog[2, -((f*Sqrt[x])/e)])/e
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^n\right)\right)\ln\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(3/2),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/9*(18*(b*x*e^4*log(x^n) + (b*(2*n + log(c)) + a)*x*e^4)*k*log(f*sqrt(x)
+ e)/x^(3/2) + 2*(3*b*f^4*k*x^2*log(x^n) + (3*a*f^4*k + (4*f^4*k*n + 3*f^4*
k*log(c))*b)*x^2)/sqrt(x) - 9*(b*f^3*k*x^2*e*log(x^n) + (a*f^3*k + (f^3*k*n
 + f^3*k*log(c))*b)*x^2*e)/x + 18*((b*f^2*k*log(c) + a*f^2*k)*x^2*e^2 + ((2
*n*log(d) + log(c)*log(d))*b + a*log(d))*x*e^4 + (b*f^2*k*x^2*e^2 + b*x*e^4
*log(d))*log(x^n))/x^(3/2))*e^(-4) + integrate((b*f*k*x*log(x^n) + (a*f*k +
 (2*f*k*n + f*k*log(c))*b)*x)*e^(-2*log(x) - 1), x) + integrate((b*f^5*k*x*
log(x^n) + (a*f^5*k + (2*f^5*k*n + f^5*k*log(c))*b)*x)/(f*sqrt(x)*e^4 + e^5
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^2, x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2),x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(3/2), x)
```

3.136 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx$

Optimal. Leaf size=310

$$-\frac{5bfkn}{9ex}+\frac{16bf^2kn}{9e^2\sqrt{x}}-\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)}{9e^3}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}}+\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right.}{3e^3}$$

```
[Out] -5/9*b*f*k*n/e/x+2/9*b*f^3*k*n*ln(x)/e^3-1/6*b*f^3*k*n*ln(x)^2/e^3-1/3*f*k*
(a+b*ln(c*x^n))/e/x+1/3*f^3*k*ln(x)*(a+b*ln(c*x^n))/e^3-4/9*b*f^3*k*n*ln(e+
f*x^(1/2))/e^3-2/3*f^3*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^3+4/3*b*f^3*k*n*
ln(-f*x^(1/2)/e)*ln(e+f*x^(1/2))/e^3-4/9*b*n*ln(d*(e+f*x^(1/2))^k)/x^(3/2)-
2/3*(a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(3/2)+4/3*b*f^3*k*n*polylog(2,1
+f*x^(1/2)/e)/e^3+16/9*b*f^2*k*n/e^2/x^(1/2)+2/3*f^2*k*(a+b*ln(c*x^n))/e^2/
x^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2504, 2442, 46, 2423, 2441, 2352, 2338}

$$\frac{4bf^3kn\,\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{3e^3}-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{3x^{3/2}}-\frac{2f^3k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{3e^3}+\frac{f^3k\log(x)(a+b\log(cx^n))}{3e^3}+\frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}}-\frac{fk(a+b\log(cx^n))}{3ex}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}}-\frac{bf^3kn\log^2(x)}{6e^3}-\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)}{9e^3}+\frac{4bf^3kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{3e^3}+\frac{2bf^3kn\log(x)}{9e^3}+\frac{16bf^2kn}{9e^2\sqrt{x}}-\frac{5bfkn}{9ex}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

```
[Out] (-5*b*f*k*n)/(9*e*x) + (16*b*f^2*k*n)/(9*e^2*Sqrt[x]) - (4*b*f^3*k*n*Log[e
 + f*Sqrt[x]])/(9*e^3) - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/(9*x^(3/2)) + (4*b
*f^3*k*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(3*e^3) + (2*b*f^3*k*n*L
og[x])/(9*e^3) - (b*f^3*k*n*Log[x]^2)/(6*e^3) - (f*k*(a + b*Log[c*x^n]))/(3
*e*x) + (2*f^2*k*(a + b*Log[c*x^n]))/(3*e^2*Sqrt[x]) - (2*f^3*k*Log[e + f*S
qrt[x]]*(a + b*Log[c*x^n]))/(3*e^3) - (2*Log[d*(e + f*Sqrt[x])^k]*(a + b*Lo
g[c*x^n]))/(3*x^(3/2)) + (f^3*k*Log[x]*(a + b*Log[c*x^n]))/(3*e^3) + (4*b*f
^3*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(3*e^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{5/2}}\,dx = -\frac{fk(a+b\log(cx^n))}{3ex} + \frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}} - \frac{2f^3k\log\left(e+\right.}{}$$

$$= -\frac{bfkn}{3ex} + \frac{4bf^2kn}{3e^2\sqrt{x}} - \frac{fk(a+b\log(cx^n))}{3ex} + \frac{2f^2k(a+b\log(cx^n))}{3e^2\sqrt{x}}$$

$$= -\frac{bfkn}{3ex} + \frac{4bf^2kn}{3e^2\sqrt{x}} - \frac{bf^3kn\log^2(x)}{6e^3} - \frac{fk(a+b\log(cx^n))}{3ex} + 2$$

$$= -\frac{bfkn}{3ex} + \frac{4bf^2kn}{3e^2\sqrt{x}} - \frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}} + \frac{4bf^3kn\log\left(e\right.}{}$$

$$= -\frac{bfkn}{3ex} + \frac{4bf^2kn}{3e^2\sqrt{x}} - \frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{9x^{3/2}} + \frac{4bf^3kn\log\left(e\right.}{}$$

$$= -\frac{5bfkn}{9ex} + \frac{16bf^2kn}{9e^2\sqrt{x}} - \frac{4bf^3kn\log\left(e+f\sqrt{x}\right)}{9e^3} - \frac{4bn\log\left(d\left(e\right.\right.}{9x}$$

Mathematica [A]
time = 0.25, size = 326, normalized size = 1.05

$$\frac{-3ae^2fk\sqrt{x}-5be^2fkn\sqrt{x}+6aef^2kx+16bef^2knx-6ae^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)-4be^3n\log\left(d\left(e+f\sqrt{x}\right)^k\right)+3af^3kx^{3/2}\log(x)+2bf^3knx^{3/2}\log(x)-6bf^3knx^{3/2}\log\left(1+\frac{f\sqrt{x}}{e}\right)\log(x)-\frac{3}{2}bf^3knx^{3/2}\log^2(x)-3be^2fk\sqrt{x}\log(cx^n)+6bef^2kx\log(cx^n)-6be^3\log\left(d\left(e+f\sqrt{x}\right)^k\right)\log(cx^n)+3bf^3kx^{3/2}\log(x)\log(cx^n)-2f^3kx^{3/2}\log\left(e+f\sqrt{x}\right)(3a+2bn-3bn\log(x)+3b\log(cx^n))-12bf^3knx^{3/2}\text{Li}_2\left(-\frac{f\sqrt{x}}{e}\right)}{9e^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(5/2),x]
```

```
[Out] (-3*a*e^2*f*k*Sqrt[x] - 5*b*e^2*f*k*n*Sqrt[x] + 6*a*e*f^2*k*x + 16*b*e*f^2*
k*n*x - 6*a*e^3*Log[d*(e + f*Sqrt[x])^k] - 4*b*e^3*n*Log[d*(e + f*Sqrt[x])^
k] + 3*a*f^3*k*x^(3/2)*Log[x] + 2*b*f^3*k*n*x^(3/2)*Log[x] - 6*b*f^3*k*n*x^
(3/2)*Log[1 + (f*Sqrt[x])/e]*Log[x] - (3*b*f^3*k*n*x^(3/2)*Log[x]^2)/2 - 3*
b*e^2*f*k*Sqrt[x]*Log[c*x^n] + 6*b*e*f^2*k*x*Log[c*x^n] - 6*b*e^3*Log[d*(e
+ f*Sqrt[x])^k]*Log[c*x^n] + 3*b*f^3*k*x^(3/2)*Log[x]*Log[c*x^n] - 2*f^3*k*
x^(3/2)*Log[e + f*Sqrt[x]]*(3*a + 2*b*n - 3*b*n*Log[x] + 3*b*Log[c*x^n]) -
12*b*f^3*k*n*x^(3/2)*PolyLog[2, -((f*Sqrt[x])/e)])/(9*e^3*x^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\right)^k\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(5/2),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/9*(2*(b*f^6*k*x^2*log(x^n) + (b*f^6*k*log(c) + a*f^6*k)*x^2)/sqrt(x) + 2
*(3*b*x*e^6*log(x^n) + (b*(2*n + 3*log(c)) + 3*a)*x*e^6)*k*log(f*sqrt(x) +
e)/x^(5/2) - (3*b*f^5*k*x^2*e*log(x^n) + (3*a*f^5*k - (f^5*k*n - 3*f^5*k*lo
g(c))*b)*x^2*e)/x + 2*(3*b*f^4*k*x^2*e^2*log(x^n) + (3*a*f^4*k - (4*f^4*k*n
 - 3*f^4*k*log(c))*b)*x^2*e^2)/x^(3/2) - 2*((3*a*f^2*k + (8*f^2*k*n + 3*f^2
*k*log(c))*b)*x^2*e^4 - ((2*n*log(d) + 3*log(c)*log(d))*b + 3*a*log(d))*x*e
^6 + 3*(b*f^2*k*x^2*e^4 - b*x*e^6*log(d))*log(x^n))/x^(5/2))*e^(-6) + 1/9*e
^(-3)*integrate((3*b*f^3*k*x*log(x^n) + (3*a*f^3*k + (2*f^3*k*n + 3*f^3*k*l
og(c))*b)*x)/x^2, x) + 1/9*integrate((3*b*f*k*x*log(x^n) + (3*a*f*k + (2*f*
k*n + 3*f*k*log(c))*b)*x)*e^(-3*log(x) - 1), x) + integrate(1/9*(3*b*f^7*k*
x*log(x^n) + (3*a*f^7*k + (2*f^7*k*n + 3*f^7*k*log(c))*b)*x)/(f*sqrt(x)*e^6
 + e^7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^3, x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\,\sqrt{x}\,\right)^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2),x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(5/2), x)
```

3.137 $\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx$

Optimal. Leaf size=394

$$-\frac{9bfkn}{100ex^2}+\frac{32bf^2kn}{225e^2x^{3/2}}-\frac{7bf^3kn}{25e^3x}+\frac{24bf^4kn}{25e^4\sqrt{x}}-\frac{4bf^5kn\log\left(e+f\sqrt{x}\right)}{25e^5}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}}+\frac{4bf^5kn\log}{}$$

```
[Out] -9/100*b*f*k*n/e/x^2+32/225*b*f^2*k*n/e^2/x^(3/2)-7/25*b*f^3*k*n/e^3/x+2/25
*b*f^5*k*n*ln(x)/e^5-1/10*b*f^5*k*n*ln(x)^2/e^5-1/10*f*k*(a+b*ln(c*x^n))/e/
x^2+2/15*f^2*k*(a+b*ln(c*x^n))/e^2/x^(3/2)-1/5*f^3*k*(a+b*ln(c*x^n))/e^3/x+
1/5*f^5*k*ln(x)*(a+b*ln(c*x^n))/e^5-4/25*b*f^5*k*n*ln(e+f*x^(1/2))/e^5-2/5*
f^5*k*(a+b*ln(c*x^n))*ln(e+f*x^(1/2))/e^5+4/5*b*f^5*k*n*ln(-f*x^(1/2)/e)*ln
(e+f*x^(1/2))/e^5-4/25*b*n*ln(d*(e+f*x^(1/2))^k)/x^(5/2)-2/5*(a+b*ln(c*x^n)
)*ln(d*(e+f*x^(1/2))^k)/x^(5/2)+4/5*b*f^5*k*n*polylog(2,1+f*x^(1/2)/e)/e^5+
24/25*b*f^4*k*n/e^4/x^(1/2)+2/5*f^4*k*(a+b*ln(c*x^n))/e^4/x^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2504, 2442, 46, 2423, 2441, 2352, 2338}

$$\frac{4bf^5kn\,\text{PolyLog}\left(2,\frac{f\sqrt{x}}{e}+1\right)}{5e^5}-\frac{2(a+b\log(cx^n))\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{5x^{5/2}}-\frac{2f^5k\log\left(e+f\sqrt{x}\right)(a+b\log(cx^n))}{5e^5}+\frac{f^5k\log(x)(a+b\log(cx^n))}{5e^5}+\frac{2f^4k(a+b\log(cx^n))}{5e^4\sqrt{x}}-\frac{f^3k(a+b\log(cx^n))}{5e^3x}+\frac{2f^2k(a+b\log(cx^n))}{15e^2x^{3/2}}-\frac{fk(a+b\log(cx^n))}{10ex^2}-\frac{4bn\log\left(d\left(e+f\sqrt{x}\right)^k\right)}{25x^{5/2}}-\frac{bf^5kn\log^2(x)}{10e^5}-\frac{4bf^5kn\log\left(e+f\sqrt{x}\right)}{25e^5}+\frac{4bf^5kn\log\left(e+f\sqrt{x}\right)\log\left(-\frac{f\sqrt{x}}{e}\right)}{5e^5}+\frac{2bf^5kn\log(x)}{25e^5}+\frac{24bf^4kn}{25e^4\sqrt{x}}-\frac{7bf^3kn}{25e^3x}+\frac{32bf^2kn}{225e^2x^{3/2}}-\frac{9bfkn}{100ex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

```
[Out] (-9*b*f*k*n)/(100*e*x^2) + (32*b*f^2*k*n)/(225*e^2*x^(3/2)) - (7*b*f^3*k*n)
/(25*e^3*x) + (24*b*f^4*k*n)/(25*e^4*Sqrt[x]) - (4*b*f^5*k*n*Log[e + f*Sqrt
[x]])/(25*e^5) - (4*b*n*Log[d*(e + f*Sqrt[x])^k])/(25*x^(5/2)) + (4*b*f^5*k
*n*Log[e + f*Sqrt[x]]*Log[-((f*Sqrt[x])/e)])/(5*e^5) + (2*b*f^5*k*n*Log[x])
/(25*e^5) - (b*f^5*k*n*Log[x]^2)/(10*e^5) - (f*k*(a + b*Log[c*x^n]))/(10*e*
x^2) + (2*f^2*k*(a + b*Log[c*x^n]))/(15*e^2*x^(3/2)) - (f^3*k*(a + b*Log[c*
x^n]))/(5*e^3*x) + (2*f^4*k*(a + b*Log[c*x^n]))/(5*e^4*Sqrt[x]) - (2*f^5*k*
Log[e + f*Sqrt[x]]*(a + b*Log[c*x^n]))/(5*e^5) - (2*Log[d*(e + f*Sqrt[x])^k
]*(a + b*Log[c*x^n]))/(5*x^(5/2)) + (f^5*k*Log[x]*(a + b*Log[c*x^n]))/(5*e^
5) + (4*b*f^5*k*n*PolyLog[2, 1 + (f*Sqrt[x])/e])/(5*e^5)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(d\left(e+f\sqrt{x}\right)^k\right)(a+b\log(cx^n))}{x^{7/2}}\,dx = -\frac{fk(a+b\log(cx^n))}{10ex^2} + \frac{2f^2k(a+b\log(cx^n))}{15e^2x^{3/2}} - \frac{f^3k(a+b\log}{5e^3x}$$

$$= -\frac{bfkn}{20ex^2} + \frac{4bf^2kn}{45e^2x^{3/2}} - \frac{bf^3kn}{5e^3x} + \frac{4bf^4kn}{5e^4\sqrt{x}} - \frac{fk(a+b\log(cx^n}{10ex^2}$$

$$= -\frac{bfkn}{20ex^2} + \frac{4bf^2kn}{45e^2x^{3/2}} - \frac{bf^3kn}{5e^3x} + \frac{4bf^4kn}{5e^4\sqrt{x}} - \frac{bf^5kn\log^2(x)}{10e^5} -$$

$$= -\frac{bfkn}{20ex^2} + \frac{4bf^2kn}{45e^2x^{3/2}} - \frac{bf^3kn}{5e^3x} + \frac{4bf^4kn}{5e^4\sqrt{x}} - \frac{4bn\log\left(d(e+f\right.}{25x^{5/2}}$$

$$= -\frac{bfkn}{20ex^2} + \frac{4bf^2kn}{45e^2x^{3/2}} - \frac{bf^3kn}{5e^3x} + \frac{4bf^4kn}{5e^4\sqrt{x}} - \frac{4bn\log\left(d(e+f\right.}{25x^{5/2}}$$

$$= -\frac{9bfkn}{100ex^2} + \frac{32bf^2kn}{225e^2x^{3/2}} - \frac{7bf^3kn}{25e^3x} + \frac{24bf^4kn}{25e^4\sqrt{x}} - \frac{4bf^5kn\log(}{25e}$$

Mathematica [A]
time = 0.28, size = 422, normalized size = 1.07

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Log[d*(e + f*Sqrt[x])^k]*(a + b*Log[c*x^n]))/x^(7/2),x]
```

```
[Out] (-90*a*e^4*f*k*Sqrt[x] - 81*b*e^4*f*k*n*Sqrt[x] + 120*a*e^3*f^2*k*x + 128*b
*e^3*f^2*k*n*x - 180*a*e^2*f^3*k*x^(3/2) - 252*b*e^2*f^3*k*n*x^(3/2) + 360*
a*e*f^4*k*x^2 + 864*b*e*f^4*k*n*x^2 - 360*a*e^5*Log[d*(e + f*Sqrt[x])^k] -
144*b*e^5*n*Log[d*(e + f*Sqrt[x])^k] + 180*a*f^5*k*x^(5/2)*Log[x] + 72*b*f^
5*k*n*x^(5/2)*Log[x] - 360*b*f^5*k*n*x^(5/2)*Log[1 + (f*Sqrt[x])/e]*Log[x]
- 90*b*f^5*k*n*x^(5/2)*Log[x]^2 - 90*b*e^4*f*k*Sqrt[x]*Log[c*x^n] + 120*b*e
^3*f^2*k*x*Log[c*x^n] - 180*b*e^2*f^3*k*x^(3/2)*Log[c*x^n] + 360*b*e*f^4*k*
x^2*Log[c*x^n] - 360*b*e^5*Log[d*(e + f*Sqrt[x])^k]*Log[c*x^n] + 180*b*f^5*
k*x^(5/2)*Log[x]*Log[c*x^n] - 72*f^5*k*x^(5/2)*Log[e + f*Sqrt[x]]*(5*a + 2*
b*n - 5*b*n*Log[x] + 5*b*Log[c*x^n]) - 720*b*f^5*k*n*x^(5/2)*PolyLog[2, -((
f*Sqrt[x])/e)])/(900*e^5*x^(5/2))
```

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d\left(e+f\sqrt{x}\,\right)^k\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(7/2),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^(1/2))^k)/x^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="max
ima")
```

```
[Out] -1/225*(2*(15*b*f^8*k*x^2*log(x^n) + (15*a*f^8*k - (4*f^8*k*n - 15*f^8*k*lo
g(c))*b)*x^2)/sqrt(x) - 9*(5*b*f^7*k*x^2*e*log(x^n) + (5*a*f^7*k - (3*f^7*k
*n - 5*f^7*k*log(c))*b)*x^2*e)/x + 18*(5*b*f^6*k*x^2*e^2*log(x^n) + (5*a*f^
6*k - (8*f^6*k*n - 5*f^6*k*log(c))*b)*x^2*e^2)/x^(3/2) + 18*(5*b*x*e^8*log(
x^n) + (b*(2*n + 5*log(c)) + 5*a)*x*e^8)*k*log(f*sqrt(x) + e)/x^(7/2) - 18*
(5*b*f^4*k*x^2*e^4*log(x^n) + (5*a*f^4*k + (12*f^4*k*n + 5*f^4*k*log(c))*b)
*x^2*e^4)/x^(5/2) - 2*((15*a*f^2*k + (16*f^2*k*n + 15*f^2*k*log(c))*b)*x^2*
e^6 - 9*((2*n*log(d) + 5*log(c)*log(d))*b + 5*a*log(d))*x*e^8 + 15*(b*f^2*k
*x^2*e^6 - 3*b*x*e^8*log(d))*log(x^n))/x^(7/2))*e^(-8) + 1/25*e^(-5)*integr
ate((5*b*f^5*k*x*log(x^n) + (5*a*f^5*k + (2*f^5*k*n + 5*f^5*k*log(c))*b)*x)
/x^2, x) + 1/25*e^(-3)*integrate((5*b*f^3*k*x*log(x^n) + (5*a*f^3*k + (2*f^
3*k*n + 5*f^3*k*log(c))*b)*x)/x^3, x) + 1/25*integrate((5*b*f*k*x*log(x^n)
+ (5*a*f*k + (2*f*k*n + 5*f*k*log(c))*b)*x)*e^(-4*log(x) - 1), x) + integra
te(1/25*(5*b*f^9*k*x*log(x^n) + (5*a*f^9*k + (2*f^9*k*n + 5*f^9*k*log(c))*b
)*x)/(f*sqrt(x)*e^8 + e^9), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="fri
cas")
```

```
[Out] integral((b*sqrt(x)*log(c*x^n) + a*sqrt(x))*log((f*sqrt(x) + e)^k*d)/x^4, x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**(1/2))**k)/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^(1/2))^k)/x^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*sqrt(x) + e)^k*d)/x^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\left(e+f\sqrt{x}\right)^k\right)\,\left(a+b\ln\left(c\,x^n\right)\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2),x)
```

```
[Out] int((log(d*(e + f*x^(1/2))^k)*(a + b*log(c*x^n)))/x^(7/2), x)
```

3.138 $\int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx$

Optimal. Leaf size=31

$$\text{Int}\left((gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right), x\right)$$

```
[Out] Unintegrable((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] Defer[Int][(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx = \int (gx)^q (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(31) = 62.
time = 0.21, size = 304, normalized size = 9.81

$$\frac{x(gx)^q \left(-akm+2bkmn-akmq-bkmn\,{}_3F_2\left(1,\frac{1}{m}+\frac{q}{m},\frac{1}{m}+\frac{q}{m};1+\frac{1}{m}+\frac{q}{m},1+\frac{1}{m}+\frac{q}{m};-\frac{fx^m}{e}\right)-bkm\log(cx^n)-bkmq\log(cx^n)+km\,{}_2F_1\left(1,\frac{1+q}{m};\frac{1+m+q}{m};-\frac{fx^m}{e}\right)(a-bn+aq+b(1+q)\log(cx^n))+a\log\left(d(e+fx^m)^k\right)-bn\log\left(d(e+fx^m)^k\right)+2aq\log\left(d(e+fx^m)^k\right)-bnq\log\left(d(e+fx^m)^k\right)+aq^2\log\left(d(e+fx^m)^k\right)+b\log(cx^n)\log\left(d(e+fx^m)^k\right)+2bq\log(cx^n)\log\left(d(e+fx^m)^k\right)+bq^2\log(cx^n)\log\left(d(e+fx^m)^k\right)\right)}{(1+q)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] (x*(g*x)^q*(-(a*k*m) + 2*b*k*m*n - a*k*m*q - b*k*m*n*HypergeometricPFQ[{1,
m^(-1) + q/m, m^(-1) + q/m}, {1 + m^(-1) + q/m, 1 + m^(-1) + q/m}, -((f*x^m
)/e)] - b*k*m*Log[c*x^n] - b*k*m*q*Log[c*x^n] + k*m*Hypergeometric2F1[1, (1
 + q)/m, (1 + m + q)/m, -((f*x^m)/e)]*(a - b*n + a*q + b*(1 + q)*Log[c*x^n]
) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e + f*x^m)^k] + 2*a*q*Log[d*(e + f*
x^m)^k] - b*n*q*Log[d*(e + f*x^m)^k] + a*q^2*Log[d*(e + f*x^m)^k] + b*Log[c
```

```
*x^n]*Log[d*(e + f*x^m)^k] + 2*b*q*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*q^2*
Log[c*x^n]*Log[d*(e + f*x^m)^k]))/(1 + q)^3
```

Maple [**A**]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^q (a + b\ln(c\,x^n)) \ln\left(d(e + f\,x^m)^k\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^q*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima"
)
```

```
[Out] (b*g^q*(q + 1)*x*x^q*log(x^n) + (a*g^q*(q + 1) + (g^q*(q + 1)*log(c) - g^q*
n)*b)*x*x^q)*log((f*x^m + e)^k)/(q^2 + 2*q + 1) + integrate((((q^2 + 2*q +
1)*b*g^q*e*log(d) - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*b*x^
m)*x^q*log(x^n) - (((f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*a -
(f*g^q*k*m*n - (f*g^q*k*m*(q + 1) - (q^2 + 2*q + 1)*f*g^q*log(d))*log(c))*b
)*x^m - ((q^2 + 2*q + 1)*b*g^q*log(c)*log(d) + (q^2 + 2*q + 1)*a*g^q*log(d)
)*e)*x^q)/((q^2 + 2*q + 1)*f*x^m + (q^2 + 2*q + 1)*e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas"
)
```

```
[Out] integral(((g*x)^q*b*log(c*x^n) + (g*x)^q*a)*log((f*x^m + e)^k*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**q*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^q*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^q*log((f*x^m + e)^k*d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln \left(d\,{\left(e+f\,x^m\right)}^k\right)\,{\left(g\,x\right)}^q\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^q*(a + b*log(c*x^n)), x)
```

3.139 $\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx$

Optimal. Leaf size=185

$$\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{r(a+b\log(cx^n))^4\log\left(1+\frac{fx^m}{e}\right)}{4bn}-\frac{r(a+b\log(cx^n))^3\operatorname{Li}_2\left(-\frac{fx^m}{e}\right)}{m}+\frac{3bn$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4*ln(d*(e+f*x^m)^r)/b/n-1/4*r*(a+b*ln(c*x^n))^4*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))^3*polylog(2,-f*x^m/e)/m+3*b*n*r*(a+b*ln(c*x^n)
)^2*polylog(3,-f*x^m/e)/m^2-6*b^2*n^2*r*(a+b*ln(c*x^n))*polylog(4,-f*x^m/e)
/m^3+6*b^3*n^3*r*polylog(5,-f*x^m/e)/m^4
```

Rubi [A]
time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$-\frac{6b^2n^2r\text{PolyLog}\left(4,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m^3}+\frac{3bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m^2}-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^3}{m}+\frac{6b^3n^3r\text{PolyLog}\left(5,-\frac{fx^m}{e}\right)}{m^4}+\frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn}-\frac{r\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^4*Log[d*(e + f*x^m)^r])/(4*b*n) - (r*(a + b*Log[c*x^n])
^4*Log[1 + (f*x^m)/e])/(4*b*n) - (r*(a + b*Log[c*x^n])^3*PolyLog[2, -((f*x^
m)/e)])/m + (3*b*n*r*(a + b*Log[c*x^n])^2*PolyLog[3, -((f*x^m)/e)])/m^2 - (
6*b^2*n^2*r*(a + b*Log[c*x^n])*PolyLog[4, -((f*x^m)/e)])/m^3 + (6*b^3*n^3*r
*PolyLog[5, -((f*x^m)/e)])/m^4
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{(fmr)\int \frac{x^{-1+m}(a+b\log(cx^n))}{e+fx^m}}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log(1-}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log(1-}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log(1-}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log(1-}{4bn}$$

$$= \frac{(a+b\log(cx^n))^4\log(d(e+fx^m)^r)}{4bn} - \frac{r(a+b\log(cx^n))^4\log(1-}{4bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1395 vs. $2(185) = 370$.
time = 0.38, size = 1395, normalized size = 7.54

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/2*(a^2*b*m*n*r*Log[x]^3) + (3*a*b^2*m*n^2*r*Log[x]^4)/4 - (3*b^3*m*n^3*r
*Log[x]^5)/10 - a*b^2*m*n*r*Log[x]^3*Log[c*x^n] + (3*b^3*m*n^2*r*Log[x]^4*L
og[c*x^n])/4 - (b^3*m*n*r*Log[x]^3*Log[c*x^n]^2)/2 - (3*a^2*b*n*r*Log[x]^2*
Log[1 + e/(f*x^m)])/2 + 2*a*b^2*n^2*r*Log[x]^3*Log[1 + e/(f*x^m)] - (3*b^3*
n^3*r*Log[x]^4*Log[1 + e/(f*x^m)])/4 - 3*a*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[
1 + e/(f*x^m)] + 2*b^3*n^2*r*Log[x]^3*Log[c*x^n]*Log[1 + e/(f*x^m)] - (3*b^
3*n*r*Log[x]^2*Log[c*x^n]^2*Log[1 + e/(f*x^m)])/2 - a^3*r*Log[x]*Log[e + f*
x^m] + 3*a^2*b*n*r*Log[x]^2*Log[e + f*x^m] - 3*a*b^2*n^2*r*Log[x]^3*Log[e +
 f*x^m] + b^3*n^3*r*Log[x]^4*Log[e + f*x^m] + (a^3*r*Log[-((f*x^m)/e)]*Log[
e + f*x^m])/m - (3*a^2*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m + (
3*a*b^2*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - (b^3*n^3*r*Log
[x]^3*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - 3*a^2*b*r*Log[x]*Log[c*x^n]*Log
[e + f*x^m] + 6*a*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] - 3*b^3*n^2*r*
Log[x]^3*Log[c*x^n]*Log[e + f*x^m] + (3*a^2*b*r*Log[-((f*x^m)/e)]*Log[c*x^n
]*Log[e + f*x^m])/m - (6*a*b^2*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*x^n]*Log[
e + f*x^m])/m + (3*b^3*n^2*r*Log[x]^2*Log[-((f*x^m)/e)]*Log[c*x^n]*Log[e +
f*x^m])/m - 3*a*b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m] + 3*b^3*n*r*Log[x]
^2*Log[c*x^n]^2*Log[e + f*x^m] + (3*a*b^2*r*Log[-((f*x^m)/e)]*Log[c*x^n]^2*
Log[e + f*x^m])/m - (3*b^3*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*x^n]^2*Log[e
+ f*x^m])/m - b^3*r*Log[x]*Log[c*x^n]^3*Log[e + f*x^m] + (b^3*r*Log[-((f*x^
m)/e)]*Log[c*x^n]^3*Log[e + f*x^m])/m + a^3*Log[x]*Log[d*(e + f*x^m)^r] - (
3*a^2*b*n*Log[x]^2*Log[d*(e + f*x^m)^r])/2 + a*b^2*n^2*Log[x]^3*Log[d*(e +
f*x^m)^r] - (b^3*n^3*Log[x]^4*Log[d*(e + f*x^m)^r])/4 + 3*a^2*b*Log[x]*Log[
c*x^n]*Log[d*(e + f*x^m)^r] - 3*a*b^2*n*Log[x]^2*Log[c*x^n]*Log[d*(e + f*x^
m)^r] + b^3*n^2*Log[x]^3*Log[c*x^n]*Log[d*(e + f*x^m)^r] + 3*a*b^2*Log[x]*L
og[c*x^n]^2*Log[d*(e + f*x^m)^r] - (3*b^3*n*Log[x]^2*Log[c*x^n]^2*Log[d*(e
+ f*x^m)^r])/2 + b^3*Log[x]*Log[c*x^n]^3*Log[d*(e + f*x^m)^r] + (b*n*r*Log[
x]*(b^2*n^2*Log[x]^2 - 3*b*n*Log[x]*(a + b*Log[c*x^n]) + 3*(a + b*Log[c*x^n
])^2)*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n])^3*Po
lyLog[2, 1 + (f*x^m)/e])/m + (3*a^2*b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2 + (
6*a*b^2*n*r*Log[c*x^n]*PolyLog[3, -(e/(f*x^m))])/m^2 + (3*b^3*n*r*Log[c*x^n
]^2*PolyLog[3, -(e/(f*x^m))])/m^2 + (6*a*b^2*n^2*r*PolyLog[4, -(e/(f*x^m))]
)/m^3 + (6*b^3*n^2*r*Log[c*x^n]*PolyLog[4, -(e/(f*x^m))])/m^3 + (6*b^3*n^3*
r*PolyLog[5, -(e/(f*x^m))])/m^4
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{3} \ln\left(d\left(e+f\,x^{m}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*n^3*log(x)^4 - 4*b^3*log(x)*log(x^n)^3 - 4*(b^3*n^2*log(c) + a*b^
2*n^2)*log(x)^3 + 6*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2
+ 6*(b^3*n*log(x)^2 - 2*(b^3*log(c) + a*b^2)*log(x))*log(x^n)^2 - 4*(b^3*n^
2*log(x)^3 - 3*(b^3*n*log(c) + a*b^2*n)*log(x)^2 + 3*(b^3*log(c)^2 + 2*a*b^
2*log(c) + a^2*b)*log(x))*log(x^n) - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3
*a^2*b*log(c) + a^3)*log(x))*log((f*x^m + e)^r) - integrate(-1/4*(4*(b^3*e*
log(d) - (b^3*f*m*r*log(x) - b^3*f*log(d))*x^m)*log(x^n)^3 + 6*((b^3*f*m*n*
r*log(x)^2 + 2*b^3*f*log(c)*log(d) + 2*a*b^2*f*log(d) - 2*(b^3*f*m*r*log(c)
 + a*b^2*f*m*r)*log(x))*x^m + 2*(b^3*log(c)*log(d) + a*b^2*log(d))*e)*log(x
^n)^2 + (b^3*f*m*n^3*r*log(x)^4 + 4*b^3*f*log(c)^3*log(d) + 12*a*b^2*f*log(
c)^2*log(d) + 12*a^2*b*f*log(c)*log(d) + 4*a^3*f*log(d) - 4*(b^3*f*m*n^2*r*
log(c) + a*b^2*f*m*n^2*r)*log(x)^3 + 6*(b^3*f*m*n*r*log(c)^2 + 2*a*b^2*f*m*
n*r*log(c) + a^2*b*f*m*n*r)*log(x)^2 - 4*(b^3*f*m*r*log(c)^3 + 3*a*b^2*f*m*
r*log(c)^2 + 3*a^2*b*f*m*r*log(c) + a^3*f*m*r)*log(x))*x^m + 4*(b^3*log(c)^
3*log(d) + 3*a*b^2*log(c)^2*log(d) + 3*a^2*b*log(c)*log(d) + a^3*log(d))*e
- 4*((b^3*f*m*n^2*r*log(x)^3 - 3*b^3*f*log(c)^2*log(d) - 6*a*b^2*f*log(c)*l
og(d) - 3*a^2*b*f*log(d) - 3*(b^3*f*m*n*r*log(c) + a*b^2*f*m*n*r)*log(x)^2
+ 3*(b^3*f*m*r*log(c)^2 + 2*a*b^2*f*m*r*log(c) + a^2*b*f*m*r)*log(x))*x^m -
 3*(b^3*log(c)^2*log(d) + 2*a*b^2*log(c)*log(d) + a^2*b*log(d))*e)*log(x^n)
)/(f*x*x^m + x*e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(176) = 352$.
time = 0.38, size = 763, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*m^4*n^3*log(d)*log(x)^4 + 24*b^3*n^3*r*polylog(5, -f*x^m*e^(-1)) +
 4*(b^3*m^4*n^2*log(c) + a*b^2*m^4*n^2)*log(d)*log(x)^3 + 6*(b^3*m^4*n*log(
c)^2 + 2*a*b^2*m^4*n*log(c) + a^2*b*m^4*n)*log(d)*log(x)^2 + 4*(b^3*m^4*log
(c)^3 + 3*a*b^2*m^4*log(c)^2 + 3*a^2*b*m^4*log(c) + a^3*m^4)*log(d)*log(x)
```

```
- 4*(b^3*m^3*n^3*r*log(x)^3 + b^3*m^3*r*log(c)^3 + 3*a*b^2*m^3*r*log(c)^2 +
 3*a^2*b*m^3*r*log(c) + a^3*m^3*r + 3*(b^3*m^3*n^2*r*log(c) + a*b^2*m^3*n^2
*r)*log(x)^2 + 3*(b^3*m^3*n*r*log(c)^2 + 2*a*b^2*m^3*n*r*log(c) + a^2*b*m^3
*n*r)*log(x))*dilog(-(f*x^m + e)*e^(-1) + 1) + (b^3*m^4*n^3*r*log(x)^4 + 4*
(b^3*m^4*n^2*r*log(c) + a*b^2*m^4*n^2*r)*log(x)^3 + 6*(b^3*m^4*n*r*log(c)^2
 + 2*a*b^2*m^4*n*r*log(c) + a^2*b*m^4*n*r)*log(x)^2 + 4*(b^3*m^4*r*log(c)^3
 + 3*a*b^2*m^4*r*log(c)^2 + 3*a^2*b*m^4*r*log(c) + a^3*m^4*r)*log(x))*log(f
*x^m + e) - (b^3*m^4*n^3*r*log(x)^4 + 4*(b^3*m^4*n^2*r*log(c) + a*b^2*m^4*n
^2*r)*log(x)^3 + 6*(b^3*m^4*n*r*log(c)^2 + 2*a*b^2*m^4*n*r*log(c) + a^2*b*m
^4*n*r)*log(x)^2 + 4*(b^3*m^4*r*log(c)^3 + 3*a*b^2*m^4*r*log(c)^2 + 3*a^2*b
*m^4*r*log(c) + a^3*m^4*r)*log(x))*log((f*x^m + e)*e^(-1)) - 24*(b^3*m*n^3*
r*log(x) + b^3*m*n^2*r*log(c) + a*b^2*m*n^2*r)*polylog(4, -f*x^m*e^(-1)) +
12*(b^3*m^2*n^3*r*log(x)^2 + b^3*m^2*n*r*log(c)^2 + 2*a*b^2*m^2*n*r*log(c)
+ a^2*b*m^2*n*r + 2*(b^3*m^2*n^2*r*log(c) + a*b^2*m^2*n^2*r)*log(x))*polylo
g(3, -f*x^m*e^(-1)))/m^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*log((f*x^m + e)^r*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^3)/x, x)
```

3.140 $\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx$

Optimal. Leaf size=150

$$\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^m)^r\right)}{3bn}-\frac{r(a+b\log(cx^n))^3\log\left(1+\frac{fx^m}{e}\right)}{3bn}-\frac{r(a+b\log(cx^n))^2\operatorname{Li}_2\left(-\frac{fx^m}{e}\right)}{m}+\frac{2bnr}{}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3*ln(d*(e+f*x^m)^r)/b/n-1/3*r*(a+b*ln(c*x^n))^3*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))^2*polylog(2,-f*x^m/e)/m+2*b*n*r*(a+b*ln(c*x^n)
)*polylog(3,-f*x^m/e)/m^2-2*b^2*n^2*r*polylog(4,-f*x^m/e)/m^3
```

Rubi [A]
time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2422, 2375, 2421, 2430, 6724}

$$\frac{2bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m^2}-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))^2}{m}-\frac{2b^2n^2r\text{PolyLog}\left(4,-\frac{fx^m}{e}\right)}{m^3}+\frac{(a+b\log(cx^n))^3\log\left(d(e+fx^m)^r\right)}{3bn}-\frac{r\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*Log[d*(e + f*x^m)^r])/(3*b*n) - (r*(a + b*Log[c*x^n])
^3*Log[1 + (f*x^m)/e])/(3*b*n) - (r*(a + b*Log[c*x^n])^2*PolyLog[2, -((f*x^
m)/e)])/m + (2*b*n*r*(a + b*Log[c*x^n])*PolyLog[3, -((f*x^m)/e)])/m^2 - (2*
b^2*n^2*r*PolyLog[4, -((f*x^m)/e)])/m^3
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
```

```
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{(fmr)\int \frac{x^{-1+m}(a+b\log(cx^n}{e+fx^m}}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{r(a+b\log(cx^n))^3\log(1}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{r(a+b\log(cx^n))^3\log(1}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{r(a+b\log(cx^n))^3\log(1}{3bn}$$

$$= \frac{(a+b\log(cx^n))^3\log(d(e+fx^m)^r)}{3bn} - \frac{r(a+b\log(cx^n))^3\log(1}{3bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 741 vs. 2(150) = 300.
time = 0.21, size = 741, normalized size = 4.94

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/3*(a*b*m*n*r*Log[x]^3) + (b^2*m*n^2*r*Log[x]^4)/4 - (b^2*m*n*r*Log[x]^3*
Log[c*x^n])/3 - a*b*n*r*Log[x]^2*Log[1 + e/(f*x^m)] + (2*b^2*n^2*r*Log[x]^3
```

```
*Log[1 + e/(f*x^m)])/3 - b^2*n*r*Log[x]^2*Log[c*x^n]*Log[1 + e/(f*x^m)] - a
^2*r*Log[x]*Log[e + f*x^m] + 2*a*b*n*r*Log[x]^2*Log[e + f*x^m] - b^2*n^2*r*
Log[x]^3*Log[e + f*x^m] + (a^2*r*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m - (2*a
*b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m + (b^2*n^2*r*Log[x]^2*Log
[-((f*x^m)/e)]*Log[e + f*x^m])/m - 2*a*b*r*Log[x]*Log[c*x^n]*Log[e + f*x^m]
 + 2*b^2*n*r*Log[x]^2*Log[c*x^n]*Log[e + f*x^m] + (2*a*b*r*Log[-((f*x^m)/e)
]*Log[c*x^n]*Log[e + f*x^m])/m - (2*b^2*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[c*
x^n]*Log[e + f*x^m])/m - b^2*r*Log[x]*Log[c*x^n]^2*Log[e + f*x^m] + (b^2*r*
Log[-((f*x^m)/e)]*Log[c*x^n]^2*Log[e + f*x^m])/m + a^2*Log[x]*Log[d*(e + f*
x^m)^r] - a*b*n*Log[x]^2*Log[d*(e + f*x^m)^r] + (b^2*n^2*Log[x]^3*Log[d*(e
+ f*x^m)^r])/3 + 2*a*b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r] - b^2*n*Log[x
]^2*Log[c*x^n]*Log[d*(e + f*x^m)^r] + b^2*Log[x]*Log[c*x^n]^2*Log[d*(e + f*
x^m)^r] + (b*n*r*Log[x]*(-(b*n*Log[x]) + 2*(a + b*Log[c*x^n]))*PolyLog[2, -
(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n])^2*PolyLog[2, 1 + (f*x^
m)/e])/m + (2*a*b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b^2*n*r*Log[c*x^n]
*PolyLog[3, -(e/(f*x^m))])/m^2 + (2*b^2*n^2*r*PolyLog[4, -(e/(f*x^m))])/m^3
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}\ln\left(d\left(e+f\,x^{m}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*n^2*log(x)^3 + 3*b^2*log(x)*log(x^n)^2 - 3*(b^2*n*log(c) + a*b*n)*
log(x)^2 - 3*(b^2*n*log(x)^2 - 2*(b^2*log(c) + a*b)*log(x))*log(x^n) + 3*(b
^2*log(c)^2 + 2*a*b*log(c) + a^2)*log(x))*log((f*x^m + e)^r) - integrate(-1
/3*(3*(b^2*e*log(d) - (b^2*f*m*r*log(x) - b^2*f*log(d))*x^m)*log(x^n)^2 - (
b^2*f*m*n^2*r*log(x)^3 - 3*b^2*f*log(c)^2*log(d) - 6*a*b*f*log(c)*log(d) -
3*a^2*f*log(d) - 3*(b^2*f*m*n*r*log(c) + a*b*f*m*n*r)*log(x)^2 + 3*(b^2*f*m
*r*log(c)^2 + 2*a*b*f*m*r*log(c) + a^2*f*m*r)*log(x))*x^m + 3*(b^2*log(c)^2
*log(d) + 2*a*b*log(c)*log(d) + a^2*log(d))*e + 3*((b^2*f*m*n*r*log(x)^2 +
2*b^2*f*log(c)*log(d) + 2*a*b*f*log(d) - 2*(b^2*f*m*r*log(c) + a*b*f*m*r)*l
```

```
og(x))*x^m + 2*(b^2*log(c)*log(d) + a*b*log(d))*e)*log(x^n))/(f*x*x^m + x*e
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(142) = 284$.
time = 0.37, size = 405, normalized size = 2.70

$$\frac{b^2m^3n^2\log(d)\log(x)^3 - 6\,b^2n^2r\mathrm{polylog}\left(4, -fx^me^{(-1)}\right) + 3\left(b^2m^3n\log(c) + abm^3n\right)\log(d)\log(x)^2 + 3\left(b^2m^3\log(c)^2 + 2\,abm^3\log(c) + a^2m^3\right)\log(d)\log(x) - 3\left(b^2m^2n^2r\log(x)^2 + b^2m^2r\log(c)^2 + 2\,abm^2r\log(c) + a^2m^2r + 2\left(b^2m^2nr\log(c) + abm^2nr\right)\log(x)\right){\rm Li}_2\left(-(fx^m + e)e^{(-1)} + 1\right) + \left(b^2m^3n^2r\log(x)^3 + 3\left(b^2m^3nr\log(c) + abm^3nr\right)\log(x)^2 + 3\left(b^2m^3r\log(c)^2 + 2\,abm^3r\log(c) + a^2m^3r\right)\log(x)\right)\log(fx^m + e) - \left(b^2m^3n^2r\log(x)^3 + 3\left(b^2m^3nr\log(c) + abm^3nr\right)\log(x)^2 + 3\left(b^2m^3r\log(c)^2 + 2\,abm^3r\log(c) + a^2m^3r\right)\log(x)\right)\log\left((fx^m + e)e^{(-1)}\right) + 6\left(b^2mn^2r\log(x) + b^2mnr\log(c) + abmnr\right)\mathrm{polylog}\left(3, -fx^me^{(-1)}\right)}{3\,m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*m^3*n^2*log(d)*log(x)^3 - 6*b^2*n^2*r*polylog(4, -f*x^m*e^(-1)) +
3*(b^2*m^3*n*log(c) + a*b*m^3*n)*log(d)*log(x)^2 + 3*(b^2*m^3*log(c)^2 + 2*
a*b*m^3*log(c) + a^2*m^3)*log(d)*log(x) - 3*(b^2*m^2*n^2*r*log(x)^2 + b^2*m
^2*r*log(c)^2 + 2*a*b*m^2*r*log(c) + a^2*m^2*r + 2*(b^2*m^2*n*r*log(c) + a*
b*m^2*n*r)*log(x))*dilog(-(f*x^m + e)*e^(-1) + 1) + (b^2*m^3*n^2*r*log(x)^3
 + 3*(b^2*m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)^2 +
2*a*b*m^3*r*log(c) + a^2*m^3*r)*log(x))*log(f*x^m + e) - (b^2*m^3*n^2*r*log
(x)^3 + 3*(b^2*m^3*n*r*log(c) + a*b*m^3*n*r)*log(x)^2 + 3*(b^2*m^3*r*log(c)
^2 + 2*a*b*m^3*r*log(c) + a^2*m^3*r)*log(x))*log((f*x^m + e)*e^(-1)) + 6*(b
^2*m*n^2*r*log(x) + b^2*m*n*r*log(c) + a*b*m*n*r)*polylog(3, -f*x^m*e^(-1))
)/m^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*log((f*x^m + e)^r*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^r\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x,x)
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n))^2)/x, x)
```

3.141 $\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx$

Optimal. Leaf size=114

$$\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{r(a+b\log(cx^n))^2\log\left(1+\frac{fx^m}{e}\right)}{2bn}-\frac{r(a+b\log(cx^n))\,\text{Li}_2\left(-\frac{fx^m}{e}\right)}{m}+\frac{bnr\text{]}}{}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^r)/b/n-1/2*r*(a+b*ln(c*x^n))^2*ln(1+f*
x^m/e)/b/n-r*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*n*r*polylog(3,-f*x^m/e
)/m^2
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2422, 2375, 2421, 6724}

$$-\frac{r\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}+\frac{bnr\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}+\frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn}-\frac{r\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^r])/(2*b*n) - (r*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^m)/e])/(2*b*n) - (r*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^m)
/e)])/m + (b*n*r*PolyLog[3, -((f*x^m)/e)])/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
```

```
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log(d(e+fx^m)^r)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn} - \frac{(fmr)\int \frac{x^{-1+m}(a+b\log(cx^n))^2}{e+fx^m}$$

$$= \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn} - \frac{r(a+b\log(cx^n))^2\log(1+$$

$$= \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn} - \frac{r(a+b\log(cx^n))^2\log(1+$$

$$= \frac{(a+b\log(cx^n))^2\log(d(e+fx^m)^r)}{2bn} - \frac{r(a+b\log(cx^n))^2\log(1+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(114) = 228.
time = 0.12, size = 277, normalized size = 2.43

$$-\frac{1}{6}bmnr\log^3(x)-\frac{1}{2}bnr\log^2(x)\log\left(1+\frac{ex^{-m}}{f}\right)+bnr\log^2(x)\log(e+fx^m)-\frac{bnr\log(x)\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{m}-br\log(x)\log(cx^n)\log(e+fx^m)+\frac{br\log\left(-\frac{fx^m}{e}\right)\log(cx^n)\log(e+fx^m)}{m}-\frac{1}{2}bn\log^2(x)\log(d(e+fx^m)^r)+\frac{a\log\left(-\frac{fx^m}{e}\right)\log(d(e+fx^m)^r)}{m}+b\log(x)\log(cx^n)\log(d(e+fx^m)^r)+\frac{bnr\log(x)\text{Li}_2\left(-\frac{ex^{-m}}{f}\right)}{m}+\frac{r(a-bn\log(x)+b\log(cx^n))\text{Li}_2\left(1+\frac{fx^m}{e}\right)}{m}+\frac{bnr\text{Li}_3\left(-\frac{ex^{-m}}{f}\right)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^r])/x,x]
```

```
[Out] -1/6*(b*m*n*r*Log[x]^3) - (b*n*r*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*n*r*Log
[x]^2*Log[e + f*x^m] - (b*n*r*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m -
b*r*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*r*Log[-((f*x^m)/e)]*Log[c*x^n]*Lo
g[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^r])/2 + (a*Log[-((f*x^m)/
e)]*Log[d*(e + f*x^m)^r])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^r] + (b
*n*r*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (r*(a - b*n*Log[x] + b*Log[c*x^n]
)*PolyLog[2, 1 + (f*x^m)/e])/m + (b*n*r*PolyLog[3, -(e/(f*x^m))])/m^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln(d(e+f\,x^m)^r)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^r)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^r)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x^m + e)^r) - integrate(-1/2*((b*f*m*n*r*log(x)^2 + 2*b*f*log(c)*log(d) + 2
*a*f*log(d) - 2*(b*f*m*r*log(c) + a*f*m*r)*log(x))*x^m + 2*(b*log(c)*log(d)
 + a*log(d))*e + 2*(b*e*log(d) - (b*f*m*r*log(x) - b*f*log(d))*x^m)*log(x^n
))/(f*x*x^m + x*e), x)
```

Fricas [A]
time = 0.36, size = 173, normalized size = 1.52

$$\frac{bm^2n\log(d)\log(x)^2 + 2\,bnr\mathrm{polylog}\left(3, -fx^m e^{(-1)}\right) + 2\left(bm^2\log(c) + am^2\right)\log(d)\log(x) - 2\left(bmnr\log(x) + bmr\log(c) + amr\right)\mathrm{Li}_2\left(-(fx^m + e)e^{(-1)} + 1\right) + \left(bm^2nr\log(x)^2 + 2\left(bm^2r\log(c) + am^2r\right)\log(x)\right)\log\left(fx^m + e\right) - \left(bm^2nr\log(x)^2 + 2\left(bm^2r\log(c) + am^2r\right)\log(x)\right)\log\left((fx^m + e)e^{(-1)}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*n*r*polylog(3, -f*x^m*e^(-1)) + 2*(b*m^2
*log(c) + a*m^2)*log(d)*log(x) - 2*(b*m*n*r*log(x) + b*m*r*log(c) + a*m*r)*
dilog(-(f*x^m + e)*e^(-1) + 1) + (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) +
a*m^2*r)*log(x))*log(f*x^m + e) - (b*m^2*n*r*log(x)^2 + 2*(b*m^2*r*log(c) +
 a*m^2*r)*log(x))*log((f*x^m + e)*e^(-1)))/m^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**r)/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^r)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^r*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(e+f\,x^m\right)^r\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^r)*(a + b*log(c*x^n)))/x, x)
```

3.142 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)},x\right)$$

```
[Out] Unintegrable(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])),x]
[Out] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(e+f\,x^m)^r\right)}{x\left(a+b\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
[Out] int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(log((f*x^m + e)^r*d)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^r\right)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))),x)
[Out] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))), x)
```

3.143 $\int \frac{\log(d(e+fx^m)^r)}{x(a+b\log(cx^n))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Defer[Int][Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.79, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d(e+fx^m)^r\right)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2),x]
[Out] Integrate[Log[d*(e + f*x^m)^r]/(x*(a + b*Log[c*x^n])^2), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(e+f\,x^m)^r\right)}{x\left(a+b\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(ln(d*(e+f*x^m)^r)/x/(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] f*m*r*integrate(x^m/((b^2*f*n*log(c) + a*b*f*n)*x*x^m + (b^2*n*log(c) + a*b
*n)*x*e + (b^2*f*n*x*x^m + b^2*n*x*e)*log(x^n)), x) - (log((f*x^m + e)^r) +
 log(d))/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((f*x^m + e)^r*d)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^
2*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(e+f*x**m)**r)/x/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(e+f*x^m)^r)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((f*x^m + e)^r*d)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(d \left(e+f x^m\right)^r\right)}{x \left(a+b \ln \left(c x^n\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2),x)
[Out] int(log(d*(e + f*x^m)^r)/(x*(a + b*log(c*x^n))^2), x)
```

3.144 $\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] Defer[Int][x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int x^2(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(29) = 58.
time = 0.13, size = 292, normalized size = 10.07

$$-\frac{x^3\left(-6bekmn-2bekm^2n+9afkmx^m\,{}_2F_1\left(1,\frac{3+m}{m};2+\frac{3}{m};-\frac{fx^m}{e}\right)+bekm(3+m)n\,{}_3F_2\left(1,\frac{3}{m},\frac{3}{m};1+\frac{3}{m},1+\frac{3}{m};-\frac{fx^m}{e}\right)+bekm(3+m)\,{}_2F_1\left(1,\frac{3}{m};\frac{3+m}{m};-\frac{fx^m}{e}\right)(n-3\log(cx^n))+9bekm\log(cx^n)+3bekm^2\log(cx^n)-27ae\log\left(d(e+fx^m)^k\right)-9aem\log\left(d(e+fx^m)^k\right)+9ben\log\left(d(e+fx^m)^k\right)+3bemn\log\left(d(e+fx^m)^k\right)-27be\log(cx^n)\log\left(d(e+fx^m)^k\right)-9bem\log(cx^n)\log\left(d(e+fx^m)^k\right)\right)}{27e(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -1/27*(x^3*(-6*b*e*k*m*n - 2*b*e*k*m^2*n + 9*a*f*k*m*x^m*Hypergeometric2F1[
1, (3 + m)/m, 2 + 3/m, -((f*x^m)/e)] + b*e*k*m*(3 + m)*n*HypergeometricPFQ[
{1, 3/m, 3/m}, {1 + 3/m, 1 + 3/m}, -((f*x^m)/e)] + b*e*k*m*(3 + m)*Hypergeo
metric2F1[1, 3/m, (3 + m)/m, -((f*x^m)/e)]*(n - 3*Log[c*x^n]) + 9*b*e*k*m*L
og[c*x^n] + 3*b*e*k*m^2*Log[c*x^n] - 27*a*e*Log[d*(e + f*x^m)^k] - 9*a*e*m*
Log[d*(e + f*x^m)^k] + 9*b*e*n*Log[d*(e + f*x^m)^k] + 3*b*e*m*n*Log[d*(e +
```

```
f*x^m)^k] - 27*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 9*b*e*m*Log[c*x^n]*Log
[d*(e + f*x^m)^k]))/(e*(3 + m))
```

Maple [**A**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\ln(c\,x^{n}))\ln\left(d(e+f\,x^{m})^{k}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
[Out] int(x^2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
[Out] 1/9*(3*b*x^3*log(x^n) - (b*(n - 3*log(c)) - 3*a)*x^3)*log((f*x^m + e)^k) +
integrate(-1/9*((3*(f*k*m - 3*f*log(d))*a - (f*k*m*n - 3*(f*k*m - 3*f*log(d
))*log(c))*b)*x^2*x^m - 9*(b*log(c)*log(d) + a*log(d))*x^2*e + 3*((f*k*m -
3*f*log(d))*b*x^2*x^m - 3*b*x^2*e*log(d))*log(x^n))/(f*x^m + e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
[Out] integral((b*x^2*log(c*x^n) + a*x^2)*log((f*x^m + e)^k*d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x^2*log((f*x^m + e)^k*d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \ln\left(d\,(e+f\,x^m)^k\right)\,(a+b\,\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
[Out] int(x^2*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

3.145 $\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] Defer[Int][x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx=\int x(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(27) = 54.
time = 0.13, size = 292, normalized size = 10.81

$$-\frac{x^2\left(-4bekmn-2bekm^2n+4afkmx^m\,{}_2F_1\left(1,\frac{2+m}{m};2+\frac{2}{m};-\frac{fx^m}{e}\right)+bekm(2+m)n\,{}_3F_2\left(1,\frac{2}{m},\frac{2}{m};1+\frac{2}{m},1+\frac{2}{m};-\frac{fx^m}{e}\right)+bekm(2+m)\,{}_2F_1\left(1,\frac{2}{m};\frac{2+m}{m};-\frac{fx^m}{e}\right)(n-2\log(cx^n))+4bekm\log(cx^n)+2bekm^2\log(cx^n)-8ae\log\left(d(e+fx^m)^k\right)-4aem\log\left(d(e+fx^m)^k\right)+4ben\log\left(d(e+fx^m)^k\right)+2bemn\log\left(d(e+fx^m)^k\right)-8be\log(cx^n)\log\left(d(e+fx^m)^k\right)-4bem\log(cx^n)\log\left(d(e+fx^m)^k\right)\right)}{8e(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
[Out] -1/8*(x^2*(-4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1
, (2 + m)/m, 2 + 2/m, -((f*x^m)/e)] + b*e*k*m*(2 + m)*n*HypergeometricPFQ[{
1, 2/m, 2/m}, {1 + 2/m, 1 + 2/m}, -((f*x^m)/e)] + b*e*k*m*(2 + m)*Hypergeom
etric2F1[1, 2/m, (2 + m)/m, -((f*x^m)/e)]*(n - 2*Log[c*x^n]) + 4*b*e*k*m*Lo
g[c*x^n] + 2*b*e*k*m^2*Log[c*x^n] - 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Lo
g[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] + 2*b*e*m*n*Log[d*(e + f*
```

```
x^m)^k] - 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*
(e + f*x^m)^k]))/(e*(2 + m))
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int x(a + b\ln(c\,x^n))\ln\left(d(e + f\,x^m)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2*log(x^n) - (b*(n - 2*log(c)) - 2*a)*x^2)*log((f*x^m + e)^k) +
integrate(-1/4*((2*(f*k*m - 2*f*log(d))*a - (f*k*m*n - 2*(f*k*m - 2*f*log(d
))*log(c))*b)*x*x^m - 4*(b*log(c)*log(d) + a*log(d))*x*e + 2*((f*k*m - 2*f*
log(d))*b*x*x^m - 2*b*x*e*log(d))*log(x^n))/(f*x^m + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

```
[Out] integral((b*x*log(c*x^n) + a*x)*log((f*x^m + e)^k*d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\log(cx^n))\log\left(d(e + fx^m)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))*log(d*(e + f*x**m)**k), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*x*log((f*x^m + e)^k*d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x\,\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
[Out] int(x*log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

3.146 $\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left((a+b\log(cx^n))\log\left(d(e+fx^m)^k\right),x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] Defer[Int][(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k], x]
```

Rubi steps

$$\int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = \int (a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(26) = 52.
time = 0.12, size = 165, normalized size = 6.35

$$bkmnx-kmx(a+b(-n\log(x)+\log(cx^n)))+x\left(bkmn-bkmn\,{}_3F_2\left(1,\frac{1}{m},\frac{1}{m};1+\frac{1}{m},1+\frac{1}{m};-\frac{fx^m}{e}\right)-bkmn\log(x)+km\,{}_2F_1\left(1,\frac{1}{m};1+\frac{1}{m};-\frac{fx^m}{e}\right)(a-bn+b\log(cx^n))+a\log\left(d(e+fx^m)^k\right)-bn\log\left(d(e+fx^m)^k\right)+b\log(cx^n)\log\left(d(e+fx^m)^k\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] b*k*m*n*x - k*m*x*(a + b*(-(n*Log[x]) + Log[c*x^n])) + x*(b*k*m*n - b*k*m*n
*HypergeometricPFQ[{1, m^(-1), m^(-1)}, {1 + m^(-1), 1 + m^(-1)}, -((f*x^m)
/e)] - b*k*m*n*Log[x] + k*m*Hypergeometric2F1[1, m^(-1), 1 + m^(-1), -((f*x
^m)/e)]*(a - b*n + b*Log[c*x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e +
f*x^m)^k] + b*Log[c*x^n]*Log[d*(e + f*x^m)^k])
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\,x^{n}\right)\right) \ln \left(d\left(e+f\,x^{m}\right)^{k}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="maxima")
```

```
[Out] (b*x*log(x^n) - (b*(n - log(c)) - a)*x)*log((f*x^m + e)^k) + integrate(-(((
f*k*m - f*log(d))*a - (f*k*m*n - (f*k*m - f*log(d))*log(c))*b)*x^m - (b*log
(c)*log(d) + a*log(d))*e + ((f*k*m - f*log(d))*b*x^m - b*e*log(d))*log(x^n)
)/(f*x^m + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log \left(cx^{n}\right)\right) \log \left(d\left(e+fx^{m}\right)^{k}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Integral((a + b*log(c*x**n))*log(d*(e + f*x**m)**k), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)), x)
```

3.147 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx$

Optimal. Leaf size=114

$$\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn}-\frac{k(a+b\log(cx^n))^2\log\left(1+\frac{fx^m}{e}\right)}{2bn}-\frac{k(a+b\log(cx^n))\,\mathrm{Li}_2\left(-\frac{fx^m}{e}\right)}{m}+\frac{bk\cdots}{}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2*ln(d*(e+f*x^m)^k)/b/n-1/2*k*(a+b*ln(c*x^n))^2*ln(1+f*
x^m/e)/b/n-k*(a+b*ln(c*x^n))*polylog(2,-f*x^m/e)/m+b*k*n*polylog(3,-f*x^m/e
)/m^2
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2422, 2375, 2421, 6724}

$$-\frac{k\text{PolyLog}\left(2,-\frac{fx^m}{e}\right)(a+b\log(cx^n))}{m}+\frac{bkn\text{PolyLog}\left(3,-\frac{fx^m}{e}\right)}{m^2}+\frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn}-\frac{k\log\left(\frac{fx^m}{e}+1\right)(a+b\log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*Log[d*(e + f*x^m)^k])/(2*b*n) - (k*(a + b*Log[c*x^n])
^2*Log[1 + (f*x^m)/e])/(2*b*n) - (k*(a + b*Log[c*x^n])*PolyLog[2, -((f*x^m)
/e)])/m + (b*k*n*PolyLog[3, -((f*x^m)/e)])/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
```

```
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x}\,dx = \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{(fkm)\int \frac{x^{-1+m}(a+b\log(cx^n}{e+fx^m}}{2bn}$$

$$= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{k(a+b\log(cx^n))^2\log(1}{2bn}$$

$$= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{k(a+b\log(cx^n))^2\log(1}{2bn}$$

$$= \frac{(a+b\log(cx^n))^2\log\left(d(e+fx^m)^k\right)}{2bn} - \frac{k(a+b\log(cx^n))^2\log(1}{2bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(114) = 228.
time = 0.12, size = 277, normalized size = 2.43

$$-\frac{1}{6}bkmn\log^3(x)-\frac{1}{2}bkn\log^2(x)\log\left(1+\frac{ex^{-m}}{f}\right)+bkn\log^2(x)\log(e+fx^m)-\frac{bkn\log(x)\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{m}-bk\log(x)\log(cx^n)\log(e+fx^m)+\frac{bk\log\left(-\frac{fx^m}{e}\right)\log(cx^n)\log(e+fx^m)}{m}-\frac{1}{2}bn\log^2(x)\log\left(d(e+fx^m)^k\right)+\frac{a\log\left(-\frac{fx^m}{e}\right)\log\left(d(e+fx^m)^k\right)}{m}+b\log(x)\log(cx^n)\log\left(d(e+fx^m)^k\right)+\frac{bkn\log(x)\text{Li}_2\left(-\frac{ex^{-m}}{f}\right)}{m}+\frac{k(a-bn\log(x)+b\log(cx^n))\text{Li}_2\left(1+\frac{fx^m}{e}\right)}{m}+\frac{bkn\text{Li}_3\left(-\frac{ex^{-m}}{f}\right)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x,x]
```

```
[Out] -1/6*(b*k*m*n*Log[x]^3) - (b*k*n*Log[x]^2*Log[1 + e/(f*x^m)])/2 + b*k*n*Log
[x]^2*Log[e + f*x^m] - (b*k*n*Log[x]*Log[-((f*x^m)/e)]*Log[e + f*x^m])/m -
b*k*Log[x]*Log[c*x^n]*Log[e + f*x^m] + (b*k*Log[-((f*x^m)/e)]*Log[c*x^n]*Lo
g[e + f*x^m])/m - (b*n*Log[x]^2*Log[d*(e + f*x^m)^k])/2 + (a*Log[-((f*x^m)/
e)]*Log[d*(e + f*x^m)^k])/m + b*Log[x]*Log[c*x^n]*Log[d*(e + f*x^m)^k] + (b
*k*n*Log[x]*PolyLog[2, -(e/(f*x^m))])/m + (k*(a - b*n*Log[x] + b*Log[c*x^n]
)*PolyLog[2, 1 + (f*x^m)/e])/m + (b*k*n*PolyLog[3, -(e/(f*x^m))])/m^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*log((f*
x^m + e)^k) - integrate(-1/2*((b*f*k*m*n*log(x)^2 + 2*b*f*log(c)*log(d) + 2
*a*f*log(d) - 2*(b*f*k*m*log(c) + a*f*k*m)*log(x))*x^m + 2*(b*log(c)*log(d)
 + a*log(d))*e + 2*(b*e*log(d) - (b*f*k*m*log(x) - b*f*log(d))*x^m)*log(x^n
))/(f*x*x^m + x*e), x)
```

Fricas [A]
time = 0.37, size = 173, normalized size = 1.52

$$\frac{bm^2n\log(d)\log(x)^2 + 2\,bkn\mathrm{polylog}\left(3, -fx^m e^{(-1)}\right) + 2\left(bm^2\log(c) + am^2\right)\log(d)\log(x) - 2\left(bkmn\log(x) + bkm\log(c) + akm\right)\mathrm{Li}_2\left(-(fx^m + e)e^{(-1)} + 1\right) + \left(bkm^2n\log(x)^2 + 2\left(bkm^2\log(c) + akm^2\right)\log(x)\right)\log\left(fx^m + e\right) - \left(bkm^2n\log(x)^2 + 2\left(bkm^2\log(c) + akm^2\right)\log(x)\right)\log\left((fx^m + e)e^{(-1)}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*m^2*n*log(d)*log(x)^2 + 2*b*k*n*polylog(3, -f*x^m*e^(-1)) + 2*(b*m^2
*log(c) + a*m^2)*log(d)*log(x) - 2*(b*k*m*n*log(x) + b*k*m*log(c) + a*k*m)*
dilog(-(f*x^m + e)*e^(-1) + 1) + (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) +
a*k*m^2)*log(x))*log(f*x^m + e) - (b*k*m^2*n*log(x)^2 + 2*(b*k*m^2*log(c) +
 a*k*m^2)*log(x))*log((f*x^m + e)*e^(-1)))/m^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x, x)
```

3.148 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
```

```
[Out] Defer[Int][((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx = \int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 282 vs. 2(29) = 58.
time = 0.12, size = 282, normalized size = 9.72

$$\frac{2bekmn-2bekm^2n+afkmx^m\,{}_2F_1\left(1,\frac{-1+m}{m};2-\frac{1}{m};-\frac{fx^m}{e}\right)+bek(-1+m)mn\,{}_3F_2\left(1,-\frac{1}{m},-\frac{1}{m};1-\frac{1}{m},1-\frac{1}{m};-\frac{fx^m}{e}\right)+bekm\log(cx^n)-bekm^2\log(cx^n)+bek(-1+m)m\,{}_2F_1\left(1,-\frac{1}{m};\frac{-1+m}{m};-\frac{fx^m}{e}\right)(n+\log(cx^n))+ae\log\left(d(e+fx^m)^k\right)-aem\log\left(d(e+fx^m)^k\right)+ben\log\left(d(e+fx^m)^k\right)-bemn\log\left(d(e+fx^m)^k\right)+be\log(cx^n)\log\left(d(e+fx^m)^k\right)-bem\log(cx^n)\log\left(d(e+fx^m)^k\right)}{e(-1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
```

```
[Out] (2*b*e*k*m*n - 2*b*e*k*m^2*n + a*f*k*m*x^m*Hypergeometric2F1[1, (-1 + m)/m,
 2 - m^(-1), -((f*x^m)/e)] + b*e*k*(-1 + m)*m*n*HypergeometricPFQ[{1, -m^(-
1), -m^(-1)}, {1 - m^(-1), 1 - m^(-1)}, -((f*x^m)/e)] + b*e*k*m*Log[c*x^n]
- b*e*k*m^2*Log[c*x^n] + b*e*k*(-1 + m)*m*Hypergeometric2F1[1, -m^(-1), (-1
```

```
 + m)/m, -((f*x^m)/e)]*(n + Log[c*x^n]) + a*e*Log[d*(e + f*x^m)^k] - a*e*m*
Log[d*(e + f*x^m)^k] + b*e*n*Log[d*(e + f*x^m)^k] - b*e*m*n*Log[d*(e + f*x^
m)^k] + b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - b*e*m*Log[c*x^n]*Log[d*(e + f
*x^m)^k])/(e*(-1 + m)*x)
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="maxima")
```

```
[Out] -(b*(n + log(c)) + b*log(x^n) + a)*log((f*x^m + e)^k)/x + integrate((((f*k*
m + f*log(d))*a + (f*k*m*n + (f*k*m + f*log(d))*log(c))*b)*x^m + (b*log(c)*
log(d) + a*log(d))*e + ((f*k*m + f*log(d))*b*x^m + b*e*log(d))*log(x^n))/(f
*x^2*x^m + x^2*e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*log(d*(e + f*x**m)**k)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^2, x)
```

3.149 $\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]
[Out] Defer[Int][((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx = \int \frac{(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{x^3}\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(29) = 58.
time = 0.12, size = 292, normalized size = 10.07

$$\frac{4bekmn-2bekm^2n+4afkmx^m\,{}_2F_1\left(1,\frac{-2+m}{m};2-\frac{2}{m};-\frac{fx^m}{e}\right)+bek(-2+m)mn\,{}_3F_2\left(1,-\frac{2}{m},-\frac{2}{m};1-\frac{2}{m},1-\frac{2}{m};-\frac{fx^m}{e}\right)+4bekm\log(cx^n)-2bekm^2\log(cx^n)+bek(-2+m)m\,{}_2F_1\left(1,-\frac{2}{m};\frac{-2+m}{m};-\frac{fx^m}{e}\right)(n+2\log(cx^n))+8ae\log\left(d(e+fx^m)^k\right)-4aem\log\left(d(e+fx^m)^k\right)+4ben\log\left(d(e+fx^m)^k\right)-2bemn\log\left(d(e+fx^m)^k\right)+8be\log(cx^n)\log\left(d(e+fx^m)^k\right)-4bem\log(cx^n)\log\left(d(e+fx^m)^k\right)}{8e(-2+m)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]
[Out] (4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1, (-2 + m)/
m, 2 - 2/m, -((f*x^m)/e)] + b*e*k*(-2 + m)*m*n*HypergeometricPFQ[{1, -2/m,
-2/m}, {1 - 2/m, 1 - 2/m}, -((f*x^m)/e)] + 4*b*e*k*m*Log[c*x^n] - 2*b*e*k*m
^2*Log[c*x^n] + b*e*k*(-2 + m)*m*Hypergeometric2F1[1, -2/m, (-2 + m)/m, -((
```

```
f*x^m)/e)]*(n + 2*Log[c*x^n]) + 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Log[d*
(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] - 2*b*e*m*n*Log[d*(e + f*x^m)
^k] + 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*(e +
 f*x^m)^k])/(8*e*(-2 + m)*x^2)
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^{n}))\ln\left(d(e+f\,x^{m})^{k}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(n + 2*log(c)) + 2*b*log(x^n) + 2*a)*log((f*x^m + e)^k)/x^2 + integ
rate(1/4*((2*(f*k*m + 2*f*log(d))*a + (f*k*m*n + 2*(f*k*m + 2*f*log(d))*log
(c))*b)*x^m + 4*(b*log(c)*log(d) + a*log(d))*e + 2*((f*k*m + 2*f*log(d))*b*
x^m + 2*b*e*log(d))*log(x^n))/(f*x^3*x^m + x^3*e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k)/x**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*log(d*(e+f*x^m)^k)/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*log((f*x^m + e)^k*d)/x^3, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3,x)
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/x^3, x)
```

3.150 $\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx$

Optimal. Leaf size=433

$$\frac{2bkn(gx)^{3m}}{27gm^2}+\frac{4be^2knx^{-2m}(gx)^{3m}}{9f^2gm^2}-\frac{5beknx^{-m}(gx)^{3m}}{36fgm^2}-\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm}-\frac{e^2kx^{-2m}(gx)^{3m}(a+b\log}{3f^2gm}$$

```
[Out] 2/27*b*k*n*(g*x)^(3*m)/g/m^2+4/9*b*e^2*k*n*(g*x)^(3*m)/f^2/g/m^2/(x^(2*m))-
5/36*b*e*k*n*(g*x)^(3*m)/f/g/m^2/(x^m)-1/9*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/g/
m-1/3*e^2*k*(g*x)^(3*m)*(a+b*ln(c*x^n))/f^2/g/m/(x^(2*m))+1/6*e*k*(g*x)^(3*
m)*(a+b*ln(c*x^n))/f/g/m/(x^m)-1/9*b*e^3*k*n*(g*x)^(3*m)*ln(e+f*x^m)/f^3/g/
m^2/(x^(3*m))-1/3*b*e^3*k*n*(g*x)^(3*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^3/g/m^2/
(x^(3*m))+1/3*e^3*k*(g*x)^(3*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^3/g/m/(x^(3*m
))-1/9*b*n*(g*x)^(3*m)*ln(d*(e+f*x^m)^k)/g/m^2+1/3*(g*x)^(3*m)*(a+b*ln(c*x^
n))*ln(d*(e+f*x^m)^k)/g/m-1/3*b*e^3*k*n*(g*x)^(3*m)*polylog(2,1+f*x^m/e)/f^
3/g/m^2/(x^(3*m))
```

Rubi [A]

time = 0.38, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2505, 20, 272, 45, 2423, 16, 32, 30, 19, 2504, 2441, 2352}

$$-\frac{be^3knx^{-3m}(gx)^{3m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3f^3gm^2}+\frac{(gx)^{3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{e^3kx^{-3m}(gx)^{3m}\log(e+fx^m)(a+b\log(cx^n))}{3f^3gm}-\frac{e^2kx^{-2m}(gx)^{3m}(a+b\log(cx^n))}{3f^2gm}+\frac{ekx^{-m}(gx)^{3m}(a+b\log(cx^n))}{6fgm}-\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm}-\frac{bn(gx)^{3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}-\frac{be^3knx^{-3m}(gx)^{3m}\log(e+fx^m)}{9f^3gm^2}-\frac{be^3knx^{-3m}(gx)^{3m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{3f^3gm^2}+\frac{4be^2knx^{-2m}(gx)^{3m}}{9f^2gm^2}-\frac{5beknx^{-m}(gx)^{3m}}{36fgm^2}+\frac{2bkn(gx)^{3m}}{27gm^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (2*b*k*n*(g*x)^(3*m))/(27*g*m^2) + (4*b*e^2*k*n*(g*x)^(3*m))/(9*f^2*g*m^2*x
^(2*m)) - (5*b*e*k*n*(g*x)^(3*m))/(36*f*g*m^2*x^m) - (k*(g*x)^(3*m)*(a + b*
Log[c*x^n]))/(9*g*m) - (e^2*k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(3*f^2*g*m*x^
(2*m)) + (e*k*(g*x)^(3*m)*(a + b*Log[c*x^n]))/(6*f*g*m*x^m) - (b*e^3*k*n*(g
*x)^(3*m)*Log[e + f*x^m])/(9*f^3*g*m^2*x^(3*m)) - (b*e^3*k*n*(g*x)^(3*m)*Lo
g[-((f*x^m)/e)]*Log[e + f*x^m])/(3*f^3*g*m^2*x^(3*m)) + (e^3*k*(g*x)^(3*m)*
(a + b*Log[c*x^n])*Log[e + f*x^m])/(3*f^3*g*m*x^(3*m)) - (b*n*(g*x)^(3*m)*L
og[d*(e + f*x^m)^k])/(9*g*m^2) + ((g*x)^(3*m)*(a + b*Log[c*x^n])*Log[d*(e +
 f*x^m)^k])/(3*g*m) - (b*e^3*k*n*(g*x)^(3*m)*PolyLog[2, 1 + (f*x^m)/e])/(3*
f^3*g*m^2*x^(3*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1+3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)\,dx = -\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}(a+b\log}{3f^2gm}$$

$$= -\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}(a+b\log}{3f^2gm}$$

$$= -\frac{k(gx)^{3m}(a+b\log(cx^n))}{9gm} - \frac{e^2kx^{-2m}(gx)^{3m}(a+b\log}{3f^2gm}$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} -$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} -$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} -$$

$$= \frac{bkn(gx)^{3m}}{27gm^2} + \frac{be^2knx^{-2m}(gx)^{3m}}{3f^2gm^2} - \frac{beknx^{-m}(gx)^{3m}}{12fgm^2} -$$

$$= \frac{2bkn(gx)^{3m}}{27gm^2} + \frac{4be^2knx^{-2m}(gx)^{3m}}{9f^2gm^2} - \frac{5beknx^{-m}(gx)^{3m}}{36fgm^2}$$

Mathematica [A]
time = 0.25, size = 410, normalized size = 0.95

$$\frac{x^{-3m}(gx)^{3m}\left(-36ae^2fkmx^m+48be^2fknx^m+18aef^2kmx^{2m}-15bef^2knx^{2m}-12af^3kmx^{3m}+8bf^3knx^{3m}-36be^3km^2n\log^2(x)-36be^2fkmx^m\log(cx^n)+18bef^2kmx^{2m}\log(cx^n)-12bf^3kmx^{3m}\log(cx^n)+36ae^3km\log(e-ex^m)-12be^3kn\log(e-ex^m)+36be^3km\log(cx^n)\log(e-ex^m)-36be^3kn\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)+12e^3km\log(x)\left(3am-bn+3bm\log(cx^n)-3bn\log(e-ex^m)+3bn\log(e+fx^m)\right)+36af^3mx^{3m}\log\left(d(e+fx^m)^k\right)-12bf^3nx^{3m}\log\left(d(e+fx^m)^k\right)+36bf^3mx^{3m}\log(cx^n)\log\left(d(e+fx^m)^k\right)-36be^3kn\mathrm{Li}_2\left(1+\frac{fx^m}{e}\right)\right)}{108f^3gm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] ((g*x)^(3*m)*(-36*a*e^2*f*k*m*x^m + 48*b*e^2*f*k*n*x^m + 18*a*e*f^2*k*m*x^(
2*m) - 15*b*e*f^2*k*n*x^(2*m) - 12*a*f^3*k*m*x^(3*m) + 8*b*f^3*k*n*x^(3*m)
- 36*b*e^3*k*m^2*n*Log[x]^2 - 36*b*e^2*f*k*m*x^m*Log[c*x^n] + 18*b*e*f^2*k*
m*x^(2*m)*Log[c*x^n] - 12*b*f^3*k*m*x^(3*m)*Log[c*x^n] + 36*a*e^3*k*m*Log[e
 - e*x^m] - 12*b*e^3*k*n*Log[e - e*x^m] + 36*b*e^3*k*m*Log[c*x^n]*Log[e - e
*x^m] - 36*b*e^3*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] + 12*e^3*k*m*Log[x]*(
3*a*m - b*n + 3*b*m*Log[c*x^n] - 3*b*n*Log[e - e*x^m] + 3*b*n*Log[e + f*x^m
]) + 36*a*f^3*m*x^(3*m)*Log[d*(e + f*x^m)^k] - 12*b*f^3*n*x^(3*m)*Log[d*(e
+ f*x^m)^k] + 36*b*f^3*m*x^(3*m)*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 36*b*e^3
*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(108*f^3*g*m^2*x^(3*m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^{-1+3m} (a+b\ln (c\,x^n))\ln \left(d(e+f\,x^m)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1+3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-1+3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] 1/9*(3*b*g^(3*m)*m*x^(3*m)*log(x^n) + (3*a*g^(3*m)*m + (3*g^(3*m)*m*log(c)
- g^(3*m)*n)*b)*x^(3*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/9*((3*(f
*g^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*a - (f*g^(3*m)*k*n - 3*(f*g^(3*m)*k*m
- 3*f*g^(3*m)*m*log(d))*log(c))*b)*x^(4*m) - 9*(b*g^(3*m)*m*log(c)*log(d) +
 a*g^(3*m)*m*log(d))*e^(3*m*log(x) + 1) - 3*(3*b*g^(3*m)*m*e^(3*m*log(x) +
1)*log(d) - (f*g^(3*m)*k*m - 3*f*g^(3*m)*m*log(d))*b*x^(4*m))*log(x^n))/(f*
g*m*x*x^m + g*m*x*e), x)
```

Fricas [A]
time = 0.38, size = 364, normalized size = 0.84

$$\frac{36\,bg^{3\,m-1}kmne^3\log\left((fx^m+e)e^{(-1)}\right)\log\left(x\right)+36\,bg^{3\,m-1}kn\mathrm{Li}_2\left(-(fx^m+e)e^{(-1)}+1\right)e^3-4\left(3\,bf^3km\log\left(c\right)+3\,af^3km-2\,bf^3kn-3\left(3\,bf^3m\log\left(c\right)+3\,af^3m-bf^3n\right)\log\left(d\right)+3\left(bf^3kmn-3\,bf^3mn\log\left(d\right)\right)\log\left(x\right)\right)g^{3\,m-1}x^{3\,m}+3\left(6\,bf^2kmne\log\left(x\right)+6\,bf^2kme\log\left(c\right)+(6\,af^2km-5\,bf^2kn)e\right)g^{3\,m-1}x^{2\,m}-12\left(3\,bfkmne^2\log\left(x\right)+3\,bfkme^2\log\left(c\right)+(3\,afkm-4\,bfkn)e^2\right)g^{3\,m-1}x^m+12\left(\left(3\,bf^3kmn\log\left(x\right)+3\,bf^3km\log\left(c\right)+3\,af^3km-bf^3kn\right)g^{3\,m-1}x^{3\,m}+\left(3\,bkme^3\log\left(c\right)+(3\,akm-bkn)e^3\right)g^{3\,m-1}\right)\log\left(fx^m+e\right)}{108\,f^3m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] 1/108*(36*b*g^(3*m - 1)*k*m*n*e^3*log((f*x^m + e)*e^(-1))*log(x) + 36*b*g^(
3*m - 1)*k*n*dilog(-(f*x^m + e)*e^(-1) + 1)*e^3 - 4*(3*b*f^3*k*m*log(c) + 3
*a*f^3*k*m - 2*b*f^3*k*n - 3*(3*b*f^3*m*log(c) + 3*a*f^3*m - b*f^3*n)*log(d
) + 3*(b*f^3*k*m*n - 3*b*f^3*m*n*log(d))*log(x))*g^(3*m - 1)*x^(3*m) + 3*(6
*b*f^2*k*m*n*e*log(x) + 6*b*f^2*k*m*e*log(c) + (6*a*f^2*k*m - 5*b*f^2*k*n)*
e)*g^(3*m - 1)*x^(2*m) - 12*(3*b*f*k*m*n*e^2*log(x) + 3*b*f*k*m*e^2*log(c)
+ (3*a*f*k*m - 4*b*f*k*n)*e^2)*g^(3*m - 1)*x^m + 12*((3*b*f^3*k*m*n*log(x)
+ 3*b*f^3*k*m*log(c) + 3*a*f^3*k*m - b*f^3*k*n)*g^(3*m - 1)*x^(3*m) + (3*b*
```

```
k*m*e^3*log(c) + (3*a*k*m - b*k*n)*e^3)*g^(3*m - 1))*log(f*x^m + e))/(f^3*m
^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(3*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,{\left(g\,x\right)}^{3\,m-1}\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(3*m - 1)*(a + b*log(c*x^n)), x)
```

3.151 $\int (gx)^{-1+2m} (a + b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$

Optimal. Leaf size=363

$$\frac{bkn(gx)^{2m}}{4gm^2} - \frac{3beknx^{-m}(gx)^{2m}}{4fgm^2} - \frac{k(gx)^{2m}(a+b\log(cx^n))}{4gm} + \frac{ekx^{-m}(gx)^{2m}(a+b\log(cx^n))}{2fgm} + \frac{be^2knx^{-2m}(gx)}{4f^2}$$

```
[Out] 1/4*b*k*n*(g*x)^(2*m)/g/m^2-3/4*b*e*k*n*(g*x)^(2*m)/f/g/m^2/(x^m)-1/4*k*(g*
x)^(2*m)*(a+b*ln(c*x^n))/g/m+1/2*e*k*(g*x)^(2*m)*(a+b*ln(c*x^n))/f/g/m/(x^m
)+1/4*b*e^2*k*n*(g*x)^(2*m)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))+1/2*b*e^2*k*n*(
g*x)^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/f^2/g/m^2/(x^(2*m))-1/2*e^2*k*(g*x)^(2*
m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/f^2/g/m/(x^(2*m))-1/4*b*n*(g*x)^(2*m)*ln(d*(
e+f*x^m)^k)/g/m^2+1/2*(g*x)^(2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m+1/2
*b*e^2*k*n*(g*x)^(2*m)*polylog(2,1+f*x^m/e)/f^2/g/m^2/(x^(2*m))
```

Rubi [A]
time = 0.26, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2505, 20, 272, 45, 2423, 16, 32, 30, 19, 2504, 2441, 2352}

$$\frac{be^2knx^{-2m}(gx)^{2m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2f^2gm^2} + \frac{(gx)^{2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm} - \frac{e^2kx^{-2m}(gx)^{2m}\log(e+fx^m)(a+b\log(cx^n))}{2f^2gm} + \frac{ekx^{-m}(gx)^{2m}(a+b\log(cx^n))}{2fgm} - \frac{k(gx)^{2m}(a+b\log(cx^n))}{4gm} - \frac{bn(gx)^{2m}\log\left(d(e+fx^m)^k\right)}{4gm^2} + \frac{be^2knx^{-2m}(gx)^{2m}\log(e+fx^m)}{4f^2gm^2} + \frac{be^2knx^{-2m}(gx)^{2m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{2f^2gm^2} - \frac{3beknx^{-m}(gx)^{2m}}{4fgm^2} + \frac{bkn(gx)^{2m}}{4gm^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (b*k*n*(g*x)^(2*m))/(4*g*m^2) - (3*b*e*k*n*(g*x)^(2*m))/(4*f*g*m^2*x^m) - (
k*(g*x)^(2*m)*(a + b*Log[c*x^n]))/(4*g*m) + (e*k*(g*x)^(2*m)*(a + b*Log[c*x
^n]))/(2*f*g*m*x^m) + (b*e^2*k*n*(g*x)^(2*m)*Log[e + f*x^m])/(4*f^2*g*m^2*x
^(2*m)) + (b*e^2*k*n*(g*x)^(2*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(2*f^2*g
*m^2*x^(2*m)) - (e^2*k*(g*x)^(2*m)*(a + b*Log[c*x^n])*Log[e + f*x^m])/(2*f^
2*g*m*x^(2*m)) - (b*n*(g*x)^(2*m)*Log[d*(e + f*x^m)^k])/(4*g*m^2) + ((g*x)^
(2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(2*g*m) + (b*e^2*k*n*(g*x)^(
2*m)*PolyLog[2, 1 + (f*x^m)/e])/(2*f^2*g*m^2*x^(2*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1+2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)\,dx=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\right.}{2fgm}$$

$$=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\right.}{2fgm}$$

$$=-\frac{k(gx)^{2m}\left(a+b\log\left(cx^n\right)\right)}{4gm}+\frac{ekx^{-m}(gx)^{2m}\left(a+b\log\right.}{2fgm}$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\right.}{4gm}$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\right.}{4gm}$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\right.}{4gm}$$

$$=\frac{bkn(gx)^{2m}}{8gm^2}-\frac{beknx^{-m}(gx)^{2m}}{2fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\right.}{4gm}$$

$$=\frac{bkn(gx)^{2m}}{4gm^2}-\frac{3beknx^{-m}(gx)^{2m}}{4fgm^2}-\frac{k(gx)^{2m}\left(a+b\log\right.}{4gm}$$

Mathematica [A]
time = 0.23, size = 352, normalized size = 0.97

$$\frac{x^{-2m}(gx)^{2m}\left(2aefkmx^m-3befknx^m-af^2kmx^{2m}+bf^2knx^{2m}+2be^2km^2n\log^2(x)+2befkmx^m\log(cx^n)-bf^2kmx^{2m}\log(cx^n)-2ae^2km\log(e-ex^m)+be^2kn\log(e-ex^m)-2be^2km\log(cx^n)\log(e-ex^m)+2be^2kn\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)+e^2km\log(x)\left(-2am+bn-2bm\log(cx^n)+2bn\log(e-ex^m)-2bn\log(e+fx^m)\right)+2af^2mx^{2m}\log\left(d(e+fx^m)^k\right)-bf^2nx^{2m}\log\left(d(e+fx^m)^k\right)+2bf^2mx^{2m}\log(cx^n)\log\left(d(e+fx^m)^k\right)+2be^2kn\mathrm{Li}_2\left(1+\frac{fx^m}{e}\right)\right)}{4f^2gm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] ((g*x)^(2*m)*(2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m - a*f^2*k*m*x^(2*m) + b*f^2
*k*n*x^(2*m) + 2*b*e^2*k*m^2*n*Log[x]^2 + 2*b*e*f*k*m*x^m*Log[c*x^n] - b*f^
2*k*m*x^(2*m)*Log[c*x^n] - 2*a*e^2*k*m*Log[e - e*x^m] + b*e^2*k*n*Log[e - e
*x^m] - 2*b*e^2*k*m*Log[c*x^n]*Log[e - e*x^m] + 2*b*e^2*k*n*Log[-((f*x^m)/e
)]*Log[e + f*x^m] + e^2*k*m*Log[x]*(-2*a*m + b*n - 2*b*m*Log[c*x^n] + 2*b*n
*Log[e - e*x^m] - 2*b*n*Log[e + f*x^m]) + 2*a*f^2*m*x^(2*m)*Log[d*(e + f*x^
m)^k] - b*f^2*n*x^(2*m)*Log[d*(e + f*x^m)^k] + 2*b*f^2*m*x^(2*m)*Log[c*x^n]
*Log[d*(e + f*x^m)^k] + 2*b*e^2*k*n*PolyLog[2, 1 + (f*x^m)/e]))/(4*f^2*g*m^
2*x^(2*m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^{-1+2m}\left(a+b\ln\left(c\,x^{n}\right)\right)\ln\left(d(e+f\,x^{m})^{k}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1+2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-1+2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] 1/4*(2*b*g^(2*m)*m*x^(2*m)*log(x^n) + (2*a*g^(2*m)*m + (2*g^(2*m)*m*log(c)
- g^(2*m)*n)*b)*x^(2*m))*log((f*x^m + e)^k)/(g*m^2) + integrate(-1/4*((2*(f
*g^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*a - (f*g^(2*m)*k*n - 2*(f*g^(2*m)*k*m
- 2*f*g^(2*m)*m*log(d))*log(c))*b)*x^(3*m) - 4*(b*g^(2*m)*m*log(c)*log(d) +
 a*g^(2*m)*m*log(d))*e^(2*m*log(x) + 1) - 2*(2*b*g^(2*m)*m*e^(2*m*log(x) +
1)*log(d) - (f*g^(2*m)*k*m - 2*f*g^(2*m)*m*log(d))*b*x^(3*m))*log(x^n))/(f*
g*m*x*x^m + g*m*x*e), x)
```

Fricas [A]
time = 0.37, size = 301, normalized size = 0.83

$$-\frac{2\,bg^{2\,m-1}kmne^{2}\log\left((fx^{m}+e)e^{(-1)}\right)\log\left(x\right)+2\,bg^{2\,m-1}kn\mathrm{Li}_{2}\left(-(fx^{m}+e)e^{(-1)}+1\right)e^{2}+\left(bf^{2}km\log\left(c\right)+af^{2}km-bf^{2}kn-\left(2\,bf^{2}m\log\left(c\right)+2\,af^{2}m-bf^{2}n\right)\log\left(d\right)+\left(bf^{2}kmn-2\,bf^{2}mn\log\left(d\right)\right)\log\left(x\right)\right)g^{2\,m-1}x^{2\,m}-\left(2\,bfkmne\log\left(x\right)+2\,bfkme\log\left(c\right)+\left(2\,afkm-3\,bfkn\right)e\right)g^{2\,m-1}x^{m}-\left(\left(2\,bf^{2}kmn\log\left(x\right)+2\,bf^{2}km\log\left(c\right)+2\,af^{2}km-bf^{2}kn\right)g^{2\,m-1}x^{2\,m}-\left(2\,bkme^{2}\log\left(c\right)+\left(2\,akm-bkn\right)e^{2}\right)g^{2\,m-1}\right)\log\left(fx^{m}+e\right)}{4\,f^{2}m^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] -1/4*(2*b*g^(2*m - 1)*k*m*n*e^2*log((f*x^m + e)*e^(-1))*log(x) + 2*b*g^(2*m
 - 1)*k*n*dilog(-(f*x^m + e)*e^(-1) + 1)*e^2 + (b*f^2*k*m*log(c) + a*f^2*k*
m - b*f^2*k*n - (2*b*f^2*m*log(c) + 2*a*f^2*m - b*f^2*n)*log(d) + (b*f^2*k*
m*n - 2*b*f^2*m*n*log(d))*log(x))*g^(2*m - 1)*x^(2*m) - (2*b*f*k*m*n*e*log(
x) + 2*b*f*k*m*e*log(c) + (2*a*f*k*m - 3*b*f*k*n)*e)*g^(2*m - 1)*x^m - ((2*
b*f^2*k*m*n*log(x) + 2*b*f^2*k*m*log(c) + 2*a*f^2*k*m - b*f^2*k*n)*g^(2*m -
 1)*x^(2*m) - (2*b*k*m*e^2*log(c) + (2*a*k*m - b*k*n)*e^2)*g^(2*m - 1))*log
(f*x^m + e))/(f^2*m^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(2*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,{\left(g\,x\right)}^{2\,m-1}\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(2*m - 1)*(a + b*log(c*x^n)), x)
```

3.152 $\int (gx)^{-1+m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$

Optimal. Leaf size=255

$$\frac{2bkn(gx)^m}{gm^2}-\frac{k(gx)^m(a+b\log(cx^n))}{gm}-\frac{beknx^{-m}(gx)^m\log(e+fx^m)}{fgm^2}-\frac{beknx^{-m}(gx)^m\log\left(-\frac{fx^m}{e}\right)\log(e+fx}{fgm^2}$$

```
[Out] 2*b*k*n*(g*x)^m/g/m^2-k*(g*x)^m*(a+b*ln(c*x^n))/g/m-b*e*k*n*(g*x)^m*ln(e+f*
x^m)/f/g/m^2/(x^m)-b*e*k*n*(g*x)^m*ln(-f*x^m/e)*ln(e+f*x^m)/f/g/m^2/(x^m)+e
*k*(g*x)^m*(a+b*ln(c*x^n))*ln(e+f*x^m)/f/g/m/(x^m)-b*n*(g*x)^m*ln(d*(e+f*x^
m)^k)/g/m^2+(g*x)^m*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m-b*e*k*n*(g*x)^m*p
olylog(2,1+f*x^m/e)/f/g/m^2/(x^m)
```

Rubi [A]
time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2505, 20, 272, 45, 2423, 16, 32, 19, 2504, 2441, 2352}

$$-\frac{beknx^{-m}(gx)^m\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{fgm^2}+\frac{(gx)^m(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{gm}+\frac{ekx^{-m}(gx)^m\log(e+fx^m)(a+b\log(cx^n))}{fgm}-\frac{k(gx)^m(a+b\log(cx^n))}{gm}-\frac{bn(gx)^m\log\left(d(e+fx^m)^k\right)}{gm^2}-\frac{beknx^{-m}(gx)^m\log(e+fx^m)}{fgm^2}-\frac{beknx^{-m}(gx)^m\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{fgm^2}+\frac{2bkn(gx)^m}{gm^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (2*b*k*n*(g*x)^m)/(g*m^2) - (k*(g*x)^m*(a + b*Log[c*x^n]))/(g*m) - (b*e*k*n
*(g*x)^m*Log[e + f*x^m])/(f*g*m^2*x^m) - (b*e*k*n*(g*x)^m*Log[-((f*x^m)/e)]
*Log[e + f*x^m])/(f*g*m^2*x^m) + (e*k*(g*x)^m*(a + b*Log[c*x^n])*Log[e + f*
x^m])/(f*g*m*x^m) - (b*n*(g*x)^m*Log[d*(e + f*x^m)^k])/(g*m^2) + ((g*x)^m*(
a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(g*m) - (b*e*k*n*(g*x)^m*PolyLog[2,
 1 + (f*x^m)/e])/(f*g*m^2*x^m)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule steps

$$\int (gx)^{-1+m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx=-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(c\right.\right.}{fgm}$$

$$=-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(c\right.\right.}{fgm}$$

$$=-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(a+b\log\left(c\right.\right.}{fgm}$$

$$=\frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}+\frac{ekx^{-m}(gx)^m\left(\right.}{}$$

$$=\frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m}{}$$

$$=\frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m}{}$$

$$=\frac{bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx)^m}{}$$

$$=\frac{2bkn(gx)^m}{gm^2}-\frac{k(gx)^m\left(a+b\log\left(cx^n\right)\right)}{gm}-\frac{beknx^{-m}(gx}{f}$$

Mathematica [A]
time = 0.15, size = 268, normalized size = 1.05

$$-\frac{x^{-m}(gx)^m\left(afkmx^m-2bfknx^m+bekm^2n\log^2(x)+bfkmx^m\log\left(cx^n\right)-aekm\log\left(e-ex^m\right)+bekn\log\left(e-ex^m\right)-bekm\log\left(cx^n\right)\log\left(e-ex^m\right)+bekn\log\left(-\frac{fx^m}{e}\right)\log\left(e+fx^m\right)-ekm\log(x)\left(am-bn+bm\log\left(cx^n\right)-bn\log\left(e-ex^m\right)+bn\log\left(e+fx^m\right)\right)-afmx^m\log\left(d\left(e+fx^m\right)^k\right)+bfnx^m\log\left(d\left(e+fx^m\right)^k\right)-bfmx^m\log\left(cx^n\right)\log\left(d\left(e+fx^m\right)^k\right)+bekn\mathrm{Li}_2\left(1+\frac{fx^m}{e}\right)\right)}{fgm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 + m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -(((g*x)^m*(a*f*k*m*x^m - 2*b*f*k*n*x^m + b*e*k*m^2*n*Log[x]^2 + b*f*k*m*x^
m*Log[c*x^n] - a*e*k*m*Log[e - e*x^m] + b*e*k*n*Log[e - e*x^m] - b*e*k*m*Lo
g[c*x^n]*Log[e - e*x^m] + b*e*k*n*Log[-((f*x^m)/e)]*Log[e + f*x^m] - e*k*m*
Log[x]*(a*m - b*n + b*m*Log[c*x^n] - b*n*Log[e - e*x^m] + b*n*Log[e + f*x^m
]) - a*f*m*x^m*Log[d*(e + f*x^m)^k] + b*f*n*x^m*Log[d*(e + f*x^m)^k] - b*f*
m*x^m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*e*k*n*PolyLog[2, 1 + (f*x^m)/e]))
/(f*g*m^2*x^m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^{-1+m} (a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1+m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-1+m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="ma
xima")
```

```
[Out] (b*g^m*m*x^m*log(x^n) + (a*g^m*m + (g^m*m*log(c) - g^m*n)*b)*x^m)*log((f*x^
m + e)^k)/(g*m^2) + integrate(-((((f*g^m*k*m - f*g^m*m*log(d))*a - (f*g^m*k*
n - (f*g^m*k*m - f*g^m*m*log(d))*log(c))*b)*x^(2*m) - (b*g^m*m*log(c)*log(d
) + a*g^m*m*log(d))*e^(m*log(x) + 1) - (b*g^m*m*e^(m*log(x) + 1)*log(d) - (
f*g^m*k*m - f*g^m*m*log(d))*b*x^(2*m))*log(x^n))/(f*g*m*x*x^m + g*m*x*e), x
)
```

Fricas [A]
time = 0.36, size = 202, normalized size = 0.79

$$\frac{bg^{m-1}kmne\log\left((fx^m+e)e^{(-1)}\right)\log(x)+bg^{m-1}kn\mathrm{Li}_2\left(-(fx^m+e)e^{(-1)}+1\right)e-(bfkm\log(c)+afkm-2\,bfkn-(bfm\log(c)+afm-bfn)\log(d)+(bfkmn-bfmn\log(d))\log(x))g^{m-1}x^m+((bfkmn\log(x)+bfkm\log(c)+afkm-bfkn)g^{m-1}x^m+(bkme\log(c)+(akm-bkn)e)g^{m-1})\log(fx^m+e)}{fm^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fr
icas")
```

```
[Out] (b*g^(m - 1)*k*m*n*e*log((f*x^m + e)*e^(-1))*log(x) + b*g^(m - 1)*k*n*dilog
(-(f*x^m + e)*e^(-1) + 1)*e - (b*f*k*m*log(c) + a*f*k*m - 2*b*f*k*n - (b*f*
```

```
m*log(c) + a*f*m - b*f*n)*log(d) + (b*f*k*m*n - b*f*m*n*log(d))*log(x))*g^(
m - 1)*x^m + ((b*f*k*m*n*log(x) + b*f*k*m*log(c) + a*f*k*m - b*f*k*n)*g^(m
- 1)*x^m + (b*k*m*e*log(c) + (a*k*m - b*k*n)*e)*g^(m - 1))*log(f*x^m + e))/
(f*m^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1+m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1+m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="gi
ac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,{\left(g\,x\right)}^{m-1}\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)),x)
```

```
[Out] int(log(d*(e + f*x^m)^k)*(g*x)^(m - 1)*(a + b*log(c*x^n)), x)
```

3.153 $\int (gx)^{-1-m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) dx$

Optimal. Leaf size=304

$$\frac{bfknx^m(gx)^{-m}\log(x)}{egm}-\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg}+\frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{bfknx^m(gx)^{-m}\log}{egm^2}$$

```
[Out] b*f*k*n*x^m*ln(x)/e/g/m/((g*x)^m)-1/2*b*f*k*n*x^m*ln(x)^2/e/g/((g*x)^m)+f*k
*x^m*ln(x)*(a+b*ln(c*x^n))/e/g/((g*x)^m)-b*f*k*n*x^m*ln(e+f*x^m)/e/g/m^2/((
g*x)^m)+b*f*k*n*x^m*ln(-f*x^m/e)*ln(e+f*x^m)/e/g/m^2/((g*x)^m)-f*k*x^m*(a+b
*ln(c*x^n))*ln(e+f*x^m)/e/g/m/((g*x)^m)-b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^
m)-(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^m)+b*f*k*n*x^m*polylog(2,1+
f*x^m/e)/e/g/m^2/((g*x)^m)
```

Rubi [A]
time = 0.21, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2505, 19, 272, 36, 29, 31, 2423, 2338, 2504, 2441, 2352, 16}

$$\frac{bfknx^m(gx)^{-m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{egm^2}-\frac{(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)}{gm}+\frac{fkx^m\log(x)(gx)^{-m}\left(a+b\log\left(cx^n\right)\right)}{eg}-\frac{fkx^m(gx)^{-m}\log\left(e+fx^m\right)\left(a+b\log\left(cx^n\right)\right)}{egm}-\frac{bn(gx)^{-m}\log\left(d\left(e+fx^m\right)^k\right)}{gm^2}-\frac{bfknx^m(gx)^{-m}\log\left(e+fx^m\right)}{egm^2}+\frac{bfknx^m(gx)^{-m}\log\left(-\frac{fx^m}{e}\right)\log\left(e+fx^m\right)}{egm^2}-\frac{bfknx^m\log^2(x)(gx)^{-m}}{2eg}+\frac{bfknx^m\log(x)(gx)^{-m}}{egm}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (b*f*k*n*x^m*Log[x])/(e*g*m*(g*x)^m) - (b*f*k*n*x^m*Log[x]^2)/(2*e*g*(g*x)^
m) + (f*k*x^m*Log[x]*(a + b*Log[c*x^n]))/(e*g*(g*x)^m) - (b*f*k*n*x^m*Log[e
 + f*x^m])/(e*g*m^2*(g*x)^m) + (b*f*k*n*x^m*Log[-((f*x^m)/e)]*Log[e + f*x^m
])/(e*g*m^2*(g*x)^m) - (f*k*x^m*(a + b*Log[c*x^n])*Log[e + f*x^m])/(e*g*m*(
g*x)^m) - (b*n*Log[d*(e + f*x^m)^k])/(g*m^2*(g*x)^m) - ((a + b*Log[c*x^n])*
Log[d*(e + f*x^m)^k])/(g*m*(g*x)^m) + (b*f*k*n*x^m*PolyLog[2, 1 + (f*x^m)/e
])/(e*g*m^2*(g*x)^m)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-m} \left(a+b\log\left(cx^n\right)\right) \log\left(d\left(e+fx^m\right)^k\right) dx = \frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg} - \frac{fkx^m(gx)^{-m}}{}$$

$$= \frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg} - \frac{fkx^m(gx)^{-m}}{}$$

$$= \frac{fkx^m(gx)^{-m}\log(x)\left(a+b\log\left(cx^n\right)\right)}{eg} - \frac{fkx^m(gx)^{-m}}{}$$

$$= -\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} + \frac{fkx^m(gx)^{-m}\log(x)\left(a+\right.}{eg}$$

$$= -\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} + \frac{fkx^m(gx)^{-m}\log(x)\left(a+\right.}{eg}$$

$$= -\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} + \frac{fkx^m(gx)^{-m}\log(x)\left(a+\right.}{eg}$$

$$= -\frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} + \frac{fkx^m(gx)^{-m}\log(x)\left(a+\right.}{eg}$$

$$= \frac{bfknx^m(gx)^{-m}\log(x)}{egm} - \frac{bfknx^m(gx)^{-m}\log^2(x)}{2eg} + \frac{f}{}$$

Mathematica [A]

time = 0.22, size = 162, normalized size = 0.53

$$\frac{(gx)^{-m}\left(-bfkm^2nx^m\log^2(x)-2(am+bn+bm\log\left(cx^n\right))\left(fkx^m\log\left(f-fx^{-m}\right)+e\log\left(d\left(e+fx^m\right)^k\right)\right)+2fkmx^m\log(x)\left(am+bn+bm\log\left(cx^n\right)+bn\log\left(f-fx^{-m}\right)-bn\log\left(1+\frac{fx^m}{e}\right)\right)-2bfknx^m\text{Li}_2\left(-\frac{fx^m}{e}\right)\right)}{2egm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-(b*f*k*m^2*n*x^m*Log[x]^2) - 2*(a*m + b*n + b*m*Log[c*x^n])*(f*k*x^m*Log[
f - f/x^m] + e*Log[d*(e + f*x^m)^k]) + 2*f*k*m*x^m*Log[x]*(a*m + b*n + b*m*
Log[c*x^n] + b*n*Log[f - f/x^m] - b*n*Log[1 + (f*x^m)/e]) - 2*b*f*k*n*x^m*P
olyLog[2, -((f*x^m)/e)])/(2*e*g*m^2*(g*x)^m)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^{-m-1} (a+b\ln(c\,x^n))\ln\left(d(e+f\,x^m)^k\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-m-1)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-m-1)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="ma
xima")
```

```
[Out] -(b*m*log(x^n) + (m*log(c) + n)*b + a*m)*g^(-m - 1)*log((f*x^m + e)^k)/(m^2
*x^m) + integrate((((f*k*m + f*m*log(d))*a + (f*k*n + (f*k*m + f*m*log(d))*
log(c))*b)*x^m + (b*m*log(c)*log(d) + a*m*log(d))*e + (b*m*e*log(d) + (f*k*
m + f*m*log(d))*b*x^m)*log(x^n))/(f*g^(m + 1)*m*x*x^(2*m) + g^(m + 1)*m*x*e
^(m*log(x) + 1)), x)
```

Fricas [A]
time = 0.38, size = 247, normalized size = 0.81

$$-\frac{\left(2\,bfg^{-m-1}kmnx^m\log\left((fx^m+e)e^{(-1)}\right)\log(x)+2\,bfg^{-m-1}knx^m\mathrm{Li}_2\left(-(fx^m+e)e^{(-1)}+1\right)-\left(bfkm^2n\log(x)^2+2\left(bfkm^2\log(c)+afkm^2+bfkmn\right)\log(x)\right)g^{-m-1}x^m+2\left(bmne\log(d)\log(x)+\left(bme\log(c)+(am+bn)e\right)\log(d)\right)g^{-m-1}+2\left(\left(bfkm\log(c)+afkm+bfkn\right)g^{-m-1}x^m+\left(bkmne\log(x)+bkme\log(c)+(akm+bkn)e\right)g^{-m-1}\right)\log(fx^m+e)\right)e^{(-1)}}{2\,m^2x^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*b*f*g^(-m - 1)*k*m*n*x^m*log((f*x^m + e)*e^(-1))*log(x) + 2*b*f*g^(
-m - 1)*k*n*x^m*dilog(-(f*x^m + e)*e^(-1) + 1) - (b*f*k*m^2*n*log(x)^2 + 2*
(b*f*k*m^2*log(c) + a*f*k*m^2 + b*f*k*m*n)*log(x))*g^(-m - 1)*x^m + 2*(b*m*
```

```
n*e*log(d)*log(x) + (b*m*e*log(c) + (a*m + b*n)*e)*log(d))*g^(-m - 1) + 2*(
(b*f*k*m*log(c) + a*f*k*m + b*f*k*n)*g^(-m - 1)*x^m + (b*k*m*n*e*log(x) + b
*k*m*e*log(c) + (a*k*m + b*k*n)*e)*g^(-m - 1))*log(f*x^m + e))*e^(-1)/(m^2*
x^m)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="gi
ac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(g\,x\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1),x)
```

```
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(m + 1), x)
```

3.154 $\int (gx)^{-1-2m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx$

Optimal. Leaf size=414

$$-\frac{3bfknx^m(gx)^{-2m}}{4egm^2}-\frac{bf^2knx^{2m}(gx)^{-2m}\log(x)}{4e^2gm}+\frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g}-\frac{fkx^m(gx)^{-2m}(a+b\log(cx^n))}{2egm}-$$

```
[Out] -3/4*b*f*k*n*x^m/e/g/m^2/((g*x)^(2*m))-1/4*b*f^2*k*n*x^(2*m)*ln(x)/e^2/g/m/
((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(x)^2/e^2/g/((g*x)^(2*m))-1/2*f*k*x^m
*(a+b*ln(c*x^n))/e/g/m/((g*x)^(2*m))-1/2*f^2*k*x^(2*m)*ln(x)*(a+b*ln(c*x^n)
)/e^2/g/((g*x)^(2*m))+1/4*b*f^2*k*n*x^(2*m)*ln(e+f*x^m)/e^2/g/m^2/((g*x)^(2
*m))-1/2*b*f^2*k*n*x^(2*m)*ln(-f*x^m/e)*ln(e+f*x^m)/e^2/g/m^2/((g*x)^(2*m))
+1/2*f^2*k*x^(2*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/e^2/g/m/((g*x)^(2*m))-1/4*b*
n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(2*m))-1/2*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^
k)/g/m/((g*x)^(2*m))-1/2*b*f^2*k*n*x^(2*m)*polylog(2,1+f*x^m/e)/e^2/g/m^2/(
(g*x)^(2*m))
```

Rubi [A]
time = 0.32, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2505, 20, 272, 46, 2423, 30, 19, 2338, 2504, 2441, 2352, 16}

$$-\frac{bf^2knx^{2m}(gx)^{-2m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{2e^2gm^2}-\frac{(gx)^{-2m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{2gm}-\frac{f^2kx^{2m}\log(x)(gx)^{-2m}(a+b\log(cx^n))}{2e^2g}+\frac{f^2kx^{2m}(gx)^{-2m}\log(e+fx^m)(a+b\log(cx^n))}{2e^2gm}-\frac{fkx^m(gx)^{-2m}(a+b\log(cx^n))}{2egm}-\frac{bn(gx)^{-2m}\log\left(d(e+fx^m)^k\right)}{4gm^2}+\frac{bf^2knx^{2m}(gx)^{-2m}\log(e+fx^m)}{4e^2gm^2}-\frac{bf^2knx^{2m}(gx)^{-2m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{2e^2gm^2}+\frac{bf^2knx^{2m}\log^2(x)(gx)^{-2m}}{4e^2g}-\frac{bf^2knx^{2m}\log(x)(gx)^{-2m}}{4e^2gm}-\frac{3bfknx^m(gx)^{-2m}}{4egm^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-3*b*f*k*n*x^m)/(4*e*g*m^2*(g*x)^(2*m)) - (b*f^2*k*n*x^(2*m)*Log[x])/(4*e^
2*g*m*(g*x)^(2*m)) + (b*f^2*k*n*x^(2*m)*Log[x]^2)/(4*e^2*g*(g*x)^(2*m)) - (
f*k*x^m*(a + b*Log[c*x^n]))/(2*e*g*m*(g*x)^(2*m)) - (f^2*k*x^(2*m)*Log[x]*(
a + b*Log[c*x^n]))/(2*e^2*g*(g*x)^(2*m)) + (b*f^2*k*n*x^(2*m)*Log[e + f*x^m
])/(4*e^2*g*m^2*(g*x)^(2*m)) - (b*f^2*k*n*x^(2*m)*Log[-((f*x^m)/e)]*Log[e +
 f*x^m])/(2*e^2*g*m^2*(g*x)^(2*m)) + (f^2*k*x^(2*m)*(a + b*Log[c*x^n])*Log[
e + f*x^m])/(2*e^2*g*m*(g*x)^(2*m)) - (b*n*Log[d*(e + f*x^m)^k])/(4*g*m^2*(
g*x)^(2*m)) - ((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/(2*g*m*(g*x)^(2*m))
 - (b*f^2*k*n*x^(2*m)*PolyLog[2, 1 + (f*x^m)/e])/(2*e^2*g*m^2*(g*x)^(2*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-2m}\left(a+b\log\left(cx^n\right)\right)\log\left(d(e+fx^m)^k\right)\,dx = -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\ \cdots}{}$$

$$= -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\ \cdots}{}$$

$$= -\frac{fkx^m(gx)^{-2m}\left(a+b\log\left(cx^n\right)\right)}{2egm} - \frac{f^2kx^{2m}(gx)^{-2m}\ \cdots}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{f\ \cdots}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{f\ \cdots}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{f\ \cdots}{}$$

$$= -\frac{bfknx^m(gx)^{-2m}}{2egm^2} + \frac{bf^2knx^{2m}(gx)^{-2m}\log^2(x)}{4e^2g} - \frac{f\ \cdots}{}$$

$$= -\frac{3bfknx^m(gx)^{-2m}}{4egm^2} - \frac{bf^2knx^{2m}(gx)^{-2m}\log(x)}{4e^2gm} + \frac{b\ \cdots}{}$$

Mathematica [A]
time = 0.23, size = 302, normalized size = 0.73

$$\frac{(gx)^{-2m}\left(-2aefkmx^m-3befknx^m+bf^2km^2nx^{2m}\log^2(x)-2befkmx^m\log\left(cx^n\right)+2af^2kmx^{2m}\log\left(f-fx^{-m}\right)+bf^2knx^{2m}\log\left(f-fx^{-m}\right)+2bf^2kmx^{2m}\log\left(cx^n\right)\log\left(f-fx^{-m}\right)-2ae^2m\log\left(d(e+fx^m)^k\right)-be^2n\log\left(d(e+fx^m)^k\right)-2be^2m\log\left(cx^n\right)\log\left(d(e+fx^m)^k\right)-f^2kmx^{2m}\log(x)\left(2am+bn+2bm\log\left(cx^n\right)+2bn\log\left(f-fx^{-m}\right)-2bn\log\left(1+\frac{fx^m}{e}\right)\right)+2bf^2knx^{2m}\text{Li}_2\left(-\frac{fx^m}{e}\right)\right)}{4e^2gm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - 2*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-2*a*e*f*k*m*x^m - 3*b*e*f*k*n*x^m + b*f^2*k*m^2*n*x^(2*m)*Log[x]^2 - 2*b*
e*f*k*m*x^m*Log[c*x^n] + 2*a*f^2*k*m*x^(2*m)*Log[f - f/x^m] + b*f^2*k*n*x^(
2*m)*Log[f - f/x^m] + 2*b*f^2*k*m*x^(2*m)*Log[c*x^n]*Log[f - f/x^m] - 2*a*e
^2*m*Log[d*(e + f*x^m)^k] - b*e^2*n*Log[d*(e + f*x^m)^k] - 2*b*e^2*m*Log[c*
x^n]*Log[d*(e + f*x^m)^k] - f^2*k*m*x^(2*m)*Log[x]*(2*a*m + b*n + 2*b*m*Log
[c*x^n] + 2*b*n*Log[f - f/x^m] - 2*b*n*Log[1 + (f*x^m)/e]) + 2*b*f^2*k*n*x^
(2*m)*PolyLog[2, -((f*x^m)/e)])/(4*e^2*g*m^2*(g*x)^(2*m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(gx\right)^{-1-2m}\left(a+b\ln\left(c\,x^n\right)\right)\ln\left(d\left(e+f\,x^m\right)^k\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1-2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-1-2*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] -1/4*(2*b*m*log(x^n) + (2*m*log(c) + n)*b + 2*a*m)*g^(-2*m - 1)*log((f*x^m
+ e)^k)/(m^2*x^(2*m)) + integrate(1/4*((2*(f*k*m + 2*f*m*log(d))*a + (f*k*n
 + 2*(f*k*m + 2*f*m*log(d))*log(c))*b)*x^m + 4*(b*m*log(c)*log(d) + a*m*log
(d))*e + 2*(2*b*m*e*log(d) + (f*k*m + 2*f*m*log(d))*b*x^m)*log(x^n))/(f*g^(
2*m + 1)*m*x*x^(3*m) + g^(2*m + 1)*m*x*e^(2*m*log(x) + 1)), x)
```

Fricas [A]
time = 0.38, size = 334, normalized size = 0.81

$$\frac{\left(2\,bf^{2}g^{-2\,m-1}kmnx^{2\,m}\log\left((fx^{m}+e)e^{(-1)}\right)\log\left(x\right)+2\,bf^{2}g^{-2\,m-1}knx^{2\,m}\mathrm{Li}_2\left(-(fx^{m}+e)e^{(-1)}+1\right)-\left(bf^{2}km^{2}n\log\left(x\right)^{2}+\left(2\,bf^{2}km^{2}\log\left(c\right)+2\,af^{2}km^{2}+bf^{2}kmn\right)\log\left(x\right)\right)g^{-2\,m-1}x^{2\,m}-\left(2\,bfkmne\log\left(x\right)+2\,bfkme\log\left(c\right)+\left(2\,afkm+3\,bfkn\right)e\right)g^{-2\,m-1}x^{m}-\left(2\,bmne^{2}\log\left(d\right)\log\left(x\right)+\left(2\,bme^{2}\log\left(c\right)+\left(2\,am+bn\right)e^{2}\right)\log\left(d\right)\right)g^{-2\,m-1}+\left(\left(2\,bf^{2}km\log\left(c\right)+2\,af^{2}km+bf^{2}kn\right)g^{-2\,m-1}x^{2\,m}-\left(2\,bkmne^{2}\log\left(x\right)+2\,bkme^{2}\log\left(c\right)+\left(2\,akm+bkn\right)e^{2}\right)g^{-2\,m-1}\right)\log\left(fx^{m}+e\right)\right)e^{(-2)}}{4\,m^{2}x^{2\,m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*b*f^2*g^(-2*m - 1)*k*m*n*x^(2*m)*log((f*x^m + e)*e^(-1))*log(x) + 2*
b*f^2*g^(-2*m - 1)*k*n*x^(2*m)*dilog(-(f*x^m + e)*e^(-1) + 1) - (b*f^2*k*m^
2*n*log(x)^2 + (2*b*f^2*k*m^2*log(c) + 2*a*f^2*k*m^2 + b*f^2*k*m*n)*log(x))
*g^(-2*m - 1)*x^(2*m) - (2*b*f*k*m*n*e*log(x) + 2*b*f*k*m*e*log(c) + (2*a*f
*k*m + 3*b*f*k*n)*e)*g^(-2*m - 1)*x^m - (2*b*m*n*e^2*log(d)*log(x) + (2*b*m
*e^2*log(c) + (2*a*m + b*n)*e^2)*log(d))*g^(-2*m - 1) + ((2*b*f^2*k*m*log(c
) + 2*a*f^2*k*m + b*f^2*k*n)*g^(-2*m - 1)*x^(2*m) - (2*b*k*m*n*e^2*log(x) +
 2*b*k*m*e^2*log(c) + (2*a*k*m + b*k*n)*e^2)*g^(-2*m - 1))*log(f*x^m + e))*
e^(-2)/(m^2*x^(2*m))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-2*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-2*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-2*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(g\,x\right)}^{2\,m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1),x)
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(2*m + 1), x)
```

3.155 $\int (gx)^{-1-3m} (a+b\log(cx^n)) \log\left(d(e+fx^m)^k\right) \, dx$

Optimal. Leaf size=484

$$-\frac{5bfknx^m(gx)^{-3m}}{36egm^2}+\frac{4bf^2knx^{2m}(gx)^{-3m}}{9e^2gm^2}+\frac{bf^3knx^{3m}(gx)^{-3m}\log(x)}{9e^3gm}-\frac{bf^3knx^{3m}(gx)^{-3m}\log^2(x)}{6e^3g}-\frac{fkx^m(gx)}{}$$

```
[Out] -5/36*b*f*k*n*x^m/e/g/m^2/((g*x)^(3*m))+4/9*b*f^2*k*n*x^(2*m)/e^2/g/m^2/((g
*x)^(3*m))+1/9*b*f^3*k*n*x^(3*m)*ln(x)/e^3/g/m/((g*x)^(3*m))-1/6*b*f^3*k*n*
x^(3*m)*ln(x)^2/e^3/g/((g*x)^(3*m))-1/6*f*k*x^m*(a+b*ln(c*x^n))/e/g/m/((g*x
)^(3*m))+1/3*f^2*k*x^(2*m)*(a+b*ln(c*x^n))/e^2/g/m/((g*x)^(3*m))+1/3*f^3*k*
x^(3*m)*ln(x)*(a+b*ln(c*x^n))/e^3/g/((g*x)^(3*m))-1/9*b*f^3*k*n*x^(3*m)*ln(
e+f*x^m)/e^3/g/m^2/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(3*m)*ln(-f*x^m/e)*ln(e+f*
x^m)/e^3/g/m^2/((g*x)^(3*m))-1/3*f^3*k*x^(3*m)*(a+b*ln(c*x^n))*ln(e+f*x^m)/
e^3/g/m/((g*x)^(3*m))-1/9*b*n*ln(d*(e+f*x^m)^k)/g/m^2/((g*x)^(3*m))-1/3*(a+
b*ln(c*x^n))*ln(d*(e+f*x^m)^k)/g/m/((g*x)^(3*m))+1/3*b*f^3*k*n*x^(3*m)*poly
log(2,1+f*x^m/e)/e^3/g/m^2/((g*x)^(3*m))
```

Rubi [A]
time = 0.42, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2505, 20, 272, 46, 2423, 30, 19, 2338, 2504, 2441, 2352, 16}

$$\frac{bf^3knx^{3m}(gx)^{-3m}\text{PolyLog}\left(2,\frac{fx^m}{e}+1\right)}{3e^3gm^2}-\frac{(gx)^{-3m}(a+b\log(cx^n))\log\left(d(e+fx^m)^k\right)}{3gm}+\frac{f^3kx^{3m}\log(x)(gx)^{-3m}(a+b\log(cx^n))}{3e^3g}-\frac{f^3kx^{3m}(gx)^{-3m}\log(e+fx^m)(a+b\log(cx^n))}{3e^3gm}+\frac{f^2kx^{2m}(gx)^{-3m}(a+b\log(cx^n))}{3e^2gm}-\frac{fkx^m(gx)^{-3m}(a+b\log(cx^n))}{6egm}-\frac{bn(gx)^{-3m}\log\left(d(e+fx^m)^k\right)}{9gm^2}-\frac{bf^3knx^{3m}(gx)^{-3m}\log(e+fx^m)}{9e^3gm^2}+\frac{bf^3knx^{3m}(gx)^{-3m}\log\left(-\frac{fx^m}{e}\right)\log(e+fx^m)}{3e^3gm^2}-\frac{bf^3knx^{3m}\log^2(x)(gx)^{-3m}}{6e^3g}+\frac{bf^3knx^{3m}\log(x)(gx)^{-3m}}{9e^3gm}+\frac{4bf^2knx^{2m}(gx)^{-3m}}{9e^2gm^2}-\frac{5bfknx^m(gx)^{-3m}}{36egm^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-5*b*f*k*n*x^m)/(36*e*g*m^2*(g*x)^(3*m)) + (4*b*f^2*k*n*x^(2*m))/(9*e^2*g*
m^2*(g*x)^(3*m)) + (b*f^3*k*n*x^(3*m)*Log[x])/(9*e^3*g*m*(g*x)^(3*m)) - (b*
f^3*k*n*x^(3*m)*Log[x]^2)/(6*e^3*g*(g*x)^(3*m)) - (f*k*x^m*(a + b*Log[c*x^n
]))/(6*e*g*m*(g*x)^(3*m)) + (f^2*k*x^(2*m)*(a + b*Log[c*x^n]))/(3*e^2*g*m*(
g*x)^(3*m)) + (f^3*k*x^(3*m)*Log[x]*(a + b*Log[c*x^n]))/(3*e^3*g*(g*x)^(3*m
)) - (b*f^3*k*n*x^(3*m)*Log[e + f*x^m])/(9*e^3*g*m^2*(g*x)^(3*m)) + (b*f^3*
k*n*x^(3*m)*Log[-((f*x^m)/e)]*Log[e + f*x^m])/(3*e^3*g*m^2*(g*x)^(3*m)) - (
f^3*k*x^(3*m)*(a + b*Log[c*x^n])*Log[e + f*x^m])/(3*e^3*g*m*(g*x)^(3*m)) -
(b*n*Log[d*(e + f*x^m)^k])/(9*g*m^2*(g*x)^(3*m)) - ((a + b*Log[c*x^n])*Log[
d*(e + f*x^m)^k])/(3*g*m*(g*x)^(3*m)) + (b*f^3*k*n*x^(3*m)*PolyLog[2, 1 + (
f*x^m)/e])/(3*e^3*g*m^2*(g*x)^(3*m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
```

```
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (gx)^{-1-3m} \left(a+b\log \left(cx^n\right)\right) \log \left(d(e+fx^m)^k\right) \, dx = -\frac{fkx^m(gx)^{-3m} \left(a+b\log \left(cx^n\right)\right)}{6egm} + \frac{f^2kx^{2m}(gx)^{-3m} \left(\right.}{3e^2 \cdots}$$

$$= -\frac{fkx^m(gx)^{-3m} \left(a+b\log \left(cx^n\right)\right)}{6egm} + \frac{f^2kx^{2m}(gx)^{-3m} \left(\right.}{3e^2 \cdots}$$

$$= -\frac{fkx^m(gx)^{-3m} \left(a+b\log \left(cx^n\right)\right)}{6egm} + \frac{f^2kx^{2m}(gx)^{-3m} \left(\right.}{3e^2 \cdots}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2} + \frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2} - \frac{bf^3knx^{3m} \cdots}{\cdots}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2} + \frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2} - \frac{bf^3knx^{3m} \cdots}{\cdots}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2} + \frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2} - \frac{bf^3knx^{3m} \cdots}{\cdots}$$

$$= -\frac{bfknx^m(gx)^{-3m}}{12egm^2} + \frac{bf^2knx^{2m}(gx)^{-3m}}{3e^2gm^2} - \frac{bf^3knx^{3m} \cdots}{\cdots}$$

$$= -\frac{5bfknx^m(gx)^{-3m}}{36egm^2} + \frac{4bf^2knx^{2m}(gx)^{-3m}}{9e^2gm^2} + \frac{bf^3knx \cdots}{\cdots}$$

Mathematica [A]
time = 0.26, size = 358, normalized size = 0.74

$$\frac{(gx)^{-3m} \left(-6ae^2fkmx^m - 5be^2fknx^m + 12aef^2kmx^{2m} + 16bef^2knx^{2m} - 6bf^3km^2nx^{3m}\log^2(x) - 6be^2fkmx^m\log\left(cx^n\right) + 12bef^2kmx^{2m}\log\left(cx^n\right) - 12af^3kmx^{3m}\log\left(f - fx^{-m}\right) - 4bf^3knx^{3m}\log\left(f - fx^{-m}\right) - 12bf^3kmx^{3m}\log\left(cx^n\right)\log\left(f - fx^{-m}\right) - 12ae^3m\log\left(d(e+fx^m)^k\right) - 4be^3n\log\left(d(e+fx^m)^k\right) - 12be^3m\log\left(cx^n\right)\log\left(d(e+fx^m)^k\right) + 4f^3kmx^{3m}\log(x)\left(3am + bn + 3bm\log\left(cx^n\right) + 3bn\log\left(f - fx^{-m}\right) - 3bn\log\left(1 + \frac{fx^m}{e}\right)\right) - 12bf^3knx^{3m}\mathrm{Li}_2\left(-\frac{fx^m}{e}\right)\right)}{36e^3gm^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^(-1 - 3*m)*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (-6*a*e^2*f*k*m*x^m - 5*b*e^2*f*k*n*x^m + 12*a*e*f^2*k*m*x^(2*m) + 16*b*e*f
^2*k*n*x^(2*m) - 6*b*f^3*k*m^2*n*x^(3*m)*Log[x]^2 - 6*b*e^2*f*k*m*x^m*Log[c
*x^n] + 12*b*e*f^2*k*m*x^(2*m)*Log[c*x^n] - 12*a*f^3*k*m*x^(3*m)*Log[f - f/
x^m] - 4*b*f^3*k*n*x^(3*m)*Log[f - f/x^m] - 12*b*f^3*k*m*x^(3*m)*Log[c*x^n]
*Log[f - f/x^m] - 12*a*e^3*m*Log[d*(e + f*x^m)^k] - 4*b*e^3*n*Log[d*(e + f*
x^m)^k] - 12*b*e^3*m*Log[c*x^n]*Log[d*(e + f*x^m)^k] + 4*f^3*k*m*x^(3*m)*Lo
g[x]*(3*a*m + b*n + 3*b*m*Log[c*x^n] + 3*b*n*Log[f - f/x^m] - 3*b*n*Log[1 +
 (f*x^m)/e]) - 12*b*f^3*k*n*x^(3*m)*PolyLog[2, -((f*x^m)/e)])/(36*e^3*g*m^2
*(g*x)^(3*m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (gx)^{-1-3m} \left(a + b \ln \left(c\, x^{n}\right)\right) \ln \left(d {\left(e + f\, x^{m}\right)}^{k}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^(-1-3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

```
[Out] int((g*x)^(-1-3*m)*(a+b*ln(c*x^n))*ln(d*(e+f*x^m)^k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
maxima")
```

```
[Out] -1/9*(3*b*m*log(x^n) + (3*m*log(c) + n)*b + 3*a*m)*g^(-3*m - 1)*log((f*x^m
+ e)^k)/(m^2*x^(3*m)) + integrate(1/9*((3*(f*k*m + 3*f*m*log(d))*a + (f*k*n
 + 3*(f*k*m + 3*f*m*log(d))*log(c))*b)*x^m + 9*(b*m*log(c)*log(d) + a*m*log
(d))*e + 3*(3*b*m*e*log(d) + (f*k*m + 3*f*m*log(d))*b*x^m)*log(x^n))/(f*g^(
3*m + 1)*m*x*x^(4*m) + g^(3*m + 1)*m*x*e^(3*m*log(x) + 1)), x)
```

Fricas [A]
time = 0.37, size = 395, normalized size = 0.82

$$-\frac{12\,b f^{3} g^{-3\,m-1} k m n x^{3\,m} \log\left(\left(f x^{m}+e\right) e^{(-1)}\right) \log\left(x\right) + 12\,b f^{3} g^{-3\,m-1} k n x^{3\,m} \mathrm{Li}_2\left(-\left(f x^{m}+e\right) e^{(-1)}+1\right) - 2\,\left(3\,b f^{3} k m^{2} n \log\left(x\right)^{2} + 2\,\left(3\,b f^{3} k m^{2} \log\left(c\right) + 3\,a f^{3} k m^{2} + b f^{3} k m n\right) \log\left(x\right)\right) g^{-3\,m-1} x^{3\,m} - 4\,\left(3\,b f^{2} k m n e \log\left(x\right) + 3\,b f^{2} k m e \log\left(c\right) + \left(3\,a f^{2} k m + 4\,b f^{2} k n\right) e\right) g^{-3\,m-1} x^{2\,m} + \left(6\,b f k m n e^{2} \log\left(x\right) + 6\,b f k m e^{2} \log\left(c\right) + \left(6\,a f k m + 5\,b f k n\right) e^{2}\right) g^{-3\,m-1} x^{m} + 4\,\left(3\,b m n e^{3} \log\left(d\right) \log\left(x\right) + \left(3\,b m e^{3} \log\left(c\right) + \left(3\,a m + b n\right) e^{3}\right) \log\left(d\right)\right) g^{-3\,m-1} + 4\,\left(\left(3\,b f^{3} k m \log\left(c\right) + 3\,a f^{3} k m + b f^{3} k n\right) g^{-3\,m-1} x^{3\,m} + \left(3\,b k m n e^{3} \log\left(x\right) + 3\,b k m e^{3} \log\left(c\right) + \left(3\,a k m + b k n\right) e^{3}\right) g^{-3\,m-1}\right) \log\left(f x^{m}+e\right)\right) e^{(-3)}}{36\,m^{2} x^{3\,m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
fricas")
```

```
[Out] -1/36*(12*b*f^3*g^(-3*m - 1)*k*m*n*x^(3*m)*log((f*x^m + e)*e^(-1))*log(x) +
 12*b*f^3*g^(-3*m - 1)*k*n*x^(3*m)*dilog(-(f*x^m + e)*e^(-1) + 1) - 2*(3*b*
f^3*k*m^2*n*log(x)^2 + 2*(3*b*f^3*k*m^2*log(c) + 3*a*f^3*k*m^2 + b*f^3*k*m*
n)*log(x))*g^(-3*m - 1)*x^(3*m) - 4*(3*b*f^2*k*m*n*e*log(x) + 3*b*f^2*k*m*e
*log(c) + (3*a*f^2*k*m + 4*b*f^2*k*n)*e)*g^(-3*m - 1)*x^(2*m) + (6*b*f*k*m*
n*e^2*log(x) + 6*b*f*k*m*e^2*log(c) + (6*a*f*k*m + 5*b*f*k*n)*e^2)*g^(-3*m
- 1)*x^m + 4*(3*b*m*n*e^3*log(d)*log(x) + (3*b*m*e^3*log(c) + (3*a*m + b*n)
*e^3)*log(d))*g^(-3*m - 1) + 4*((3*b*f^3*k*m*log(c) + 3*a*f^3*k*m + b*f^3*k
*n)*g^(-3*m - 1)*x^(3*m) + (3*b*k*m*n*e^3*log(x) + 3*b*k*m*e^3*log(c) + (3*
a*k*m + b*k*n)*e^3)*g^(-3*m - 1))*log(f*x^m + e))*e^(-3)/(m^2*x^(3*m))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**(-1-3*m)*(a+b*ln(c*x**n))*ln(d*(e+f*x**m)**k),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^(-1-3*m)*(a+b*log(c*x^n))*log(d*(e+f*x^m)^k),x, algorithm="
giac")
[Out] integrate((b*log(c*x^n) + a)*(g*x)^(-3*m - 1)*log((f*x^m + e)^k*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(e+f\,x^m\right)}^k\right)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{{\left(g\,x\right)}^{3\,m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1),x)
[Out] int((log(d*(e + f*x^m)^k)*(a + b*log(c*x^n)))/(g*x)^(3*m + 1), x)
```

3.156 $\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal. Leaf size=84

$$\frac{1}{27}benrx^3-\frac{1}{27}erx^3(3a-bn+3b\log(cx^n))-\frac{1}{9}bnx^3(d+e\log(fx^r))+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))$$

```
[Out] 1/27*b*e*n*r*x^3-1/27*e*r*x^3*(3*a-b*n+3*b*ln(c*x^n))-1/9*b*n*x^3*(d+e*ln(f
*x^r))+1/3*x^3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341, 2413, 12}

$$\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{27}erx^3(3a+3b\log(cx^n)-bn)-\frac{1}{9}bnx^3(d+e\log(fx^r))+\frac{1}{27}benrx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (b*e*n*r*x^3)/27 - (e*r*x^3*(3*a - b*n + 3*b*Log[c*x^n]))/27 - (b*n*x^3*(d
+ e*Log[f*x^r]))/9 + (x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int x^2(a+b\log(cx^n))(d+e\log(fx^r))\,dx = -\frac{1}{9}bnx^3(d+e\log(fx^r))+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r$$
$$= -\frac{1}{9}bnx^3(d+e\log(fx^r))+\frac{1}{3}x^3(a+b\log(cx^n))(d+e\log(fx^r$$
$$= \frac{1}{27}benrx^3-\frac{1}{27}erx^3(3a-bn+3b\log(cx^n))-\frac{1}{9}bnx^3(d+e\log$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.85

$$\frac{1}{27}x^3(9ad-3bdn-3aer+2benr+(9ae-3ben)\log(fx^r)+3b\log(cx^n)(3d-er+3e\log(fx^r)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^3*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r]
+ 3*b*Log[c*x^n]*(3*d - e*r + 3*e*Log[f*x^r])))/27
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 1640, normalized size = 19.52

method	result	size
risch	Expression too large to display	1640

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a*d+(1/3*b*e*x^3*ln(x^n)-1/6*I*Pi*b*e*x^3*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)+1/6*I*Pi*b*e*x^3*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I*Pi*b*e*x^3*csgn
(I*x^n)*csgn(I*c*x^n)^2-1/6*I*Pi*b*e*x^3*csgn(I*c*x^n)^3+1/3*ln(c)*b*e*x^3-
1/9*b*e*n*x^3+1/3*x^3*a*e)*ln(x^r)+1/6*I*ln(f)*Pi*b*e*x^3*csgn(I*x^n)*csgn(
I*c*x^n)^2-1/18*I*Pi*b*e*r*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2-1/9*a*e*r*x^3-1/
9*b*e*r*x^3*ln(x^n)+1/12*Pi^2*b*e*x^3*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*c*x^
n)^2+1/3*ln(f)*ln(c)*b*e*x^3-1/9*ln(f)*b*e*n*x^3-1/9*ln(c)*b*e*r*x^3+1/3*ln
(c)*b*d*x^3+1/3*ln(f)*a*e*x^3+1/3*b*d*x^3*ln(x^n)+1/6*I*Pi*b*e*x^3*csgn(I*x
^r)*csgn(I*f*x^r)^2*ln(x^n)+1/6*I*Pi*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2*ln(x
^n)+1/3*ln(f)*b*e*x^3*ln(x^n)+1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(I*x^r)*csgn(
I*f*x^r)*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*b*e*x^3*csgn(I*x^r)*csgn(I*f
*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*b*e*x^3*csgn(I*f*x^r)^3*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/12*Pi^2*b*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^
2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/6*I*ln(f)*Pi*b*e*x^3*csgn(I*c*x^n)^
```

```
3+1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3-1/12*Pi^2*b*e
*x^3*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/12*Pi^2*b*e*x^
3*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2+1/12*Pi^2*b*e*x^3*c
sgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*csgn(I*c*x^n)^2+2/27*b*e*n*r*x
^3-1/18*I*Pi*b*e*r*x^3*csgn(I*c)*csgn(I*c*x^n)^2+1/6*I*ln(f)*Pi*b*e*x^3*csg
n(I*c)*csgn(I*c*x^n)^2+1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2-1
/6*I*Pi*b*d*x^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/9*b*d*n*x^3-1/6*I*Pi*
b*e*x^3*csgn(I*f*x^r)^3*ln(x^n)+1/12*Pi^2*b*e*x^3*csgn(I*f*x^r)^3*csgn(I*x^
n)*csgn(I*c*x^n)^2+1/12*Pi^2*b*e*x^3*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c*x
^n)^3-1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x^n)^3
+1/18*I*Pi*b*e*r*x^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/6*I*Pi*b*d*x^3*c
sgn(I*x^n)*csgn(I*c*x^n)^2-1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*f)*csgn(I*x^r)*csg
n(I*f*x^r)+1/6*I*Pi*a*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2+1/6*I*Pi*a*e*x^3*csgn
(I*x^r)*csgn(I*f*x^r)^2+1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2+1/
18*I*Pi*b*e*n*x^3*csgn(I*f*x^r)^3+1/18*I*Pi*b*e*r*x^3*csgn(I*c*x^n)^3-1/12*
Pi^2*b*e*x^3*csgn(I*f*x^r)^3*csgn(I*c*x^n)^3-1/6*I*Pi*a*e*x^3*csgn(I*f*x^r)
^3-1/6*I*ln(c)*Pi*b*e*x^3*csgn(I*f*x^r)^3-1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(
I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-1/6*I*Pi*b*e*x^3*csgn(I*f)*csgn(I*x^r)
*csgn(I*f*x^r)*ln(x^n)-1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r
)*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/18*I*Pi*b*e*n*x^3*csgn(I*x^r)*csgn(
I*f*x^r)^2+1/18*I*Pi*b*e*n*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/6*I*Pi
*b*d*x^3*csgn(I*c*x^n)^3-1/6*I*ln(f)*Pi*b*e*x^3*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-1/18*I*Pi*b*e*n*x^3*csgn(I*f)*csgn(I*f*x^r)^2+1/6*I*Pi*b*d*x^3*csg
n(I*c)*csgn(I*c*x^n)^2-1/6*I*Pi*a*e*x^3*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)
+1/12*Pi^2*b*e*x^3*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c
*x^n)
```

Maxima [A]
time = 0.30, size = 108, normalized size = 1.29

$$-\frac{1}{9}\,bdnx^3 - \frac{1}{9}\,arx^3e + \frac{1}{3}\,bdx^3\log\left(cx^n\right) + \frac{1}{3}\,ax^3e\log\left(fx^r\right) + \frac{1}{3}\,adx^3 + \frac{1}{27}\,\left((2\,r - 3\,\log\left(f\right))x^3 - 3\,x^3\log\left(x^r\right)\right)bne - \frac{1}{9}\,\left(rx^3 - 3\,x^3\log\left(fx^r\right)\right)be\log\left(cx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -1/9*b*d*n*x^3 - 1/9*a*r*x^3*e + 1/3*b*d*x^3*log(c*x^n) + 1/3*a*x^3*e*log(f
*x^r) + 1/3*a*d*x^3 + 1/27*((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*b*n*e -
1/9*(r*x^3 - 3*x^3*log(f*x^r))*b*e*log(c*x^n)
```

Fricas [A]
time = 0.37, size = 145, normalized size = 1.73

$$\frac{1}{3}\,bnrx^3e\log\left(x\right)^2 + \frac{1}{27}\,(2\,bn - 3\,a)rx^3e - \frac{1}{9}\,(bdn - 3\,ad)x^3 - \frac{1}{9}\,\left(brx^3e - 3\,bdx^3\right)\log\left(c\right) + \frac{1}{9}\,\left(3\,bx^3e\log\left(c\right) - (bn - 3\,a)x^3e\right)\log\left(f\right) + \frac{1}{9}\,\left(3\,brx^3e\log\left(c\right) + 3\,bnx^3e\log\left(f\right) + 3\,bdnx^3 - (2\,bn - 3\,a)rx^3e\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/3*b*n*r*x^3*e*log(x)^2 + 1/27*(2*b*n - 3*a)*r*x^3*e - 1/9*(b*d*n - 3*a*d)
*x^3 - 1/9*(b*r*x^3*e - 3*b*d*x^3)*log(c) + 1/9*(3*b*x^3*e*log(c) - (b*n -
3*a)*x^3*e)*log(f) + 1/9*(3*b*r*x^3*e*log(c) + 3*b*n*x^3*e*log(f) + 3*b*d*n
*x^3 - (2*b*n - 3*a)*r*x^3*e)*log(x)
```

Sympy [A]
time = 2.40, size = 128, normalized size = 1.52

$$\frac{adx^3}{3} - \frac{aerx^3}{9} + \frac{aex^3\log\left(fx^r\right)}{3} - \frac{bdnx^3}{9} + \frac{bdx^3\log\left(cx^n\right)}{3} + \frac{2benrx^3}{27} - \frac{benx^3\log\left(fx^r\right)}{9} - \frac{berx^3\log\left(cx^n\right)}{9} + \frac{bex^3\log\left(cx^n\right)\log\left(fx^r\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x**3/3 - a*e*r*x**3/9 + a*e*x**3*log(f*x**r)/3 - b*d*n*x**3/9 + b*d*x**
3*log(c*x**n)/3 + 2*b*e*n*r*x**3/27 - b*e*n*x**3*log(f*x**r)/9 - b*e*r*x**3
*log(c*x**n)/9 + b*e*x**3*log(c*x**n)*log(f*x**r)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(79) = 158.
time = 4.66, size = 161, normalized size = 1.92

$$\frac{1}{3}bnrx^3e\log\left(x\right)^2 - \frac{2}{9}bnrx^3e\log\left(x\right) + \frac{1}{3}brx^3e\log\left(c\right)\log\left(x\right) + \frac{1}{3}bnx^3e\log\left(f\right)\log\left(x\right) + \frac{2}{27}bnrx^3e - \frac{1}{9}brx^3e\log\left(c\right) - \frac{1}{9}bnx^3e\log\left(f\right) + \frac{1}{3}bx^3e\log\left(c\right)\log\left(f\right) + \frac{1}{3}bdnx^3\log\left(x\right) + \frac{1}{3}arx^3e\log\left(x\right) - \frac{1}{9}bdnx^3 - \frac{1}{9}arx^3e + \frac{1}{3}bdx^3\log\left(c\right) + \frac{1}{3}ax^3e\log\left(f\right) + \frac{1}{3}adx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/3*b*n*r*x^3*e*log(x)^2 - 2/9*b*n*r*x^3*e*log(x) + 1/3*b*r*x^3*e*log(c)*lo
g(x) + 1/3*b*n*x^3*e*log(f)*log(x) + 2/27*b*n*r*x^3*e - 1/9*b*r*x^3*e*log(c
) - 1/9*b*n*x^3*e*log(f) + 1/3*b*x^3*e*log(c)*log(f) + 1/3*b*d*n*x^3*log(x)
 + 1/3*a*r*x^3*e*log(x) - 1/9*b*d*n*x^3 - 1/9*a*r*x^3*e + 1/3*b*d*x^3*log(c
) + 1/3*a*x^3*e*log(f) + 1/3*a*d*x^3
```

Mupad [B]
time = 4.04, size = 82, normalized size = 0.98

$$\ln\left(f\,x^r\right)\,\left(\frac{a\,e\,x^3}{3}-\frac{b\,e\,n\,x^3}{9}+\frac{b\,e\,x^3\,\ln\left(c\,x^n\right)}{3}\right)+x^3\,\left(\frac{a\,d}{3}-\frac{b\,d\,n}{9}-\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}\right)+\frac{b\,x^3\,\ln\left(c\,x^n\right)\,\left(3\,d-e\,r\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] log(f*x^r)*((a*e*x^3)/3 - (b*e*n*x^3)/9 + (b*e*x^3*log(c*x^n))/3) + x^3*((a
*d)/3 - (b*d*n)/9 - (a*e*r)/9 + (2*b*e*n*r)/27) + (b*x^3*log(c*x^n)*(3*d -
e*r))/9
```

3.157 $\int x(a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal. Leaf size=84

$$\frac{1}{8}benrx^2-\frac{1}{8}erx^2(2a-bn+2b\log(cx^n))-\frac{1}{4}bnx^2(d+e\log(fx^r))+\frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))$$

```
[Out] 1/8*b*e*n*r*x^2-1/8*e*r*x^2*(2*a-b*n+2*b*ln(c*x^n))-1/4*b*n*x^2*(d+e*ln(f*x
^r))+1/2*x^2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2341, 2413, 12}

$$\frac{1}{2}x^2(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{8}erx^2(2a+2b\log(cx^n)-bn)-\frac{1}{4}bnx^2(d+e\log(fx^r))+\frac{1}{8}benrx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (b*e*n*r*x^2)/8 - (e*r*x^2*(2*a - b*n + 2*b*Log[c*x^n]))/8 - (b*n*x^2*(d +
e*Log[f*x^r]))/4 + (x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int x(a+b\log(cx^n))\,(d+e\log(fx^r))\;dx = -\frac{1}{4}bnx^2(d+e\log(fx^r)) + \frac{1}{2}x^2(a+b\log(cx^n))\,(d+e\log(fx^r)$$
$$= -\frac{1}{4}bnx^2(d+e\log(fx^r)) + \frac{1}{2}x^2(a+b\log(cx^n))\,(d+e\log(fx^r)$$
$$= \frac{1}{8}benrx^2 - \frac{1}{8}erx^2(2a-bn+2b\log(cx^n)) - \frac{1}{4}bnx^2(d+e\log(f$$

Mathematica [**A**]
time = 0.03, size = 68, normalized size = 0.81

$$\frac{1}{4}x^2(2ad-bdn-aer+benr+e(2a-bn)\log(fx^r)+b\log(cx^n)\,(2d-er+2e\log(fx^r)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^2*(2*a*d - b*d*n - a*e*r + b*e*n*r + e*(2*a - b*n)*Log[f*x^r] + b*Log[c*
x^n]*(2*d - e*r + 2*e*Log[f*x^r])))/4
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 1640, normalized size = 19.52

method	result	size
risch	Expression too large to display	1640

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*a*d-1/8*Pi^2*b*e*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*Pi*ln(f)*b*e*x^2*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+1/8*Pi^2*b*e*x^2*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*x^n)*
csgn(I*c*x^n)+1/8*Pi^2*b*e*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+1/2*ln(c)*b*d*x^2-1/8*Pi^2*b*e*x^2*csgn(I*x^r)*csgn(I*f
*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*Pi*b*d*x^2*csgn(I*c*x^n)^3+1/2*ln
(f)*ln(c)*b*e*x^2-1/4*ln(f)*b*e*n*x^2-1/4*ln(c)*b*e*r*x^2+1/2*ln(f)*b*e*x^2
*ln(x^n)-1/4*b*e*r*x^2*ln(x^n)+1/4*I*Pi*b*d*x^2*csgn(I*c)*csgn(I*c*x^n)^2+1
/4*I*Pi*b*d*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2+(1/2*b*e*x^2*ln(x^n)-1/4*I*Pi*b
*e*x^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*I*Pi*b*e*x^2*csgn(I*c)*csgn(
I*c*x^n)^2+1/4*I*Pi*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*I*Pi*b*e*x^2*cs
gn(I*c*x^n)^3+1/2*ln(c)*b*e*x^2-1/4*b*e*n*x^2+1/2*a*e*x^2)*ln(x^r)+1/2*b*d*
x^2*ln(x^n)+1/2*ln(f)*a*e*x^2-1/4*r*a*e*x^2-1/8*I*Pi*b*e*r*x^2*csgn(I*x^n)*
csgn(I*c*x^n)^2-1/8*I*Pi*b*e*r*x^2*csgn(I*c)*csgn(I*c*x^n)^2+1/4*I*Pi*ln(c)
```

```
*b*e*x^2*csgn(I*f)*csgn(I*f*x^r)^2+1/8*I*Pi*b*e*r*x^2*csgn(I*c*x^n)^3-1/4*I
*Pi*b*d*x^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*Pi^2*b*e*x^2*csgn(I*f)*
csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x^n)^3-1/8*Pi^2*b*e*x^2*csgn(I*f)*csgn(I
*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-1/8*Pi^2*b*e*x^2*csgn(I*f)*csgn(I*f*x^r
)^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*I*Pi*b*e*n*x^2*csgn(I*f*x^r)^3-1/8*I*Pi
*b*e*n*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2+1/4*b*e*n*r*x^2-1/8*Pi^2*b*e*x^2*csg
n(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*Pi*a*e*x^2*csgn(I*
f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*b*d*n*x^2+1/4*I*Pi*b*e*x^2*csgn(I*x^r)*csg
n(I*f*x^r)^2*ln(x^n)+1/4*I*Pi*ln(f)*b*e*x^2*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I
*Pi*ln(c)*b*e*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*I*Pi*ln(f)*b*e*x^
2*csgn(I*c*x^n)^3-1/4*I*Pi*ln(c)*b*e*x^2*csgn(I*f*x^r)^3-1/4*I*Pi*b*e*x^2*c
sgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+1/8*I*Pi*b*e*r*x^2*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)+1/4*I*Pi*ln(f)*b*e*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2+1
/4*I*Pi*a*e*x^2*csgn(I*f)*csgn(I*f*x^r)^2+1/4*I*Pi*a*e*x^2*csgn(I*x^r)*csgn
(I*f*x^r)^2-1/4*I*Pi*b*e*x^2*csgn(I*f*x^r)^3*ln(x^n)-1/4*I*Pi*a*e*x^2*csgn(
I*f*x^r)^3-1/8*I*Pi*b*e*n*x^2*csgn(I*f)*csgn(I*f*x^r)^2+1/8*Pi^2*b*e*x^2*cs
gn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*x^n)*csgn(I*c*x^n)^2+1/8*Pi^2*b*e*
x^2*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3+1/8*Pi^2*b*e*x^2*csgn(I*x^r)*
csgn(I*f*x^r)^2*csgn(I*c*x^n)^3+1/8*Pi^2*b*e*x^2*csgn(I*f*x^r)^3*csgn(I*c)*
csgn(I*c*x^n)^2+1/8*Pi^2*b*e*x^2*csgn(I*f*x^r)^3*csgn(I*x^n)*csgn(I*c*x^n)^
2+1/4*I*Pi*b*e*x^2*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)-1/8*Pi^2*b*e*x^2*csgn(
I*f*x^r)^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/8*Pi^2*b*e*x^2*csgn(I*f*x^
r)^3*csgn(I*c*x^n)^3+1/8*I*Pi*b*e*n*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)
+1/8*Pi^2*b*e*x^2*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*csgn(I*c*x^
n)^2+1/4*I*Pi*ln(c)*b*e*x^2*csgn(I*x^r)*csgn(I*f*x^r)^2
```

Maxima [A]
time = 0.29, size = 106, normalized size = 1.26

$$-\frac{1}{4}\,bdnx^2 - \frac{1}{4}\,arx^2e + \frac{1}{2}\,bdx^2\log\left(cx^n\right) + \frac{1}{2}\,ax^2e\log\left(fx^r\right) + \frac{1}{2}\,adx^2 + \frac{1}{4}\,\left((r - \log\left(f\right))x^2 - x^2\log\left(x^r\right)\right)bne - \frac{1}{4}\,\left(rx^2 - 2\,x^2\log\left(fx^r\right)\right)be\log\left(cx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*n*x^2 - 1/4*a*r*x^2*e + 1/2*b*d*x^2*log(c*x^n) + 1/2*a*x^2*e*log(f
*x^r) + 1/2*a*d*x^2 + 1/4*((r - log(f))*x^2 - x^2*log(x^r))*b*n*e - 1/4*(r*
x^2 - 2*x^2*log(f*x^r))*b*e*log(c*x^n)
```

Fricas [A]
time = 0.37, size = 140, normalized size = 1.67

$$\frac{1}{2}\,bnrx^2e\log\left(x\right)^2 + \frac{1}{4}\,(bn - a)rx^2e - \frac{1}{4}\,(bdn - 2\,ad)x^2 - \frac{1}{4}\,\left(brx^2e - 2\,bdx^2\right)\log\left(c\right) + \frac{1}{4}\,\left(2\,bx^2e\log\left(c\right) - (bn - 2\,a)x^2e\right)\log\left(f\right) + \frac{1}{2}\,\left(brx^2e\log\left(c\right) + bnx^2e\log\left(f\right) + bdnx^2 - (bn - a)rx^2e\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/2*b*n*r*x^2*e*log(x)^2 + 1/4*(b*n - a)*r*x^2*e - 1/4*(b*d*n - 2*a*d)*x^2
- 1/4*(b*r*x^2*e - 2*b*d*x^2)*log(c) + 1/4*(2*b*x^2*e*log(c) - (b*n - 2*a)*
x^2*e)*log(f) + 1/2*(b*r*x^2*e*log(c) + b*n*x^2*e*log(f) + b*d*n*x^2 - (b*n
 - a)*r*x^2*e)*log(x)
```

Sympy [A]
time = 1.08, size = 126, normalized size = 1.50

$$\frac{adx^2}{2} - \frac{aerx^2}{4} + \frac{aex^2\log\left(fx^r\right)}{2} - \frac{bdnx^2}{4} + \frac{bdx^2\log\left(cx^n\right)}{2} + \frac{benrx^2}{4} - \frac{benx^2\log\left(fx^r\right)}{4} - \frac{berx^2\log\left(cx^n\right)}{4} + \frac{bex^2\log\left(cx^n\right)\log\left(fx^r\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x**2/2 - a*e*r*x**2/4 + a*e*x**2*log(f*x**r)/2 - b*d*n*x**2/4 + b*d*x**
2*log(c*x**n)/2 + b*e*n*r*x**2/4 - b*e*n*x**2*log(f*x**r)/4 - b*e*r*x**2*lo
g(c*x**n)/4 + b*e*x**2*log(c*x**n)*log(f*x**r)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(79) = 158.
time = 4.01, size = 161, normalized size = 1.92

$$\frac{1}{2}bnrx^2e\log\left(x\right)^2 - \frac{1}{2}bnrx^2e\log\left(x\right) + \frac{1}{2}brx^2e\log\left(c\right)\log\left(x\right) + \frac{1}{2}bnx^2e\log\left(f\right)\log\left(x\right) + \frac{1}{4}bnrx^2e - \frac{1}{4}brx^2e\log\left(c\right) - \frac{1}{4}bnx^2e\log\left(f\right) + \frac{1}{2}bx^2e\log\left(c\right)\log\left(f\right) + \frac{1}{2}bdnx^2\log\left(x\right) + \frac{1}{2}arx^2e\log\left(x\right) - \frac{1}{4}bdnx^2 - \frac{1}{4}arx^2e + \frac{1}{2}bdx^2\log\left(c\right) + \frac{1}{2}ax^2e\log\left(f\right) + \frac{1}{2}adx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/2*b*n*r*x^2*e*log(x)^2 - 1/2*b*n*r*x^2*e*log(x) + 1/2*b*r*x^2*e*log(c)*lo
g(x) + 1/2*b*n*x^2*e*log(f)*log(x) + 1/4*b*n*r*x^2*e - 1/4*b*r*x^2*e*log(c)
 - 1/4*b*n*x^2*e*log(f) + 1/2*b*x^2*e*log(c)*log(f) + 1/2*b*d*n*x^2*log(x)
+ 1/2*a*r*x^2*e*log(x) - 1/4*b*d*n*x^2 - 1/4*a*r*x^2*e + 1/2*b*d*x^2*log(c)
 + 1/2*a*x^2*e*log(f) + 1/2*a*d*x^2
```

Mupad [B]
time = 4.00, size = 82, normalized size = 0.98

$$\ln\left(f\,x^r\right)\,\left(\frac{a\,e\,x^2}{2}-\frac{b\,e\,n\,x^2}{4}+\frac{b\,e\,x^2\,\ln\left(c\,x^n\right)}{2}\right)+x^2\,\left(\frac{a\,d}{2}-\frac{b\,d\,n}{4}-\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}\right)+\frac{b\,x^2\,\ln\left(c\,x^n\right)\,\left(2\,d-e\,r\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] log(f*x^r)*((a*e*x^2)/2 - (b*e*n*x^2)/4 + (b*e*x^2*log(c*x^n))/2) + x^2*((a
*d)/2 - (b*d*n)/4 - (a*e*r)/4 + (b*e*n*r)/4) + (b*x^2*log(c*x^n)*(2*d - e*r
))/4
```

3.158 $\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx$

Optimal. Leaf size=77

$$benrx-e(a-bn)rx-berx\log(cx^n)+ax(d+e\log(fx^r))-bnx(d+e\log(fx^r))+bx\log(cx^n)(d+e\log(fx^r))$$

```
[Out] b*e*n*r*x-e*(-b*n+a)*r*x-b*e*r*x*ln(c*x^n)+a*x*(d+e*ln(f*x^r))-b*n*x*(d+e*l
n(f*x^r))+b*x*ln(c*x^n)*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2332, 2408}

$$-erx(a-bn)+ax(d+e\log(fx^r))+bx\log(cx^n)(d+e\log(fx^r))-berx\log(cx^n)-bnx(d+e\log(fx^r))+benrx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] b*e*n*r*x - e*(a - b*n)*r*x - b*e*r*x*Log[c*x^n] + a*x*(d + e*Log[f*x^r]) -
 b*n*x*(d + e*Log[f*x^r]) + b*x*Log[c*x^n]*(d + e*Log[f*x^r])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\log(cx^n))(d+e\log(fx^r))\,dx &= ax(d+e\log(fx^r))-bnx(d+e\log(fx^r))+bx\log(cx^n)(d+e\log\\ &= -e(a-bn)rx+ax(d+e\log(fx^r))-bnx(d+e\log(fx^r))+bx\log\\ &= benrx-e(a-bn)rx-berx\log(cx^n)+ax(d+e\log(fx^r))-bnx\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.75

$$x(ad-bdn-aer+2benr+e(a-bn)\log(fx^r)+b\log(cx^n)(d-er+e\log(fx^r)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*(d + e*Log[f*x^r]),x]
```

```
[Out] x*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b*n)*Log[f*x^r] + b*Log[c*x^n]*
(d - e*r + e*Log[f*x^r]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 1503, normalized size = 19.52

method	result	size
risch	Expression too large to display	1503

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*Pi*a*e*x*csgn(I*f*x^r)^3-1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2*cs
gn(I*c)*csgn(I*c*x^n)^2-1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*x^n
)*csgn(I*c*x^n)^2-1/4*Pi^2*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn
(I*c*x^n)^2-1/4*Pi^2*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c
*x^n)^2-1/4*Pi^2*b*e*x*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-
b*e*r*x*ln(x^n)-1/2*I*Pi*b*e*r*x*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*Pi*b*e*r*x
*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*Pi*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2*ln(x^
n)+ln(f)*b*e*x*ln(x^n)+1/2*I*Pi*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)-1
/2*I*Pi*b*e*n*x*csgn(I*x^r)*csgn(I*f*x^r)^2+(b*e*x*ln(x^n)-1/2*I*Pi*b*e*x*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*b*e*x*csgn(I*c)*csgn(I*c*x^n)^2
+1/2*I*Pi*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*b*e*x*csgn(I*c*x^n)^3+
ln(c)*b*e*x-b*e*n*x+a*e*x)*ln(x^r)-1/2*I*Pi*b*e*n*x*csgn(I*f)*csgn(I*f*x^r)
^2-1/2*I*Pi*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+1/4*Pi^2*b*e*
x*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*Pi*a*
e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*x^r
)*csgn(I*f*x^r)*csgn(I*c*x^n)^3-b*d*n*x+1/2*I*Pi*ln(c)*b*e*x*csgn(I*f)*csgn
(I*f*x^r)^2+1/2*I*Pi*ln(c)*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2+x*a*d-1/2*I*Pi
*b*d*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x+1/2*I*Pi*ln(f)*b*e*x*csgn(I*c)*c
sgn(I*c*x^n)^2+1/2*I*Pi*ln(f)*b*e*x*csgn(I*x^n)*csgn(I*c*x^n)^2-r*a*e*x+ln(
f)*ln(c)*b*e*x-ln(f)*b*e*n*x-ln(c)*b*e*r*x+b*d*x*ln(x^n)+ln(f)*a*e*x+ln(c)*
b*d*x+1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*csgn(I*c
*x^n)^2+1/2*I*Pi*b*e*r*x*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*b*e*x
*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*c*x^n)^2+1/4*Pi^2*b*e*x*csgn(I*f*x^r)^3*c
sgn(I*x^n)*csgn(I*c*x^n)^2+1/4*Pi^2*b*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*
f*x^r)*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*ln(c)*b*e*x*csgn(I*f*x^r)^3+1/2
*I*Pi*b*e*n*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+1/2*I*Pi*b*d*csgn(I*c)*cs
gn(I*c*x^n)^2*x-1/2*I*Pi*ln(c)*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+2*
b*e*n*r*x-1/2*I*Pi*b*d*csgn(I*c*x^n)^3*x+1/2*I*Pi*b*e*n*x*csgn(I*f*x^r)^3+1
/2*I*Pi*b*e*r*x*csgn(I*c*x^n)^3+1/2*I*Pi*a*e*x*csgn(I*f)*csgn(I*f*x^r)^2+1/
```

```
2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2*x-1/4*Pi^2*b*e*x*csgn(I*f*x^r)^3*csg
n(I*c*x^n)^3-1/4*Pi^2*b*e*x*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*a*e*x*csgn(I*x^r)*csgn(I*f*x^r)^2-1/2*I*P
i*b*e*x*csgn(I*f*x^r)^3*ln(x^n)-1/2*I*Pi*ln(f)*b*e*x*csgn(I*c*x^n)^3+1/4*Pi
^2*b*e*x*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3+1/4*Pi^2*b*e*x*csgn(I*x^
r)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3-1/2*I*Pi*ln(f)*b*e*x*csgn(I*c)*csgn(I*x^
n)*csgn(I*c*x^n)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.12

$$-bdnx + ((2r - \log(f))x - x\log(x^r))bne - arxe + bdx\log(cx^n) - (rx - x\log(fx^r))be\log(cx^n) + axe\log(fx^r) + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] -b*d*n*x + ((2*r - log(f))*x - x*log(x^r))*b*n*e - a*r*x*e + b*d*x*log(c*x^
n) - (r*x - x*log(f*x^r))*b*e*log(c*x^n) + a*x*e*log(f*x^r) + a*d*x
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.49

$$bnrxe\log(x)^2 + (2bn - a)rxe - (bdn - ad)x - (brxe - bdx)\log(c) + (bxe\log(c) - (bn - a)xe)\log(f) + (brxe\log(c) + bnxe\log(f) + bdnx - (2bn - a)rxe)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] b*n*r*x*e*log(x)^2 + (2*b*n - a)*r*x*e - (b*d*n - a*d)*x - (b*r*x*e - b*d*x
)*log(c) + (b*x*e*log(c) - (b*n - a)*x*e)*log(f) + (b*r*x*e*log(c) + b*n*x*
e*log(f) + b*d*n*x - (2*b*n - a)*r*x*e)*log(x)
```

Sympy [A]
time = 0.44, size = 97, normalized size = 1.26

$$adx - aerx + aex\log(fx^r) - bdnx + bdx\log(cx^n) + 2benrx - benx\log(fx^r) - berx\log(cx^n) + bex\log(cx^n)\log(fx^r)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r)),x)
```

```
[Out] a*d*x - a*e*r*x + a*e*x*log(f*x**r) - b*d*n*x + b*d*x*log(c*x**n) + 2*b*e*n
*r*x - b*e*n*x*log(f*x**r) - b*e*r*x*log(c*x**n) + b*e*x*log(c*x**n)*log(f*
x**r)
```

Giac [A]
time = 4.54, size = 122, normalized size = 1.58

$$bnrxe\log(x)^2 - 2bnrxe\log(x) + brxe\log(c)\log(x) + bnxe\log(f)\log(x) + 2bnrxe - brxe\log(c) - bnxe\log(f) + bxe\log(c)\log(f) + bdnx\log(x) + arxe\log(x) - bdnx - arxe + bdx\log(c) + axe\log(f) + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] b*n*r*x*e*log(x)^2 - 2*b*n*r*x*e*log(x) + b*r*x*e*log(c)*log(x) + b*n*x*e*l
og(f)*log(x) + 2*b*n*r*x*e - b*r*x*e*log(c) - b*n*x*e*log(f) + b*x*e*log(c)
*log(f) + b*d*n*x*log(x) + a*r*x*e*log(x) - b*d*n*x - a*r*x*e + b*d*x*log(c
) + a*x*e*log(f) + a*d*x
```

Mupad [B]
time = 3.78, size = 66, normalized size = 0.86

$$x\,\left(a\,d-b\,d\,n-a\,e\,r+2\,b\,e\,n\,r\right)+\ln\left(f\,x^r\right)\,\left(a\,e\,x-b\,e\,n\,x+b\,e\,x\,\ln\left(c\,x^n\right)\right)+b\,x\,\ln\left(c\,x^n\right)\,\left(d-e\,r\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n)),x)
```

```
[Out] x*(a*d - b*d*n - a*e*r + 2*b*e*n*r) + log(f*x^r)*(a*e*x - b*e*n*x + b*e*x*l
og(c*x^n)) + b*x*log(c*x^n)*(d - e*r)
```

3.159 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=57

$$-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}+\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}$$

```
[Out] -1/6*e*r*(a+b*ln(c*x^n))^3/b^2/n^2+1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/b/
n
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {2338, 2413, 12, 2339, 30}

$$\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}-\frac{er(a+b\log(cx^n))^3}{6b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -1/6*(e*r*(a + b*Log[c*x^n])^3)/(b^2*n^2) + ((a + b*Log[c*x^n])^2*(d + e*Lo
g[f*x^r]))/(2*b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}\,dx &= \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn} - (er)\int \frac{(a+b\log(cx^n))^2}{2bnx}\,dx \\ &= \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn} - \frac{(er)\int \frac{(a+b\log(cx^n))^2}{x}\,dx}{2bn} \\ &= \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn} - \frac{(er)\mathrm{Subst}\left(\int x^2\,dx, x, a+b\right.}{2b^2n^2} \\ &= -\frac{er(a+b\log(cx^n))^3}{6b^2n^2} + \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 1.26

$$\frac{1}{6}\log(x)\left(2benr\log^2(x)+6(a+b\log(cx^n))(d+e\log(fx^r))-3\log(x)\left(bdn+aer+ber\log(cx^n)+ben\log(fx^r)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] (Log[x]*(2*b*e*n*r*Log[x]^2 + 6*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]) - 3*L
og[x]*(b*d*n + a*e*r + b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 1597, normalized size = 28.02

method	result	size
risch	Expression too large to display	1597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*a*d+1/2*I*ln(x)*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2+1/4*I*ln(x)^2*Pi*b
*e*r*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/2*I*ln(x^n)*Pi*b*e*csgn(I*f*x^r)
^3*ln(x)+1/2*I*ln(x)*Pi*a*e*csgn(I*x^r)*csgn(I*f*x^r)^2+ln(x)*ln(c)*b*d-1/2
*b*d*n*ln(x)^2+1/4*ln(x)*Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csg
```

```
n(I*x^n)*csgn(I*c*x^n)+1/4*ln(x)*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^
r)*csgn(I*x^n)*csgn(I*c*x^n)^2+(b*e*ln(x)*ln(x^n)-1/2*b*e*n*ln(x)^2-1/2*I*l
n(x)*Pi*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(x)*Pi*b*e*csgn(I*c
)*csgn(I*c*x^n)^2+1/2*I*ln(x)*Pi*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln(x
)*Pi*b*e*csgn(I*c*x^n)^3+ln(x)*ln(c)*b*e+ln(x)*a*e)*ln(x^r)+ln(x^n)*b*d*ln(
x)+ln(x^n)*ln(f)*b*e*ln(x)-1/2*ln(x^n)*r*b*e*ln(x)^2+ln(x)*ln(f)*a*e-1/2*ln
(x)^2*a*e*r+1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f*x^r)^3-1/2*ln(x)^2*ln(f)*b*e*n-
1/2*ln(x)^2*ln(c)*b*e*r+1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f)*csgn(I*x^r)*csgn(I
*f*x^r)+ln(x)*ln(f)*ln(c)*b*e+1/3*b*e*n*r*ln(x)^3-1/2*I*ln(x)*Pi*b*d*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(x)*Pi*a*e*csgn(I*f)*csgn(I*f*x^r)^2+
1/4*I*ln(x)^2*Pi*b*e*r*csgn(I*c*x^n)^3+1/4*ln(x)*Pi^2*b*e*csgn(I*f*x^r)^3*c
sgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(x)*Pi*b*d*csgn(I*c)*csgn(I*c*x^n)^2+1/4
*ln(x)*Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3+1/4*ln(x)*Pi^2*b*
e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3+1/4*ln(x)*Pi^2*b*e*csgn(I*f*x
^r)^3*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(x)*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)-1/4*ln(x)*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*ln(x)*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*
csgn(I*f*x^r)*csgn(I*c)*csgn(I*c*x^n)^2-1/2*I*ln(x)*Pi*ln(c)*b*e*csgn(I*f*x
^r)^3-1/2*I*ln(x)*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/2*I*ln
(x)*Pi*ln(f)*b*e*csgn(I*c*x^n)^3+1/4*ln(x)*Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)
^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(x^n)*Pi*b*e*csgn(I*x^r)*csg
n(I*f*x^r)^2*ln(x)+1/2*I*ln(x)*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I
*ln(x)*Pi*ln(f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2+1/2*I*ln(x)*Pi*ln(c)*b*e*cs
gn(I*f)*csgn(I*f*x^r)^2+1/2*I*ln(x)*Pi*ln(c)*b*e*csgn(I*x^r)*csgn(I*f*x^r)^
2-1/2*I*ln(x)*Pi*a*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-1/4*ln(x)*Pi^2*b*e
*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x^n)^3-1/4*ln(x)*Pi^2*b*e*csg
n(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-1/4*ln(x)*Pi^2*b*e*csgn(I*
f)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*ln(x)*Pi^2*b*e*csgn(I*x^
r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-1/4*ln(x)*Pi^2*b*e*csgn(I*x^r)
*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-1/4*ln(x)*Pi^2*b*e*csgn(I*f*x^
r)^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*f)*c
sgn(I*f*x^r)^2-1/4*I*ln(x)^2*Pi*b*e*n*csgn(I*x^r)*csgn(I*f*x^r)^2-1/4*I*ln(
x)^2*Pi*b*e*r*csgn(I*c)*csgn(I*c*x^n)^2-1/4*I*ln(x)^2*Pi*b*e*r*csgn(I*x^n)*
csgn(I*c*x^n)^2+1/2*I*ln(x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^r)^2*ln(x)-1/4*ln
(x)*Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c*x^n)^3-1/2*I*ln(x)*Pi*a*e*csgn(I*f*x^
r)^3-1/2*I*ln(x)*Pi*b*d*csgn(I*c*x^n)^3-1/2*I*ln(x^n)*Pi*b*e*csgn(I*f)*csgn
(I*x^r)*csgn(I*f*x^r)*ln(x)
```

Maxima [A]
time = 0.29, size = 76, normalized size = 1.33

$$\frac{be\log\left(cx^{n}\right)\log\left(fx^{r}\right)^{2}}{2\,r}-\frac{bne\log\left(fx^{r}\right)^{3}}{6\,r^{2}}+\frac{bd\log\left(cx^{n}\right)^{2}}{2\,n}+\frac{ae\log\left(fx^{r}\right)^{2}}{2\,r}+ad\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

[Out] 1/2*b*e*log(c*x^n)*log(f*x^r)^2/r - 1/6*b*n*e*log(f*x^r)^3/r^2 + 1/2*b*d*log(c*x^n)^2/n + 1/2*a*e*log(f*x^r)^2/r + a*d*log(x)

Fricas [A]
time = 0.38, size = 68, normalized size = 1.19

$$\frac{1}{3}bnre\log(x)^3 + \frac{1}{2}(bre\log(c) + bne\log(f) + bdn + are)\log(x)^2 + (bd\log(c) + ad + (be\log(c) + ae)\log(f))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*n*r*e*log(x)^3 + 1/2*(b*r*e*log(c) + b*n*e*log(f) + b*d*n + a*r*e)*log(x)^2 + (b*d*log(c) + a*d + (b*e*log(c) + a*e)*log(f))*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log(cx^n))(d + e\log(fx^r))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))*(d + e*log(f*x**r))/x, x)
```

Giac [A]
time = 4.20, size = 85, normalized size = 1.49

$$\frac{1}{3}bnre\log(x)^3 + \frac{1}{2}bre\log(c)\log(x)^2 + \frac{1}{2}bne\log(f)\log(x)^2 + be\log(c)\log(f)\log(x) + \frac{1}{2}bdn\log(x)^2 + \frac{1}{2}are\log(x)^2 + bd\log(c)\log(x) + ae\log(f)\log(x) + ad\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] 1/3*b*n*r*e*log(x)^3 + 1/2*b*r*e*log(c)*log(x)^2 + 1/2*b*n*e*log(f)*log(x)^2 + b*e*log(c)*log(f)*log(x) + 1/2*b*d*n*log(x)^2 + 1/2*a*r*e*log(x)^2 + b*d*log(c)*log(x) + a*e*log(f)*log(x) + a*d*log(x)
```

Mupad [B]
time = 3.87, size = 73, normalized size = 1.28

$$a\,d\,\ln(x) + \frac{b\,d\ln(c\,x^n)^2}{2\,n} + \frac{a\,e\ln(f\,x^r)^2}{2\,r} - \frac{b\,e\,r\ln(c\,x^n)^3}{6\,n^2} + \frac{b\,e\ln(c\,x^n)^2\,\ln(f\,x^r)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x,x)
```

```
[Out] a*d*log(x) + (b*d*log(c*x^n)^2)/(2*n) + (a*e*log(f*x^r)^2)/(2*r) - (b*e*r*log(c*x^n)^3)/(6*n^2) + (b*e*log(c*x^n)^2*log(f*x^r))/(2*n)
```

3.160 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{benr}{x}-\frac{er(a+bn+b\log(cx^n))}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}$$

```
[Out] -b*e*n*r/x-e*r*(a+b*n+b*ln(c*x^n))/x-b*n*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n))*
(d+e*ln(f*x^r))/x
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2341, 2413}

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{er(a+b\log(cx^n)+bn)}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{benr}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((b*e*n*r)/x) - (e*r*(a + b*n + b*Log[c*x^n]))/x - (b*n*(d + e*Log[f*x^r])
)/x - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^2}\,dx &= -\frac{bn(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x}-(er)\int \\ &= -\frac{benr}{x}-\frac{er(a+bn+b\log(cx^n))}{x}-\frac{bn(d+e\log(fx^r))}{x}-\frac{(a+b}{}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 57, normalized size = 0.79

$$-\frac{ad + bdn + aer + 2benr + e(a + bn)\log(fx^r) + b\log(cx^n)(d + er + e\log(fx^r))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((a*d + b*d*n + a*e*r + 2*b*e*n*r + e*(a + b*n)*Log[f*x^r] + b*Log[c*x^n]*
(d + e*r + e*Log[f*x^r]))/x)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 1443, normalized size = 20.04

method	result	size
risch	Expression too large to display	1443

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(
c)+2*b*n+2*ln(x^n)*b+2*a)/x*ln(x^r)-1/4*(4*a*e*r+4*b*d*n+2*I*Pi*b*e*csgn(I*
f)*csgn(I*f*x^r)^2*ln(x^n)+2*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+4
*a*d-2*I*Pi*b*d*csgn(I*c*x^n)^3+4*d*b*ln(c)+Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*
csgn(I*f*x^r)*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)-2*I*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+4*
a*ln(f)*e+4*b*ln(c)*ln(f)*e+4*b*ln(c)*e*r+Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^
2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f
*x^r)*csgn(I*c)*csgn(I*c*x^n)^2+4*n*ln(f)*b*e+4*ln(f)*b*e*ln(x^n)+4*b*e*r*l
n(x^n)+8*b*e*n*r+2*I*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*f*x^r)^2-2*I*Pi*a*e*csgn
(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+4*b*d*ln(x^n)-Pi^2*b*e*csgn(I*f*x^r)^3*csgn
(I*c*x^n)^3-2*I*Pi*a*e*csgn(I*f*x^r)^3-Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(
I*f*x^r)*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2*I*Pi*b*e*r*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)-2*I*n*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-2*I*Pi*
b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+Pi^2*b*e*csgn(I*x^r)*csgn(I
*f*x^r)^2*csgn(I*c*x^n)^3-2*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^3-2*I*Pi*ln(c)*b*e
*csgn(I*f*x^r)^3+2*I*Pi*a*e*csgn(I*f)*csgn(I*f*x^r)^2+2*I*Pi*b*d*csgn(I*c)*
csgn(I*c*x^n)^2+2*I*Pi*ln(c)*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+2*I*Pi*b*e*r*c
sgn(I*c)*csgn(I*c*x^n)^2+2*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*c*x^n)^2+2*I*Pi*
ln(f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*cs
gn(I*c)*csgn(I*c*x^n)^2-Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn
(I*c*x^n)^2-Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-
```

```
Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2+2*I*n*Pi*b
*e*csgn(I*f)*csgn(I*f*x^r)^2+2*I*n*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2-2*I*P
i*b*d*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2*I*n*Pi*b*e*csgn(I*f*x^r)^3+Pi^2
*b*e*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*c*x^n)^2+Pi^2*b*e*csgn(I*f*x^r)^3*csg
n(I*x^n)*csgn(I*c*x^n)^2+Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3
+2*I*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)+2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2+2*I*Pi*a
*e*csgn(I*x^r)*csgn(I*f*x^r)^2-2*I*Pi*b*e*r*csgn(I*c*x^n)^3-2*I*Pi*b*e*csgn
(I*f*x^r)^3*ln(x^n)-Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x
^n)^3)/x
```

Maxima [A]
time = 0.29, size = 98, normalized size = 1.36

$$-b\left(\frac{r}{x}+\frac{\log\left(fx^{r}\right)}{x}\right)e\log\left(cx^{n}\right)-\frac{bn(2r+\log\left(f\right)+\log\left(x^{r}\right))e}{x}-\frac{bdn}{x}-\frac{are}{x}-\frac{bd\log\left(cx^{n}\right)}{x}-\frac{ae\log\left(fx^{r}\right)}{x}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

```
[Out] -b*(r/x + log(f*x^r)/x)*e*log(c*x^n) - b*n*(2*r + log(f) + log(x^r))*e/x -
b*d*n/x - a*r*e/x - b*d*log(c*x^n)/x - a*e*log(f*x^r)/x - a*d/x
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.32

$$-\frac{bnre\log\left(x\right)^{2}+bdn+(2bn+a)re+ad+(bre+bd)\log\left(c\right)+(be\log\left(c\right)+(bn+a)e)\log\left(f\right)+(bre\log\left(c\right)+bne\log\left(f\right)+bdn+(2bn+a)re)\log\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*n*r*e*log(x)^2 + b*d*n + (2*b*n + a)*r*e + a*d + (b*r*e + b*d)*log(c) +
 (b*e*log(c) + (b*n + a)*e)*log(f) + (b*r*e*log(c) + b*n*e*log(f) + b*d*n +
 (2*b*n + a)*r*e)*log(x))/x
```

Sympy [A]
time = 0.48, size = 99, normalized size = 1.38

$$-\frac{ad}{x}-\frac{aer}{x}-\frac{ae\log\left(fx^{r}\right)}{x}-\frac{bdn}{x}-\frac{bd\log\left(cx^{n}\right)}{x}-\frac{2benr}{x}-\frac{ben\log\left(fx^{r}\right)}{x}-\frac{ber\log\left(cx^{n}\right)}{x}-\frac{be\log\left(cx^{n}\right)\log\left(fx^{r}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**2,x)
```

```
[Out] -a*d/x - a*e*r/x - a*e*log(f*x**r)/x - b*d*n/x - b*d*log(c*x**n)/x - 2*b*e*
n*r/x - b*e*n*log(f*x**r)/x - b*e*r*log(c*x**n)/x - b*e*log(c*x**n)*log(f*x
**r)/x
```

Giac [A]
time = 5.15, size = 108, normalized size = 1.50

$$-\frac{bnre\log(x)^2 + 2\,bnre\log(x) + bre\log(c)\log(x) + bne\log(f)\log(x) + 2\,bnre + bre\log(c) + bne\log(f) + be\log(c)\log(f) + bdn\log(x) + are\log(x) + bdn + are + bd\log(c) + ae\log(f) + ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

```
[Out] -(b*n*r*e*log(x)^2 + 2*b*n*r*e*log(x) + b*r*e*log(c)*log(x) + b*n*e*log(f)*
log(x) + 2*b*n*r*e + b*r*e*log(c) + b*n*e*log(f) + b*e*log(c)*log(f) + b*d*
n*log(x) + a*r*e*log(x) + b*d*n + a*r*e + b*d*log(c) + a*e*log(f) + a*d)/x
```

Mupad [B]
time = 3.81, size = 75, normalized size = 1.04

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{x}+\frac{b\,e\,n}{x}+\frac{b\,e\,\ln\left(c\,x^n\right)}{x}\right)-\frac{a\,d+b\,d\,n+a\,e\,r+2\,b\,e\,n\,r}{x}-\frac{b\,\ln\left(c\,x^n\right)\,\left(d+e\,r\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] - log(f*x^r)*((a*e)/x + (b*e*n)/x + (b*e*log(c*x^n))/x) - (a*d + b*d*n + a*
e*r + 2*b*e*n*r)/x - (b*log(c*x^n)*(d + e*r))/x
```

3.161 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=83

$$-\frac{benr}{8x^2}-\frac{er(2a+bn+2b\log(cx^n))}{8x^2}-\frac{bn(d+e\log(fx^r))}{4x^2}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}$$

```
[Out] -1/8*b*e*n*r/x^2-1/8*e*r*(2*a+b*n+2*b*ln(c*x^n))/x^2-1/4*b*n*(d+e*ln(f*x^r)
)/x^2-1/2*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341, 2413, 12}

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\frac{er(2a+2b\log(cx^n)+bn)}{8x^2}-\frac{bn(d+e\log(fx^r))}{4x^2}-\frac{benr}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/8*(b*e*n*r)/x^2 - (e*r*(2*a + b*n + 2*b*Log[c*x^n]))/(8*x^2) - (b*n*(d +
 e*Log[f*x^r]))/(4*x^2) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^3}\,dx = -\frac{bn(d+e\log(fx^r))}{4x^2} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2} - (er)$$
$$= -\frac{bn(d+e\log(fx^r))}{4x^2} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2} - \frac{1}{4}(er$$
$$= -\frac{benr}{8x^2} - \frac{er(2a+bn+2b\log(cx^n))}{8x^2} - \frac{bn(d+e\log(fx^r))}{4x^2} - \frac{(a}{}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.77

$$-\frac{2ad+bdn+aer+benr+e(2a+bn)\log(fx^r)+b\log(cx^n)(2d+er+2e\log(fx^r))}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/4*(2*a*d + b*d*n + a*e*r + b*e*n*r + e*(2*a + b*n)*Log[f*x^r] + b*Log[c*
x^n]*(2*d + e*r + 2*e*Log[f*x^r]))/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 1442, normalized size = 17.37

method	result	size
risch	Expression too large to display	1442

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*e*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(
c)+b*n+2*ln(x^n)*b+2*a)/x^2*ln(x^r)-1/8*(2*a*e*r+2*b*d*n+2*I*Pi*b*e*csgn(I*
f)*csgn(I*f*x^r)^2*ln(x^n)+2*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+4
*a*d-2*I*Pi*b*d*csgn(I*c*x^n)^3+4*d*b*ln(c)+Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*
csgn(I*f*x^r)*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)-2*I*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+4*
a*ln(f)*e+4*b*ln(c)*ln(f)*e+2*b*ln(c)*e*r+Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^
2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f
*x^r)*csgn(I*c)*csgn(I*c*x^n)^2+2*n*ln(f)*b*e+4*ln(f)*b*e*ln(x^n)+2*b*e*r*l
n(x^n)+2*b*e*n*r+2*I*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*f*x^r)^2-2*I*Pi*a*e*csgn
(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+4*b*d*ln(x^n)-I*n*Pi*b*e*csgn(I*f)*csgn(I*x
^r)*csgn(I*f*x^r)-I*Pi*b*e*r*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-Pi^2*b*e*c
```

```
sgn(I*f*x^r)^3*csgn(I*c*x^n)^3-2*I*Pi*a*e*csgn(I*f*x^r)^3-Pi^2*b*e*csgn(I*f
)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2*I*Pi*b*e*
csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*ln(x^n)+Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x
^r)^2*csgn(I*c*x^n)^3-2*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^3-2*I*Pi*ln(c)*b*e*csg
n(I*f*x^r)^3+2*I*Pi*a*e*csgn(I*f)*csgn(I*f*x^r)^2+2*I*Pi*b*d*csgn(I*c)*csgn
(I*c*x^n)^2+2*I*Pi*ln(c)*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+I*n*Pi*b*e*csgn(I*
f)*csgn(I*f*x^r)^2+I*n*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+I*Pi*b*e*r*csgn(I
*x^n)*csgn(I*c*x^n)^2+2*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*c*x^n)^2+2*I*Pi*ln(
f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2-Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(
I*c)*csgn(I*c*x^n)^2-Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*
c*x^n)^2-Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-Pi^
2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*b*d*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I
*c*x^n)^2+Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*x^n)*csgn(I*c*x^n)^2+Pi^2*b*e*csg
n(I*f)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3-Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)-I*n*Pi*b*e*csgn(I*f*x^r)^3-I*Pi*b*e*r*csgn(I*c*x^n
)^3+2*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2+I*Pi*b*e*r*csgn(I*c)*csgn(I*c*x^
n)^2+2*I*Pi*a*e*csgn(I*x^r)*csgn(I*f*x^r)^2-2*I*Pi*b*e*csgn(I*f*x^r)^3*ln(x
^n)-Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x^n)^3)/x^2
```

Maxima [A]
time = 0.30, size = 97, normalized size = 1.17

$$-\frac{1}{4}\,b\left(\frac{r}{x^2}+\frac{2\,\log\left(fx^r\right)}{x^2}\right)e\log\left(cx^n\right)-\frac{bn(r+\log\left(f\right)+\log\left(x^r\right))e}{4\,x^2}-\frac{bdn}{4\,x^2}-\frac{are}{4\,x^2}-\frac{bd\log\left(cx^n\right)}{2\,x^2}-\frac{ae\log\left(fx^r\right)}{2\,x^2}-\frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(r/x^2 + 2*log(f*x^r)/x^2)*e*log(c*x^n) - 1/4*b*n*(r + log(f) + log(
x^r))*e/x^2 - 1/4*b*d*n/x^2 - 1/4*a*r*e/x^2 - 1/2*b*d*log(c*x^n)/x^2 - 1/2*
a*e*log(f*x^r)/x^2 - 1/2*a*d/x^2
```

Fricas [A]
time = 0.36, size = 100, normalized size = 1.20

$$-\frac{2\,bnre\log\left(x\right)^2+bdn+(bn+a)re+2\,ad+(bre+2\,bd)\log\left(c\right)+(2\,be\log\left(c\right)+(bn+2\,a)e)\log\left(f\right)+2\,(bre\log\left(c\right)+bne\log\left(f\right)+bdn+(bn+a)re)\log\left(x\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*n*r*e*log(x)^2 + b*d*n + (b*n + a)*r*e + 2*a*d + (b*r*e + 2*b*d)*
log(c) + (2*b*e*log(c) + (b*n + 2*a)*e)*log(f) + 2*(b*r*e*log(c) + b*n*e*lo
g(f) + b*d*n + (b*n + a)*r*e)*log(x))/x^2
```

Sympy [A]
time = 1.23, size = 128, normalized size = 1.54

$$-\frac{ad}{2x^2}-\frac{aer}{4x^2}-\frac{ae\log\left(fx^r\right)}{2x^2}-\frac{bdn}{4x^2}-\frac{bd\log\left(cx^n\right)}{2x^2}-\frac{benr}{4x^2}-\frac{ben\log\left(fx^r\right)}{4x^2}-\frac{ber\log\left(cx^n\right)}{4x^2}-\frac{be\log\left(cx^n\right)\log\left(fx^r\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] -a*d/(2*x**2) - a*e*r/(4*x**2) - a*e*log(f*x**r)/(2*x**2) - b*d*n/(4*x**2)
- b*d*log(c*x**n)/(2*x**2) - b*e*n*r/(4*x**2) - b*e*n*log(f*x**r)/(4*x**2)
- b*e*r*log(c*x**n)/(4*x**2) - b*e*log(c*x**n)*log(f*x**r)/(2*x**2)
```

Giac [A]
time = 3.86, size = 116, normalized size = 1.40

$$-\frac{2\,bnre\log\left(x\right)^{2}+2\,bnre\log\left(x\right)+2\,bre\log\left(c\right)\log\left(x\right)+2\,bne\log\left(f\right)\log\left(x\right)+bnre+bre\log\left(c\right)+bne\log\left(f\right)+2\,be\log\left(c\right)\log\left(f\right)+2\,bdn\log\left(x\right)+2\,are\log\left(x\right)+bdn+are+2\,bd\log\left(c\right)+2\,ae\log\left(f\right)+2\,ad}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*n*r*e*log(x)^2 + 2*b*n*r*e*log(x) + 2*b*r*e*log(c)*log(x) + 2*b*n
*e*log(f)*log(x) + b*n*r*e + b*r*e*log(c) + b*n*e*log(f) + 2*b*e*log(c)*log
(f) + 2*b*d*n*log(x) + 2*a*r*e*log(x) + b*d*n + a*r*e + 2*b*d*log(c) + 2*a*
e*log(f) + 2*a*d)/x^2
```

Mupad [B]
time = 3.94, size = 83, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{2\,x^2}+\frac{b\,e\,n}{4\,x^2}+\frac{b\,e\,\ln\left(c\,x^n\right)}{2\,x^2}\right)-\frac{\frac{a\,d}{2}+\frac{b\,d\,n}{4}+\frac{a\,e\,r}{4}+\frac{b\,e\,n\,r}{4}}{x^2}-\frac{b\,\ln\left(c\,x^n\right)\,\left(2\,d+e\,r\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] - log(f*x^r)*((a*e)/(2*x^2) + (b*e*n)/(4*x^2) + (b*e*log(c*x^n))/(2*x^2)) -
 ((a*d)/2 + (b*d*n)/4 + (a*e*r)/4 + (b*e*n*r)/4)/x^2 - (b*log(c*x^n)*(2*d +
 e*r))/(4*x^2)
```

3.162 $\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{benr}{27x^3}-\frac{er(3a+bn+3b\log(cx^n))}{27x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}$$

```
[Out] -1/27*b*e*n*r/x^3-1/27*e*r*(3*a+b*n+3*b*ln(c*x^n))/x^3-1/9*b*n*(d+e*ln(f*x^
r))/x^3-1/3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341, 2413, 12}

$$-\frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3}-\frac{er(3a+3b\log(cx^n)+bn)}{27x^3}-\frac{bn(d+e\log(fx^r))}{9x^3}-\frac{benr}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -1/27*(b*e*n*r)/x^3 - (e*r*(3*a + b*n + 3*b*Log[c*x^n]))/(27*x^3) - (b*n*(d
 + e*Log[f*x^r]))/(9*x^3) - ((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{x^4}\,dx = -\frac{bn(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3} - (er)$$

$$= -\frac{bn(d+e\log(fx^r))}{9x^3} - \frac{(a+b\log(cx^n))(d+e\log(fx^r))}{3x^3} - \frac{1}{9}(er$$

$$= -\frac{benr}{27x^3} - \frac{er(3a+bn+3b\log(cx^n))}{27x^3} - \frac{bn(d+e\log(fx^r))}{9x^3} - \frac{(a}{}$$

<u>**Mathematica**</u> [**A**]
time = 0.04, size = 69, normalized size = 0.83

$$-\frac{9ad+3bdn+3aer+2benr+3e(3a+bn)\log(fx^r)+3b\log(cx^n)(3d+er+3e\log(fx^r))}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -1/27*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r] +
 3*b*Log[c*x^n]*(3*d + e*r + 3*e*Log[f*x^r]))/x^3
```

<u>**Maple**</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 1451, normalized size = 17.48

method	result	size
risch	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*e*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*c
sgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^
3+6*b*ln(c)+2*b*n+6*ln(x^n)*b+6*a)/x^3*ln(x^r)-1/108*(12*a*e*r+12*b*d*n+36*
a*d-6*I*Pi*b*e*r*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-6*I*n*Pi*b*e*csgn(I*f)
*csgn(I*x^r)*csgn(I*f*x^r)-18*I*Pi*b*d*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+
36*d*b*ln(c)+9*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*x^n)*csg
n(I*c*x^n)^2-18*I*Pi*b*d*csgn(I*c*x^n)^3+6*I*n*Pi*b*e*csgn(I*f)*csgn(I*f*x^
r)^2+18*I*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*ln(x^n)+36*a*ln(f)*e+36*b*ln(c
)*ln(f)*e+12*b*ln(c)*e*r+9*Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)+9*Pi^2*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c)*csg
n(I*x^n)*csgn(I*c*x^n)+9*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(
I*c)*csgn(I*c*x^n)^2+12*n*ln(f)*b*e+36*ln(f)*b*e*ln(x^n)+12*b*e*r*ln(x^n)+1
8*I*Pi*ln(f)*b*e*csgn(I*x^n)*csgn(I*c*x^n)^2+18*I*Pi*ln(c)*b*e*csgn(I*f)*cs
gn(I*f*x^r)^2+6*I*n*Pi*b*e*csgn(I*x^r)*csgn(I*f*x^r)^2+8*b*e*n*r+36*b*d*ln(
```

```
x^n)-18*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-9*Pi^2*b*e*csgn(
I*f*x^r)^3*csgn(I*c*x^n)^3-9*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-18*I*Pi*a*e*csgn(I*f*x^r)^3+9*Pi^2*b*e*c
sgn(I*x^r)*csgn(I*f*x^r)^2*csgn(I*c*x^n)^3-18*I*Pi*ln(c)*b*e*csgn(I*f*x^r)^
3+18*I*Pi*a*e*csgn(I*f)*csgn(I*f*x^r)^2+18*I*Pi*a*e*csgn(I*x^r)*csgn(I*f*x^
r)^2-18*I*Pi*ln(c)*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)-18*I*Pi*a*e*csgn
(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+18*I*Pi*ln(c)*b*e*csgn(I*x^r)*csgn(I*f*x^r)
^2-9*Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-9*Pi^2*b*
e*csgn(I*f)*csgn(I*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-9*Pi^2*b*e*csgn(I*x
^r)*csgn(I*f*x^r)^2*csgn(I*c)*csgn(I*c*x^n)^2-9*Pi^2*b*e*csgn(I*x^r)*csgn(I
*f*x^r)^2*csgn(I*x^n)*csgn(I*c*x^n)^2-18*I*Pi*b*e*csgn(I*f)*csgn(I*x^r)*csg
n(I*f*x^r)*ln(x^n)-6*I*Pi*b*e*r*csgn(I*c*x^n)^3-18*I*Pi*b*e*csgn(I*f*x^r)^3
*ln(x^n)+9*Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*c*x^n)^2+9*Pi^2*b*e*cs
gn(I*f*x^r)^3*csgn(I*x^n)*csgn(I*c*x^n)^2+9*Pi^2*b*e*csgn(I*f)*csgn(I*f*x^r
)^2*csgn(I*c*x^n)^3-9*Pi^2*b*e*csgn(I*f*x^r)^3*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)-6*I*n*Pi*b*e*csgn(I*f*x^r)^3+18*I*Pi*b*d*csgn(I*c)*csgn(I*c*x^n)^2+
18*I*Pi*b*d*csgn(I*x^n)*csgn(I*c*x^n)^2-18*I*Pi*ln(f)*b*e*csgn(I*c*x^n)^3-9
*Pi^2*b*e*csgn(I*f)*csgn(I*x^r)*csgn(I*f*x^r)*csgn(I*c*x^n)^3+18*I*Pi*b*e*c
sgn(I*f)*csgn(I*f*x^r)^2*ln(x^n)+6*I*Pi*b*e*r*csgn(I*x^n)*csgn(I*c*x^n)^2+6
*I*Pi*b*e*r*csgn(I*c)*csgn(I*c*x^n)^2+18*I*Pi*ln(f)*b*e*csgn(I*c)*csgn(I*c*
x^n)^2)/x^3
```

Maxima [**A**]
time = 0.30, size = 103, normalized size = 1.24

$$-\frac{1}{9}\,b\left(\frac{r}{x^3}+\frac{3\,\log\left(fx^r\right)}{x^3}\right)e\log\left(cx^n\right)-\frac{bn(2\,r+3\,\log\left(f\right)+3\,\log\left(x^r\right))e}{27\,x^3}-\frac{bdn}{9\,x^3}-\frac{are}{9\,x^3}-\frac{bd\log\left(cx^n\right)}{3\,x^3}-\frac{ae\log\left(fx^r\right)}{3\,x^3}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*b*(r/x^3 + 3*log(f*x^r)/x^3)*e*log(c*x^n) - 1/27*b*n*(2*r + 3*log(f) +
 3*log(x^r))*e/x^3 - 1/9*b*d*n/x^3 - 1/9*a*r*e/x^3 - 1/3*b*d*log(c*x^n)/x^3
 - 1/3*a*e*log(f*x^r)/x^3 - 1/3*a*d/x^3
```

Fricas [**A**]
time = 0.38, size = 112, normalized size = 1.35

$$-\frac{9\,bnre\log\left(x\right)^2+3\,bdn+(2\,bn+3\,a)re+9\,ad+3\,(bre+3\,bd)\log\left(c\right)+3\,(3\,be\log\left(c\right)+(bn+3\,a)e)\log\left(f\right)+3\,(3\,bre\log\left(c\right)+3\,bne\log\left(f\right)+3\,bdn+(2\,bn+3\,a)re)\log\left(x\right)}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b*n*r*e*log(x)^2 + 3*b*d*n + (2*b*n + 3*a)*r*e + 9*a*d + 3*(b*r*e
+ 3*b*d)*log(c) + 3*(3*b*e*log(c) + (b*n + 3*a)*e)*log(f) + 3*(3*b*r*e*log(
c) + 3*b*n*e*log(f) + 3*b*d*n + (2*b*n + 3*a)*r*e)*log(x))/x^3
```

Sympy [A]
time = 2.66, size = 129, normalized size = 1.55

$$-\frac{ad}{3x^3}-\frac{aer}{9x^3}-\frac{ae\log\left(fx^r\right)}{3x^3}-\frac{bdn}{9x^3}-\frac{bd\log\left(cx^n\right)}{3x^3}-\frac{2benr}{27x^3}-\frac{ben\log\left(fx^r\right)}{9x^3}-\frac{ber\log\left(cx^n\right)}{9x^3}-\frac{be\log\left(cx^n\right)\log\left(fx^r\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*(d+e*ln(f*x**r))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e*r/(9*x**3) - a*e*log(f*x**r)/(3*x**3) - b*d*n/(9*x**3)
- b*d*log(c*x**n)/(3*x**3) - 2*b*e*n*r/(27*x**3) - b*e*n*log(f*x**r)/(9*x**
3) - b*e*r*log(c*x**n)/(9*x**3) - b*e*log(c*x**n)*log(f*x**r)/(3*x**3)
```

Giac [A]
time = 6.28, size = 121, normalized size = 1.46

$$-\frac{9\,bnre\log\left(x\right)^{2}+6\,bnre\log\left(x\right)+9\,bre\log\left(c\right)\log\left(x\right)+9\,bne\log\left(f\right)\log\left(x\right)+2\,bnre+3\,bre\log\left(c\right)+3\,bne\log\left(f\right)+9\,be\log\left(c\right)\log\left(f\right)+9\,bdn\log\left(x\right)+9\,are\log\left(x\right)+3\,bdn+3\,are+9\,bd\log\left(c\right)+9\,ae\log\left(f\right)+9\,ad}{27\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

```
[Out] -1/27*(9*b*n*r*e*log(x)^2 + 6*b*n*r*e*log(x) + 9*b*r*e*log(c)*log(x) + 9*b*
n*e*log(f)*log(x) + 2*b*n*r*e + 3*b*r*e*log(c) + 3*b*n*e*log(f) + 9*b*e*log
(c)*log(f) + 9*b*d*n*log(x) + 9*a*r*e*log(x) + 3*b*d*n + 3*a*r*e + 9*b*d*lo
g(c) + 9*a*e*log(f) + 9*a*d)/x^3
```

Mupad [B]
time = 3.93, size = 83, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\,\left(\frac{a\,e}{3\,x^3}+\frac{b\,e\,n}{9\,x^3}+\frac{b\,e\,\ln\left(c\,x^n\right)}{3\,x^3}\right)-\frac{\frac{a\,d}{3}+\frac{b\,d\,n}{9}+\frac{a\,e\,r}{9}+\frac{2\,b\,e\,n\,r}{27}}{x^3}-\frac{b\,\ln\left(c\,x^n\right)\,\left(3\,d+e\,r\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n)))/x^4,x)
```

```
[Out] - log(f*x^r)*((a*e)/(3*x^3) + (b*e*n)/(9*x^3) + (b*e*log(c*x^n))/(3*x^3)) -
 ((a*d)/3 + (b*d*n)/9 + (a*e*r)/9 + (2*b*e*n*r)/27)/x^3 - (b*log(c*x^n)*(3*
d + e*r))/(9*x^3)
```

3.163 $\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$

Optimal. Leaf size=207

$$-\frac{2}{81}b^2en^2rx^3+\frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e\left(9a^2-6abn+2b^2n^2\right)rx^3+\frac{2}{27}b^2enrx^3\log(cx^n)-\frac{2}{27}be(3a-bn)rx^3\log$$

```
[Out] -2/81*b^2*e*n^2*r*x^3+2/81*b*e*n*(-b*n+3*a)*r*x^3-1/81*e*(2*b^2*n^2-6*a*b*n
+9*a^2)*r*x^3+2/27*b^2*e*n*r*x^3*ln(c*x^n)-2/27*b*e*(-b*n+3*a)*r*x^3*ln(c*x
^n)-1/9*b^2*e*r*x^3*ln(c*x^n)^2+2/27*b^2*n^2*x^3*(d+e*ln(f*x^r))-2/9*b*n*x^
3*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))+1/3*x^3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.14, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2342, 2341, 2413, 12, 14}

$$-\frac{1}{81}erx^3\left(9a^2-6abn+2b^2n^2\right)+\frac{1}{3}x^3(a+b\log(cx^n))^2(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log(fx^r))-\frac{2}{27}berx^3(3a-bn)\log(cx^n)+\frac{2}{81}benrx^3(3a-bn)-\frac{1}{9}b^2erx^3\log^2(cx^n)+\frac{2}{27}b^2enrx^3\log(cx^n)+\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))-\frac{2}{81}b^2en^2rx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (-2*b^2*e*n^2*r*x^3)/81 + (2*b*e*n*(3*a - b*n)*r*x^3)/81 - (e*(9*a^2 - 6*a*
b*n + 2*b^2*n^2)*r*x^3)/81 + (2*b^2*e*n*r*x^3*Log[c*x^n])/27 - (2*b*e*(3*a
- b*n)*r*x^3*Log[c*x^n])/27 - (b^2*e*r*x^3*Log[c*x^n]^2)/9 + (2*b^2*n^2*x^3
*(d + e*Log[f*x^r]))/27 - (2*b*n*x^3*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))
/9 + (x^3*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx &= \frac{2}{27}b^2n^2x^3(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log\\ &= \frac{2}{27}b^2n^2x^3(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log\\ &= \frac{2}{27}b^2n^2x^3(d+e\log(fx^r))-\frac{2}{9}bnx^3(a+b\log(cx^n))(d+e\log\\ &= -\frac{1}{81}e(9a^2-6abn+2b^2n^2)rx^3+\frac{2}{27}b^2n^2x^3(d+e\log(fx^r))-\\ &= \frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e(9a^2-6abn+2b^2n^2)rx^3-\frac{2}{27}be(3\\ &= -\frac{2}{81}b^2en^2rx^3+\frac{2}{81}ben(3a-bn)rx^3-\frac{1}{81}e(9a^2-6abn+2b^2r\end{aligned}$$

Mathematica [A]
time = 0.07, size = 157, normalized size = 0.76

$$\frac{1}{27}x^3\left(9a^2d-6abdn+2b^2dn^2-3a^2er+4abenr-2b^2en^2r+e\left(9a^2-6abn+2b^2n^2\right)\log(fx^r)+3b^2\log^2(cx^n)(3d-er+3e\log(fx^r))+2b\log(cx^n)(9ad-3bdn-3aer+2benr+(9ae-3ben)\log(fx^r))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^3*(9*a^2*d - 6*a*b*d*n + 2*b^2*d*n^2 - 3*a^2*e*r + 4*a*b*e*n*r - 2*b^2*e
*n^2*r + e*(9*a^2 - 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2*(3
*d - e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d - 3*b*d*n - 3*a*e*r + 2*
b*e*n*r + (9*a*e - 3*b*e*n)*Log[f*x^r])))/27
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 9271, normalized size = 44.79

method	result	size
risch	Expression too large to display	9271

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 256, normalized size = 1.24

$$\frac{1}{3}b^2dx^3\log(cx^n)^2 - \frac{2}{9}abdnx^3 - \frac{1}{9}a^2rx^3e + \frac{2}{3}abdx^3\log(cx^n) + \frac{1}{3}a^2x^3e\log(fx^r) + \frac{1}{3}a^2dx^3 - \frac{1}{9}(rx^3 - 3x^3\log(fx^r))b^2e\log(cx^n)^2 + \frac{2}{27}((2r - 3\log(f))x^3 - 3x^3\log(x^r))abne - \frac{2}{9}(rx^3 - 3x^3\log(fx^r))abe\log(cx^n) + \frac{2}{27}(n^2x^3 - 3nx^3\log(cx^n))b^2d - \frac{2}{27}(((r - \log(f))x^3 - x^3\log(x^r))n^2 - ((2r - 3\log(f))x^3 - 3x^3\log(x^r))n\log(cx^n))b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*d*x^3*log(c*x^n)^2 - 2/9*a*b*d*n*x^3 - 1/9*a^2*r*x^3*e + 2/3*a*b*d*
x^3*log(c*x^n) + 1/3*a^2*x^3*e*log(f*x^r) + 1/3*a^2*d*x^3 - 1/9*(r*x^3 - 3*
x^3*log(f*x^r))*b^2*e*log(c*x^n)^2 + 2/27*((2*r - 3*log(f))*x^3 - 3*x^3*log
(x^r))*a*b*n*e - 2/9*(r*x^3 - 3*x^3*log(f*x^r))*a*b*e*log(c*x^n) + 2/27*(n^
2*x^3 - 3*n*x^3*log(c*x^n))*b^2*d - 2/27*(((r - log(f))*x^3 - x^3*log(x^r))
*n^2 - ((2*r - 3*log(f))*x^3 - 3*x^3*log(x^r))*n*log(c*x^n))*b^2*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(196) = 392$.
time = 0.36, size = 407, normalized size = 1.97

$$\frac{1}{3}b^2n^2rx^3e\log(x)^3 - \frac{1}{27}(2b^2n^2 - 4abn + 3a^2)rx^3e + \frac{1}{27}(2b^2dn^2 - 6abdn + 9a^2d)x^3 - \frac{1}{9}(b^2rx^3e - 3b^2dx^3)\log(c)^2 + \frac{1}{3}(2b^2nrx^3e\log(c) + b^2n^2x^3e\log(f) + b^2dn^2x^3 - (b^2n^2 - 2abn)rx^3e)\log(x)^2 + \frac{2}{27}((2b^2n - 3ab)rx^3e - 3(b^2dn - 3abd)x^3)\log(c) + \frac{1}{27}(9b^2x^3e\log(c)^2 - 6(b^2n - 3ab)x^3e\log(c) + (2b^2n^2 - 6abn + 9a^2)x^3e)\log(f) + \frac{1}{9}(3b^2rx^3e\log(c)^2 + (2b^2n^2 - 4abn + 3a^2)rx^3e - 2(b^2dn^2 - 3abdn)x^3 + 2(3b^2dnx^3 - (2b^2n - 3ab)rx^3e)\log(c) + 2(3b^2nx^3e\log(c) - (b^2n^2 - 3abn)x^3e)\log(f))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/3*b^2*n^2*r*x^3*e*log(x)^3 - 1/27*(2*b^2*n^2 - 4*a*b*n + 3*a^2)*r*x^3*e +
 1/27*(2*b^2*d*n^2 - 6*a*b*d*n + 9*a^2*d)*x^3 - 1/9*(b^2*r*x^3*e - 3*b^2*d*
x^3)*log(c)^2 + 1/3*(2*b^2*n*r*x^3*e*log(c) + b^2*n^2*x^3*e*log(f) + b^2*d*
n^2*x^3 - (b^2*n^2 - 2*a*b*n)*r*x^3*e)*log(x)^2 + 2/27*((2*b^2*n - 3*a*b)*r
*x^3*e - 3*(b^2*d*n - 3*a*b*d)*x^3)*log(c) + 1/27*(9*b^2*x^3*e*log(c)^2 - 6
*(b^2*n - 3*a*b)*x^3*e*log(c) + (2*b^2*n^2 - 6*a*b*n + 9*a^2)*x^3*e)*log(f)
 + 1/9*(3*b^2*r*x^3*e*log(c)^2 + (2*b^2*n^2 - 4*a*b*n + 3*a^2)*r*x^3*e - 2*
(b^2*d*n^2 - 3*a*b*d*n)*x^3 + 2*(3*b^2*d*n*x^3 - (2*b^2*n - 3*a*b)*r*x^3*e)
*log(c) + 2*(3*b^2*n*x^3*e*log(c) - (b^2*n^2 - 3*a*b*n)*x^3*e)*log(f))*log(
x)
```

Sympy [A]
time = 5.38, size = 340, normalized size = 1.64

$$\frac{a^2dx^3}{3} - \frac{a^2erx^3}{9} + \frac{a^2ex^3\log(fx^r)}{3} - \frac{2abdnx^3}{9} + \frac{2abdx^3\log(cx^n)}{3} + \frac{4abenrx^3}{27} - \frac{2abenx^3\log(fx^r)}{9} - \frac{2aberx^3\log(cx^n)}{9} + \frac{2abex^3\log(cx^n)\log(fx^r)}{3} + \frac{2b^2dn^2x^3}{27} - \frac{2b^2dnx^3\log(cx^n)}{9} + \frac{b^2dx^3\log(cx^n)^2}{3} - \frac{2b^2en^2rx^3}{27} + \frac{2b^2en^2x^3\log(fx^r)}{27} + \frac{4b^2enrx^3\log(cx^n)}{27} - \frac{2b^2enx^3\log(cx^n)\log(fx^r)}{9} - \frac{b^2erx^3\log(cx^n)^2}{9} + \frac{b^2ex^3\log(cx^n)^2\log(fx^r)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x**3/3 - a**2*e*r*x**3/9 + a**2*e*x**3*log(f*x**r)/3 - 2*a*b*d*n*x**
3/9 + 2*a*b*d*x**3*log(c*x**n)/3 + 4*a*b*e*n*r*x**3/27 - 2*a*b*e*n*x**3*log
(f*x**r)/9 - 2*a*b*e*r*x**3*log(c*x**n)/9 + 2*a*b*e*x**3*log(c*x**n)*log(f*
x**r)/3 + 2*b**2*d*n**2*x**3/27 - 2*b**2*d*n*x**3*log(c*x**n)/9 + b**2*d*x*
*3*log(c*x**n)**2/3 - 2*b**2*e*n**2*r*x**3/27 + 2*b**2*e*n**2*x**3*log(f*x*
*r)/27 + 4*b**2*e*n*r*x**3*log(c*x**n)/27 - 2*b**2*e*n*x**3*log(c*x**n)*log
(f*x**r)/9 - b**2*e*r*x**3*log(c*x**n)**2/9 + b**2*e*x**3*log(c*x**n)**2*lo
g(f*x**r)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(196) = 392$.
time = 5.41, size = 506, normalized size = 2.44

$$\frac{1}{3} b^{2} n^{2} r x^{3} e \log\left(x\right)^{3} - \frac{1}{3} b^{2} n^{2} r x^{3} e \log\left(x\right)^{2} + \frac{2}{3} b^{2} n r x^{3} e \log\left(c\right) \log\left(x\right)^{2} + \frac{1}{3} b^{2} n^{2} x^{3} e \log\left(f\right) \log\left(x\right)^{2} + \frac{2}{9} b^{2} n^{2} r x^{3} e \log\left(x\right) - \frac{4}{9} b^{2} n r x^{3} e \log\left(c\right) \log\left(x\right) + \frac{1}{3} b^{2} r x^{3} e \log\left(c\right)^{2} \log\left(x\right) - \frac{2}{9} b^{2} n^{2} x^{3} e \log\left(f\right) \log\left(x\right) + \frac{2}{3} b^{2} n x^{3} e \log\left(c\right) \log\left(f\right) \log\left(x\right) + \frac{1}{3} b^{2} d n^{2} x^{3} \log\left(x\right)^{2} + \frac{2}{3} a b n r x^{3} e \log\left(x\right)^{2} - \frac{2}{27} b^{2} n^{2} r x^{3} e + \frac{4}{27} b^{2} n r x^{3} e \log\left(c\right) - \frac{1}{9} b^{2} r x^{3} e \log\left(c\right)^{2} + \frac{2}{27} b^{2} n^{2} x^{3} e \log\left(f\right) - \frac{2}{9} b^{2} n x^{3} e \log\left(c\right) \log\left(f\right) + \frac{1}{3} b^{2} x^{3} e \log\left(c\right)^{2} \log\left(f\right) - \frac{2}{9} b^{2} d n^{2} x^{3} \log\left(x\right) - \frac{4}{9} a b n r x^{3} e \log\left(x\right) + \frac{2}{3} b^{2} d n x^{3} \log\left(c\right) \log\left(x\right) + \frac{2}{3} a b r x^{3} e \log\left(c\right) \log\left(x\right) + \frac{2}{3} a b n x^{3} e \log\left(f\right) \log\left(x\right) + \frac{2}{27} b^{2} d n^{2} x^{3} + \frac{4}{27} a b n r x^{3} e - \frac{2}{9} b^{2} d n x^{3} \log\left(c\right) - \frac{2}{9} a b r x^{3} e \log\left(c\right) + \frac{1}{3} b^{2} d x^{3} \log\left(c\right)^{2} - \frac{2}{9} a b n x^{3} e \log\left(f\right) + \frac{2}{3} a b x^{3} e \log\left(c\right) \log\left(f\right) + \frac{2}{3} a b d n x^{3} \log\left(x\right) + \frac{1}{3} a^{2} r x^{3} e \log\left(x\right) - \frac{2}{9} a b d n x^{3} - \frac{1}{9} a^{2} r x^{3} e + \frac{2}{3} a b d x^{3} \log\left(c\right) + \frac{1}{3} a^{2} x^{3} e \log\left(f\right) + \frac{1}{3} a^{2} d x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/3*b^2*n^2*r*x^3*e*log(x)^3 - 1/3*b^2*n^2*r*x^3*e*log(x)^2 + 2/3*b^2*n*r*x
^3*e*log(c)*log(x)^2 + 1/3*b^2*n^2*x^3*e*log(f)*log(x)^2 + 2/9*b^2*n^2*r*x^
3*e*log(x) - 4/9*b^2*n*r*x^3*e*log(c)*log(x) + 1/3*b^2*r*x^3*e*log(c)^2*log
(x) - 2/9*b^2*n^2*x^3*e*log(f)*log(x) + 2/3*b^2*n*x^3*e*log(c)*log(f)*log(x
) + 1/3*b^2*d*n^2*x^3*log(x)^2 + 2/3*a*b*n*r*x^3*e*log(x)^2 - 2/27*b^2*n^2*
r*x^3*e + 4/27*b^2*n*r*x^3*e*log(c) - 1/9*b^2*r*x^3*e*log(c)^2 + 2/27*b^2*n
^2*x^3*e*log(f) - 2/9*b^2*n*x^3*e*log(c)*log(f) + 1/3*b^2*x^3*e*log(c)^2*lo
g(f) - 2/9*b^2*d*n^2*x^3*log(x) - 4/9*a*b*n*r*x^3*e*log(x) + 2/3*b^2*d*n*x^
3*log(c)*log(x) + 2/3*a*b*r*x^3*e*log(c)*log(x) + 2/3*a*b*n*x^3*e*log(f)*lo
g(x) + 2/27*b^2*d*n^2*x^3 + 4/27*a*b*n*r*x^3*e - 2/9*b^2*d*n*x^3*log(c) - 2
/9*a*b*r*x^3*e*log(c) + 1/3*b^2*d*x^3*log(c)^2 - 2/9*a*b*n*x^3*e*log(f) + 2
/3*a*b*x^3*e*log(c)*log(f) + 2/3*a*b*d*n*x^3*log(x) + 1/3*a^2*r*x^3*e*log(x
) - 2/9*a*b*d*n*x^3 - 1/9*a^2*r*x^3*e + 2/3*a*b*d*x^3*log(c) + 1/3*a^2*x^3*
e*log(f) + 1/3*a^2*d*x^3
```

Mupad [B]
time = 3.99, size = 189, normalized size = 0.91

$$\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{2\,a\,b\,e\,x^3}{3}-\frac{2\,b^2\,e\,n\,x^3}{9}\right)+\frac{a^2\,e\,x^3}{3}+\frac{2\,b^2\,e\,n^2\,x^3}{27}+\frac{b^2\,e\,x^3\,{\ln\left(c\,x^n\right)}^2}{3}-\frac{2\,a\,b\,e\,n\,x^3}{9}\right)+x^3\,\left(\frac{a^2\,d}{3}+\frac{2\,b^2\,d\,n^2}{27}-\frac{a^2\,e\,r}{9}-\frac{2\,b^2\,e\,n^2\,r}{27}-\frac{2\,a\,b\,d\,n}{9}+\frac{4\,a\,b\,e\,n\,r}{27}\right)+\frac{b^2\,x^3\,{\ln\left(c\,x^n\right)}^2\,\left(3\,d-e\,r\right)}{9}+\frac{2\,b\,x^3\,\ln\left(c\,x^n\right)\,\left(9\,a\,d-3\,b\,d\,n-3\,a\,e\,r+2\,b\,e\,n\,r\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] log(f*x^r)*(log(c*x^n)*((2*a*b*e*x^3)/3 - (2*b^2*e*n*x^3)/9) + (a^2*e*x^3)/
3 + (2*b^2*e*n^2*x^3)/27 + (b^2*e*x^3*log(c*x^n)^2)/3 - (2*a*b*e*n*x^3)/9)
```

```
+ x^3*((a^2*d)/3 + (2*b^2*d*n^2)/27 - (a^2*e*r)/9 - (2*b^2*e*n^2*r)/27 - (2
*a*b*d*n)/9 + (4*a*b*e*n*r)/27) + (b^2*x^3*log(c*x^n)^2*(3*d - e*r))/9 + (2
*b*x^3*log(c*x^n)*(9*a*d - 3*b*d*n - 3*a*e*r + 2*b*e*n*r))/27
```

3.164 $\int x(a+b\log(cx^n))^2(d+e\log(fx^r))\,dx$

Optimal. Leaf size=206

$$-\frac{1}{8}b^2en^2rx^2+\frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2+\frac{1}{4}b^2enrx^2\log(cx^n)-\frac{1}{4}be(2a-bn)rx^2\log(cx^n)$$

```
[Out] -1/8*b^2*e*n^2*r*x^2+1/8*b*e*n*(-b*n+2*a)*r*x^2-1/8*e*(b^2*n^2-2*a*b*n+2*a^
2)*r*x^2+1/4*b^2*e*n*r*x^2*ln(c*x^n)-1/4*b*e*(-b*n+2*a)*r*x^2*ln(c*x^n)-1/4
*b^2*e*r*x^2*ln(c*x^n)^2+1/4*b^2*n^2*x^2*(d+e*ln(f*x^r))-1/2*b*n*x^2*(a+b*l
n(c*x^n))*(d+e*ln(f*x^r))+1/2*x^2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.12, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2342, 2341, 2413, 12, 14}

$$-\frac{1}{8}erx^2\left(2a^2-2abn+b^2n^2\right)+\frac{1}{2}x^2(a+b\log(cx^n))^2(d+e\log(fx^r))-\frac{1}{2}bnx^2(a+b\log(cx^n))(d+e\log(fx^r))-\frac{1}{4}berx^2(2a-bn)\log(cx^n)+\frac{1}{8}benrx^2(2a-bn)-\frac{1}{4}b^2erx^2\log^2(cx^n)+\frac{1}{4}b^2enrx^2\log(cx^n)+\frac{1}{4}b^2n^2x^2(d+e\log(fx^r))-\frac{1}{8}b^2en^2rx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] -1/8*(b^2*e*n^2*r*x^2) + (b*e*n*(2*a - b*n)*r*x^2)/8 - (e*(2*a^2 - 2*a*b*n
+ b^2*n^2)*r*x^2)/8 + (b^2*e*n*r*x^2*Log[c*x^n])/4 - (b*e*(2*a - b*n)*r*x^2
*Log[c*x^n])/4 - (b^2*e*r*x^2*Log[c*x^n]^2)/4 + (b^2*n^2*x^2*(d + e*Log[f*x
^r]))/4 - (b*n*x^2*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/2 + (x^2*(a + b*L
og[c*x^n])^2*(d + e*Log[f*x^r]))/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+b\log(cx^n))^2\,(d+e\log(fx^r))\;dx &= \frac{1}{4}b^2n^2x^2(d+e\log(fx^r))-\frac{1}{2}bnx^2(a+b\log(cx^n))\,(d+e\log(fx\\
&= \frac{1}{4}b^2n^2x^2(d+e\log(fx^r))-\frac{1}{2}bnx^2(a+b\log(cx^n))\,(d+e\log(fx\\
&= \frac{1}{4}b^2n^2x^2(d+e\log(fx^r))-\frac{1}{2}bnx^2(a+b\log(cx^n))\,(d+e\log(fx\\
&= -\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2+\frac{1}{4}b^2n^2x^2(d+e\log(fx^r))-\frac{1}{2}bnx\\
&= \frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2-\frac{1}{4}be(2a-bn)\\
&= -\frac{1}{8}b^2en^2rx^2+\frac{1}{8}ben(2a-bn)rx^2-\frac{1}{8}e\left(2a^2-2abn+b^2n^2\right)rx^2\end{aligned}$$

Mathematica [A]
time = 0.07, size = 154, normalized size = 0.75

$$\frac{1}{8}x^2\left(4a^2d-4abdn+2b^2dn^2-2a^2er+4abenr-3b^2en^2r+2e\left(2a^2-2abn+b^2n^2\right)\log(fx^r)+2b^2\log^2(cx^n)\,(2d-er+2e\log(fx^r))-4b\log(cx^n)\,(-2ad+bdn+aer-benr+(-2ae+ben)\log(fx^r))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] (x^2*(4*a^2*d - 4*a*b*d*n + 2*b^2*d*n^2 - 2*a^2*e*r + 4*a*b*e*n*r - 3*b^2*e
*n^2*r + 2*e*(2*a^2 - 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*(2
*d - e*r + 2*e*Log[f*x^r]) - 4*b*Log[c*x^n]*(-2*a*d + b*d*n + a*e*r - b*e*n
*r + (-2*a*e + b*e*n)*Log[f*x^r])))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 9262, normalized size = 44.96

method	result	size
risch	Expression too large to display	9262

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**A**]
time = 0.31, size = 253, normalized size = 1.23

$$\frac{1}{2}b^2dx^2\log(cx^n)^2-\frac{1}{2}abdnx^2-\frac{1}{4}a^2rx^2e+abdx^2\log(cx^n)-\frac{1}{4}\left(rx^2-2x^2\log(fx^r)\right)b^2e\log(cx^n)^2+\frac{1}{2}a^2x^2e\log(fx^r)+\frac{1}{2}a^2dx^2+\frac{1}{2}\left((r-\log(f))x^2-x^2\log(x^r)\right)abne-\frac{1}{2}\left(rx^2-2x^2\log(fx^r)\right)abe\log(cx^n)+\frac{1}{4}\left(n^2x^2-2nx^2\log(cx^n)\right)b^2d-\frac{1}{8}\left(\left((3r-2\log(f))x^2-2x^2\log(x^r)\right)n^2-4\left((r-\log(f))x^2-x^2\log(x^r)\right)n\log(cx^n)\right)b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*d*x^2*log(c*x^n)^2 - 1/2*a*b*d*n*x^2 - 1/4*a^2*r*x^2*e + a*b*d*x^2*
log(c*x^n) - 1/4*(r*x^2 - 2*x^2*log(f*x^r))*b^2*e*log(c*x^n)^2 + 1/2*a^2*x^
2*e*log(f*x^r) + 1/2*a^2*d*x^2 + 1/2*((r - log(f))*x^2 - x^2*log(x^r))*a*b*
n*e - 1/2*(r*x^2 - 2*x^2*log(f*x^r))*a*b*e*log(c*x^n) + 1/4*(n^2*x^2 - 2*n*
x^2*log(c*x^n))*b^2*d - 1/8*(((3*r - 2*log(f))*x^2 - 2*x^2*log(x^r))*n^2 -
4*((r - log(f))*x^2 - x^2*log(x^r))*n*log(c*x^n))*b^2*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(195) = 390$.
time = 0.35, size = 405, normalized size = 1.97

$$\frac{1}{2}b^2n^2rx^2e\log(x)^3-\frac{1}{8}\left(3b^2n^2-4abn+2a^2\right)rx^2e+\frac{1}{4}\left(b^2dn^2-2abdn+2a^2d\right)x^2-\frac{1}{4}\left(b^2rx^2e-2b^2dx^2\right)\log(c)^2+\frac{1}{4}\left(4b^2nrx^2e\log(c)+2b^2n^2x^2e\log(f)+2b^2dn^2x^2-\left(3b^2n^2-4abn\right)rx^2e\right)\log(x)^2+\frac{1}{2}\left(\left(b^2n-ab\right)rx^2e-\left(b^2dn-2abd\right)x^2\right)\log(c)+\frac{1}{4}\left(2b^2x^2e\log(c)^2-2\left(b^2n-2ab\right)x^2e\log(c)+\left(b^2n^2-2abn+2a^2\right)x^2e\right)\log(f)+\frac{1}{4}\left(2b^2rx^2e\log(c)^2+\left(3b^2n^2-4abn+2a^2\right)rx^2e-2\left(b^2dn^2-2abdn\right)x^2+4\left(b^2dnx^2-\left(b^2n-ab\right)rx^2e\right)\log(c)+2\left(2b^2nx^2e\log(c)-\left(b^2n^2-2abn\right)x^2e\right)\log(f)\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] 1/2*b^2*n^2*r*x^2*e*log(x)^3 - 1/8*(3*b^2*n^2 - 4*a*b*n + 2*a^2)*r*x^2*e +
1/4*(b^2*d*n^2 - 2*a*b*d*n + 2*a^2*d)*x^2 - 1/4*(b^2*r*x^2*e - 2*b^2*d*x^2)
*log(c)^2 + 1/4*(4*b^2*n*r*x^2*e*log(c) + 2*b^2*n^2*x^2*e*log(f) + 2*b^2*d*
n^2*x^2 - (3*b^2*n^2 - 4*a*b*n)*r*x^2*e)*log(x)^2 + 1/2*((b^2*n - a*b)*r*x^
2*e - (b^2*d*n - 2*a*b*d)*x^2)*log(c) + 1/4*(2*b^2*x^2*e*log(c)^2 - 2*(b^2*
n - 2*a*b)*x^2*e*log(c) + (b^2*n^2 - 2*a*b*n + 2*a^2)*x^2*e)*log(f) + 1/4*(
2*b^2*r*x^2*e*log(c)^2 + (3*b^2*n^2 - 4*a*b*n + 2*a^2)*r*x^2*e - 2*(b^2*d*n
^2 - 2*a*b*d*n)*x^2 + 4*(b^2*d*n*x^2 - (b^2*n - a*b)*r*x^2*e)*log(c) + 2*(2
*b^2*n*x^2*e*log(c) - (b^2*n^2 - 2*a*b*n)*x^2*e)*log(f))*log(x)
```

Sympy [**A**]
time = 2.45, size = 318, normalized size = 1.54

$$\frac{a^2dx^2}{2}-\frac{a^2erx^2}{4}+\frac{a^2ex^2\log(fx^r)}{2}-\frac{abdnx^2}{2}+abdx^2\log(cx^n)+\frac{abenrx^2}{2}-\frac{abenx^2\log(fx^r)}{2}-\frac{aberx^2\log(cx^n)}{2}+abex^2\log(cx^n)\log(fx^r)+\frac{b^2dn^2x^2}{4}-\frac{b^2dnx^2\log(cx^n)}{2}+\frac{b^2dx^2\log(cx^n)^2}{2}-\frac{3b^2en^2rx^2}{8}+\frac{b^2en^2x^2\log(fx^r)}{4}+\frac{b^2enrx^2\log(cx^n)}{2}-\frac{b^2enx^2\log(cx^n)\log(fx^r)}{2}-\frac{b^2erx^2\log(cx^n)^2}{4}+\frac{b^2ex^2\log(cx^n)^2\log(fx^r)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x**2/2 - a**2*e*r*x**2/4 + a**2*e*x**2*log(f*x**r)/2 - a*b*d*n*x**2/
2 + a*b*d*x**2*log(c*x**n) + a*b*e*n*r*x**2/2 - a*b*e*n*x**2*log(f*x**r)/2
- a*b*e*r*x**2*log(c*x**n)/2 + a*b*e*x**2*log(c*x**n)*log(f*x**r) + b**2*d*
n**2*x**2/4 - b**2*d*n*x**2*log(c*x**n)/2 + b**2*d*x**2*log(c*x**n)**2/2 -
3*b**2*e*n**2*r*x**2/8 + b**2*e*n**2*x**2*log(f*x**r)/4 + b**2*e*n*r*x**2*l
og(c*x**n)/2 - b**2*e*n*x**2*log(c*x**n)*log(f*x**r)/2 - b**2*e*r*x**2*log(
c*x**n)**2/4 + b**2*e*x**2*log(c*x**n)**2*log(f*x**r)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(195) = 390.
time = 5.34, size = 497, normalized size = 2.41

$$\frac{1}{2}b^2n^2rx^2e\log(x)^3-\frac{3}{4}b^2n^2rx^2e\log(x)^2+b^2nrx^2e\log(c)\log(x)^2+\frac{1}{2}b^2n^2x^2e\log(f)\log(x)^2+\frac{3}{4}b^2n^2rx^2e\log(x)-b^2nrx^2e\log(c)\log(x)+\frac{1}{2}b^2rx^2e\log(c)^2\log(x)-\frac{1}{2}b^2n^2x^2e\log(f)\log(x)+b^2nx^2e\log(c)\log(f)\log(x)+\frac{1}{2}b^2dn^2x^2\log(x)^2+abnrx^2e\log(x)^2-\frac{3}{8}b^2n^2rx^2e+\frac{1}{2}b^2nrx^2e\log(c)-\frac{1}{4}b^2rx^2e\log(c)^2+\frac{1}{4}b^2n^2x^2e\log(f)-\frac{1}{2}b^2nx^2e\log(c)\log(f)+\frac{1}{2}b^2x^2e\log(c)^2\log(f)-\frac{1}{2}b^2dn^2x^2\log(x)-abnrx^2e\log(x)+b^2dnx^2\log(c)\log(x)+abrx^2e\log(c)\log(x)+abnx^2e\log(f)\log(x)+\frac{1}{4}b^2dn^2x^2+\frac{1}{2}abnrx^2e-\frac{1}{2}b^2dnx^2\log(c)-\frac{1}{2}abrx^2e\log(c)+\frac{1}{2}b^2dx^2\log(c)^2-\frac{1}{2}abnx^2e\log(f)+abx^2e\log(c)\log(f)+abdnx^2\log(x)+\frac{1}{2}a^2rx^2e\log(x)-\frac{1}{2}abdnx^2-\frac{1}{4}a^2rx^2e+abdx^2\log(c)+\frac{1}{2}a^2x^2e\log(f)+\frac{1}{2}a^2dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] 1/2*b^2*n^2*r*x^2*e*log(x)^3 - 3/4*b^2*n^2*r*x^2*e*log(x)^2 + b^2*n*r*x^2*e
*log(c)*log(x)^2 + 1/2*b^2*n^2*x^2*e*log(f)*log(x)^2 + 3/4*b^2*n^2*r*x^2*e*
log(x) - b^2*n*r*x^2*e*log(c)*log(x) + 1/2*b^2*r*x^2*e*log(c)^2*log(x) - 1/
2*b^2*n^2*x^2*e*log(f)*log(x) + b^2*n*x^2*e*log(c)*log(f)*log(x) + 1/2*b^2*
d*n^2*x^2*log(x)^2 + a*b*n*r*x^2*e*log(x)^2 - 3/8*b^2*n^2*r*x^2*e + 1/2*b^2
*n*r*x^2*e*log(c) - 1/4*b^2*r*x^2*e*log(c)^2 + 1/4*b^2*n^2*x^2*e*log(f) - 1
/2*b^2*n*x^2*e*log(c)*log(f) + 1/2*b^2*x^2*e*log(c)^2*log(f) - 1/2*b^2*d*n^
2*x^2*log(x) - a*b*n*r*x^2*e*log(x) + b^2*d*n*x^2*log(c)*log(x) + a*b*r*x^2
*e*log(c)*log(x) + a*b*n*x^2*e*log(f)*log(x) + 1/4*b^2*d*n^2*x^2 + 1/2*a*b*
n*r*x^2*e - 1/2*b^2*d*n*x^2*log(c) - 1/2*a*b*r*x^2*e*log(c) + 1/2*b^2*d*x^2
*log(c)^2 - 1/2*a*b*n*x^2*e*log(f) + a*b*x^2*e*log(c)*log(f) + a*b*d*n*x^2*
log(x) + 1/2*a^2*r*x^2*e*log(x) - 1/2*a*b*d*n*x^2 - 1/4*a^2*r*x^2*e + a*b*d
*x^2*log(c) + 1/2*a^2*x^2*e*log(f) + 1/2*a^2*d*x^2
```

Mupad [B]
time = 4.15, size = 187, normalized size = 0.91

$$\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(a\,b\,e\,x^2-\frac{b^2\,e\,n\,x^2}{2}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,e\,n^2\,x^2}{4}+\frac{b^2\,e\,x^2\,{\ln\left(c\,x^n\right)}^2}{2}-\frac{a\,b\,e\,n\,x^2}{2}\right)+x^2\,\left(\frac{a^2\,d}{2}+\frac{b^2\,d\,n^2}{4}-\frac{a^2\,e\,r}{4}-\frac{3\,b^2\,e\,n^2\,r}{8}-\frac{a\,b\,d\,n}{2}+\frac{a\,b\,e\,n\,r}{2}\right)+\frac{b^2\,x^2\,{\ln\left(c\,x^n\right)}^2\,\left(2\,d-e\,r\right)}{4}+\frac{b\,x^2\,\ln\left(c\,x^n\right)\,\left(2\,a\,d-b\,d\,n-a\,e\,r+b\,e\,n\,r\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] log(f*x^r)*(log(c*x^n)*(a*b*e*x^2 - (b^2*e*n*x^2)/2) + (a^2*e*x^2)/2 + (b^2
*e*n^2*x^2)/4 + (b^2*e*x^2*log(c*x^n)^2)/2 - (a*b*e*n*x^2)/2) + x^2*((a^2*d
)/2 + (b^2*d*n^2)/4 - (a^2*e*r)/4 - (3*b^2*e*n^2*r)/8 - (a*b*d*n)/2 + (a*b*
e*n*r)/2) + (b^2*x^2*log(c*x^n)^2*(2*d - e*r))/4 + (b*x^2*log(c*x^n)*(2*a*d
 - b*d*n - a*e*r + b*e*n*r))/2
```

3.165 $\int (a+b\log(cx^n))^2 (d+e\log(fx^r))\,dx$

Optimal. Leaf size=147

$$2abenrx-4b^2en^2rx+2ben(a-bn)rx+4b^2enrx\log(cx^n)-erx(a+b\log(cx^n))^2-2abnx(d+e\log(fx^r))+2b^2n$$

```
[Out] 2*a*b*e*n*r*x-4*b^2*e*n^2*r*x+2*b*e*n*(-b*n+a)*r*x+4*b^2*e*n*r*x*ln(c*x^n)-
e*r*x*(a+b*ln(c*x^n))^2-2*a*b*n*x*(d+e*ln(f*x^r))+2*b^2*n^2*x*(d+e*ln(f*x^r
))-2*b^2*n*x*ln(c*x^n)*(d+e*ln(f*x^r))+x*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))
```

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2333, 2332, 2408}

$$x(a+b\log(cx^n))^2(d+e\log(fx^r))-erx(a+b\log(cx^n))^2-2abnx(d+e\log(fx^r))+2abenrx+2benrx(a-bn)-2b^2nx\log(cx^n)(d+e\log(fx^r))+4b^2enrx\log(cx^n)+2b^2n^2x(d+e\log(fx^r))-4b^2en^2rx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] 2*a*b*e*n*r*x - 4*b^2*e*n^2*r*x + 2*b*e*n*(a - b*n)*r*x + 4*b^2*e*n*r*x*Log
[c*x^n] - e*r*x*(a + b*Log[c*x^n])^2 - 2*a*b*n*x*(d + e*Log[f*x^r]) + 2*b^2
*n^2*x*(d + e*Log[f*x^r]) - 2*b^2*n*x*Log[c*x^n]*(d + e*Log[f*x^r]) + x*(a
+ b*Log[c*x^n])^2*(d + e*Log[f*x^r])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rubi steps

$$\int (a+b\log(cx^n))^2(d+e\log(fx^r))\,dx = -2abnx(d+e\log(fx^r))+2b^2n^2x(d+e\log(fx^r))-2b^2nx\log(c$$

$$= 2ben(a-bn)rx-2abnx(d+e\log(fx^r))+2b^2n^2x(d+e\log(fx^r$$

$$= -2b^2en^2rx+2ben(a-bn)rx+2b^2enrx\log(cx^n)-erx(a+b\log$$

$$= 2abenrx-2b^2en^2rx+2ben(a-bn)rx+2b^2enrx\log(cx^n)-erx$$

$$= 2abenrx-4b^2en^2rx+2ben(a-bn)rx+4b^2enrx\log(cx^n)-erx$$

Mathematica [A]
time = 0.05, size = 141, normalized size = 0.96

$$x(a^2d-2abdn+2b^2dn^2-a^2er+4abenr-6b^2en^2r+e(a^2-2abn+2b^2n^2)\log(fx^r)+b^2\log^2(cx^n)(d-er+e\log(fx^r))+2b\log(cx^n)(ad-bdn-aer+2benr+e(a-bn)\log(fx^r)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]),x]
```

```
[Out] x*(a^2*d - 2*a*b*d*n + 2*b^2*d*n^2 - a^2*e*r + 4*a*b*e*n*r - 6*b^2*e*n^2*r
+ e*(a^2 - 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d - e*r + e*
Log[f*x^r]) + 2*b*Log[c*x^n]*(a*d - b*d*n - a*e*r + 2*b*e*n*r + e*(a - b*n)
*Log[f*x^r]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 8701, normalized size = 59.19

method	result	size
risch	Expression too large to display	8701

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 219, normalized size = 1.49

$$b^2dx\log(cx^n)^2-(rx-x\log(fx^r))b^2e\log(cx^n)^2-2\,abdnx+2\,((2\,r-\log(f))x-x\log(x^r))abne-a^2rxe+2\,abdx\log(cx^n)-2\,(rx-x\log(fx^r))abe\log(cx^n)+a^2xe\log(fx^r)+2\,(n^2x-nx\log(cx^n))b^2d+a^2dx-2\,(((3\,r-\log(f))x-x\log(x^r))n^2-((2\,r-\log(f))x-x\log(x^r))n\log(cx^n))b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] b^2*d*x*log(c*x^n)^2 - (r*x - x*log(f*x^r))*b^2*e*log(c*x^n)^2 - 2*a*b*d*n*
x + 2*((2*r - log(f))*x - x*log(x^r))*a*b*n*e - a^2*r*x*e + 2*a*b*d*x*log(c
*x^n) - 2*(r*x - x*log(f*x^r))*a*b*e*log(c*x^n) + a^2*x*e*log(f*x^r) + 2*(n
^2*x - n*x*log(c*x^n))*b^2*d + a^2*d*x - 2*(((3*r - log(f))*x - x*log(x^r))
*n^2 - ((2*r - log(f))*x - x*log(x^r))*n*log(c*x^n))*b^2*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(157) = 314$.
time = 0.35, size = 350, normalized size = 2.38

$$b^2n^2rxe\log(x)^3 - (6\,b^2n^2 - 4\,abn + a^2)rxe - (b^2rxe - b^2dx)\log(c)^2 + (2\,b^2nrxe\log(c) + b^2n^2xe\log(f) + b^2dn^2x - (3\,b^2n^2 - 2\,abn)rxe)\log(x)^2 + (2\,b^2dn^2 - 2\,abdn + a^2d)x + 2\,((2\,b^2n - ab)rxe - (b^2dn - abd)x)\log(c) + (b^2xe\log(c)^2 - 2\,(b^2n - ab)xe\log(c) + (2\,b^2n^2 - 2\,abn + a^2)xe)\log(f) + (b^2rxe\log(c)^2 + (6\,b^2n^2 - 4\,abn + a^2)rxe - 2\,(b^2dn^2 - abdn)x + 2\,(b^2dnx - (2\,b^2n - ab)rxe)\log(c) + 2\,(b^2nxe\log(c) - (b^2n^2 - abn)xe)\log(f))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] b^2*n^2*r*x*e*log(x)^3 - (6*b^2*n^2 - 4*a*b*n + a^2)*r*x*e - (b^2*r*x*e - b
^2*d*x)*log(c)^2 + (2*b^2*n*r*x*e*log(c) + b^2*n^2*x*e*log(f) + b^2*d*n^2*x
 - (3*b^2*n^2 - 2*a*b*n)*r*x*e)*log(x)^2 + (2*b^2*d*n^2 - 2*a*b*d*n + a^2*d
)*x + 2*((2*b^2*n - a*b)*r*x*e - (b^2*d*n - a*b*d)*x)*log(c) + (b^2*x*e*log
(c)^2 - 2*(b^2*n - a*b)*x*e*log(c) + (2*b^2*n^2 - 2*a*b*n + a^2)*x*e)*log(f
) + (b^2*r*x*e*log(c)^2 + (6*b^2*n^2 - 4*a*b*n + a^2)*r*x*e - 2*(b^2*d*n^2
- a*b*d*n)*x + 2*(b^2*d*n*x - (2*b^2*n - a*b)*r*x*e)*log(c) + 2*(b^2*n*x*e*
log(c) - (b^2*n^2 - a*b*n)*x*e)*log(f))*log(x)
```

Sympy [**A**]
time = 1.10, size = 279, normalized size = 1.90

$$a^2dx - a^2erx + a^2ex\log(fx^r) - 2abdnx + 2abdx\log(cx^n) + 4abenrx - 2abenx\log(fx^r) - 2aberx\log(cx^n) + 2abex\log(cx^n)\log(fx^r) + 2b^2dn^2x - 2b^2dnx\log(cx^n) + b^2dx\log(cx^n)^2 - 6b^2en^2rx + 2b^2en^2x\log(fx^r) + 4b^2enrx\log(cx^n) - 2b^2enx\log(cx^n)\log(fx^r) - b^2erx\log(cx^n)^2 + b^2ex\log(cx^n)^2\log(fx^r)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r)),x)
```

```
[Out] a**2*d*x - a**2*e*r*x + a**2*e*x*log(f*x**r) - 2*a*b*d*n*x + 2*a*b*d*x*log(
c*x**n) + 4*a*b*e*n*r*x - 2*a*b*e*n*x*log(f*x**r) - 2*a*b*e*r*x*log(c*x**n)
 + 2*a*b*e*x*log(c*x**n)*log(f*x**r) + 2*b**2*d*n**2*x - 2*b**2*d*n*x*log(c
*x**n) + b**2*d*x*log(c*x**n)**2 - 6*b**2*e*n**2*r*x + 2*b**2*e*n**2*x*log(
f*x**r) + 4*b**2*e*n*r*x*log(c*x**n) - 2*b**2*e*n*x*log(c*x**n)*log(f*x**r)
 - b**2*e*r*x*log(c*x**n)**2 + b**2*e*x*log(c*x**n)**2*log(f*x**r)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(157) =$ 314.
time = 7.32, size = 425, normalized size = 2.89

$$b^2n^2rxe\log(x)^3 - 3\,b^2n^2rxe\log(x)^2 + 2\,b^2nrxe\log(c)\log(x)^2 + b^2n^2xe\log(f)\log(x)^2 + 6\,b^2n^2rxe\log(x) - 4\,b^2nrxe\log(c)\log(x) + b^2rxe\log(c)^2\log(x) - 2\,b^2n^2xe\log(f)\log(x) + 2\,b^2nxe\log(c)\log(f)\log(x) + b^2dn^2x\log(x)^2 + 2\,abnrxe\log(x)^2 - 6\,b^2n^2rxe + 4\,b^2nrxe\log(c) - b^2rxe\log(c)^2 + 2\,b^2n^2xe\log(f) - 2\,b^2nxe\log(c)\log(f) + b^2xe\log(c)^2\log(f) - 2\,b^2dn^2x\log(x) - 4\,abnrxe\log(x) + 2\,b^2dnx\log(c)\log(x) + 2\,abrxe\log(c)\log(x) + 2\,abnxe\log(f)\log(x) + 2\,b^2dn^2x + 4\,abnrxe - 2\,b^2dnx\log(c) - 2\,abrxe\log(c) + b^2dx\log(c)^2 - 2\,abnxe\log(f) + 2\,abxe\log(c)\log(f) + 2\,abdnx\log(x) + a^2rxe\log(x) - 2\,abdnx - a^2rxe + 2\,abdx\log(c) + a^2xe\log(f) + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] b^2*n^2*r*x*e*log(x)^3 - 3*b^2*n^2*r*x*e*log(x)^2 + 2*b^2*n*r*x*e*log(c)*lo
g(x)^2 + b^2*n^2*x*e*log(f)*log(x)^2 + 6*b^2*n^2*r*x*e*log(x) - 4*b^2*n*r*x
*e*log(c)*log(x) + b^2*r*x*e*log(c)^2*log(x) - 2*b^2*n^2*x*e*log(f)*log(x)
+ 2*b^2*n*x*e*log(c)*log(f)*log(x) + b^2*d*n^2*x*log(x)^2 + 2*a*b*n*r*x*e*l
og(x)^2 - 6*b^2*n^2*r*x*e + 4*b^2*n*r*x*e*log(c) - b^2*r*x*e*log(c)^2 + 2*b
^2*n^2*x*e*log(f) - 2*b^2*n*x*e*log(c)*log(f) + b^2*x*e*log(c)^2*log(f) - 2
*b^2*d*n^2*x*log(x) - 4*a*b*n*r*x*e*log(x) + 2*b^2*d*n*x*log(c)*log(x) + 2*
a*b*r*x*e*log(c)*log(x) + 2*a*b*n*x*e*log(f)*log(x) + 2*b^2*d*n^2*x + 4*a*b
*n*r*x*e - 2*b^2*d*n*x*log(c) - 2*a*b*r*x*e*log(c) + b^2*d*x*log(c)^2 - 2*a
*b*n*x*e*log(f) + 2*a*b*x*e*log(c)*log(f) + 2*a*b*d*n*x*log(x) + a^2*r*x*e*
log(x) - 2*a*b*d*n*x - a^2*r*x*e + 2*a*b*d*x*log(c) + a^2*x*e*log(f) + a^2*
d*x
```

Mupad [B]
time = 3.88, size = 165, normalized size = 1.12

$$x\left(a^2 d+2 b^2 d n^2-a^2 e r-6 b^2 e n^2 r-2 a b d n+4 a b e n r\right)+\ln\left(f x^r\right)\left(a^2 e x-\ln\left(c x^n\right)\left(2 b^2 e n x-2 a b e x\right)+2 b^2 e n^2 x+b^2 e x \ln\left(c x^n\right)^2-2 a b e n x\right)+2 b x \ln\left(c x^n\right)\left(a d-b d n-a e r+2 b e n r\right)+b^2 x \ln\left(c x^n\right)^2 (d-e r)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^2,x)
```

```
[Out] x*(a^2*d + 2*b^2*d*n^2 - a^2*e*r - 6*b^2*e*n^2*r - 2*a*b*d*n + 4*a*b*e*n*r)
 + log(f*x^r)*(a^2*e*x - log(c*x^n)*(2*b^2*e*n*x - 2*a*b*e*x) + 2*b^2*e*n^2
*x + b^2*e*x*log(c*x^n)^2 - 2*a*b*e*n*x) + 2*b*x*log(c*x^n)*(a*d - b*d*n -
a*e*r + 2*b*e*n*r) + b^2*x*log(c*x^n)^2*(d - e*r)
```

3.166 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=57

$$-\frac{er(a+b\log(cx^n))^4}{12b^2n^2}+\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}$$

```
[Out] -1/12*e*r*(a+b*ln(c*x^n))^4/b^2/n^2+1/3*(a+b*ln(c*x^n))^3*(d+e*ln(f*x^r))/b
/n
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2339, 30, 2413, 12}

$$\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{er(a+b\log(cx^n))^4}{12b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -1/12*(e*r*(a + b*Log[c*x^n])^4)/(b^2*n^2) + ((a + b*Log[c*x^n])^3*(d + e*L
og[f*x^r]))/(3*b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
```

```
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}\,dx &= \frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-(er)\int \frac{(a+b\log(cx^n))^3}{3bnx}\,dx\\ &= \frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{(er)\int \frac{(a+b\log(cx^n))^3}{x}\,dx}{3bn}\\ &= \frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}-\frac{(er)\text{Subst}\left(\int x^3\,dx,x,a+b\,\text{l}\right.}{3b^2n^2}\\ &= -\frac{er(a+b\log(cx^n))^4}{12b^2n^2}+\frac{(a+b\log(cx^n))^3(d+e\log(fx^r))}{3bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(57) = 114.
time = 0.07, size = 129, normalized size = 2.26

$$\frac{1}{12}\log(x)\left(-3b^2en^2r\log^3(x)+12(a+b\log(cx^n))^2(d+e\log(fx^r))+4bn\log^2(x)(bdn+2aer+2ber\log(cx^n)+ben\log(fx^r))-6\log(x)(a+b\log(cx^n))(2bdn+aer+ber\log(cx^n)+2ben\log(fx^r))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x,x]
[Out] (Log[x]*(-3*b^2*e*n^2*r*Log[x]^3 + 12*(a + b*Log[c*x^n])^2*(d + e*Log[f*x^r
]) + 4*b*n*Log[x]^2*(b*d*n + 2*a*e*r + 2*b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r
]) - 6*Log[x]*(a + b*Log[c*x^n])*(2*b*d*n + a*e*r + b*e*r*Log[c*x^n] + 2*b*
e*n*Log[f*x^r])))/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 9164, normalized size = 160.77

method	result	size
risch	Expression too large to display	9164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(55) = 110.
time = 0.29, size = 168, normalized size = 2.95

$$\frac{b^2e\log(cx^n)^2\log(fx^r)^2}{2\,r}+\frac{b^2d\log(cx^n)^3}{3\,n}+\frac{abe\log(cx^n)\log(fx^r)^2}{r}-\frac{abne\log(fx^r)^3}{3\,r^2}-\frac{1}{12}\left(\frac{4\,n\log(cx^n)\log(fx^r)^3}{r^2}-\frac{n^2\log(fx^r)^4}{r^3}\right)b^2e+\frac{abd\log(cx^n)^2}{n}+\frac{a^2e\log(fx^r)^2}{2\,r}+a^2d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*log(c*x^n)^2*log(f*x^r)^2/r + 1/3*b^2*d*log(c*x^n)^3/n + a*b*e*lo
g(c*x^n)*log(f*x^r)^2/r - 1/3*a*b*n*e*log(f*x^r)^3/r^2 - 1/12*(4*n*log(c*x^
n)*log(f*x^r)^3/r^2 - n^2*log(f*x^r)^4/r^3)*b^2*e + a*b*d*log(c*x^n)^2/n +
1/2*a^2*e*log(f*x^r)^2/r + a^2*d*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(55) = 110$.
time = 0.35, size = 182, normalized size = 3.19

$$\frac{1}{4}\,b^2n^2re\log\left(x\right)^4+\frac{1}{3}\left(2\,b^2nre\log\left(c\right)+b^2n^2e\log\left(f\right)+b^2dn^2+2\,abnre\right)\log\left(x\right)^3+\frac{1}{2}\left(b^2re\log\left(c\right)^2+2\,abdn+a^2re+2\left(b^2dn+abre\right)\log\left(c\right)+2\left(b^2ne\log\left(c\right)+abne\right)\log\left(f\right)\right)\log\left(x\right)^2+\left(b^2d\log\left(c\right)^2+2\,abd\log\left(c\right)+a^2d+\left(b^2e\log\left(c\right)^2+2\,abe\log\left(c\right)+a^2e\right)\log\left(f\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*n^2*r*e*log(x)^4 + 1/3*(2*b^2*n*r*e*log(c) + b^2*n^2*e*log(f) + b^2
*d*n^2 + 2*a*b*n*r*e)*log(x)^3 + 1/2*(b^2*r*e*log(c)^2 + 2*a*b*d*n + a^2*r*
e + 2*(b^2*d*n + a*b*r*e)*log(c) + 2*(b^2*n*e*log(c) + a*b*n*e)*log(f))*log
(x)^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d + (b^2*e*log(c)^2 + 2*a*b*
e*log(c) + a^2*e)*log(f))*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^2\left(d+e\log\left(fx^r\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*(d + e*log(f*x**r))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(55) = 110$.
time = 4.62, size = 223, normalized size = 3.91

$$\frac{1}{4}\,b^2n^2re\log\left(x\right)^4+\frac{2}{3}\,b^2nre\log\left(c\right)\log\left(x\right)^3+\frac{1}{3}\,b^2n^2e\log\left(f\right)\log\left(x\right)^3+\frac{1}{2}\,b^2re\log\left(c\right)^2\log\left(x\right)^2+b^2ne\log\left(c\right)\log\left(f\right)\log\left(x\right)^2+\frac{1}{3}\,b^2dn^2\log\left(x\right)^3+\frac{2}{3}\,abnre\log\left(x\right)^3+b^2e\log\left(c\right)^2\log\left(f\right)\log\left(x\right)+b^2dn\log\left(c\right)\log\left(x\right)^2+abre\log\left(c\right)\log\left(x\right)^2+abne\log\left(f\right)\log\left(x\right)^2+b^2d\log\left(c\right)^2\log\left(x\right)+2\,abe\log\left(c\right)\log\left(f\right)\log\left(x\right)+abdn\log\left(x\right)^2+\frac{1}{2}\,a^2re\log\left(x\right)^2+2\,abd\log\left(c\right)\log\left(x\right)+a^2e\log\left(f\right)\log\left(x\right)+a^2d\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*n^2*r*e*log(x)^4 + 2/3*b^2*n*r*e*log(c)*log(x)^3 + 1/3*b^2*n^2*e*lo
g(f)*log(x)^3 + 1/2*b^2*r*e*log(c)^2*log(x)^2 + b^2*n*e*log(c)*log(f)*log(x
)^2 + 1/3*b^2*d*n^2*log(x)^3 + 2/3*a*b*n*r*e*log(x)^3 + b^2*e*log(c)^2*log(
```

```
f)*log(x) + b^2*d*n*log(c)*log(x)^2 + a*b*r*e*log(c)*log(x)^2 + a*b*n*e*log
(f)*log(x)^2 + b^2*d*log(c)^2*log(x) + 2*a*b*e*log(c)*log(f)*log(x) + a*b*d
*n*log(x)^2 + 1/2*a^2*r*e*log(x)^2 + 2*a*b*d*log(c)*log(x) + a^2*e*log(f)*l
og(x) + a^2*d*log(x)
```

Mupad [B]
time = 3.99, size = 124, normalized size = 2.18

$$\ln(f\,x^r)\,\left(\frac{b^2\,e\ln(c\,x^n)^3}{3\,n}+\frac{a\,b\,e\ln(c\,x^n)^2}{n}\right)+\frac{\ln(c\,x^n)^3\,(b^2\,d\,n-a\,b\,e\,r)}{3\,n^2}+a^2\,d\,\ln(x)+\frac{a^2\,e\ln(f\,x^r)^2}{2\,r}+\frac{a\,b\,d\ln(c\,x^n)^2}{n}-\frac{b^2\,e\,r\ln(c\,x^n)^4}{12\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] log(f*x^r)*((b^2*e*log(c*x^n)^3)/(3*n) + (a*b*e*log(c*x^n)^2)/n) + (log(c*x
^n)^3*(b^2*d*n - a*b*e*r))/(3*n^2) + a^2*d*log(x) + (a^2*e*log(f*x^r)^2)/(2
*r) + (a*b*d*log(c*x^n)^2)/n - (b^2*e*r*log(c*x^n)^4)/(12*n^2)
```

3.167 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=181

$$-\frac{2b^2en^2r}{x}-\frac{2ben(a+bn)r}{x}-\frac{e(a^2+2abn+2b^2n^2)\,r}{x}-\frac{2b^2enr\log(cx^n)}{x}-\frac{2be(a+bn)r\log(cx^n)}{x}-\frac{b^2er\log^2(c}{x}$$

```
[Out] -2*b^2*e*n^2*r/x-2*b*e*n*(b*n+a)*r/x-e*(2*b^2*n^2+2*a*b*n+a^2)*r/x-2*b^2*e*
n*r*ln(c*x^n)/x-2*b*e*(b*n+a)*r*ln(c*x^n)/x-b^2*e*r*ln(c*x^n)^2/x-2*b^2*n^2
*(d+e*ln(f*x^r))/x-2*b*n*(a+b*ln(c*x^n))*(d+e*ln(f*x^r))/x-(a+b*ln(c*x^n))^
2*(d+e*ln(f*x^r))/x
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2342, 2341, 2413, 14}

$$-\frac{er(a^2+2abn+2b^2n^2)}{x}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x}-\frac{2ber(a+bn)\log(cx^n)}{x}-\frac{2benr(a+bn)}{x}-\frac{b^2er\log^2(cx^n)}{x}-\frac{2b^2enr\log(cx^n)}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x}-\frac{2b^2en^2r}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] (-2*b^2*e*n^2*r)/x - (2*b*e*n*(a + b*n)*r)/x - (e*(a^2 + 2*a*b*n + 2*b^2*n^
2)*r)/x - (2*b^2*e*n*r*Log[c*x^n])/x - (2*b*e*(a + b*n)*r*Log[c*x^n])/x - (
b^2*e*r*Log[c*x^n]^2)/x - (2*b^2*n^2*(d + e*Log[f*x^r]))/x - (2*b*n*(a + b*
Log[c*x^n])*(d + e*Log[f*x^r]))/x - ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]
))/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^2}\,dx &= -\frac{2b^2n^2(d+e\log(fx^r))}{x}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x} \\ &= -\frac{2b^2n^2(d+e\log(fx^r))}{x}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{x} \\ &= -\frac{e(a^2+2abn+2b^2n^2)r}{x}-\frac{2b^2n^2(d+e\log(fx^r))}{x}-\frac{2bn(a+b\lo \\ &= -\frac{2ben(a+bn)r}{x}-\frac{e(a^2+2abn+2b^2n^2)r}{x}-\frac{2be(a+bn)r\log(cx \\ &= -\frac{2b^2en^2r}{x}-\frac{2ben(a+bn)r}{x}-\frac{e(a^2+2abn+2b^2n^2)r}{x}-\frac{2b^2enr\,lo}{x}\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 138, normalized size = 0.76

$$-\frac{a^2d+2abdn+2b^2dn^2+a^2er+4abenr+6b^2en^2r+e(a^2+2abn+2b^2n^2)\log(fx^r)+b^2\log^2(cx^n)(d+er+e\log(fx^r))+2b\log(cx^n)(a(d+er)+bn(d+2er)+e(a+bn)\log(fx^r))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((a^2*d + 2*a*b*d*n + 2*b^2*d*n^2 + a^2*e*r + 4*a*b*e*n*r + 6*b^2*e*n^2*r
+ e*(a^2 + 2*a*b*n + 2*b^2*n^2)*Log[f*x^r] + b^2*Log[c*x^n]^2*(d + e*r + e*
Log[f*x^r]) + 2*b*Log[c*x^n]*(a*(d + e*r) + b*n*(d + 2*e*r) + e*(a + b*n)*L
og[f*x^r]))/x)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.47, size = 8407, normalized size = 46.45

method	result	size
risch	Expression too large to display	8407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 227, normalized size = 1.25

$$-b^2\left(\frac{r}{x}+\frac{\log(fx^r)}{x}\right)e\log(cx^n)^2-2\,ab\left(\frac{r}{x}+\frac{\log(fx^r)}{x}\right)e\log(cx^n)-2\,b^2d\left(\frac{n^2}{x}+\frac{n\log(cx^n)}{x}\right)-2\left(\frac{(r\log(x)+3\,r+\log(f))n^2}{x}+\frac{n(2\,r+\log(f)+\log(x^r))\log(cx^n)}{x}\right)b^2e-\frac{2\,abn(2\,r+\log(f)+\log(x^r))e}{x}-\frac{b^2d\log(cx^n)^2}{x}-\frac{2\,abdn}{x}-\frac{a^2re}{x}-\frac{2\,abd\log(cx^n)}{x}-\frac{a^2e\log(fx^r)}{x}-\frac{a^2d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*(r/x + log(f*x^r)/x)*e*log(c*x^n)^2 - 2*a*b*(r/x + log(f*x^r)/x)*e*log
(c*x^n) - 2*b^2*d*(n^2/x + n*log(c*x^n)/x) - 2*((r*log(x) + 3*r + log(f))*n
^2/x + n*(2*r + log(f) + log(x^r))*log(c*x^n)/x)*b^2*e - 2*a*b*n*(2*r + log
(f) + log(x^r))*e/x - b^2*d*log(c*x^n)^2/x - 2*a*b*d*n/x - a^2*r*e/x - 2*a*
b*d*log(c*x^n)/x - a^2*e*log(f*x^r)/x - a^2*d/x
```

Fricas [A]
time = 0.36, size = 316, normalized size = 1.75

$$-\frac{b^2n^2re\log(x)^3+2\,b^2dn^2+2\,abdn+a^2d+(6\,b^2n^2+4\,abn+a^2)re+(b^2re+b^2d)\log(c)^2+(2\,b^2nre\log(c)+b^2n^2e\log(f)+b^2dn^2+(3\,b^2n^2+2\,abn)re)\log(x)^2+2\,(b^2dn+abd+(2\,b^2n+ab)re)\log(c)+(b^2e\log(c)^2+2\,(b^2n+ab)e\log(c)+(2\,b^2n^2+2\,abn+a^2)e)\log(f)+(b^2re\log(c)^2+2\,b^2dn^2+2\,abdn+(6\,b^2n^2+4\,abn+a^2)re+2\,(b^2dn+(2\,b^2n+ab)re)\log(c)+2\,(b^2ne\log(c)+(b^2n^2+abn)e)\log(f))\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*n^2*r*e*log(x)^3 + 2*b^2*d*n^2 + 2*a*b*d*n + a^2*d + (6*b^2*n^2 + 4*a
*b*n + a^2)*r*e + (b^2*r*e + b^2*d)*log(c)^2 + (2*b^2*n*r*e*log(c) + b^2*n^
2*e*log(f) + b^2*d*n^2 + (3*b^2*n^2 + 2*a*b*n)*r*e)*log(x)^2 + 2*(b^2*d*n +
 a*b*d + (2*b^2*n + a*b)*r*e)*log(c) + (b^2*e*log(c)^2 + 2*(b^2*n + a*b)*e*
log(c) + (2*b^2*n^2 + 2*a*b*n + a^2)*e)*log(f) + (b^2*r*e*log(c)^2 + 2*b^2*
d*n^2 + 2*a*b*d*n + (6*b^2*n^2 + 4*a*b*n + a^2)*r*e + 2*(b^2*d*n + (2*b^2*n
 + a*b)*r*e)*log(c) + 2*(b^2*n*e*log(c) + (b^2*n^2 + a*b*n)*e)*log(f))*log(
x))/x
```

Sympy [A]
time = 1.21, size = 280, normalized size = 1.55

$$-\frac{a^2d}{x}-\frac{a^2er}{x}-\frac{a^2e\log(fx^r)}{x}-\frac{2abdn}{x}-\frac{2abd\log(cx^n)}{x}-\frac{4abenr}{x}-\frac{2aben\log(fx^r)}{x}-\frac{2aber\log(cx^n)}{x}-\frac{2abe\log(cx^n)\log(fx^r)}{x}-\frac{2b^2dn^2}{x}-\frac{2b^2dn\log(cx^n)}{x}-\frac{b^2d\log(cx^n)^2}{x}-\frac{6b^2en^2r}{x}-\frac{2b^2en^2\log(fx^r)}{x}-\frac{4b^2enr\log(cx^n)}{x}-\frac{2b^2en\log(cx^n)\log(fx^r)}{x}-\frac{b^2er\log(cx^n)^2}{x}-\frac{b^2e\log(cx^n)^2\log(fx^r)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**2,x)
```

```
[Out] -a**2*d/x - a**2*e*r/x - a**2*e*log(f*x**r)/x - 2*a*b*d*n/x - 2*a*b*d*log(c
*x**n)/x - 4*a*b*e*n*r/x - 2*a*b*e*n*log(f*x**r)/x - 2*a*b*e*r*log(c*x**n)/
x - 2*a*b*e*log(c*x**n)*log(f*x**r)/x - 2*b**2*d*n**2/x - 2*b**2*d*n*log(c*
x**n)/x - b**2*d*log(c*x**n)**2/x - 6*b**2*e*n**2*r/x - 2*b**2*e*n**2*log(f
```

```
*x**r)/x - 4*b**2*e*n*r*log(c*x**n)/x - 2*b**2*e*n*log(c*x**n)*log(f*x**r)/
x - b**2*e*r*log(c*x**n)**2/x - b**2*e*log(c*x**n)**2*log(f*x**r)/x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(190) = 380.
time = 6.80, size = 392, normalized size = 2.17

$$-\frac{b^2n^2re\log(x)^3 + 3b^2n^2re\log(x)^2 + 2b^2nre\log(c)\log(x)^2 + b^2n^2e\log(f)\log(x)^2 + 6b^2n^2re\log(x) + 4b^2nre\log(c)\log(x) + b^2re\log(c)^2\log(x) + 2b^2n^2e\log(f)\log(x) + 2b^2ne\log(c)\log(f)\log(x) + b^2dn^2\log(x)^2 + 2abnre\log(x)^2 + 6b^2n^2re + 4b^2nre\log(c) + b^2re\log(c)^2 + 2b^2n^2e\log(f) + 2b^2ne\log(c)\log(f) + b^2e\log(c)^2\log(f) + 2b^2dn^2\log(x) + 4abnre\log(x) + 2b^2dn\log(c)\log(x) + 2abre\log(c)\log(x) + 2abne\log(f)\log(x) + 2b^2dn^2 + 4abnre + 2b^2dn\log(c) + 2abre\log(c) + b^2d\log(c)^2 + 2abne\log(f) + 2abe\log(c)\log(f) + 2abdn\log(x) + a^2re\log(x) + 2abdn + a^2re + 2abd\log(c) + a^2e\log(f) + a^2d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

```
[Out] -(b^2*n^2*r*e*log(x)^3 + 3*b^2*n^2*r*e*log(x)^2 + 2*b^2*n*r*e*log(c)*log(x)
^2 + b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 4*b^2*n*r*e*log(c)*
log(x) + b^2*r*e*log(c)^2*log(x) + 2*b^2*n^2*e*log(f)*log(x) + 2*b^2*n*e*lo
g(c)*log(f)*log(x) + b^2*d*n^2*log(x)^2 + 2*a*b*n*r*e*log(x)^2 + 6*b^2*n^2*
r*e + 4*b^2*n*r*e*log(c) + b^2*r*e*log(c)^2 + 2*b^2*n^2*e*log(f) + 2*b^2*n*
e*log(c)*log(f) + b^2*e*log(c)^2*log(f) + 2*b^2*d*n^2*log(x) + 4*a*b*n*r*e*
log(x) + 2*b^2*d*n*log(c)*log(x) + 2*a*b*r*e*log(c)*log(x) + 2*a*b*n*e*log(
f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 2*b^2*d*n*log(c) + 2*a*b*r*e*log(c)
 + b^2*d*log(c)^2 + 2*a*b*n*e*log(f) + 2*a*b*e*log(c)*log(f) + 2*a*b*d*n*lo
g(x) + a^2*r*e*log(x) + 2*a*b*d*n + a^2*r*e + 2*a*b*d*log(c) + a^2*e*log(f)
 + a^2*d)/x
```

Mupad [B]
time = 4.01, size = 181, normalized size = 1.00

$$-\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{2\,a\,b\,e}{x}+\frac{2\,b^2\,e\,n}{x}\right)+\frac{a^2\,e}{x}+\frac{2\,b^2\,e\,n^2}{x}+\frac{b^2\,e\,{\ln\left(c\,x^n\right)}^2}{x}+\frac{2\,a\,b\,e\,n}{x}\right)-\frac{a^2\,d+2\,b^2\,d\,n^2+a^2\,e\,r+6\,b^2\,e\,n^2\,r+2\,a\,b\,d\,n+4\,a\,b\,e\,n\,r}{x}-\frac{2\,b\,\ln\left(c\,x^n\right)\,\left(a\,d+b\,d\,n+a\,e\,r+2\,b\,e\,n\,r\right)}{x}-\frac{b^2\,{\ln\left(c\,x^n\right)}^2\,\left(d+e\,r\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^2,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((2*a*b*e)/x + (2*b^2*e*n)/x) + (a^2*e)/x + (2*b^2
*e*n^2)/x + (b^2*e*log(c*x^n)^2)/x + (2*a*b*e*n)/x) - (a^2*d + 2*b^2*d*n^2
+ a^2*e*r + 6*b^2*e*n^2*r + 2*a*b*d*n + 4*a*b*e*n*r)/x - (2*b*log(c*x^n)*(a
*d + b*d*n + a*e*r + 2*b*e*n*r))/x - (b^2*log(c*x^n)^2*(d + e*r))/x
```

3.168 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=204

$$-\frac{b^2en^2r}{8x^2}-\frac{ben(2a+bn)r}{8x^2}-\frac{e(2a^2+2abn+b^2n^2)\,r}{8x^2}-\frac{b^2enr\log\left(cx^n\right)}{4x^2}-\frac{be(2a+bn)r\log\left(cx^n\right)}{4x^2}-\frac{b^2er\log^2\left(cx^n\right.}{4x^2}$$

```
[Out] -1/8*b^2*e*n^2*r/x^2-1/8*b*e*n*(b*n+2*a)*r/x^2-1/8*e*(b^2*n^2+2*a*b*n+2*a^2
)*r/x^2-1/4*b^2*e*n*r*ln(c*x^n)/x^2-1/4*b*e*(b*n+2*a)*r*ln(c*x^n)/x^2-1/4*b
^2*e*r*ln(c*x^n)^2/x^2-1/4*b^2*n^2*(d+e*ln(f*x^r))/x^2-1/2*b*n*(a+b*ln(c*x^
n))*(d+e*ln(f*x^r))/x^2-1/2*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2342, 2341, 2413, 12, 14}

$$-\frac{er(2a^2+2abn+b^2n^2)}{8x^2}-\frac{bn(a+b\log\left(cx^n\right))\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{(a+b\log\left(cx^n\right))^2\left(d+e\log\left(fx^r\right)\right)}{2x^2}-\frac{ber(2a+bn)\log\left(cx^n\right)}{4x^2}-\frac{benr(2a+bn)}{8x^2}-\frac{b^2er\log^2\left(cx^n\right)}{4x^2}-\frac{b^2enr\log\left(cx^n\right)}{4x^2}-\frac{b^2n^2(d+e\log\left(fx^r\right))}{4x^2}-\frac{b^2en^2r}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/8*(b^2*e*n^2*r)/x^2 - (b*e*n*(2*a + b*n)*r)/(8*x^2) - (e*(2*a^2 + 2*a*b*
n + b^2*n^2)*r)/(8*x^2) - (b^2*e*n*r*Log[c*x^n])/(4*x^2) - (b*e*(2*a + b*n)
*r*Log[c*x^n])/(4*x^2) - (b^2*e*r*Log[c*x^n]^2)/(4*x^2) - (b^2*n^2*(d + e*L
og[f*x^r]))/(4*x^2) - (b*n*(a + b*Log[c*x^n])*(d + e*Log[f*x^r]))/(2*x^2) -
 ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^3}\,dx &= -\frac{b^2n^2(d+e\log(fx^r))}{4x^2}-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\ldots\\
&= -\frac{b^2n^2(d+e\log(fx^r))}{4x^2}-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\ldots\\
&= -\frac{b^2n^2(d+e\log(fx^r))}{4x^2}-\frac{bn(a+b\log(cx^n))(d+e\log(fx^r))}{2x^2}-\ldots\\
&= -\frac{e(2a^2+2abn+b^2n^2)r}{8x^2}-\frac{b^2n^2(d+e\log(fx^r))}{4x^2}-\frac{bn(a+b\log(\ldots}{\ldots}\\
&= -\frac{ben(2a+bn)r}{8x^2}-\frac{e(2a^2+2abn+b^2n^2)r}{8x^2}-\frac{be(2a+bn)r\log(c\ldots}{4x^2}\\
&= -\frac{b^2en^2r}{8x^2}-\frac{ben(2a+bn)r}{8x^2}-\frac{e(2a^2+2abn+b^2n^2)r}{8x^2}-\frac{b^2enr\log\ldots}{4x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 151, normalized size = 0.74

$$-\frac{4a^2d+4abdn+2b^2dn^2+2a^2er+4abenr+3b^2en^2r+2e(2a^2+2abn+b^2n^2)\log(fx^r)+2b^2\log^2(cx^n)(2d+er+2e\log(fx^r))+4b\log(cx^n)(2ad+bdn+aer+benr+e(2a+bn)\log(fx^r))}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -1/8*(4*a^2*d + 4*a*b*d*n + 2*b^2*d*n^2 + 2*a^2*e*r + 4*a*b*e*n*r + 3*b^2*e
*n^2*r + 2*e*(2*a^2 + 2*a*b*n + b^2*n^2)*Log[f*x^r] + 2*b^2*Log[c*x^n]^2*(2
*d + e*r + 2*e*Log[f*x^r]) + 4*b*Log[c*x^n]*(2*a*d + b*d*n + a*e*r + b*e*n*
r + e*(2*a + b*n)*Log[f*x^r]))/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 8407, normalized size = 41.21

method	result	size
risch	Expression too large to display	8407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 230, normalized size = 1.13

$$-\frac{1}{4}\,b^{2}\left(\frac{r}{x^{2}}+\frac{2\,\log\left(fx^{r}\right)}{x^{2}}\right)e\log\left(cx^{n}\right)^{2}-\frac{1}{2}\,ab\left(\frac{r}{x^{2}}+\frac{2\,\log\left(fx^{r}\right)}{x^{2}}\right)e\log\left(cx^{n}\right)-\frac{1}{4}\,b^{2}d\left(\frac{n^{2}}{x^{2}}+\frac{2\,n\log\left(cx^{n}\right)}{x^{2}}\right)-\frac{1}{8}\,b^{2}\left(\frac{(2\,r\log\left(x\right)+3\,r+2\,\log\left(f\right))n^{2}}{x^{2}}+\frac{4\,n(r+\log\left(f\right)+\log\left(x^{r}\right))\log\left(cx^{n}\right)}{x^{2}}\right)e-\frac{abn(r+\log\left(f\right)+\log\left(x^{r}\right))e}{2\,x^{2}}-\frac{b^{2}d\log\left(cx^{n}\right)^{2}}{2\,x^{2}}-\frac{abdn}{2\,x^{2}}-\frac{a^{2}re}{4\,x^{2}}-\frac{abd\log\left(cx^{n}\right)}{x^{2}}-\frac{a^{2}e\log\left(fx^{r}\right)}{2\,x^{2}}-\frac{a^{2}d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(r/x^2 + 2*log(f*x^r)/x^2)*e*log(c*x^n)^2 - 1/2*a*b*(r/x^2 + 2*log
(f*x^r)/x^2)*e*log(c*x^n) - 1/4*b^2*d*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 1/8*
b^2*((2*r*log(x) + 3*r + 2*log(f))*n^2/x^2 + 4*n*(r + log(f) + log(x^r))*lo
g(c*x^n)/x^2)*e - 1/2*a*b*n*(r + log(f) + log(x^r))*e/x^2 - 1/2*b^2*d*log(c
*x^n)^2/x^2 - 1/2*a*b*d*n/x^2 - 1/4*a^2*r*e/x^2 - a*b*d*log(c*x^n)/x^2 - 1/
2*a^2*e*log(f*x^r)/x^2 - 1/2*a^2*d/x^2
```

Fricas [A]
time = 0.35, size = 334, normalized size = 1.64

$$-\frac{4\,b^{2}n^{2}re\log\left(x\right)^{3}+2\,b^{2}dn^{2}+4\,abdn+4\,a^{2}d+(3\,b^{2}n^{2}+4\,abn+2\,a^{2})re+2\,(b^{2}re+2\,b^{2}d)\log\left(c\right)^{2}+2\,(4\,b^{2}nre\log\left(c\right)+2\,b^{2}n^{2}e\log\left(f\right)+2\,b^{2}dn^{2}+(3\,b^{2}n^{2}+4\,abn)re)\log\left(x\right)^{2}+4\,(b^{2}dn+2\,abd+(b^{2}n+ab)re)\log\left(c\right)+2\,(2\,b^{2}e\log\left(c\right)^{2}+2\,(b^{2}n+2\,ab)e\log\left(c\right)+(b^{2}n^{2}+2\,abn+2\,a^{2})e)\log\left(f\right)+2\,(2\,b^{2}re\log\left(c\right)^{2}+2\,b^{2}dn^{2}+4\,abdn+(3\,b^{2}n^{2}+4\,abn+2\,a^{2})re+4\,(b^{2}dn+(b^{2}n+ab)re)\log\left(c\right)+2\,(2\,b^{2}ne\log\left(c\right)+(b^{2}n^{2}+2\,abn)e)\log\left(f\right))\log\left(x\right)}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^2*n^2*r*e*log(x)^3 + 2*b^2*d*n^2 + 4*a*b*d*n + 4*a^2*d + (3*b^2*n
^2 + 4*a*b*n + 2*a^2)*r*e + 2*(b^2*r*e + 2*b^2*d)*log(c)^2 + 2*(4*b^2*n*r*e
*log(c) + 2*b^2*n^2*e*log(f) + 2*b^2*d*n^2 + (3*b^2*n^2 + 4*a*b*n)*r*e)*log
(x)^2 + 4*(b^2*d*n + 2*a*b*d + (b^2*n + a*b)*r*e)*log(c) + 2*(2*b^2*e*log(c
)^2 + 2*(b^2*n + 2*a*b)*e*log(c) + (b^2*n^2 + 2*a*b*n + 2*a^2)*e)*log(f) +
2*(2*b^2*r*e*log(c)^2 + 2*b^2*d*n^2 + 4*a*b*d*n + (3*b^2*n^2 + 4*a*b*n + 2*
a^2)*r*e + 4*(b^2*d*n + (b^2*n + a*b)*r*e)*log(c) + 2*(2*b^2*n*e*log(c) + (
b^2*n^2 + 2*a*b*n)*e)*log(f))*log(x))/x^2
```

Sympy [A]
time = 1.25, size = 320, normalized size = 1.57

$$-\frac{a^{2}d}{2x^{2}}-\frac{a^{2}er}{4x^{2}}-\frac{a^{2}e\log\left(fx^{r}\right)}{2x^{2}}-\frac{abdn}{2x^{2}}-\frac{abd\log\left(cx^{n}\right)}{x^{2}}-\frac{abenr}{2x^{2}}-\frac{aben\log\left(fx^{r}\right)}{2x^{2}}-\frac{aber\log\left(cx^{n}\right)}{2x^{2}}-\frac{abe\log\left(cx^{n}\right)\log\left(fx^{r}\right)}{x^{2}}-\frac{b^{2}dn^{2}}{4x^{2}}-\frac{b^{2}dn\log\left(cx^{n}\right)}{2x^{2}}-\frac{b^{2}d\log\left(cx^{n}\right)^{2}}{2x^{2}}-\frac{3b^{2}en^{2}r}{8x^{2}}-\frac{b^{2}en^{2}\log\left(fx^{r}\right)}{4x^{2}}-\frac{b^{2}enr\log\left(cx^{n}\right)}{2x^{2}}-\frac{b^{2}en\log\left(cx^{n}\right)\log\left(fx^{r}\right)}{2x^{2}}-\frac{b^{2}er\log\left(cx^{n}\right)^{2}}{4x^{2}}-\frac{b^{2}e\log\left(cx^{n}\right)^{2}\log\left(fx^{r}\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] -a**2*d/(2*x**2) - a**2*e*r/(4*x**2) - a**2*e*log(f*x**r)/(2*x**2) - a*b*d*
n/(2*x**2) - a*b*d*log(c*x**n)/x**2 - a*b*e*n*r/(2*x**2) - a*b*e*n*log(f*x*
*r)/(2*x**2) - a*b*e*r*log(c*x**n)/(2*x**2) - a*b*e*log(c*x**n)*log(f*x**r)
/x**2 - b**2*d*n**2/(4*x**2) - b**2*d*n*log(c*x**n)/(2*x**2) - b**2*d*log(c
*x**n)**2/(2*x**2) - 3*b**2*e*n**2*r/(8*x**2) - b**2*e*n**2*log(f*x**r)/(4*
x**2) - b**2*e*n*r*log(c*x**n)/(2*x**2) - b**2*e*n*log(c*x**n)*log(f*x**r)/
(2*x**2) - b**2*e*r*log(c*x**n)**2/(4*x**2) - b**2*e*log(c*x**n)**2*log(f*x
**r)/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(195) = 390.
time = 5.27, size = 403, normalized size = 1.98

$$-\frac{4b^2n^2re\log(x)^3 + 6b^2n^2re\log(x)^2 + 8b^2nre\log(c)\log(x)^2 + 4b^2n^2e\log(f)\log(x)^2 + 6b^2n^2re\log(x) + 8b^2nre\log(c)\log(x) + 4b^2re\log(c)^2\log(x) + 4b^2n^2e\log(f)\log(x) + 8b^2ne\log(c)\log(f)\log(x) + 4b^2dn^2\log(x)^2 + 8abnre\log(x)^2 + 3b^2n^2re + 4b^2nre\log(c) + 2b^2re\log(c)^2 + 2b^2n^2e\log(f) + 4b^2ne\log(c)\log(f) + 4b^2e\log(c)^2\log(f) + 4b^2dn^2\log(x) + 8abnre\log(x) + 8b^2dn\log(c)\log(x) + 8abre\log(c)\log(x) + 8abne\log(f)\log(x) + 2b^2dn^2 + 4abnre + 4b^2dn\log(c) + 4abre\log(c) + 4b^2d\log(c)^2 + 4abne\log(f) + 8abe\log(c)\log(f) + 8abdn\log(x) + 4a^2re\log(x) + 4abdn + 2a^2re + 8abd\log(c) + 4a^2e\log(f) + 4a^2d}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*n^2*r*e*log(x)^3 + 6*b^2*n^2*r*e*log(x)^2 + 8*b^2*n*r*e*log(c)*
log(x)^2 + 4*b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 8*b^2*n*r*e
*log(c)*log(x) + 4*b^2*r*e*log(c)^2*log(x) + 4*b^2*n^2*e*log(f)*log(x) + 8*
b^2*n*e*log(c)*log(f)*log(x) + 4*b^2*d*n^2*log(x)^2 + 8*a*b*n*r*e*log(x)^2
 + 3*b^2*n^2*r*e + 4*b^2*n*r*e*log(c) + 2*b^2*r*e*log(c)^2 + 2*b^2*n^2*e*log
(f) + 4*b^2*n*e*log(c)*log(f) + 4*b^2*e*log(c)^2*log(f) + 4*b^2*d*n^2*log(x
) + 8*a*b*n*r*e*log(x) + 8*b^2*d*n*log(c)*log(x) + 8*a*b*r*e*log(c)*log(x)
 + 8*a*b*n*e*log(f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 4*b^2*d*n*log(c) +
4*a*b*r*e*log(c) + 4*b^2*d*log(c)^2 + 4*a*b*n*e*log(f) + 8*a*b*e*log(c)*log
(f) + 8*a*b*d*n*log(x) + 4*a^2*r*e*log(x) + 4*a*b*d*n + 2*a^2*r*e + 8*a*b*d
*log(c) + 4*a^2*e*log(f) + 4*a^2*d)/x^2
```

Mupad [B]
time = 4.08, size = 186, normalized size = 0.91

$$-\ln(f\,x^r)\left(\ln(c\,x^n)\left(\frac{a\,b\,e}{x^2}+\frac{b^2\,e\,n}{2\,x^2}\right)+\frac{a^2\,e}{2\,x^2}+\frac{b^2\,e\,n^2}{4\,x^2}+\frac{b^2\,e\ln(c\,x^n)^2}{2\,x^2}+\frac{a\,b\,e\,n}{2\,x^2}\right)-\frac{\frac{a^2\,d}{2}+\frac{b^2\,d\,n^2}{4}+\frac{a^2\,e\,r}{4}+\frac{3\,b^2\,e\,n^2\,r}{8}+\frac{a\,b\,d\,n}{2}+\frac{a\,b\,e\,n\,r}{2}}{x^2}-\frac{b^2\ln(c\,x^n)^2\,(2\,d+e\,r)}{4\,x^2}-\frac{b\ln(c\,x^n)\,(2\,a\,d+b\,d\,n+a\,e\,r+b\,e\,n\,r)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^3,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((a*b*e)/x^2 + (b^2*e*n)/(2*x^2)) + (a^2*e)/(2*x^2
) + (b^2*e*n^2)/(4*x^2) + (b^2*e*log(c*x^n)^2)/(2*x^2) + (a*b*e*n)/(2*x^2))
 - ((a^2*d)/2 + (b^2*d*n^2)/4 + (a^2*e*r)/4 + (3*b^2*e*n^2*r)/8 + (a*b*d*n)
/2 + (a*b*e*n*r)/2)/x^2 - (b^2*log(c*x^n)^2*(2*d + e*r))/(4*x^2) - (b*log(c
*x^n)*(2*a*d + b*d*n + a*e*r + b*e*n*r))/(2*x^2)
```

3.169 $\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=205

$$-\frac{2b^2en^2r}{81x^3}-\frac{2ben(3a+bn)r}{81x^3}-\frac{e(9a^2+6abn+2b^2n^2)\,r}{81x^3}-\frac{2b^2enr\log(cx^n)}{27x^3}-\frac{2be(3a+bn)r\log(cx^n)}{27x^3}-\frac{b^2er\log}{9x}$$

```
[Out] -2/81*b^2*e*n^2*r/x^3-2/81*b*e*n*(b*n+3*a)*r/x^3-1/81*e*(2*b^2*n^2+6*a*b*n+
9*a^2)*r/x^3-2/27*b^2*e*n*r*ln(c*x^n)/x^3-2/27*b*e*(b*n+3*a)*r*ln(c*x^n)/x^
3-1/9*b^2*e*r*ln(c*x^n)^2/x^3-2/27*b^2*n^2*(d+e*ln(f*x^r))/x^3-2/9*b*n*(a+b
*ln(c*x^n))*(d+e*ln(f*x^r))/x^3-1/3*(a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^3
```

Rubi [A]
time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2342, 2341, 2413, 12, 14}

$$-\frac{er(9a^2+6abn+2b^2n^2)}{81x^3}-\frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}-\frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{3x^3}-\frac{2ber(3a+bn)\log(cx^n)}{27x^3}-\frac{2benr(3a+bn)}{81x^3}-\frac{b^2er\log^2(cx^n)}{9x^3}-\frac{2b^2enr\log(cx^n)}{27x^3}-\frac{2b^2n^2(d+e\log(fx^r))}{27x^3}-\frac{2b^2en^2r}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] (-2*b^2*e*n^2*r)/(81*x^3) - (2*b*e*n*(3*a + b*n)*r)/(81*x^3) - (e*(9*a^2 +
6*a*b*n + 2*b^2*n^2)*r)/(81*x^3) - (2*b^2*e*n*r*Log[c*x^n])/(27*x^3) - (2*b
*e*(3*a + b*n)*r*Log[c*x^n])/(27*x^3) - (b^2*e*r*Log[c*x^n]^2)/(9*x^3) - (2
*b^2*n^2*(d + e*Log[f*x^r]))/(27*x^3) - (2*b*n*(a + b*Log[c*x^n])*(d + e*Lo
g[f*x^r]))/(9*x^3) - ((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^2(d+e\log(fx^r))}{x^4}\,dx = -\frac{2b^2n^2(d+e\log(fx^r))}{27x^3} - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}$$

$$= -\frac{2b^2n^2(d+e\log(fx^r))}{27x^3} - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}$$

$$= -\frac{2b^2n^2(d+e\log(fx^r))}{27x^3} - \frac{2bn(a+b\log(cx^n))(d+e\log(fx^r))}{9x^3}$$

$$= -\frac{e(9a^2+6abn+2b^2n^2)r}{81x^3} - \frac{2b^2n^2(d+e\log(fx^r))}{27x^3} - \frac{2bn(a+b\,\mathrm{l}}{}$$

$$= -\frac{2ben(3a+bn)r}{81x^3} - \frac{e(9a^2+6abn+2b^2n^2)r}{81x^3} - \frac{2be(3a+bn)r\,\mathrm{log}}{27x^3}$$

$$= -\frac{2b^2en^2r}{81x^3} - \frac{2ben(3a+bn)r}{81x^3} - \frac{e(9a^2+6abn+2b^2n^2)r}{81x^3} - \frac{2b^2en}{}$$

Mathematica [A]
time = 0.08, size = 155, normalized size = 0.76

$$-\frac{9a^2d+6abdn+2b^2dn^2+3a^2er+4abenr+2b^2en^2r+e(9a^2+6abn+2b^2n^2)\log(fx^r)+3b^2\log^2(cx^n)(3d+er+3e\log(fx^r))+2b\log(cx^n)(9ad+3bdn+3aer+2benr+3e(3a+bn)\log(fx^r))}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -1/27*(9*a^2*d + 6*a*b*d*n + 2*b^2*d*n^2 + 3*a^2*e*r + 4*a*b*e*n*r + 2*b^2*
e*n^2*r + e*(9*a^2 + 6*a*b*n + 2*b^2*n^2)*Log[f*x^r] + 3*b^2*Log[c*x^n]^2*(
3*d + e*r + 3*e*Log[f*x^r]) + 2*b*Log[c*x^n]*(9*a*d + 3*b*d*n + 3*a*e*r + 2
*b*e*n*r + 3*e*(3*a + b*n)*Log[f*x^r]))/x^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 8407, normalized size = 41.01

method	result	size
risch	Expression too large to display	8407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*(d+e*ln(f*x^r))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 236, normalized size = 1.15

$$-\frac{1}{9}b^2\left(\frac{r}{x^3}+\frac{3\log\left(fx^r\right)}{x^3}\right)e\log\left(cx^n\right)^2-\frac{2}{9}ab\left(\frac{r}{x^3}+\frac{3\log\left(fx^r\right)}{x^3}\right)e\log\left(cx^n\right)-\frac{2}{27}b^2d\left(\frac{n^2}{x^3}+\frac{3n\log\left(cx^n\right)}{x^3}\right)-\frac{2}{27}b^2\left(\frac{(r\log(x)+r+\log(f))n^2}{x^3}+\frac{n(2r+3\log(f)+3\log(x^r))\log\left(cx^n\right)}{x^3}\right)e-\frac{2abn(2r+3\log(f)+3\log(x^r))e}{27x^3}-\frac{b^2d\log\left(cx^n\right)^2}{3x^3}-\frac{2abdn}{9x^3}-\frac{a^2re}{9x^3}-\frac{2abd\log\left(cx^n\right)}{3x^3}-\frac{a^2e\log\left(fx^r\right)}{3x^3}-\frac{a^2d}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*b^2*(r/x^3 + 3*log(f*x^r)/x^3)*e*log(c*x^n)^2 - 2/9*a*b*(r/x^3 + 3*log
(f*x^r)/x^3)*e*log(c*x^n) - 2/27*b^2*d*(n^2/x^3 + 3*n*log(c*x^n)/x^3) - 2/2
7*b^2*((r*log(x) + r + log(f))*n^2/x^3 + n*(2*r + 3*log(f) + 3*log(x^r))*lo
g(c*x^n)/x^3)*e - 2/27*a*b*n*(2*r + 3*log(f) + 3*log(x^r))*e/x^3 - 1/3*b^2*
d*log(c*x^n)^2/x^3 - 2/9*a*b*d*n/x^3 - 1/9*a^2*r*e/x^3 - 2/3*a*b*d*log(c*x^
n)/x^3 - 1/3*a^2*e*log(f*x^r)/x^3 - 1/3*a^2*d/x^3
```

Fricas [A]
time = 0.36, size = 337, normalized size = 1.64

$$-\frac{9b^2n^2re\log(x)^3+2b^2dn^2+6abdn+9a^2d+(2b^2n^2+4abn+3a^2)re+3(b^2re+3b^2d)\log(c)^2+9(2b^2nre\log(c)+b^2n^2e\log(f)+b^2dn^2+(b^2n^2+2abn)re)\log(x)^2+2(3b^2dn+9abd+(2b^2n+3ab)re)\log(c)+(9b^2e\log(c)^2+6(b^2n+3ab)e\log(c)+(2b^2n^2+6abn+9a^2)e)\log(f)+3(3b^2re\log(c)^2+2b^2dn^2+6abdn+(2b^2n^2+4abn+3a^2)re+2(3b^2dn+(2b^2n+3ab)re)\log(c)+2(3b^2ne\log(c)+(b^2n^2+3abn)e)\log(f))\log(x)}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b^2*n^2*r*e*log(x)^3 + 2*b^2*d*n^2 + 6*a*b*d*n + 9*a^2*d + (2*b^2*
n^2 + 4*a*b*n + 3*a^2)*r*e + 3*(b^2*r*e + 3*b^2*d)*log(c)^2 + 9*(2*b^2*n*r*
e*log(c) + b^2*n^2*e*log(f) + b^2*d*n^2 + (b^2*n^2 + 2*a*b*n)*r*e)*log(x)^2
 + 2*(3*b^2*d*n + 9*a*b*d + (2*b^2*n + 3*a*b)*r*e)*log(c) + (9*b^2*e*log(c)
^2 + 6*(b^2*n + 3*a*b)*e*log(c) + (2*b^2*n^2 + 6*a*b*n + 9*a^2)*e)*log(f) +
 3*(3*b^2*r*e*log(c)^2 + 2*b^2*d*n^2 + 6*a*b*d*n + (2*b^2*n^2 + 4*a*b*n + 3
*a^2)*r*e + 2*(3*b^2*d*n + (2*b^2*n + 3*a*b)*r*e)*log(c) + 2*(3*b^2*n*e*log
(c) + (b^2*n^2 + 3*a*b*n)*e)*log(f))*log(x))/x^3
```

Sympy [A]
time = 2.72, size = 342, normalized size = 1.67

$$-\frac{a^2d}{3x^3}-\frac{a^2er}{9x^3}-\frac{a^2e\log\left(fx^r\right)}{3x^3}-\frac{2abdn}{9x^3}-\frac{2abd\log\left(cx^n\right)}{3x^3}-\frac{4abenr}{27x^3}-\frac{2aben\log\left(fx^r\right)}{9x^3}-\frac{2aber\log\left(cx^n\right)}{9x^3}-\frac{2abe\log\left(cx^n\right)\log\left(fx^r\right)}{3x^3}-\frac{2b^2dn^2}{27x^3}-\frac{2b^2dn\log\left(cx^n\right)}{9x^3}-\frac{b^2d\log\left(cx^n\right)^2}{3x^3}-\frac{2b^2en^2r}{27x^3}-\frac{2b^2en^2\log\left(fx^r\right)}{27x^3}-\frac{4b^2enr\log\left(cx^n\right)}{27x^3}-\frac{2b^2en\log\left(cx^n\right)\log\left(fx^r\right)}{9x^3}-\frac{b^2er\log\left(cx^n\right)^2}{9x^3}-\frac{b^2e\log\left(cx^n\right)^2\log\left(fx^r\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*(d+e*ln(f*x**r))/x**4,x)
```

```
[Out] -a**2*d/(3*x**3) - a**2*e*r/(9*x**3) - a**2*e*log(f*x**r)/(3*x**3) - 2*a*b*
d*n/(9*x**3) - 2*a*b*d*log(c*x**n)/(3*x**3) - 4*a*b*e*n*r/(27*x**3) - 2*a*b
*e*n*log(f*x**r)/(9*x**3) - 2*a*b*e*r*log(c*x**n)/(9*x**3) - 2*a*b*e*log(c*
x**n)*log(f*x**r)/(3*x**3) - 2*b**2*d*n**2/(27*x**3) - 2*b**2*d*n*log(c*x**
n)/(9*x**3) - b**2*d*log(c*x**n)**2/(3*x**3) - 2*b**2*e*n**2*r/(27*x**3) -
2*b**2*e*n**2*log(f*x**r)/(27*x**3) - 4*b**2*e*n*r*log(c*x**n)/(27*x**3) -
2*b**2*e*n*log(c*x**n)*log(f*x**r)/(9*x**3) - b**2*e*r*log(c*x**n)**2/(9*x*
*3) - b**2*e*log(c*x**n)**2*log(f*x**r)/(3*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(196) = 392.
time = 6.98, size = 403, normalized size = 1.97

$$-\frac{9\,b^2n^2re\log\left(x\right)^3+9\,b^2n^2re\log\left(x\right)^2+18\,b^2nre\log\left(c\right)\log\left(x\right)^2+9\,b^2n^2e\log\left(f\right)\log\left(x\right)^2+6\,b^2n^2re\log\left(x\right)+12\,b^2nre\log\left(c\right)\log\left(x\right)+9\,b^2re\log\left(c\right)^2\log\left(x\right)+6\,b^2n^2e\log\left(f\right)\log\left(x\right)+18\,b^2ne\log\left(c\right)\log\left(f\right)\log\left(x\right)+9\,b^2dn^2\log\left(x\right)^2+18\,abnre\log\left(x\right)^2+2\,b^2n^2re+4\,b^2nre\log\left(c\right)+3\,b^2re\log\left(c\right)^2+2\,b^2n^2e\log\left(f\right)+6\,b^2ne\log\left(c\right)\log\left(f\right)+9\,b^2e\log\left(c\right)^2\log\left(f\right)+6\,b^2dn^2\log\left(x\right)+12\,abnre\log\left(x\right)+18\,b^2dn\log\left(c\right)\log\left(x\right)+18\,abre\log\left(c\right)\log\left(x\right)+18\,abne\log\left(f\right)\log\left(x\right)+2\,b^2dn^2+4\,abnre+6\,b^2dn\log\left(c\right)+6\,abre\log\left(c\right)+9\,b^2d\log\left(c\right)^2+6\,abne\log\left(f\right)+18\,abe\log\left(c\right)\log\left(f\right)+18\,abdn\log\left(x\right)+9\,a^2re\log\left(x\right)+6\,abdn+3\,a^2re+18\,abd\log\left(c\right)+9\,a^2e\log\left(f\right)+9\,a^2d}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
```

```
[Out] -1/27*(9*b^2*n^2*r*e*log(x)^3 + 9*b^2*n^2*r*e*log(x)^2 + 18*b^2*n*r*e*log(c
)*log(x)^2 + 9*b^2*n^2*e*log(f)*log(x)^2 + 6*b^2*n^2*r*e*log(x) + 12*b^2*n*
r*e*log(c)*log(x) + 9*b^2*r*e*log(c)^2*log(x) + 6*b^2*n^2*e*log(f)*log(x) +
 18*b^2*n*e*log(c)*log(f)*log(x) + 9*b^2*d*n^2*log(x)^2 + 18*a*b*n*r*e*log(
x)^2 + 2*b^2*n^2*r*e + 4*b^2*n*r*e*log(c) + 3*b^2*r*e*log(c)^2 + 2*b^2*n^2*
e*log(f) + 6*b^2*n*e*log(c)*log(f) + 9*b^2*e*log(c)^2*log(f) + 6*b^2*d*n^2*
log(x) + 12*a*b*n*r*e*log(x) + 18*b^2*d*n*log(c)*log(x) + 18*a*b*r*e*log(c)
*log(x) + 18*a*b*n*e*log(f)*log(x) + 2*b^2*d*n^2 + 4*a*b*n*r*e + 6*b^2*d*n*
log(c) + 6*a*b*r*e*log(c) + 9*b^2*d*log(c)^2 + 6*a*b*n*e*log(f) + 18*a*b*e*
log(c)*log(f) + 18*a*b*d*n*log(x) + 9*a^2*r*e*log(x) + 6*a*b*d*n + 3*a^2*r*
e + 18*a*b*d*log(c) + 9*a^2*e*log(f) + 9*a^2*d)/x^3
```

Mupad [B]
time = 4.20, size = 190, normalized size = 0.93

$$-\ln\left(f\,x^r\right)\,\left(\ln\left(c\,x^n\right)\,\left(\frac{2\,a\,b\,e}{3\,x^3}+\frac{2\,b^2\,e\,n}{9\,x^3}\right)+\frac{a^2\,e}{3\,x^3}+\frac{2\,b^2\,e\,n^2}{27\,x^3}+\frac{b^2\,e\,{\ln\left(c\,x^n\right)}^2}{3\,x^3}+\frac{2\,a\,b\,e\,n}{9\,x^3}\right)-\frac{\frac{a^2\,d}{3}+\frac{2\,b^2\,d\,n^2}{27}+\frac{a^2\,e\,r}{9}+\frac{2\,b^2\,e\,n^2\,r}{27}+\frac{2\,a\,b\,d\,n}{9}+\frac{4\,a\,b\,e\,n\,r}{27}}{x^3}-\frac{b^2\,{\ln\left(c\,x^n\right)}^2\,\left(3\,d+e\,r\right)}{9\,x^3}-\frac{2\,b\,\ln\left(c\,x^n\right)\,\left(9\,a\,d+3\,b\,d\,n+3\,a\,e\,r+2\,b\,e\,n\,r\right)}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^2)/x^4,x)
```

```
[Out] - log(f*x^r)*(log(c*x^n)*((2*a*b*e)/(3*x^3) + (2*b^2*e*n)/(9*x^3)) + (a^2*e
)/(3*x^3) + (2*b^2*e*n^2)/(27*x^3) + (b^2*e*log(c*x^n)^2)/(3*x^3) + (2*a*b*
e*n)/(9*x^3)) - ((a^2*d)/3 + (2*b^2*d*n^2)/27 + (a^2*e*r)/9 + (2*b^2*e*n^2*
r)/27 + (2*a*b*d*n)/9 + (4*a*b*e*n*r)/27)/x^3 - (b^2*log(c*x^n)^2*(3*d + e*
r))/(9*x^3) - (2*b*log(c*x^n)*(9*a*d + 3*b*d*n + 3*a*e*r + 2*b*e*n*r))/(27*
x^3)
```

3.170 $\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$

Optimal. Leaf size=141

$$\frac{bnx^3}{3em}-\frac{be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\,\mathrm{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2}+\frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\,\mathrm{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+\cdots}{em}$$

```
[Out] 1/3*b*n*x^3/e/m-b*n*x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(3
*d/e/m)/m^2/((f*x^m)^(3/m))+x^3*Ei(3*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e
/exp(3*d/e/m)/m/((f*x^m)^(3/m))
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2347, 2209, 2413, 12, 15, 6617}

$$\frac{x^3e^{-\frac{3d}{em}}(fx^m)^{-3/m}(a+b\log(cx^n))\,\mathrm{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^3e^{-\frac{3d}{em}}(fx^m)^{-3/m}(d+e\log(fx^m))\,\mathrm{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)}{e^2m^2}+\frac{bnx^3}{3em}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x^3)/(3*e*m) - (b*n*x^3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(d
 + e*Log[f*x^m]))/(e^2*E^((3*d)/(e*m))*m^2*(f*x^m)^(3/m)) + (x^3*ExpIntegra
lEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((3*d)/(e*m))*m*
(f*x^m)^(3/m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6617

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - (bn)\int \frac{e^{-\frac{3d}{em}}x^2(fx^m)^{-3/\cdots}}{\cdots}$$

$$= \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{3d}{em}}n\right)\int x^2(fx^m)^{-3/\cdots}}{em\cdots}$$

$$= \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\right)\cdots}{em\cdots}$$

$$= \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\right)\cdots}{\cdots}$$

$$= \frac{e^{-\frac{3d}{em}}x^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\right)\cdots}{\cdots}$$

$$= \frac{bnx^3}{3em} - \frac{be^{-\frac{3d}{em}}nx^3(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3d}{em}+\frac{3\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{em} + \frac{e^{-\frac{3d}{em}}x^3(fx^m\cdots}{\cdots}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.66

$$\frac{x^3\left(bemn+3e^{-\frac{3d}{em}}(fx^m)^{-3/m}\operatorname{Ei}\left(\frac{3(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\right)}{3e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (x^3*(b*e*m*n + (3*ExpIntegralEi[(3*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*d
*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((3*d)/(e*m))*(f*x^m)^(3/m)))
)/(3*e^2*m^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 2350, normalized size = 16.67

method	result	size
risch	Expression too large to display	2350

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)/e/m*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2
*I*d)/e/m)-b/e/m*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csg
n(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*c
sgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn
(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*l
n(x))+2*I*d)/e/m)*ln(x^n)+1/3*b*n*x^3/e/m-1/2*I*b*n/e/m^2*x^3*f^(-3/m)*(x^m
)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*
f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^
3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)
-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(
I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csg
n(I*x^m)*csgn(I*f*x^m)+1/2*I*b*n/e/m^2*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(
-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*
e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,
-3*ln(x)+3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csg
n(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln
(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*
b*n/e/m^2*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*c
sgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x
^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2
*I*d)/e/m)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*I*b*n/e/m^2*x^3*f^(-3/m)*(x^m
)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*
```

```
f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^
3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)
-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(
I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f*x^m)
^3+b*n/e/m^2*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(
I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x
^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x)
)+2*I*d)/e/m)*ln(f)+b*n/e/m^2*x^3*f^(-3/m)*(x^m)^(-3/m)*exp(-3/2*(-I*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-3*ln(x)+
3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m
)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e
*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^m)+b*n/e^2/m^2*x^3*f^(-3/m)*(x^m)^(-3/m
)*exp(-3/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn
(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d
)/e/m)*Ei(1,-3*ln(x)+3/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*c
sgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m
)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x^2/(e*log(f*x^m) + d), x)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.67

$$\frac{\left(bmnx^3 e^{\left(\frac{3\,(e\log(f)+d)e^{(-1)}}{m}+1\right)} + 3\,(bme\log(c) - bne\log(f) - bdn + ame)\,\mathrm{log\_integral}\left(x^3 e^{\left(\frac{3\,(e\log(f)+d)e^{(-1)}}{m}\right)}\right)\right)e^{\left(-\frac{3\,(e\log(f)+d)e^{(-1)}}{m}-2\right)}}{3\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
[Out] 1/3*(b*m*n*x^3*e^(3*(e*log(f) + d)*e^(-1)/m + 1) + 3*(b*m*e*log(c) - b*n*e*
log(f) - b*d*n + a*m*e)*log_integral(x^3*e^(3*(e*log(f) + d)*e^(-1)/m)))*e^
(-3*(e*log(f) + d)*e^(-1)/m - 2)/m^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\log(cx^n))}{d + e\log(fx^m)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Giac [A]
time = 3.84, size = 206, normalized size = 1.46

$$\frac{bnx^3e^{(-1)}}{3\,m} - \frac{bdn\mathrm{Ei}\left(\frac{3\,de^{(-1)}}{m} + \frac{3\,\log(f)}{m} + 3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m} - 2\right)}}{f^{\frac{3}{m}}m^2} + \frac{b\mathrm{Ei}\left(\frac{3\,de^{(-1)}}{m} + \frac{3\,\log(f)}{m} + 3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m} - 1\right)}\log(c)}{f^{\frac{3}{m}}m} - \frac{bn\mathrm{Ei}\left(\frac{3\,de^{(-1)}}{m} + \frac{3\,\log(f)}{m} + 3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m} - 1\right)}\log(f)}{f^{\frac{3}{m}}m^2} + \frac{a\mathrm{Ei}\left(\frac{3\,de^{(-1)}}{m} + \frac{3\,\log(f)}{m} + 3\,\log(x)\right)e^{\left(-\frac{3\,de^{(-1)}}{m} - 1\right)}}{f^{\frac{3}{m}}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] 1/3*b*n*x^3*e^(-1)/m - b*d*n*Ei(3*d*e^(-1)/m + 3*log(f)/m + 3*log(x))*e^(-3
*d*e^(-1)/m - 2)/(f^(3/m)*m^2) + b*Ei(3*d*e^(-1)/m + 3*log(f)/m + 3*log(x))
*e^(-3*d*e^(-1)/m - 1)*log(c)/(f^(3/m)*m) - b*n*Ei(3*d*e^(-1)/m + 3*log(f)/
m + 3*log(x))*e^(-3*d*e^(-1)/m - 1)*log(f)/(f^(3/m)*m^2) + a*Ei(3*d*e^(-1)/
m + 3*log(f)/m + 3*log(x))*e^(-3*d*e^(-1)/m - 1)/(f^(3/m)*m)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{d+e\,\ln\left(f\,x^m\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

```
[Out] int((x^2*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

3.171 $\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx$

Optimal. Leaf size=141

$$\frac{bnx^2}{2em}-\frac{be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2}+\frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b}{em}$$

```
[Out] 1/2*b*n*x^2/e/m-b*n*x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(2
*d/e/m)/m^2/((f*x^m)^(2/m))+x^2*Ei(2*(d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e
/exp(2*d/e/m)/m/((f*x^m)^(2/m))
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2347, 2209, 2413, 12, 15, 6617}

$$\frac{x^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}(a+b\log(cx^n))\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{em}-\frac{bnx^2e^{-\frac{2d}{em}}(fx^m)^{-2/m}(d+e\log(fx^m))\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)}{e^2m^2}+\frac{bnx^2}{2em}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x^2)/(2*e*m) - (b*n*x^2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(d
 + e*Log[f*x^m]))/(e^2*E^((2*d)/(e*m))*m^2*(f*x^m)^(2/m)) + (x^2*ExpIntegra
lEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log[c*x^n]))/(e*E^((2*d)/(e*m))*m*
(f*x^m)^(2/m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6617

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x(a+b\log(cx^n))}{d+e\log(fx^m)}\,dx = \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - (bn)\int \frac{e^{-\frac{2d}{em}}x(fx^{-2/m}}{}$$

$$= \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{2d}{em}}n\right)\int x(fx^m)^{-2/}}{em}$$

$$= \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\right)}{em}$$

$$= \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\right)}{}$$

$$= \frac{e^{-\frac{2d}{em}}x^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\right)}{}$$

$$= \frac{bnx^2}{2em} - \frac{be^{-\frac{2d}{em}}nx^2(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2d}{em}+\frac{2\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{em} + \frac{e^{-\frac{2d}{em}}x^2(fx^}{}$$

Mathematica [A]
time = 0.10, size = 93, normalized size = 0.66

$$\frac{x^2\left(bemn+2e^{-\frac{2d}{em}}(fx^m)^{-2/m}\operatorname{Ei}\left(\frac{2(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\right)}{2e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*Log[f*x^m]),x]
```

```
[Out] (x^2*(b*e*m*n + (2*ExpIntegralEi[(2*(d + e*Log[f*x^m]))/(e*m)]*(a*e*m - b*d
*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^((2*d)/(e*m))*(f*x^m)^(2/m)))
)/(2*e^2*m^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 2350, normalized size = 16.67

method	result	size
risch	Expression too large to display	2350

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)/e/m*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(
I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^
2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/
m)-b/e/m*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I
*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2
*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*
x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m
)*ln(x^n)+1/2*b*n*x^2/e/m-1/2*I*b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-
I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e
+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-
2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f
*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2
*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1
/2*I*b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*
csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*
x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*c
sgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*cs
gn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*
d)/e/m)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2
/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I
*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/
e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*
f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*
I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^
2-1/2*I*b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^
m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I
```

```
*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2
*I*d)/e/m)*Pi*csgn(I*f*x^m)^3+b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2/m)*exp(-(-I*
Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I
*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-2*
ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x
^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I
*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(f)+b*n/e/m^2*x^2*f^(-2/m)*(x^m)^(-2/m)*
exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x
^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)
*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*c
sgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*
ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^m)+b*n/e^2/m^2*x^2*f^(-2/m)*
(x^m)^(-2/m)*exp(-(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*
f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^
3*e+2*d)/e/m)*Ei(1,-2*ln(x)+I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*P
i*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*
x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
[Out] integrate((b*log(c*x^n) + a)*x/(e*log(f*x^m) + d), x)
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.67

$$\frac{\left(bmnx^2e^{\left(\frac{2\,(e\log(f)+d)e^{(-1)}}{m}+1\right)} + 2\,(bme\log\left(c\right) - bne\log\left(f\right) - bdn + ame)\,\mathrm{log\_integral}\left(x^2e^{\left(\frac{2\,(e\log(f)+d)e^{(-1)}}{m}\right)}\right)\right)e^{\left(-\frac{2\,(e\log(f)+d)e^{(-1)}}{m}-2\right)}}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
[Out] 1/2*(b*m*n*x^2*e^(2*(e*log(f) + d)*e^(-1)/m + 1) + 2*(b*m*e*log(c) - b*n*e*
log(f) - b*d*n + a*m*e)*log_integral(x^2*e^(2*(e*log(f) + d)*e^(-1)/m)))*e^
(-2*(e*log(f) + d)*e^(-1)/m - 2)/m^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\log\left(cx^n\right))}{d + e\log\left(fx^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Giac [A]
time = 4.76, size = 206, normalized size = 1.46

$$\frac{bnx^2e^{(-1)}}{2\,m} - \frac{bdn\mathrm{Ei}\left(\frac{2\,de^{(-1)}}{m} + \frac{2\,\log(f)}{m} + 2\,\log(x)\right)e^{\left(-\frac{2\,de^{(-1)}}{m} - 2\right)}}{f^{\frac{2}{m}}m^2} + \frac{b\mathrm{Ei}\left(\frac{2\,de^{(-1)}}{m} + \frac{2\,\log(f)}{m} + 2\,\log(x)\right)e^{\left(-\frac{2\,de^{(-1)}}{m} - 1\right)}\log(c)}{f^{\frac{2}{m}}m} - \frac{bn\mathrm{Ei}\left(\frac{2\,de^{(-1)}}{m} + \frac{2\,\log(f)}{m} + 2\,\log(x)\right)e^{\left(-\frac{2\,de^{(-1)}}{m} - 1\right)}\log(f)}{f^{\frac{2}{m}}m^2} + \frac{a\mathrm{Ei}\left(\frac{2\,de^{(-1)}}{m} + \frac{2\,\log(f)}{m} + 2\,\log(x)\right)e^{\left(-\frac{2\,de^{(-1)}}{m} - 1\right)}}{f^{\frac{2}{m}}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] 1/2*b*n*x^2*e^(-1)/m - b*d*n*Ei(2*d*e^(-1)/m + 2*log(f)/m + 2*log(x))*e^(-2
*d*e^(-1)/m - 2)/(f^(2/m)*m^2) + b*Ei(2*d*e^(-1)/m + 2*log(f)/m + 2*log(x))
*e^(-2*d*e^(-1)/m - 1)*log(c)/(f^(2/m)*m) - b*n*Ei(2*d*e^(-1)/m + 2*log(f)/
m + 2*log(x))*e^(-2*d*e^(-1)/m - 1)*log(f)/(f^(2/m)*m^2) + a*Ei(2*d*e^(-1)/
m + 2*log(f)/m + 2*log(x))*e^(-2*d*e^(-1)/m - 1)/(f^(2/m)*m)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\,\ln(c\,x^n))}{d + e\,\ln(f\,x^m)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)),x)
```

```
[Out] int((x*(a + b*log(c*x^n)))/(d + e*log(f*x^m)), x)
```

3.172 $\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx$

Optimal. Leaf size=130

$$\frac{bnx}{em}-\frac{be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\,\mathrm{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(d+e\log(fx^m))}{e^2m^2}+\frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\,\mathrm{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(c$$

```
[Out] b*n*x/e/m-b*n*x*Ei((d+e*ln(f*x^m))/e/m)*(d+e*ln(f*x^m))/e^2/exp(d/e/m)/m^2/
((f*x^m)^(1/m))+x*Ei((d+e*ln(f*x^m))/e/m)*(a+b*ln(c*x^n))/e/exp(d/e/m)/m/((
f*x^m)^(1/m))
```

Rubi [A]

time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2337, 2209, 2408, 12, 15, 6617}

$$\frac{xe^{-\frac{d}{em}}(fx^m)^{-1/m}(a+b\log(cx^n))\,\mathrm{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)}{em}-\frac{bnxe^{-\frac{d}{em}}(fx^m)^{-1/m}(d+e\log(fx^m))\,\mathrm{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)}{e^2m^2}+\frac{bnx}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

```
[Out] (b*n*x)/(e*m) - (b*n*x*ExpIntegralEi[(d + e*Log[f*x^m])/(e*m)]*(d + e*Log[f
*x^m]))/(e^2*E^(d/(e*m))*m^2*(f*x^m)^m^(-1)) + (x*ExpIntegralEi[(d + e*Log[
f*x^m])/(e*m)]*(a + b*Log[c*x^n]))/(e*E^(d/(e*m))*m*(f*x^m)^m^(-1))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rule 6617

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\log(cx^n)}{d+e\log(fx^m)}\,dx &= \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em} - (bn)\int \frac{e^{-\frac{d}{em}}(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e
\\
&= \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{d}{em}}n\right)\int (fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e}{}\right.}{em}
\\
&= \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\right)\int \frac{\operatorname{Ei}\left(\frac{d+}{}\right.}{}}{em}
\\
&= \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\right)\operatorname{Subst}\left(\right.}{em^2}
\\
&= \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{em} - \frac{\left(be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\right)\operatorname{Subst}\left(\right.}{em}
\\
&= \frac{bnx}{em} - \frac{be^{-\frac{d}{em}}nx(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{em} + \frac{e^{-\frac{d}{em}}x(fx^m)^{-1/m}\operatorname{E}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.66

$$
\frac{x\left(bemn+e^{-\frac{d}{em}}(fx^m)^{-1/m}\operatorname{Ei}\left(\frac{d+e\log(fx^m)}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\right)}{e^2m^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*Log[f*x^m]),x]
```

```
[Out] (x*(b*e*m*n + (ExpIntegralEi[(d + e*Log[f*x^m])/(e*m)]*(a*e*m - b*d*n - b*e
*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(E^(d/(e*m))*(f*x^m)^m^(-1))))/(e^2*m^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2329, normalized size = 17.92

method	result	size
risch	Expression too large to display	2329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)/e/m*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csg
n(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m
)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*cs
gn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csg
n(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d
)/e/m)-b/e/m*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*
csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*
x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*
csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*
I*d)/e/m)*ln(x^n)+b*n*x/e/m-1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/
2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)
^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei
(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*cs
gn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*l
n(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*
x^m)+1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn
(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*c
sgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*c
sgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn
(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*l
n(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*b*n/e/m^2*x*(x^m)^(-1/
m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn
(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^
m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m
)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn
(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*x^m)*
csgn(I*f*x^m)^2-1/2*I*b*n/e/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/
2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^
```

```
2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(
ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f*x^m)^3+b*n/e/m^2*x*(x^m)^(-1/m)*f^
(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)
*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*
e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*P
i*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*
x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(f)+b*n/e/m^2*x*(x
^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I
*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csg
n(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn
(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e
*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^m)
+b*n/e^2/m^2*x*(x^m)^(-1/m)*f^(-1/m)*exp(-1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*
csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*
x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,-ln(x)+1/2*I*(e*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*
csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*
I*d)/e/m)*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)/(e*log(f*x^m) + d), x)
```

Fricas [A]
time = 0.35, size = 87, normalized size = 0.67

$$\frac{\left(bmnxe^{\left(\frac{(e\log(f)+d)e^{(-1)}}{m}+1\right)} + (bme\log{(c)} - bne\log{(f)} - bdn + ame)\,\mathrm{log\_integral}\left(xe^{\left(\frac{(e\log(f)+d)e^{(-1)}}{m}\right)}\right)\right)e^{\left(-\frac{(e\log(f)+d)e^{(-1)}}{m}-2\right)}}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] (b*m*n*x*e^((e*log(f) + d)*e^(-1)/m + 1) + (b*m*e*log(c) - b*n*e*log(f) - b
*d*n + a*m*e)*log_integral(x*e^((e*log(f) + d)*e^(-1)/m)))*e^(-(e*log(f) +
d)*e^(-1)/m - 2)/m^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log{(cx^n)}}{d + e\log{(fx^m)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*log(f*x**m)), x)
```

Giac [A]
time = 3.66, size = 179, normalized size = 1.38

$$\frac{bnxe^{(-1)}}{m} - \frac{bdn\mathrm{Ei}\left(\frac{de^{(-1)}}{m} + \frac{\log(f)}{m} + \log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-2\right)}}{f^{\left(\frac{1}{m}\right)}m^2} + \frac{b\mathrm{Ei}\left(\frac{de^{(-1)}}{m} + \frac{\log(f)}{m} + \log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-1\right)}\log(c)}{f^{\left(\frac{1}{m}\right)}m} - \frac{bn\mathrm{Ei}\left(\frac{de^{(-1)}}{m} + \frac{\log(f)}{m} + \log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-1\right)}\log(f)}{f^{\left(\frac{1}{m}\right)}m^2} + \frac{a\mathrm{Ei}\left(\frac{de^{(-1)}}{m} + \frac{\log(f)}{m} + \log(x)\right)e^{\left(-\frac{de^{(-1)}}{m}-1\right)}}{f^{\left(\frac{1}{m}\right)}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] b*n*x*e^(-1)/m - b*d*n*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m -
2)/(f^(1/m)*m^2) + b*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m - 1)
*log(c)/(f^(1/m)*m) - b*n*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(-1)/m
 - 1)*log(f)/(f^(1/m)*m^2) + a*Ei(d*e^(-1)/m + log(f)/m + log(x))*e^(-d*e^(
-1)/m - 1)/(f^(1/m)*m)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{d+e\,\ln\left(f\,x^m\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*log(f*x^m)),x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*log(f*x^m)), x)
```

3.173 $\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx$

Optimal. Leaf size=71

$$\frac{bn\log(x)}{em}-\frac{bn(d+e\log(fx^m))\log(d+e\log(fx^m))}{e^2m^2}+\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}$$

```
[Out] b*n*ln(x)/e/m-b*n*(d+e*ln(f*x^m))*ln(d+e*ln(f*x^m))/e^2/m^2+(a+b*ln(c*x^n))
*ln(d+e*ln(f*x^m))/e/m
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2339, 29, 2413, 12, 2436, 2332}

$$\frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{bn(d+e\log(fx^m))\log(d+e\log(fx^m))}{e^2m^2}+\frac{bn\log(x)}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

```
[Out] (b*n*Log[x])/(e*m) - (b*n*(d + e*Log[f*x^m])*Log[d + e*Log[f*x^m]])/(e^2*m^
2) + ((a + b*Log[c*x^n])*Log[d + e*Log[f*x^m]])/(e*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
```

```
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log(cx^n)}{x(d+e\log(fx^m))}\,dx &= \frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-(bn)\int \frac{\log(d+e\log(fx^m))}{emx}\,dx\\
&= \frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{(bn)\int \frac{\log(d+e\log(fx^m))}{x}\,dx}{em}\\
&= \frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{(bn)\text{Subst}\left(\int \log(d+ex)\,dx,x,\log(fx\right.}{em^2}\\
&= \frac{(a+b\log(cx^n))\log(d+e\log(fx^m))}{em}-\frac{(bn)\text{Subst}\left(\int \log(x)\,dx,x,d+e\log(fx\right.}{e^2m^2}\\
&= \frac{bn\log(x)}{em}-\frac{bn(d+e\log(fx^m))\log(d+e\log(fx^m))}{e^2m^2}+\frac{(a+b\log(cx^n))\log(d}{em}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.82

$$\frac{bemn\log(x)+(aem-bdn-ben\log(fx^m)+bem\log(cx^n))\log(d+e\log(fx^m))}{e^2m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x*(d + e*Log[f*x^m])),x]
```

```
[Out] (b*e*m*n*Log[x] + (a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n])*Log
[d + e*Log[f*x^m]])/(e^2*m^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 1744, normalized size = 24.56

method	result	size
risch	Expression too large to display	1744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/x/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/m*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*
f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+
2*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I/m*ln(
e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*P
i*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*
e+2*I*d)/e*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I/m*ln(e*Pi*csgn(I*f)*csgn(I*
x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-1/2*I/m*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e
*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*
f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*b*Pi*csgn(I*c*x^n)^3+1/m*ln(e*P
i*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*c
sgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*ln(x^m)*e+2
*I*d)/e*b*ln(c)+1/m*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I
*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2
*I*e*ln(f)+2*I*ln(x^m)*e+2*I*d)/e*a+b*n*ln(x)/e/m+1/2*I*b/e/m^2*ln(e*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I
*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*
(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I*b/e
/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^
m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*
I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*f)*csgn(I*f*x^m)^2-1/2
*I*b/e/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(
I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*
e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*x^m)*csgn(I*f*x^
m)^2+1/2*I*b/e/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*
f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*
I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*Pi*n*csgn(I*f*x^m)^3
-b/e/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*
f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*
m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*ln(f)*n+b/e/m*ln(e*Pi*csgn(I*f
)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)
*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x
^m)-m*ln(x))+2*I*d)*ln(x^n)-b/e/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*
x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*c
sgn(I*f*x^m)^3+2*I*ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*n*l
n(x^m)-b/e^2/m^2*ln(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)
*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*
ln(x)*e*m+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)*d*n
```

Maxima [A]
time = 0.30, size = 120, normalized size = 1.69

$$\frac{be^{(-1)}\log\left(cx^n\right)\log\left((e\log\left(f\right)+e\log\left(x^m\right)+d)e^{(-1)}\right)}{m}-\frac{((e\log\left(f\right)+e\log\left(x^m\right)+d)e^{(-1)}\log\left((e\log\left(f\right)+e\log\left(x^m\right)+d)e^{(-1)}\right)-(e\log\left(f\right)+e\log\left(x^m\right)+d)e^{(-1)})bne^{(-1)}}{m^2}+\frac{ae^{(-1)}\log\left((e\log\left(f\right)+e\log\left(x^m\right)+d)e^{(-1)}\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] b*e^(-1)*log(c*x^n)*log((e*log(f) + e*log(x^m) + d)*e^(-1))/m - ((e*log(f)
+ e*log(x^m) + d)*e^(-1)*log((e*log(f) + e*log(x^m) + d)*e^(-1)) - (e*log(f
) + e*log(x^m) + d)*e^(-1))*b*n*e^(-1)/m^2 + a*e^(-1)*log((e*log(f) + e*log
(x^m) + d)*e^(-1))/m
```

Fricas [A]
time = 0.37, size = 56, normalized size = 0.79

$$\frac{(bmne\log(x) + (bme\log(c) - bne\log(f) - bdn + ame)\log(me\log(x) + e\log(f) + d))e^{(-2)}}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] (b*m*n*e*log(x) + (b*m*e*log(c) - b*n*e*log(f) - b*d*n + a*m*e)*log(m*e*log
(x) + e*log(f) + d))*e^(-2)/m^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log(cx^n)}{x(d + e\log(fx^m))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x*(d + e*log(f*x**m))), x)
```

Giac [A]
time = 5.49, size = 85, normalized size = 1.20

$$\frac{bne^{(-1)}\log(x)}{m} + \frac{(bme\log(c) - bne\log(f) - bdn + ame)e^{(-2)}\log\left(\frac{1}{4}(\pi m(\operatorname{sgn}(x) - 1)e + \pi(\operatorname{sgn}(f) - 1)e)^2 + (me\log(|x|) + e\log(|f|) + d)^2\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] b*n*e^(-1)*log(x)/m + 1/2*(b*m*e*log(c) - b*n*e*log(f) - b*d*n + a*m*e)*e^(
-2)*log(1/4*(pi*m*(sgn(x) - 1)*e + pi*(sgn(f) - 1)*e)^2 + (m*e*log(abs(x))
+ e*log(abs(f)) + d)^2)/m^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\ln(c\,x^n)}{x\,(d + e\,\ln(f\,x^m))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))),x)
[Out] int((a + b*log(c*x^n))/(x*(d + e*log(f*x^m))), x)
```

3.174 $\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx$

Optimal. Leaf size=133

$$-\frac{bn}{emx}-\frac{be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(d+e\log(fx^m))}{e^2m^2x}+\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx}$$

```
[Out] -b*n/e/m/x-b*exp(d/e/m)*n*(f*x^m)^(1/m)*Ei((-d-e*ln(f*x^m))/e/m)*(d+e*ln(f*
x^m))/e^2/m^2/x+exp(d/e/m)*(f*x^m)^(1/m)*Ei((-d-e*ln(f*x^m))/e/m)*(a+b*ln(c
*x^n))/e/m/x
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2347, 2209, 2413, 12, 15, 6617}

$$\frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(a+b\log(cx^n))\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)}{emx}-\frac{bne^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}(d+e\log(fx^m))\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)}{e^2m^2x}-\frac{bn}{emx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

```
[Out] -((b*n)/(e*m*x)) - (b*E^(d/(e*m))*n*(f*x^m)^m^(-1)*ExpIntegralEi[-((d + e*L
og[f*x^m])/(e*m))]*(d + e*Log[f*x^m]))/(e^2*m^2*x) + (E^(d/(e*m))*(f*x^m)^m
^(-1)*ExpIntegralEi[-((d + e*Log[f*x^m])/(e*m))]*(a + b*Log[c*x^n]))/(e*m*x
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6617

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x^2(d+e\log(fx^m))}\,dx = \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx} - (bn)\int \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log\right.}{}}{emx^2}$$

$$= \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx} - \frac{\left(be^{\frac{d}{em}}n\right)\int \frac{(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(f}{em}\right.}{x^2}}{em}$$

$$= \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx} - \frac{\left(be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\right)\int \frac{\operatorname{Ei}\left(-\frac{d+e\log}{em}\right.}{x}}{emx}$$

$$= \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx} - \frac{\left(be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\right)\operatorname{Subst}\left(\int \operatorname{Ei}\right.}{em^2x}$$

$$= \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(a+b\log(cx^n))}{emx} + \frac{\left(be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\right)\operatorname{Subst}\left(\int \operatorname{Ei}\right.}{emx}$$

$$= -\frac{bn}{emx} - \frac{be^{\frac{d}{em}}n(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d}{em}-\frac{\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{emx} + \frac{e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(\right.}{}$$

Mathematica [A]
time = 0.08, size = 87, normalized size = 0.65

$$\frac{-bemn+e^{\frac{d}{em}}(fx^m)^{\frac{1}{m}}\operatorname{Ei}\left(-\frac{d+e\log(fx^m)}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))}{e^2m^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*Log[f*x^m])),x]
```

```
[Out] (-(b*e*m*n) + E^(d/(e*m))*(f*x^m)^m^(-1)*ExpIntegralEi[-((d + e*Log[f*x^m])
/(e*m))]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(e^2*m^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2296, normalized size = 17.26

method	result	size
risch	Expression too large to display	2296

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/x^2/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)/e/m/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I
*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2
*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I
*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*
f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/
m)-b/e/m/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*
f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*
e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*
x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m
)*ln(x^n)-b*n/e/m/x-1/2*I*b*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*cs
gn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*c
sgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/
2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^
2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(
ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*b
*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*
f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*
e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*
x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f
*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m
)*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I*b*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/
2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)
^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei
(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csg
n(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln
(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*
I*b*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn
```

```
(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)
^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn
(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(
I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/
e/m)*Pi*csgn(I*f*x^m)^3+b*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-I*Pi*csgn
(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csg
n(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,ln(x)-1/2*
I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-
e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln
(x^m)-m*ln(x))+2*I*d)/e/m)*ln(f)+b*n/e/m^2/x*f^(1/m)*(x^m)^(1/m)*exp(1/2*(-
I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e
+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,l
n(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*
f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+
2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^m)+b*n/e^2/m^2/x*f^(1/m)*(x^m)^(1/
m)*exp(1/2*(-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn
(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d
)/e/m)*Ei(1,ln(x)-1/2*I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn
(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3
+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

Fricas [A]
time = 0.41, size = 83, normalized size = 0.62

$$-\frac{\left(bmne - (bmxe\log\left(c\right) - bnxe\log\left(f\right) - bdnx + amxe)e^{\left(\frac{(e\log(f)+d)e^{(-1)}}{m}\right)}\,\text{log\_integral}\left(\frac{e^{\left(-\frac{(e\log(f)+d)e^{(-1)}}{m}\right)}}{x}\right)\right)e^{(-2)}}{m^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] -(b*m*n*e - (b*m*x*e*log(c) - b*n*x*e*log(f) - b*d*n*x + a*m*x*e)*e^((e*log
(f) + d)*e^(-1)/m)*log_integral(e^(-(e*log(f) + d)*e^(-1)/m)/x))*e^(-2)/(m^
2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log\left(cx^n\right)}{x^2 \left(d + e \log\left(f x^m\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x**2/(d+e*ln(f*x**m)),x)
[Out] Integral((a + b*log(c*x**n))/(x**2*(d + e*log(f*x**m))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^2/(d+e*log(f*x^m)),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\ln\left(c\,x^n\right)}{x^2\,\left(d + e\,\ln\left(f\,x^m\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))),x)
[Out] int((a + b*log(c*x^n))/(x^2*(d + e*log(f*x^m))), x)
```

3.175 $\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx$

Optimal. Leaf size=141

$$-\frac{bn}{2emx^2}-\frac{be^{\frac{2d}{em}}n(fx^m)^{2/m}\operatorname{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(d+e\log(fx^m))}{e^2m^2x^2}+\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\operatorname{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log$$

```
[Out] -1/2*b*n/e/m/x^2-b*exp(2*d/e/m)*n*(f*x^m)^(2/m)*Ei(-2*(d+e*ln(f*x^m))/e/m)*
(d+e*ln(f*x^m))/e^2/m^2/x^2+exp(2*d/e/m)*(f*x^m)^(2/m)*Ei(-2*(d+e*ln(f*x^m)
)/e/m)*(a+b*ln(c*x^n))/e/m/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2347, 2209, 2413, 12, 15, 6617}

$$\frac{e^{\frac{2d}{em}}(fx^m)^{2/m}(a+b\log(cx^n))\operatorname{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{emx^2}-\frac{bne^{\frac{2d}{em}}(fx^m)^{2/m}(d+e\log(fx^m))\operatorname{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)}{e^2m^2x^2}-\frac{bn}{2emx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

```
[Out] -1/2*(b*n)/(e*m*x^2) - (b*E^((2*d)/(e*m))*n*(f*x^m)^(2/m)*ExpIntegralEi[(-2
*(d + e*Log[f*x^m]))/(e*m)]*(d + e*Log[f*x^m]))/(e^2*m^2*x^2) + (E^((2*d)/(
e*m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log[f*x^m]))/(e*m)]*(a + b*Log
[c*x^n]))/(e*m*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6617

```
Int[ExpIntegralEi[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(ExpInte
gralEi[a + b*x]/b), x] - Simp[E^(a + b*x)/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x^3(d+e\log(fx^m))}\,dx = \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2} - (bn)\int \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\Big(}{em}$$

$$= \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n\right)\int \frac{(fx^m)^{2/m}\mathrm{Ei}\big(-}{x}}{em}$$

$$= \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n(fx^m)^{2/m}\right)\int \frac{\mathrm{Ei}\big(}{}}{emx^2}$$

$$= \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2} - \frac{\left(be^{\frac{2d}{em}}n(fx^m)^{2/m}\right)\mathrm{Subst}}{}$$

$$= \frac{e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(a+b\log(cx^n))}{emx^2} + \frac{\left(be^{\frac{2d}{em}}n(fx^m)^{2/m}\right)\mathrm{Subst}}{}$$

$$= -\frac{bn}{2emx^2} - \frac{be^{\frac{2d}{em}}n(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2d}{em}-\frac{2\log(fx^m)}{m}\right)\left(\frac{d}{em}+\frac{\log(fx^m)}{m}\right)}{emx^2} + \frac{e^{\frac{2d}{em}}(fx^m}{}$$

Mathematica [A]
time = 0.08, size = 94, normalized size = 0.67

$$\frac{-bemn+2e^{\frac{2d}{em}}(fx^m)^{2/m}\,\mathrm{Ei}\left(-\frac{2(d+e\log(fx^m))}{em}\right)(aem-bdn-ben\log(fx^m)+bem\log(cx^n))}{2e^2m^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*Log[f*x^m])),x]
```

```
[Out] (-(b*e*m*n) + 2*E^((2*d)/(e*m))*(f*x^m)^(2/m)*ExpIntegralEi[(-2*(d + e*Log[
f*x^m]))/(e*m)]*(a*e*m - b*d*n - b*e*n*Log[f*x^m] + b*e*m*Log[c*x^n]))/(2*e
^2*m^2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2341, normalized size = 16.60

method	result	size
risch	Expression too large to display	2341

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/x^3/(d+e*ln(f*x^m)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*a)/e/m/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f
*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e
-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^
m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)-b
/e/m/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)
*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi
*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csg
n(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+
e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*ln(x^n
)-1/2*b*n/e/m/x^2-1/2*I*b*n/e/m^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I
*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(
I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e
*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi
*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m
)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I*b*n/e/m
^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e
+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*c
sgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(
I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*
Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(
I*f)*csgn(I*f*x^m)^2+1/2*I*b*n/e/m^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csg
n(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*cs
gn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I
*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e
*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(
x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*I*b*n/e/m^2/x^
```

```
2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi
*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I
*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x
^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2+e*Pi*cs
gn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/e/m)*Pi*csgn(I*f*x
^m)^3+b*n/e/m^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^m)*cs
gn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I*f*x^
m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)*csgn
(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*csgn(
I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*I*d)/
e/m)*ln(f)+b*n/e/m^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*csgn(I*x^
m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^m)*csgn(I
*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csgn(I*x^m)*
csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*ln(x))+2*
I*d)/e/m)*ln(x^m)+b*n/e^2/m^2/x^2*f^(2/m)*(x^m)^(2/m)*exp((-I*Pi*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)*e+I*Pi*csgn(I*f)*csgn(I*f*x^m)^2*e+I*Pi*csgn(I*x^
m)*csgn(I*f*x^m)^2*e-I*Pi*csgn(I*f*x^m)^3*e+2*d)/e/m)*Ei(1,2*ln(x)-I*(e*Pi*
csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-e*Pi*csgn(I*f)*csgn(I*f*x^m)^2-e*Pi*csg
n(I*x^m)*csgn(I*f*x^m)^2+e*Pi*csgn(I*f*x^m)^3+2*I*e*ln(f)+2*I*e*(ln(x^m)-m*
ln(x))+2*I*d)/e/m)*d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

Fricas [A]
time = 0.37, size = 92, normalized size = 0.65

$$-\frac{\left(bmne - 2\left(bmx^2e\log\left(c\right) - bnx^2e\log\left(f\right) - bdnx^2 + amx^2e\right)e^{\left(\frac{2\left(e\log\left(f\right)+d\right)e^{(-1)}}{m}\right)}\operatorname{log\_integral}\left(\frac{e^{\left(-\frac{2\left(e\log\left(f\right)+d\right)e^{(-1)}}{m}\right)}}{x^2}\right)\right)e^{(-2)}}{2\,m^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*m*n*e - 2*(b*m*x^2*e*log(c) - b*n*x^2*e*log(f) - b*d*n*x^2 + a*m*x^
2*e)*e^(2*(e*log(f) + d)*e^(-1)/m)*log_integral(e^(-2*(e*log(f) + d)*e^(-1)
/m)/x^2))*e^(-2)/(m^2*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log(cx^n)}{x^3(d + e\log(fx^m))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x**3/(d+e*ln(f*x**m)),x)
```

```
[Out] Integral((a + b*log(c*x**n))/(x**3*(d + e*log(f*x**m))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x^3/(d+e*log(f*x^m)),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)/((e*log(f*x^m) + d)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\ln(c\,x^n)}{x^3\,\left(d + e\,\ln(f\,x^m)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))),x)
```

```
[Out] int((a + b*log(c*x^n))/(x^3*(d + e*log(f*x^m))), x)
```

3.176 $\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx$

Optimal. Leaf size=89

$$\frac{e^{-\frac{d}{en}}(-bd+ae+ben)x(cx^n)^{-1/n}\,\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{(bd-ae)x}{e^2n\,(d+e\log(cx^n))}$$

```
[Out] (b*e*n+a*e-b*d)*x*Ei((d+e*ln(c*x^n))/e/n)/e^3/exp(d/e/n)/n^2/((c*x^n)^(1/n)
)+(-a*e+b*d)*x/e^2/n/(d+e*ln(c*x^n))
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.52, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2407, 2334, 2337, 2209}

$$-\frac{x(cx^n)^{-1/n}\,e^{-\frac{d}{en}}(bd-ae)\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{x(bd-ae)}{e^2n\,(e\log(cx^n)+d)}+\frac{bx(cx^n)^{-1/n}\,e^{-\frac{d}{en}}\text{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

```
[Out] -(((b*d - a*e)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^3*E^(d/(e*n))*
n^2*(c*x^n)^n^(-1))) + (b*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(e^2*E
^(d/(e*n))*n*(c*x^n)^n^(-1)) + ((b*d - a*e)*x)/(e^2*n*(d + e*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2407

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log(cx^n)}{(d+e\log(cx^n))^2}\,dx &= \int \left(\frac{-bd+ae}{e\,(d+e\log(cx^n))^2}+\frac{b}{e\,(d+e\log(cx^n))}\right)dx\\ &= \frac{b\int \frac{1}{d+e\log(cx^n)}\,dx}{e}+\frac{(-bd+ae)\int \frac{1}{(d+e\log(cx^n))^2}\,dx}{e}\\ &= \frac{(bd-ae)x}{e^2n\,(d+e\log(cx^n))}-\frac{(bd-ae)\int \frac{1}{d+e\log(cx^n)}\,dx}{e^2n}+\frac{\left(bx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{n}}}{d+ex}\,d\right.}{en}\\ &= \frac{be^{-\frac{d}{en}}x(cx^n)^{-1/n}\operatorname{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}+\frac{(bd-ae)x}{e^2n\,(d+e\log(cx^n))}-\frac{\left((bd-ae)x(cx^n)^{-1/n}\right)}{}\\ &= -\frac{(bd-ae)e^{-\frac{d}{en}}x(cx^n)^{-1/n}\operatorname{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^3n^2}+\frac{be^{-\frac{d}{en}}x(cx^n)^{-1/n}\operatorname{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)}{e^2n}+\frac{}{e}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 87, normalized size = 0.98

$$\frac{e^{-\frac{d}{en}}(-bd+ae+ben)x(cx^n)^{-1/n}\operatorname{Ei}\left(\frac{d+e\log(cx^n)}{en}\right)-\frac{e(-bd+ae)nx}{d+e\log(cx^n)}}{e^3n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*Log[c*x^n])^2,x]
```

```
[Out] (((-(b*d) + a*e + b*e*n)*x*ExpIntegralEi[(d + e*Log[c*x^n])/(e*n)])/(E^(d/(
e*n))*(c*x^n)^n^(-1)) - (e*(-(b*d) + a*e)*n*x)/(d + e*Log[c*x^n]))/(e^3*n^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 370, normalized size = 4.16

method	result
risch	$-\frac{2x(ae-bd)}{e^2n\left(2d+2e\ln(c)+2e\ln(x^n)-ie\pi\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)+ie\pi\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2+ie\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2-ie\pi\operatorname{csgn}(icx^n\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/(d+e*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/e^2/n*x*(a*e-b*d)/(2*d+2*e*ln(c)+2*e*ln(x^n)-I*e*Pi*csgn(I*c)*csgn(I*x^n
)*csgn(I*c*x^n)+I*e*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*e*Pi*csgn(I*x^n)*csgn(I*
c*x^n)^2-I*e*Pi*csgn(I*c*x^n)^3)-(b*e*n+a*e-b*d)/e^3/n^2*x*(x^n)^(-1/n)*c^(
-1/n)*exp(-1/2*(-I*e*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*e*Pi*csgn(I*c
)*csgn(I*c*x^n)^2+I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*csgn(I*c*x^n)^3
+2*d)/e/n)*Ei(1,-ln(x)-1/2*(-I*e*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*e
*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*e*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*e*Pi*csg
n(I*c*x^n)^3+2*e*ln(c)+2*e*(ln(x^n)-n*ln(x))+2*d)/e/n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*d - (b*n + a)*e)*integrate(1/(d*n*e^2 + n*e^3*log(c) + n*e^3*log(x^n)),
 x) + (b*d - a*e)*x/(d*n*e^2 + n*e^3*log(c) + n*e^3*log(x^n))
```

Fricas [A]
time = 0.36, size = 151, normalized size = 1.70

$$\frac{\left((bdnxe - anxe^2)e^{\left(\frac{(e\log(c)+d)e^{(-1)}}{n}\right)} - (bd^2 - (bdn + ad)e + (bde - (bn + a)e^2)\log(c) + (bdne - (bn^2 + an)e^2)\log(x))\,\mathrm{log\_integral}\left(xe^{\left(\frac{(e\log(c)+d)e^{(-1)}}{n}\right)}\right)\right)e^{\left(-\frac{(e\log(c)+d)e^{(-1)}}{n}\right)}}{n^3e^4\log(x) + dn^2e^3 + n^2e^4\log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] ((b*d*n*x*e - a*n*x*e^2)*e^((e*log(c) + d)*e^(-1)/n) - (b*d^2 - (b*d*n + a*
d)*e + (b*d*e - (b*n + a)*e^2)*log(c) + (b*d*n*e - (b*n^2 + a*n)*e^2)*log(x
))*log_integral(x*e^((e*log(c) + d)*e^(-1)/n)))*e^(-(e*log(c) + d)*e^(-1)/n
)/(n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log(cx^n)}{(d + e\log(cx^n))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/(d+e*ln(c*x**n))**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))/(d + e*log(c*x**n))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(89) = 178.
time = 4.72, size = 661, normalized size = 7.43

$$\frac{bdnxe}{n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c)}+\frac{bn^2\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+2\right)}\log(x)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}-\frac{bdn\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+1\right)}\log(x)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}-\frac{anxe^2}{n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c)}+\frac{bdn\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+1\right)}}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}-\frac{bd^2\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}\right)}}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}+\frac{bn\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+2\right)}\log(c)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}-\frac{bd\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+1\right)}\log(c)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}+\frac{an\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+2\right)}\log(x)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}+\frac{ad\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+1\right)}}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}+\frac{a\mathrm{Ei}\left(\frac{de^{(-1)}}{n}+\frac{\log(c)}{n}+\log(x)\right)e^{\left(-\frac{de^{(-1)}}{n}+2\right)}\log(c)}{(n^3e^4\log(x)+dn^2e^3+n^2e^4\log(c))c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d+e*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b*d*n*x*e/(n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)) + b*n^2*Ei(d*e^(-1)
/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 2)*log(x)/((n^3*e^4*log(x) + d*n^2
*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*
e^(-d*e^(-1)/n + 1)*log(x)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c
^(1/n)) - a*n*x*e^2/(n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)) + b*d*n*E
i(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 1)/((n^3*e^4*log(x) + d*
n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d^2*Ei(d*e^(-1)/n + log(c)/n + log(x
))*e^(-d*e^(-1)/n)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n))
+ b*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 2)*log(c)/((n^3*e
^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) - b*d*Ei(d*e^(-1)/n + log(
c)/n + log(x))*e^(-d*e^(-1)/n + 1)*log(c)/((n^3*e^4*log(x) + d*n^2*e^3 + n^
2*e^4*log(c))*c^(1/n)) + a*n*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1
)/n + 2)*log(x)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) + a
*d*Ei(d*e^(-1)/n + log(c)/n + log(x))*e^(-d*e^(-1)/n + 1)/((n^3*e^4*log(x)
+ d*n^2*e^3 + n^2*e^4*log(c))*c^(1/n)) + a*Ei(d*e^(-1)/n + log(c)/n + log(x
))*e^(-d*e^(-1)/n + 2)*log(c)/((n^3*e^4*log(x) + d*n^2*e^3 + n^2*e^4*log(c)
)*c^(1/n))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\ln(c\,x^n)}{\left(d+e\ln(c\,x^n)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2,x)
```

```
[Out] int((a + b*log(c*x^n))/(d + e*log(c*x^n))^2, x)
```

3.177 $\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx$

Optimal. Leaf size=29

$$bn\log(x) - bn\log(x)\log(\log(x)) + (a + b\log(cx^n))\log(\log(x))$$

```
[Out] b*n*ln(x)-b*n*ln(x)*ln(ln(x))+(a+b*ln(c*x^n))*ln(ln(x))
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2339, 29, 2413, 2601}

$$\log(\log(x))\,(a + b\log(cx^n)) + bn\log(x) - bn\log(\log(x))\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

```
[Out] b*n*Log[x] - b*n*Log[x]*Log[Log[x]] + (a + b*Log[c*x^n])*Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{a+b\log(cx^n)}{x\log(x)}\,dx = (a+b\log(cx^n))\log(\log(x)) - (bn)\int \frac{\log(\log(x))}{x}\,dx$$
$$= bn\log(x) - bn\log(x)\log(\log(x)) + (a+b\log(cx^n))\log(\log(x))$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.97

$$bn\log(x) + a\log(\log(x)) + b(-n\log(x) + \log(cx^n))\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])/(x*Log[x]),x]
```

```
[Out] b*n*Log[x] + a*Log[Log[x]] + b*(-(n*Log[x]) + Log[c*x^n])*Log[Log[x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 131, normalized size = 4.52

method	result
risch	$-bn\ln(x)\ln(\ln(x)) + \ln(\ln(x))\ln(x^n)\,b + \ln(x)\,bn - \frac{i\ln(\ln(x))b\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(icx^n)}{2} + \frac{i\ln(\ln(x))}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))/x/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -b*n*ln(x)*ln(ln(x))+ln(ln(x))*ln(x^n)*b+ln(x)*b*n-1/2*I*ln(ln(x))*b*Pi*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*ln(ln(x))*b*Pi*csgn(I*c)*csgn(I*c*x^
n)^2+1/2*I*ln(ln(x))*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*ln(ln(x))*b*Pi*
csgn(I*c*x^n)^3+ln(ln(x))*b*ln(c)+ln(ln(x))*a
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.10

$$-(\log(x)\log(\log(x)) - \log(x))bn + b\log(cx^n)\log(\log(x)) + a\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="maxima")
```

```
[Out] -(log(x)*log(log(x)) - log(x))*b*n + b*log(c*x^n)*log(log(x)) + a*log(log(x
))
```

Fricas [A]
time = 0.36, size = 16, normalized size = 0.55

$$bn\log(x) + (b\log(c) + a)\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="fricas")
```

```
[Out] b*n*log(x) + (b*log(c) + a)*log(log(x))
```

Sympy [A]
time = 5.21, size = 32, normalized size = 1.10

$$a\log\left(\log\left(x\right)\right) - b(n(\log\left(x\right)\log\left(\log\left(x\right)\right) - \log\left(x\right)) - \log\left(cx^{n}\right)\log\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x/ln(x),x)
```

```
[Out] a*log(log(x)) - b*(n*(log(x)*log(log(x)) - log(x)) - log(c*x**n)*log(log(x)
))
```

Giac [A]
time = 3.92, size = 17, normalized size = 0.59

$$bn\log\left(x\right) + \left(b\log\left(c\right) + a\right)\log\left(\left|\log\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/x/log(x),x, algorithm="giac")
```

```
[Out] b*n*log(x) + (b*log(c) + a)*log(abs(log(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\ln\left(c\,x^n\right)}{x\,\ln\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/(x*log(x)),x)
```

```
[Out] int((a + b*log(c*x^n))/(x*log(x)), x)
```

3.178 $\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx$

Optimal. Leaf size=347

$$-\frac{ee^{-\frac{a(1+m)}{bn}}rx(gx)^m(cx^n)^{-\frac{1+m}{n}}\Gamma\left(2+p,-\frac{a(1+m)}{bn}-\frac{(1+m)\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{(1+m)^2}-$$

```
[Out] -e*r*x*(g*x)^m*GAMMA(2+p,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/
exp(a*(1+m)/b/n)/(1+m)^2/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^
p)-e*r*x*(g*x)^m*GAMMA(1+p,-a*(1+m)/b/n-(1+m)*ln(c*x^n)/n)*(a+b*ln(c*x^n))^
(1+p)/b/exp(a*(1+m)/b/n)/(1+m)/n/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n
))/b/n)^p)+(g*x)^(1+m)*GAMMA(1+p,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n)
)^p*(d+e*ln(f*x^r))/exp(a*(1+m)/b/n)/g/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(
a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A]

time = 0.25, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$\frac{(gx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{g(m+1)}-\frac{erx(gx)^me^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\text{Gamma}\left(p+2,-\frac{a(m+1)}{bn}-\frac{(m+1)\log(cx^n)}{n}\right)}{(m+1)^2}-\frac{erx(gx)^me^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^{p+1}\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a(m+1)}{bn}-\frac{(m+1)\log(cx^n)}{n}\right)}{b(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((e*r*x*(g*x)^m*Gamma[2 + p, -((a*(1 + m))/(b*n)) - ((1 + m)*Log[c*x^n])/n
]*(a + b*Log[c*x^n])^p)/(E^((a*(1 + m))/(b*n))*(1 + m)^2*(c*x^n)^((1 + m)/n
)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p)) - (e*r*x*(g*x)^m*Gamma[1 + p,
 -((a*(1 + m))/(b*n)) - ((1 + m)*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))
/(b*E^((a*(1 + m))/(b*n))*(1 + m)*n*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*
Log[c*x^n]))/(b*n)))^p) + ((g*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log
[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((a*(1 + m))/
(b*n))*g*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n))
)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int (gx)^m (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx = \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(gx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a}{g(1+}$$

$$= -\frac{ee^{-\frac{a(1+m)}{bn}}rx(gx)^m(cx^n)^{-\frac{1+m}{n}}\Gamma\left(2+p,-\frac{a(1+m)}{bn}-\frac{(1+m)\log(}{n}\right.}{(1+m)^2}$$

Mathematica [A]
time = 0.67, size = 179, normalized size = 0.52

$$-\frac{e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-m}(gx)^m(a+b\log(cx^n))^{-1+p}\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)+(1+m)\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)}{(1+m)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -(((g*x)^m*(a + b*Log[c*x^n])^(-1 + p)*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n
)))^(1 - p)*(-(b*e*n*r*Gamma[2 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))])
 + (1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(b*d*n - a*e
*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/(E^(((1 + m)*(a - b*n*Log[x] +
b*Log[c*x^n]))/(b*n))*(1 + m)^3*x^m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (gx)^m (a+b\ln(c\,x^n))^p (d+e\ln(f\,x^r))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

```
[Out] int((g*x)^m*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas"
)
```

```
[Out] integral(((g*x)^m*e*log(f*x^r) + (g*x)^m*d)*(b*log(c*x^n) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming t_nostep near 0Simplification ass
uming sageVARg near 0Simplification assuming t_nostep near 0Simplification
assuming
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d+e\ln (f\,x^r))\ (g\,x)^m\,(a+b\ln (c\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p,x)
```

```
[Out] int((d + e*log(f*x^r))*(g*x)^m*(a + b*log(c*x^n))^p, x)
```

3.179 $\int x^2(a+b\log(cx^n))^p (d+e\log(fx^r))\,dx$

Optimal. Leaf size=298

$$-3^{-2-p}ee^{-\frac{3a}{bn}}rx^3(cx^n)^{-3/n}\,\Gamma\left(2+p,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}-\frac{3^{-1-p}ee^{-}}{}$$

```
[Out] -3^(-2-p)*e*r*x^3*GAMMA(2+p,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(3
*a/b/n)/((c*x^n)^(3/n))/(((-a-b*ln(c*x^n))/b/n)^p)-3^(-1-p)*e*r*x^3*GAMMA(1
+p,-3*a/b/n-3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/exp(3*a/b/n)/n/((c*x^n)^
(3/n))/(((-a-b*ln(c*x^n))/b/n)^p)+3^(-1-p)*x^3*GAMMA(1+p,-3*(a+b*ln(c*x^n))
/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(3*a/b/n)/((c*x^n)^(3/n))/(((-a-
b*ln(c*x^n))/b/n)^p)
```

Rubi [A]

time = 0.17, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)+e\left(-3^{-p-2}\right)rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)-\frac{e3^{-p-1}rx^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^{p+1}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((3^(-2 - p)*e*r*x^3*Gamma[2 + p, (-3*a)/(b*n) - (3*Log[c*x^n])/n]*(a + b*
Log[c*x^n])^p)/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(-((a + b*Log[c*x^n])/(b*n)))
^p)) - (3^(-1 - p)*e*r*x^3*Gamma[1 + p, (-3*a)/(b*n) - (3*Log[c*x^n])/n]*(a
 + b*Log[c*x^n])^(1 + p))/(b*E^((3*a)/(b*n))*n*(c*x^n)^(3/n)*(-((a + b*Log[
c*x^n])/(b*n)))^p) + (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*x^n]))/(
b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((3*a)/(b*n))*(c*x^n)^(3/
n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx &= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}\\
&= -3^{-2-p}ee^{-\frac{3a}{bn}}rx^3(cx^n)^{-3/n}\,\Gamma\left(2+p,-\frac{3a}{bn}-\frac{3\log(cx^n)}{n}\right)(a+
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 156, normalized size = 0.52

$$
-3^{-2-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{3(a+b\log(cx^n))}{bn}\right)+3\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((3^(-2 - p)*x^3*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))
^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]) + 3*Gamma[
1 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b
*e*n*Log[f*x^r])))/(E^((3*a)/(b*n))*(c*x^n)^(3/n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int x^2(a+b\ln(c\,x^n))^p(d+e\ln(f\,x^r))\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
[Out] integral((x^2*e*log(f*x^r) + d*x^2)*(b*log(c*x^n) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
[Out] int(x^2*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

3.180 $\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx$

Optimal. Leaf size=298

$$-2^{-2-p}ee^{-\frac{2a}{bn}}rx^2(cx^n)^{-2/n}\Gamma\left(2+p,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}-\frac{2^{-1-p}ee^{-}}{}$$

```
[Out] -2^(-2-p)*e*r*x^2*GAMMA(2+p,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(2
*a/b/n)/((c*x^n)^(2/n))/(((-a-b*ln(c*x^n))/b/n)^p)-2^(-1-p)*e*r*x^2*GAMMA(1
+p,-2*a/b/n-2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/exp(2*a/b/n)/n/((c*x^n)^
(2/n))/(((-a-b*ln(c*x^n))/b/n)^p)+2^(-1-p)*x^2*GAMMA(1+p,-2*(a+b*ln(c*x^n))
/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(2*a/b/n)/((c*x^n)^(2/n))/(((-a-
b*ln(c*x^n))/b/n)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)+e\left(-2^{-p-2}\right)rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)-\frac{e2^{-p-1}rx^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^{p+1}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((2^(-2 - p)*e*r*x^2*Gamma[2 + p, (-2*a)/(b*n) - (2*Log[c*x^n])/n]*(a + b*
Log[c*x^n])^p)/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(-((a + b*Log[c*x^n])/(b*n)))
^p)) - (2^(-1 - p)*e*r*x^2*Gamma[1 + p, (-2*a)/(b*n) - (2*Log[c*x^n])/n]*(a
 + b*Log[c*x^n])^(1 + p))/(b*E^((2*a)/(b*n))*n*(c*x^n)^(2/n)*(-((a + b*Log[
c*x^n])/(b*n)))^p) + (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*x^n]))/(
b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^((2*a)/(b*n))*(c*x^n)^(2/
n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int x(a+b\log(cx^n))^p(d+e\log(fx^r))\,dx = 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log$$

$$= -2^{-2-p}ee^{-\frac{2a}{bn}}rx^2(cx^n)^{-2/n}\Gamma\left(2+p,-\frac{2a}{bn}-\frac{2\log(cx^n)}{n}\right)(a+b$$

Mathematica [A]
time = 0.26, size = 156, normalized size = 0.52

$$-2^{-2-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{2(a+b\log(cx^n))}{bn}\right)+2\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((2^(-2 - p)*x^2*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))
^(1 - p)*(-(b*e*n*r*Gamma[2 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]) + 2*Gamma[
1 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b
*e*n*Log[f*x^r])))/(E^((2*a)/(b*n))*(c*x^n)^(2/n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))^p\,(d+e\ln(f\,x^r))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
[Out] integral((x*e*log(f*x^r) + d*x)*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\log{(cx^{n})})^{p} (d + e\log{(fx^{r})})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
[Out] Integral(x*(a + b*log(c*x**n))**p*(d + e*log(f*x**r)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
[Out] int(x*(d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

3.181 $\int (a+b\log(cx^n))^p (d+e\log(fx^r))\,dx$

Optimal. Leaf size=271

$$-ee^{-\frac{a}{bn}}rx(cx^n)^{-1/n}\Gamma\left(2+p,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}-\frac{ee^{-\frac{a}{bn}}rx(cx^n)^{-1/}}{}$$

```
[Out] -e*r*x*GAMMA(2+p,-a/b/n-ln(c*x^n)/n)*(a+b*ln(c*x^n))^p/exp(a/b/n)/((c*x^n)^
(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)-e*r*x*GAMMA(1+p,-a/b/n-ln(c*x^n)/n)*(a+b*
ln(c*x^n))^(1+p)/b/exp(a/b/n)/n/((c*x^n)^(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)+
x*GAMMA(1+p,(-a-b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/exp(a/b
/n)/((c*x^n)^(1/n))/(((-a-b*ln(c*x^n))/b/n)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2337, 2212, 2408, 12, 15, 19, 6692}

$$xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)-erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)-\frac{erxe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^{p+1}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((e*r*x*Gamma[2 + p, -(a/(b*n)) - Log[c*x^n]/n]*(a + b*Log[c*x^n])^p)/(E^(
a/(b*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)) - (e*r*x*Gamma[1
+ p, -(a/(b*n)) - Log[c*x^n]/n]*(a + b*Log[c*x^n])^(1 + p))/(b*E^(a/(b*n))*
n*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p) + (x*Gamma[1 + p, -((a +
b*Log[c*x^n])/(b*n))]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(E^(a/(b*n))
*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\log(cx^n))^p\,(d+e\log(fx^r))\,dx &= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\right.\\
&= -ee^{-\frac{a}{bn}}rx(cx^n)^{-1/n}\,\Gamma\left(2+p,-\frac{a}{bn}-\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n)
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 146, normalized size = 0.54

$$-e^{-\frac{a}{bn}}x(cx^n)^{-1/n}(a+b\log(cx^n))^{-1+p}\left(-\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(-benr\Gamma\left(2+p,-\frac{a+b\log(cx^n)}{bn}\right)+\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]),x]
```

```
[Out] -((x*(a + b*Log[c*x^n])^(-1 + p)*(-((a + b*Log[c*x^n])/(b*n)))^(1 - p)*(-(b
*e*n*r*Gamma[2 + p, -((a + b*Log[c*x^n])/(b*n))]) + Gamma[1 + p, -((a + b*L
og[c*x^n])/(b*n))]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/
(E^(a/(b*n))*(c*x^n)^n^(-1)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (a+b\ln(c\,x^n))^p\,(d+e\ln(f\,x^r))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

```
[Out] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.09, size = 134, normalized size = 0.49

$$-\frac{(bre\log(c) - bne\log(f) - bdn + (bnp + bn + a)re)e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1, -\frac{bn\log(x)+b\log(c)+a}{bn}\right) - (bnrxe\log(x) + brxe\log(c) + arxe)(bn\log(x) + b\log(c) + a)^p}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="fricas")
```

```
[Out] -((b*r*e*log(c) - b*n*e*log(f) - b*d*n + (b*n*p + b*n + a)*r*e)*e^(-(b*n*p*
log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(x) + b*log(c) +
 a)/(b*n)) - (b*n*r*x*e*log(x) + b*r*x*e*log(c) + a*r*x*e)*(b*n*log(x) + b*
log(c) + a)^p)/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\log(cx^n))^p (d + e\log(fx^r))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r)),x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r)),x, algorithm="giac")
```

```
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (d + e\ln(f\,x^r))\,(a + b\ln(c\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p,x)
[Out] int((d + e*log(f*x^r))*(a + b*log(c*x^n))^p, x)
```

3.182 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx$

Optimal. Leaf size=71

$$-\frac{er(a+b\log(cx^n))^{2+p}}{b^2n^2(1+p)(2+p)}+\frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)}$$

```
[Out] -e*r*(a+b*ln(c*x^n))^(2+p)/b^2/n^2/(1+p)/(2+p)+(a+b*ln(c*x^n))^(1+p)*(d+e*l
n(f*x^r))/b/n/(1+p)
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2339, 30, 2413, 12}

$$\frac{(d+e\log(fx^r))(a+b\log(cx^n))^{p+1}}{bn(p+1)}-\frac{er(a+b\log(cx^n))^{p+2}}{b^2n^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] -((e*r*(a + b*Log[c*x^n])^(2 + p))/(b^2*n^2*(1 + p)*(2 + p))) + ((a + b*Log
[c*x^n])^(1 + p)*(d + e*Log[f*x^r]))/(b*n*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
```

```
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x}\,dx = \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - (er)\int \frac{(a+b\log(cx^n))^{1}}{bn(1+p)x}$$

$$= \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - \frac{(er)\int \frac{(a+b\log(cx^n))^{1+p}}{x}\,dx}{bn(1+p)}$$

$$= \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)} - \frac{(er)\text{Subst}\left(\int x^{1+p}\,dx, x, \right.}{b^2n^2(1+p}$$

$$= -\frac{er(a+b\log(cx^n))^{2+p}}{b^2n^2(1+p)(2+p)} + \frac{(a+b\log(cx^n))^{1+p}(d+e\log(fx^r))}{bn(1+p)}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 1.00

$$\frac{(a+b\log(cx^n))^{1+p}(2bdn+bdnp-aer-ber\log(cx^n)+ben(2+p)\log(fx^r))}{b^2n^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^(1 + p)*(2*b*d*n + b*d*n*p - a*e*r - b*e*r*Log[c*x^n] +
 b*e*n*(2 + p)*Log[f*x^r]))/(b^2*n^2*(1 + p)*(2 + p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 854, normalized size = 12.03

method	result	size
risch	Expression too large to display	854

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(
I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*e*Pi*csg
n(I*f)*csgn(I*x^r)*csgn(I*f*x^r)+1/2*I*(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x
)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*
x^n))))^(1+p)/b/n/(1+p)*e*Pi*csgn(I*f)*csgn(I*f*x^r)^2+1/2*I*(ln(x)*b*n+a+b
*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(
```

```
-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*e*Pi*csgn(I*x^r)*csgn(I*f*x^r
)^2-1/2*I*(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-cs
gn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*e*Pi*
csgn(I*f*x^r)^3+(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n
)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)
*e*r*ln(x)+(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-c
sgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*e*ln
(f)+(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c
*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*e*(ln(x^r)-
r*ln(x))+(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n)*(-csg
n(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(1+p)/b/n/(1+p)*d-1/b^
2/n^2/(1+p)*e*r*(ln(x)*b*n+a+b*(ln(c)+ln(x^n)-n*ln(x)-1/2*I*Pi*csgn(I*c*x^n
)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n))))^(2+p)/(2+p)
```

Maxima [A]
time = 0.29, size = 97, normalized size = 1.37

$$\frac{(b\log(cx^n)+a)^{p+1}e\log(fx^r)}{bn(p+1)}+\frac{(b\log(cx^n)+a)^{p+1}d}{bn(p+1)}-\frac{(b\log(cx^n)+a)^{p+2}re}{b^2n^2(p+2)(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="maxima")
```

```
[Out] (b*log(c*x^n) + a)^(p + 1)*e*log(f*x^r)/(b*n*(p + 1)) + (b*log(c*x^n) + a)^
(p + 1)*d/(b*n*(p + 1)) - (b*log(c*x^n) + a)^(p + 2)*r*e/(b^2*n^2*(p + 2)*(
p + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(73) = 146.
time = 0.36, size = 229, normalized size = 3.23

$$-\frac{\left(b^2re\log(c)^2-abdnp-(b^2n^2p+b^2n^2)re\log(x)^2-2\,abdn+a^2re-(b^2dnp+2\,b^2dn-2\,abre)\log(c)-((b^2np+2\,b^2n)e\log(c)+(abnp+2\,abn)e)\log(f)-(b^2npre\log(c)+b^2dn^2p+abnpre+2\,b^2dn^2+(b^2n^2p+2\,b^2n^2)e\log(f))\log(x)\right)(bn\log(x)+b\log(c)+a)^p}{b^2n^2p^2+3\,b^2n^2p+2\,b^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="fricas")
```

```
[Out] -(b^2*r*e*log(c)^2 - a*b*d*n*p - (b^2*n^2*p + b^2*n^2)*r*e*log(x)^2 - 2*a*b
*d*n + a^2*r*e - (b^2*d*n*p + 2*b^2*d*n - 2*a*b*r*e)*log(c) - ((b^2*n*p + 2
*b^2*n)*e*log(c) + (a*b*n*p + 2*a*b*n)*e)*log(f) - (b^2*n*p*r*e*log(c) + b^
2*d*n^2*p + a*b*n*p*r*e + 2*b^2*d*n^2 + (b^2*n^2*p + 2*b^2*n^2)*e*log(f))*l
og(x))*(b*n*log(x) + b*log(c) + a)^p/(b^2*n^2*p^2 + 3*b^2*n^2*p + 2*b^2*n^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\log(cx^n))^p\,(d+e\log(fx^r))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(73) = 146.
time = 5.01, size = 246, normalized size = 3.46

$$\frac{\frac{(bn\log(x)+b\log(c)+a)^{p+1}e\log(f)}{p+1}+\frac{(bn\log(x)+b\log(c)+a)^{p+1}d}{p+1}-\frac{\left((bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^{p}bp\log(c)-(bn\log(x)+b\log(c)+a)^{2}(bn\log(x)+b\log(c)+a)^{p}p+(bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^{p}ap+2\,(bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^{p}b\log(c)-(bn\log(x)+b\log(c)+a)^{2}(bn\log(x)+b\log(c)+a)^{p}+2\,(bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^{p}a\right)re}{(p^{2}+3\,p+2)bn}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x,x, algorithm="giac")
```

```
[Out] ((b*n*log(x) + b*log(c) + a)^(p + 1)*e*log(f)/(p + 1) + (b*n*log(x) + b*log
(c) + a)^(p + 1)*d/(p + 1) - ((b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*l
og(c) + a)^p*b*p*log(c) - (b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*log
(c) + a)^p*p + (b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*a*
p + 2*(b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*b*log(c) -
(b*n*log(x) + b*log(c) + a)^2*(b*n*log(x) + b*log(c) + a)^p + 2*(b*n*log(x)
 + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p*a)*r*e/((p^2 + 3*p + 2)*b*n)
)/(b*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+e\,\ln\left(f\,x^r\right)\right)\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^p}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x,x)
```

```
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x, x)
```

3.183 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^2}\,dx$

Optimal. Leaf size=260

$$-\frac{ee^{\frac{a}{bn}}r(cx^n)^{\frac{1}{n}}\,\Gamma\left(2+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}+\frac{ee^{\frac{a}{bn}}r(cx^n)^{\frac{1}{n}}\,\Gamma\left(1+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)(}{bn}$$

```
[Out] -e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(2+p,a/b/n+ln(c*x^n)/n)*(a+b*ln(c*x^n))^
p/x/(((a+b*ln(c*x^n))/b/n)^p)+e*exp(a/b/n)*r*(c*x^n)^(1/n)*GAMMA(1+p,a/b/n+
ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x/(((a+b*ln(c*x^n))/b/n)^p)-exp(a/b/
n)*(c*x^n)^(1/n)*GAMMA(1+p,(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f
*x^r))/x/(((a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}-\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)}{x}+\frac{ere^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^{p+1}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)}{bnx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((e*E^(a/(b*n))*r*(c*x^n)^n^(-1)*Gamma[2 + p, a/(b*n) + Log[c*x^n]/n]*(a +
 b*Log[c*x^n])^p)/(x*((a + b*Log[c*x^n])/(b*n))^p)) + (e*E^(a/(b*n))*r*(c*x
^n)^n^(-1)*Gamma[1 + p, a/(b*n) + Log[c*x^n]/n]*(a + b*Log[c*x^n])^(1 + p))
/(b*n*x*((a + b*Log[c*x^n])/(b*n))^p) - (E^(a/(b*n))*(c*x^n)^n^(-1)*Gamma[1
 + p, (a + b*Log[c*x^n])/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x
*((a + b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p (d+e\log(fx^r))}{x^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-}}{x}$$

$$= -\frac{ee^{\frac{a}{bn}}r(cx^n)^{\frac{1}{n}}\Gamma\left(2+p,\frac{a}{bn}+\frac{\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right.}{x}$$

Mathematica [A]
time = 0.24, size = 141, normalized size = 0.54

$$-\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{a+b\log(cx^n)}{bn}\right)+\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^2,x]
```

```
[Out] -((E^(a/(b*n))*(c*x^n)^n^(-1)*(a + b*Log[c*x^n])^(-1 + p)*((a + b*Log[c*x^n
])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (a + b*Log[c*x^n])/(b*n)] + Gamma[1
 + p, (a + b*Log[c*x^n])/(b*n)]*(b*d*n - a*e*r - b*e*r*Log[c*x^n] + b*e*n*L
og[f*x^r])))/x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^p (d+e\ln(f\,x^r))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="fricas")
```

```
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log{(cx^n)})^p (d + e\log{(fx^r)})}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2,x)
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^2, x)
```

3.184 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^3}\,dx$

Optimal. Leaf size=295

$$-\frac{2^{-2-p}ee^{\frac{2a}{bn}}r(cx^n)^{2/n}\,\Gamma\left(2+p,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}+\frac{2^{-1-p}ee^{\frac{2a}{bn}}r(cx^n)^{2/n}\,\Gamma\Big(1+$$

```
[Out] -2^(-2-p)*e*exp(2*a/b/n)*r*(c*x^n)^(2/n)*GAMMA(2+p,2*a/b/n+2*ln(c*x^n)/n)*(
a+b*ln(c*x^n))^p/x^2/(((a+b*ln(c*x^n))/b/n)^p)+2^(-1-p)*e*exp(2*a/b/n)*r*(c
*x^n)^(2/n)*GAMMA(1+p,2*a/b/n+2*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x^2/
(((a+b*ln(c*x^n))/b/n)^p)-2^(-1-p)*exp(2*a/b/n)*(c*x^n)^(2/n)*GAMMA(1+p,2*(
a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^2/(((a+b*ln(c*x^n))
/b/n)^p)
```

Rubi [A]
time = 0.16, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$-\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}-\frac{e2^{-p-2}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)}{x^2}+\frac{e2^{-p-1}re^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^{p+1}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)}{bnx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -((2^(-2 - p)*e*E^((2*a)/(b*n))*r*(c*x^n)^(2/n)*Gamma[2 + p, (2*a)/(b*n) +
(2*Log[c*x^n])/n]*(a + b*Log[c*x^n])^p)/(x^2*((a + b*Log[c*x^n])/(b*n))^p))
 + (2^(-1 - p)*e*E^((2*a)/(b*n))*r*(c*x^n)^(2/n)*Gamma[1 + p, (2*a)/(b*n) +
 (2*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))/(b*n*x^2*((a + b*Log[c*x^n])
/(b*n))^p) - (2^(-1 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[1 + p, (2*(a +
 b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^2*((a +
b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p\,(d+e\log(fx^r))}{x^3}\,dx = -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+\cdots}{\cdots}\right.}{x^2}$$

$$= -\frac{2^{-2-p}ee^{\frac{2a}{bn}}r(cx^n)^{2/n}\,\Gamma\left(2+p,\frac{2a}{bn}+\frac{2\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\cdots}{x^2}$$

Mathematica [A]
time = 0.26, size = 154, normalized size = 0.52

$$-\frac{2^{-2-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{2(a+b\log(cx^n))}{bn}\right)+2\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^3,x]
```

```
[Out] -((2^(-2 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*(a + b*Log[c*x^n])^(-1 + p)*((a
 + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (2*(a + b*Log[c*x^n])
)/(b*n)] + 2*Gamma[1 + p, (2*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*
e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^p\,(d+e\ln(f\,x^r))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="fricas")
```

```
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log{(cx^n)})^p \, (d + e \log{(fx^r)})}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*(d + e*log(f*x**r))/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,\ln(f\,x^r))\,\,(a+b\,\ln(c\,x^n))^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3,x)
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^3, x)
```

3.185 $\int \frac{(a+b\log(cx^n))^p(d+e\log(fx^r))}{x^4}\,dx$

Optimal. Leaf size=295

$$-\frac{3^{-2-p}ee^{\frac{3a}{bn}}r(cx^n)^{3/n}\,\Gamma\left(2+p,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}+\frac{3^{-1-p}ee^{\frac{3a}{bn}}r(cx^n)^{3/n}\,\Gamma\Big(1+p}{}$$

```
[Out] -3^(-2-p)*e*exp(3*a/b/n)*r*(c*x^n)^(3/n)*GAMMA(2+p,3*a/b/n+3*ln(c*x^n)/n)*(
a+b*ln(c*x^n))^p/x^3/(((a+b*ln(c*x^n))/b/n)^p)+3^(-1-p)*e*exp(3*a/b/n)*r*(c
*x^n)^(3/n)*GAMMA(1+p,3*a/b/n+3*ln(c*x^n)/n)*(a+b*ln(c*x^n))^(1+p)/b/n/x^3/
(((a+b*ln(c*x^n))/b/n)^p)-3^(-1-p)*exp(3*a/b/n)*(c*x^n)^(3/n)*GAMMA(1+p,3*(
a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^3/(((a+b*ln(c*x^n))
/b/n)^p)
```

Rubi [A]
time = 0.16, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2347, 2212, 2413, 12, 15, 19, 6692}

$$-\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}(d+e\log(fx^r))(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}-\frac{e3^{-p-2}re^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+2,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)}{x^3}+\frac{e3^{-p-1}re^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^{p+1}\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\text{Gamma}\left(p+1,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)}{bnx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -((3^(-2 - p)*e*E^((3*a)/(b*n))*r*(c*x^n)^(3/n)*Gamma[2 + p, (3*a)/(b*n) +
(3*Log[c*x^n])/n]*(a + b*Log[c*x^n])^p)/(x^3*((a + b*Log[c*x^n])/(b*n))^p))
 + (3^(-1 - p)*e*E^((3*a)/(b*n))*r*(c*x^n)^(3/n)*Gamma[1 + p, (3*a)/(b*n) +
 (3*Log[c*x^n])/n]*(a + b*Log[c*x^n])^(1 + p))/(b*n*x^3*((a + b*Log[c*x^n])
/(b*n))^p) - (3^(-1 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*Gamma[1 + p, (3*(a +
 b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/(x^3*((a +
b*Log[c*x^n])/(b*n))^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p\,(d+e\log(fx^r))}{x^4}\,dx = -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b}{\cdots}\right.}{x^3}$$

$$= -\frac{3^{-2-p}ee^{\frac{3a}{bn}}r(cx^n)^{3/n}\,\Gamma\left(2+p,\frac{3a}{bn}+\frac{3\log(cx^n)}{n}\right)(a+b\log(cx^n))^p\left(\cdots\right.}{x^3}$$

Mathematica [A]
time = 0.26, size = 154, normalized size = 0.52

$$-\frac{3^{-2-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^{-1+p}\left(\frac{a+b\log(cx^n)}{bn}\right)^{1-p}\left(benr\Gamma\left(2+p,\frac{3(a+b\log(cx^n))}{bn}\right)+3\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(bdn-aer-ber\log(cx^n)+ben\log(fx^r))\right)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^p*(d + e*Log[f*x^r]))/x^4,x]
```

```
[Out] -((3^(-2 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*(a + b*Log[c*x^n])^(-1 + p)*((a
 + b*Log[c*x^n])/(b*n))^(1 - p)*(b*e*n*r*Gamma[2 + p, (3*(a + b*Log[c*x^n])
)/(b*n)] + 3*Gamma[1 + p, (3*(a + b*Log[c*x^n]))/(b*n)]*(b*d*n - a*e*r - b*
e*r*Log[c*x^n] + b*e*n*Log[f*x^r])))/x^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^p\,(d+e\ln(f\,x^r))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^4,x)
[Out] int((a+b*ln(c*x^n))^p*(d+e*ln(f*x^r))/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="fricas")
[Out] integral((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*(d+e*ln(f*x**r))/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*(d+e*log(f*x^r))/x^4,x, algorithm="giac")
[Out] integrate((e*log(f*x^r) + d)*(b*log(c*x^n) + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,\ln(f\,x^r))\,(a+b\,\ln(c\,x^n))^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4,x)
[Out] int(((d + e*log(f*x^r))*(a + b*log(c*x^n))^p)/x^4, x)
```

3.186 $\int (d+ex^2) \sin^{-1}(ax) \log(cx^n) \, dx$

Optimal. Leaf size=246

$$-\frac{dn\sqrt{1-a^2x^2}}{a}-\frac{(3a^2d+e)\,n\sqrt{1-a^2x^2}}{3a^3}+\frac{2en(1-a^2x^2)^{3/2}}{27a^3}-dnx\sin^{-1}(ax)-\frac{1}{9}enx^3\sin^{-1}(ax)-\frac{en\tanh^{-1}}{}$$

```
[Out] 2/27*e*n*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsin(a*x)-1/9*e*n*x^3*arcsin(a*x)-1
/9*e*n*arctanh((-a^2*x^2+1)^(1/2))/a^3+1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2+
1)^(1/2))/a^3-1/9*e*(-a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arcsin(a*x)*ln(c*x
^n)+1/3*e*x^3*arcsin(a*x)*ln(c*x^n)-d*n*(-a^2*x^2+1)^(1/2)/a-1/3*(3*a^2*d+e
)*n*(-a^2*x^2+1)^(1/2)/a^3+1/3*(3*a^2*d+e)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A]
time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {4755, 455, 45, 2434, 272, 52, 65, 214, 4715, 267, 4723}

$$-\frac{dn\sqrt{1-a^2x^2}}{a}+\frac{\sqrt{1-a^2x^2}\,(3a^2d+e)\log(cx^n)}{3a^3}-\frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3}-\frac{n\sqrt{1-a^2x^2}\,(3a^2d+e)}{3a^3}+\frac{n(3a^2d+e)\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{3a^3}+\frac{2en(1-a^2x^2)^{3/2}}{27a^3}-\frac{en\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{9a^3}+dx\text{ArcSin}(ax)\log(cx^n)+\frac{1}{3}ex^3\text{ArcSin}(ax)\log(cx^n)-dnx\text{ArcSin}(ax)-\frac{1}{9}enx^3\text{ArcSin}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

```
[Out] -((d*n*Sqrt[1 - a^2*x^2])/a) - ((3*a^2*d + e)*n*Sqrt[1 - a^2*x^2])/(3*a^3)
+ (2*e*n*(1 - a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcSin[a*x] - (e*n*x^3*ArcSi
n[a*x])/9 - (e*n*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) + ((3*a^2*d + e)*n*Arc
Tanh[Sqrt[1 - a^2*x^2]])/(3*a^3) + ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x
^n])/(3*a^3) - (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSin[a*x]
*Log[c*x^n] + (e*x^3*ArcSin[a*x]*Log[c*x^n])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4755

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[c*x], u, x] -
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0]
)
```

Rubi steps

$$\int \left(d+ex^2\right)\sin^{-1}(ax)\log\left(cx^n\right)\,dx = \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sin^{-}$$

$$= \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right)}{3a^3} - \frac{e\left(1-a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3} + dx\sin^{-}$$

$$= -dnx\sin^{-1}(ax) - \frac{1}{9}enx^3\sin^{-1}(ax) + \frac{\left(3a^2d+e\right)\sqrt{1-a^2x^2}\,\log\left(cx^n\right.}{3a^3}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dn$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} + \frac{en\sqrt{1-a^2x^2}}{9a^3} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{en(1}{}$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dn$$

$$= -\frac{dn\sqrt{1-a^2x^2}}{a} - \frac{\left(3a^2d+e\right)n\sqrt{1-a^2x^2}}{3a^3} + \frac{2en\left(1-a^2x^2\right)^{3/2}}{27a^3} - dn$$

Mathematica [A]
time = 0.12, size = 248, normalized size = 1.01

$$\frac{-54a^2dn\sqrt{1-a^2x^2}-7en\sqrt{1-a^2x^2}-2a^2enx^2\sqrt{1-a^2x^2}-3\left(9a^2d+2e\right)n\log(x)+27a^2d\sqrt{1-a^2x^2}\,\log\left(cx^n\right)+6e\sqrt{1-a^2x^2}\,\log\left(cx^n\right)+3a^2ex^2\sqrt{1-a^2x^2}\,\log\left(cx^n\right)-3a^3x\sin^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)+27a^2dn\log\left(1+\sqrt{1-a^2x^2}\right)+6en\log\left(1+\sqrt{1-a^2x^2}\right)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSin[a*x]*Log[c*x^n],x]
```

```
[Out] (-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n*x^2*Sq
rt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^2*x^2]*L
og[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1 - a^2*x^2
]*Log[c*x^n] - 3*a^3*x*ArcSin[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c
*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1 - a^2
*x^2]])/(27*a^3)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.98, size = 6894, normalized size = 28.02

method	result	size
default	Expression too large to display	6894

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsin(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^2
*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5)*e - 16
2*a^2*n*e*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integrat
e(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a^3 +
 log(a*x - 1)/a^3)*log(c) - 3*a^2*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a
^5 + 3*log(a*x - 1)/a^5)*e*log(c))*a^3 - 2*(-2*I*a^3*n*e + 3*I*a^3*e*log(c)
)*x^3 - 54*a^3*integrate(-1/9*((a*n*e - 3*a*e*log(c))*x^3 + 9*(a*d*n - a*d*
log(c))*x - 3*(a*x^3*e + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a
^2*x^2 - 1), x) - 9*(3*I*a^2*d + I*e)*n*dilog(a*x) - 9*(-3*I*a^2*d - I*e)*n
*dilog(-a*x) - 6*(9*I*a^3*d*log(c) + 3*I*a*e*log(c) + 2*(-9*I*a^3*d - 2*I*a
*e)*n)*x + 6*((a^3*n*e - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*d*log(c))*x
)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)) - 3*(-9*I*a^2*d*log(c) + (9*I*
a^2*d + I*e)*n - 3*I*e*log(c))*log(a*x + 1) - 3*(9*I*a^2*d*log(c) + (-9*I*a
^2*d - I*e)*n + 3*I*e*log(c))*log(a*x - 1) - 3*(2*I*a^3*x^3*e + 6*(3*I*a^3*
d + I*a*e)*x + 6*(a^3*x^3*e + 3*a^3*d*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a
```

```
*x + 1)) + 3*(-3*I*a^2*d - I*e)*log(a*x + 1) + 3*(3*I*a^2*d + I*e)*log(-a*x
 + 1))*log(x^n))/a^3
```

Fricas [A]
time = 0.57, size = 231, normalized size = 0.94

$$\frac{18\,(a^3x^3e+3\,a^3dx)\arcsin(ax)\log(c)+18\,(a^3nx^3e+3\,a^3dnx)\arcsin(ax)\log(x)-6\,(a^3nx^3e+9\,a^3dnx)\arcsin(ax)+3\,(9\,a^2dn+2\,ne)\log\left(\sqrt{-a^2x^2+1}+1\right)-3\,(9\,a^2dn+2\,ne)\log\left(\sqrt{-a^2x^2+1}-1\right)-2\,(54\,a^2dn+(2\,a^2nx^2+7\,n)e-3\,(9\,a^2d+(a^2x^2+2)e)\log(c)-3\,(9\,a^2dn+(a^2nx^2+2\,n)e)\log(x))\sqrt{-a^2x^2+1}}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] 1/54*(18*(a^3*x^3*e + 3*a^3*d*x)*arcsin(a*x)*log(c) + 18*(a^3*n*x^3*e + 3*a
^3*d*n*x)*arcsin(a*x)*log(x) - 6*(a^3*n*x^3*e + 9*a^3*d*n*x)*arcsin(a*x) +
3*(9*a^2*d*n + 2*n*e)*log(sqrt(-a^2*x^2 + 1) + 1) - 3*(9*a^2*d*n + 2*n*e)*l
og(sqrt(-a^2*x^2 + 1) - 1) - 2*(54*a^2*d*n + (2*a^2*n*x^2 + 7*n)*e - 3*(9*a
^2*d + (a^2*x^2 + 2)*e)*log(c) - 3*(9*a^2*d*n + (a^2*n*x^2 + 2*n)*e)*log(x)
)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d+ex^2\right)\log\left(cx^n\right)\operatorname{asin}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asin(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asin(a*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5311 vs. $2(224) = 448$.
time = 6.32, size = 5311, normalized size = 21.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] 1/54*(54*a^3*d*n*x*arcsin(a*x)*log(a*x) - 54*a^3*d*n*x*arcsin(a*x)*log(a) +
 54*a^3*d*x*arcsin(a*x)*log(c) - 108*a^3*d*n*x*arcsin(a*x)/(sqrt(-a^2*x^2 +
 1)*a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2
 + sqrt(-a^2*x^2 + 1) + 1) - 54*a^4*d*n*x^2*log(abs(a)*abs(x))/((a^2*x^2/(s
qrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^2*x^2 + 1) + 1)^2) + 18*(a^2*x^2 - 1
)*a*x*arcsin(a*x)*e*log(c) + 54*a^4*d*n*x^2*log(sqrt(-a^2*x^2 + 1) + 1)/((a
^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^2*x^2 + 1) + 1)^2) + 54*sqr
t(-a^2*x^2 + 1)*a^2*d*n*log(a*x) - 54*sqrt(-a^2*x^2 + 1)*a^2*d*n*log(a) + 1
```

```
08*a^4*d*n*x^2/((a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)*(sqrt(-a^2*x^2 + 1
) + 1)^2) + 18*a*x*arcsin(a*x)*e*log(c) + 54*sqrt(-a^2*x^2 + 1)*a^2*d*log(c
) - 54*a^2*d*n*log(abs(a)*abs(x))/(a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^2 + 1)
+ 54*a^2*d*n*log(sqrt(-a^2*x^2 + 1) + 1)/(a^2*x^2/(sqrt(-a^2*x^2 + 1) + 1)^
2 + 1) - 6*(-a^2*x^2 + 1)^(3/2)*e*log(c) - 108*a^2*d*n/(a^2*x^2/(sqrt(-a^2*
x^2 + 1) + 1)^2 + 1) + (18*(a^2*x^2 - 1)*a*x*arcsin(a*x)*log(a*x) - 18*(a^2
*x^2 - 1)*a*x*arcsin(a*x)*log(a) + 18*a*x*arcsin(a*x)*log(a*x) - 18*a*x*arc
sin(a*x)*log(a) - 6*(-a^2*x^2 + 1)^(3/2)*log(a*x) + 6*(-a^2*x^2 + 1)^(3/2)*
log(a) + 18*sqrt(-a^2*x^2 + 1)*log(a*x) - 18*sqrt(-a^2*x^2 + 1)*log(a) - (1
92*(a^2*x^2 - 1)^2*a^8*x^8*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2) - 3*
sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) - 192*(a^2*x^2 - 1)^2
*a^8*x^8*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2
*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) - 224*(a^2*x^2 - 1)^2*a^8*x^8/
((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1)
+ 1)^6) + 144*(a^2*x^2 - 1)^2*a^7*x^7*arcsin(a*x)/((4*(-a^2*x^2 + 1)^(3/2)
- 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^5) + 96*(a^2*x^2 - 1
)*a^8*x^8*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1
) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) - 96*(a^2*x^2 - 1)*a^8*x^8*log(sqrt(-a
^2*x^2 + 1) + 1)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sq
rt(-a^2*x^2 + 1) + 1)^6) - 112*(a^2*x^2 - 1)*a^8*x^8/((4*(-a^2*x^2 + 1)^(3/
2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) + 12*(a^2*x^2
- 1)*a^7*x^7*arcsin(a*x)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) +
1)*(sqrt(-a^2*x^2 + 1) + 1)^6) + 72*(a^2*x^2 - 1)*a^7*x^7*arcsin(a*x)/((4*(
-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^
5) + 12*a^8*x^8*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x
^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) + 576*(a^2*x^2 - 1)^2*a^6*x^6*lo
g(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sq
rt(-a^2*x^2 + 1) + 1)^4) - 12*a^8*x^8*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2
*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^6) -
 576*(a^2*x^2 - 1)^2*a^6*x^6*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2*x^2 + 1)
^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) - 14*a^8*x
^8/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 +
1) + 1)^6) - 336*(a^2*x^2 - 1)^2*a^6*x^6/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(
-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) + 3*a^7*x^7*arcsin(a*x)/((
4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)*(sqrt(-a^2*x^2 + 1) + 1)
^6) + 9*a^7*x^7*arcsin(a*x)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1)
 + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^5) + 288*(a^2*x^2 - 1)^2*a^5*x^5*arcsin(a*
x)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 +
1) + 1)^3) + 288*(a^2*x^2 - 1)*a^6*x^6*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1
)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) - 288*(a^
2*x^2 - 1)*a^6*x^6*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2*x^2 + 1)^(3/2) - 3
*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) - 168*(a^2*x^2 - 1)*
a^6*x^6/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x
^2 + 1) + 1)^4) + 36*(a^2*x^2 - 1)*a^5*x^5*arcsin(a*x)/((4*(-a^2*x^2 + 1)^(
3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)*(sqrt(-a^2*x^2 + 1) + 1)^4) + 144*(a^2*x^2
```

```
 - 1)*a^5*x^5*arcsin(a*x)/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) +
 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^3) + 12*a^6*x^6*log(abs(a)*abs(x))/(sqrt(-a^
2*x^2 + 1) + 1)^6 + 36*a^6*x^6*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2)
- 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) + 576*(a^2*x^2 -
1)^2*a^4*x^4*log(abs(a)*abs(x))/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2
+ 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^2) - 12*a^6*x^6*log(sqrt(-a^2*x^2 + 1)
 + 1)/(sqrt(-a^2*x^2 + 1) + 1)^6 - 36*a^6*x^6*log(sqrt(-a^2*x^2 + 1) + 1)/(
(4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) +
 1)^4) - 576*(a^2*x^2 - 1)^2*a^4*x^4*log(sqrt(-a^2*x^2 + 1) + 1)/((4*(-a^2*
x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^2) -
15*a^6*x^6/(sqrt(-a^2*x^2 + 1) + 1)^6 - 21*a^6*x^6/((4*(-a^2*x^2 + 1)^(3/2)
 - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(-a^2*x^2 + 1) + 1)^4) + 384*(a^2*x^2 -
 1)^2*a^4*x^4/((4*(-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2 + 1) + 1)^2*(sqrt(
-a^2*x^2 + 1) + 1)^2) + 9*a^5*x^5*arcsin(a*x)/(...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\,x^n\right)\,\mathrm{asin}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asin(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*asin(a*x)*(d + e*x^2), x)
```

3.187 $\int (d+ex^2)\cos^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=245

$$\frac{dn\sqrt{1-a^2x^2}}{a}+\frac{(3a^2d+e)\,n\sqrt{1-a^2x^2}}{3a^3}-\frac{2en(1-a^2x^2)^{3/2}}{27a^3}-dnx\cos^{-1}(ax)-\frac{1}{9}enx^3\cos^{-1}(ax)+\frac{en\tanh^{-1}\left(\right.}{9}$$

```
[Out] -2/27*e*n*(-a^2*x^2+1)^(3/2)/a^3-d*n*x*arccos(a*x)-1/9*e*n*x^3*arccos(a*x)+
1/9*e*n*arctanh((-a^2*x^2+1)^(1/2))/a^3-1/3*(3*a^2*d+e)*n*arctanh((-a^2*x^2
+1)^(1/2))/a^3+1/9*e*(-a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arccos(a*x)*ln(c*
x^n)+1/3*e*x^3*arccos(a*x)*ln(c*x^n)+d*n*(-a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d+
e)*n*(-a^2*x^2+1)^(1/2)/a^3-1/3*(3*a^2*d+e)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^
3
```

Rubi [A]
time = 0.16, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.611$, Rules used = {4756, 455, 45, 2434, 272, 52, 65, 214, 4716, 267, 4724}

$$\frac{dn\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}\,(3a^2d+e)\log(cx^n)}{3a^3}+\frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3}+\frac{n\sqrt{1-a^2x^2}\,(3a^2d+e)}{3a^3}-\frac{n(3a^2d+e)\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{3a^3}-\frac{2en(1-a^2x^2)^{3/2}}{27a^3}+\frac{en\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{9a^3}+dx\text{ArcCos}(ax)\log(cx^n)+\frac{1}{3}ex^3\text{ArcCos}(ax)\log(cx^n)-dnx\text{ArcCos}(ax)-\frac{1}{9}enx^3\text{ArcCos}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[1 - a^2*x^2])/a + ((3*a^2*d + e)*n*Sqrt[1 - a^2*x^2])/(3*a^3) - (
2*e*n*(1 - a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcCos[a*x] - (e*n*x^3*ArcCos[a
*x])/9 + (e*n*ArcTanh[Sqrt[1 - a^2*x^2]])/(9*a^3) - ((3*a^2*d + e)*n*ArcTan
h[Sqrt[1 - a^2*x^2]])/(3*a^3) - ((3*a^2*d + e)*Sqrt[1 - a^2*x^2]*Log[c*x^n]
)/(3*a^3) + (e*(1 - a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcCos[a*x]*Lo
g[c*x^n] + (e*x^3*ArcCos[a*x]*Log[c*x^n])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4756

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[c*x], u, x] +
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0]
)
```

Rubi steps

$$\int (d+ex^2)\cos^{-1}(ax)\log(cx^n)\,dx = -\frac{(3a^2d+e)\sqrt{1-a^2x^2}\log(cx^n)}{3a^3} + \frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3} + dx\cos$$

$$= -\frac{(3a^2d+e)\sqrt{1-a^2x^2}\log(cx^n)}{3a^3} + \frac{e(1-a^2x^2)^{3/2}\log(cx^n)}{9a^3} + dx\cos$$

$$= -dnx\cos^{-1}(ax) - \frac{1}{9}enx^3\cos^{-1}(ax) - \frac{(3a^2d+e)\sqrt{1-a^2x^2}\log(cx^n)}{3a^3}$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a} + \frac{(3a^2d+e)n\sqrt{1-a^2x^2}}{3a^3} - \frac{en(1-a^2x^2)^{3/2}}{27a^3} - dnx\,\mathrm{co}$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a} - \frac{en\sqrt{1-a^2x^2}}{9a^3} + \frac{(3a^2d+e)n\sqrt{1-a^2x^2}}{3a^3} - \frac{en(1-}{27}$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a} + \frac{(3a^2d+e)n\sqrt{1-a^2x^2}}{3a^3} - \frac{2en(1-a^2x^2)^{3/2}}{27a^3} - dnx\,\mathrm{c}$$

$$= \frac{dn\sqrt{1-a^2x^2}}{a} + \frac{(3a^2d+e)n\sqrt{1-a^2x^2}}{3a^3} - \frac{2en(1-a^2x^2)^{3/2}}{27a^3} - dnx\,\mathrm{c}$$

Mathematica [A]
time = 0.13, size = 248, normalized size = 1.01

$$-\frac{-54a^2dn\sqrt{1-a^2x^2} - 7en\sqrt{1-a^2x^2} - 2a^2enx^2\sqrt{1-a^2x^2} - 3(9a^2d+2e)n\log(x) + 27a^2d\sqrt{1-a^2x^2}\log(cx^n) + 6e\sqrt{1-a^2x^2}\log(cx^n) + 3a^2ex^2\sqrt{1-a^2x^2}\log(cx^n) + 3a^3x\cos^{-1}(ax)\left(n(9d+ex^2) - 3(3d+ex^2)\log(cx^n)\right) + 27a^2dn\log\left(1+\sqrt{1-a^2x^2}\right) + 6en\log\left(1+\sqrt{1-a^2x^2}\right)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCos[a*x]*Log[c*x^n],x]
```

```
[Out] -1/27*(-54*a^2*d*n*Sqrt[1 - a^2*x^2] - 7*e*n*Sqrt[1 - a^2*x^2] - 2*a^2*e*n*
x^2*Sqrt[1 - a^2*x^2] - 3*(9*a^2*d + 2*e)*n*Log[x] + 27*a^2*d*Sqrt[1 - a^2*
x^2]*Log[c*x^n] + 6*e*Sqrt[1 - a^2*x^2]*Log[c*x^n] + 3*a^2*e*x^2*Sqrt[1 - a
^2*x^2]*Log[c*x^n] + 3*a^3*x*ArcCos[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)
*Log[c*x^n]) + 27*a^2*d*n*Log[1 + Sqrt[1 - a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1
 - a^2*x^2]])/a^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.76, size = 5619, normalized size = 22.93

method	result	size
default	Expression too large to display	5619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccos(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/54*(-I*(27*a^2*d*n*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3) + a^2
*n*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5)*e - 16
2*a^2*n*e*integrate(1/9*x^4*log(x)/(a^2*x^2 - 1), x) - 486*a^2*d*n*integrat
e(1/9*x^2*log(x)/(a^2*x^2 - 1), x) - 27*a^2*d*(2*x/a^2 - log(a*x + 1)/a^3 +
 log(a*x - 1)/a^3)*log(c) - 3*a^2*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a
^5 + 3*log(a*x - 1)/a^5)*e*log(c))*a^3 - 2*(-2*I*a^3*n*e + 3*I*a^3*e*log(c)
)*x^3 + 54*a^3*integrate(-1/9*((a*n*e - 3*a*e*log(c))*x^3 + 9*(a*d*n - a*d*
log(c))*x - 3*(a*x^3*e + 3*a*d*x)*log(x^n))*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a
^2*x^2 - 1), x) - 9*(3*I*a^2*d + I*e)*n*dilog(a*x) - 9*(-3*I*a^2*d - I*e)*n
*dilog(-a*x) - 6*(9*I*a^3*d*log(c) + 3*I*a*e*log(c) + 2*(-9*I*a^3*d - 2*I*a
*e)*n)*x + 6*((a^3*n*e - 3*a^3*e*log(c))*x^3 + 9*(a^3*d*n - a^3*d*log(c))*x
)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - 3*(-9*I*a^2*d*log(c) + (9*I*
a^2*d + I*e)*n - 3*I*e*log(c))*log(a*x + 1) - 3*(9*I*a^2*d*log(c) + (-9*I*a
^2*d - I*e)*n + 3*I*e*log(c))*log(a*x - 1) - 3*(2*I*a^3*x^3*e + 6*(3*I*a^3*
d + I*a*e)*x + 6*(a^3*x^3*e + 3*a^3*d*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
```

```
1), a*x) + 3*(-3*I*a^2*d - I*e)*log(a*x + 1) + 3*(3*I*a^2*d + I*e)*log(-a*x
 + 1))*log(x^n))/a^3
```

Fricas [A]
time = 0.63, size = 320, normalized size = 1.31

$$\frac{18\,(3\,a^3dx-3\,a^3d+(a^3x^3-a^3)e)\arccos(ax)\log(c)+18\,(a^3nx^3e+3\,a^3dnx)\arccos(ax)\log(x)-6\,(9\,a^3dnx-9\,a^3dn+(a^3nx^3-a^3n)e)\arccos(ax)-6\,(9\,a^3dn+a^3ne-3\,(3\,a^3d+a^3e)\log(c))\arctan\left(\frac{\sqrt{-a^2x^2+1}\,ax}{a^2x^2-1}\right)-3\,(9\,a^2dn+2\,ne)\log\left(\sqrt{-a^2x^2+1}+1\right)+3\,(9\,a^2dn+2\,ne)\log\left(\sqrt{-a^2x^2+1}-1\right)+2\,(54\,a^2dn+(2\,a^2nx^2+7\,n)e-3\,(9\,a^2d+(a^2x^2+2)e)\log(c)-3\,(9\,a^2dn+(a^2nx^2+2\,n)e)\log(x))\sqrt{-a^2x^2+1}}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] 1/54*(18*(3*a^3*d*x - 3*a^3*d + (a^3*x^3 - a^3)*e)*arccos(a*x)*log(c) + 18*
(a^3*n*x^3*e + 3*a^3*d*n*x)*arccos(a*x)*log(x) - 6*(9*a^3*d*n*x - 9*a^3*d*n
 + (a^3*n*x^3 - a^3*n)*e)*arccos(a*x) - 6*(9*a^3*d*n + a^3*n*e - 3*(3*a^3*d
 + a^3*e)*log(c))*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) - 3*(9*a^2*d
*n + 2*n*e)*log(sqrt(-a^2*x^2 + 1) + 1) + 3*(9*a^2*d*n + 2*n*e)*log(sqrt(-a
^2*x^2 + 1) - 1) + 2*(54*a^2*d*n + (2*a^2*n*x^2 + 7*n)*e - 3*(9*a^2*d + (a^
2*x^2 + 2)*e)*log(c) - 3*(9*a^2*d*n + (a^2*n*x^2 + 2*n)*e)*log(x))*sqrt(-a^
2*x^2 + 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)\log\left(cx^n\right)\operatorname{acos}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acos(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acos(a*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2136 vs. 2(223) = 446.
time = 7.26, size = 2136, normalized size = 8.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*arccos(a*x)*e*log(a*x) - 1/3*n*x^3*arccos(a*x)*e*log(a) + 1/3*x^3
*arccos(a*x)*e*log(c) + d*n*x*arccos(a*x)*log(a*x) - d*n*x*arccos(a*x)*log(
a) - 1/9*sqrt(-a^2*x^2 + 1)*n*x^2*e*log(a*x)/a + 1/9*sqrt(-a^2*x^2 + 1)*n*x
^2*e*log(a)/a + d*x*arccos(a*x)*log(c) - 1/9*sqrt(-a^2*x^2 + 1)*x^2*e*log(c
)/a - sqrt(-a^2*x^2 + 1)*d*n*log(a*x)/a + sqrt(-a^2*x^2 + 1)*d*n*log(a)/a -
 sqrt(-a^2*x^2 + 1)*d*log(c)/a + d*n*arccos(a*x)/(a*((a^2*x^2 - 1)/(a*x + 1
```

```
)^2 - 1)) + d*n*log(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/(a*((a^2*x^2 - 1)/(a
*x + 1)^2 - 1)) - d*n*log(abs(-a*x + sqrt(-a^2*x^2 + 1) - 1))/(a*((a^2*x^2
- 1)/(a*x + 1)^2 - 1)) + 4*sqrt(-a^2*x^2 + 1)*d*n/((a*x - (a^2*x^2 - 1)*a*x
/(a*x + 1)^2 - (a^2*x^2 - 1)/(a*x + 1)^2 + 1)*a) - 2/9*sqrt(-a^2*x^2 + 1)*n
*e*log(a*x)/a^3 + 2/9*sqrt(-a^2*x^2 + 1)*n*e*log(a)/a^3 + (a^2*x^2 - 1)*d*n
*arccos(a*x)/((a*x + 1)^2*a*((a^2*x^2 - 1)/(a*x + 1)^2 - 1)) - (a^2*x^2 - 1
)*d*n*log(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/((a*x + 1)^2*a*((a^2*x^2 - 1)/
(a*x + 1)^2 - 1)) + (a^2*x^2 - 1)*d*n*log(abs(-a*x + sqrt(-a^2*x^2 + 1) - 1
))/((a*x + 1)^2*a*((a^2*x^2 - 1)/(a*x + 1)^2 - 1)) - 2/9*sqrt(-a^2*x^2 + 1)
*e*log(c)/a^3 + 1/9*n*arccos(a*x)*e/(a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(
a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 2/9*n*e*lo
g(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/(a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*
(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) - 2/9*n*e*l
og(abs(-a*x + sqrt(-a^2*x^2 + 1) - 1))/(a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 -
3*(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 2/3*sqr
t(-a^2*x^2 + 1)*n*e/((a*x - 3*(a^2*x^2 - 1)*a*x/(a*x + 1)^2 + 3*(a^2*x^2 -
1)^2*a*x/(a*x + 1)^4 - 3*(a^2*x^2 - 1)/(a*x + 1)^2 - (a^2*x^2 - 1)^3*a*x/(a
*x + 1)^6 + 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 - (a^2*x^2 - 1)^3/(a*x + 1)^6 + 1
)*a^3) + 1/3*(a^2*x^2 - 1)*n*arccos(a*x)*e/((a*x + 1)^2*a^3*(3*(a^2*x^2 - 1
)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6
 - 1)) - 2/3*(a^2*x^2 - 1)*n*e*log(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/((a*x
 + 1)^2*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 +
(a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 2/3*(a^2*x^2 - 1)*n*e*log(abs(-a*x + sq
rt(-a^2*x^2 + 1) - 1))/((a*x + 1)^2*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a
^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 1/3*(a^2*x^
2 - 1)^2*n*arccos(a*x)*e/((a*x + 1)^4*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*
(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 2/3*(a^2*
x^2 - 1)^2*n*e*log(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/((a*x + 1)^4*a^3*(3*(
a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/
(a*x + 1)^6 - 1)) - 2/3*(a^2*x^2 - 1)^2*n*e*log(abs(-a*x + sqrt(-a^2*x^2 +
1) - 1))/((a*x + 1)^4*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/
(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) - 20/27*(-a^2*x^2 + 1)^(3/2
)*n*e/((a*x + 1)^3*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*
x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) + 1/9*(a^2*x^2 - 1)^3*n*arccos
(a*x)*e/((a*x + 1)^6*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(
a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1)) - 2/9*(a^2*x^2 - 1)^3*n*e*lo
g(abs(a*x + sqrt(-a^2*x^2 + 1) + 1))/((a*x + 1)^6*a^3*(3*(a^2*x^2 - 1)/(a*x
 + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1))
 + 2/9*(a^2*x^2 - 1)^3*n*e*log(abs(-a*x + sqrt(-a^2*x^2 + 1) - 1))/((a*x +
1)^6*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1)^4 + (a^
2*x^2 - 1)^3/(a*x + 1)^6 - 1)) - 2/3*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*n*e
/((a*x + 1)^5*a^3*(3*(a^2*x^2 - 1)/(a*x + 1)^2 - 3*(a^2*x^2 - 1)^2/(a*x + 1
)^4 + (a^2*x^2 - 1)^3/(a*x + 1)^6 - 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^{n}\right) \operatorname{acos}(a\,x)\,\left(e\,x^{2}+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acos(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*acos(a*x)*(d + e*x^2), x)
```

3.188 $\int (d+ex^2)\tan^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=182

$$\frac{5enx^2}{36a}-dnx\tan^{-1}(ax)-\frac{1}{9}enx^3\tan^{-1}(ax)-\frac{ex^2\log(cx^n)}{6a}+dx\tan^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\tan^{-1}(ax)\log(cx^n)$$

```
[Out] 5/36*e*n*x^2/a-d*n*x*arctan(a*x)-1/9*e*n*x^3*arctan(a*x)-1/6*e*x^2*ln(c*x^n
)/a+d*x*arctan(a*x)*ln(c*x^n)+1/3*e*x^3*arctan(a*x)*ln(c*x^n)+1/2*d*n*ln(a^
2*x^2+1)/a-1/18*e*n*ln(a^2*x^2+1)/a^3-1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*x^2+
1)/a^3-1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5032, 1607, 455, 45, 2435, 4930, 266, 4946, 272, 2438}

$$-\frac{n(3a^2d-e)\text{PolyLog}(2,-a^2x^2)}{12a^3}+\frac{dn\log(a^2x^2+1)}{2a}-\frac{(3a^2d-e)\log(a^2x^2+1)\log(cx^n)}{6a^3}-\frac{en\log(a^2x^2+1)}{18a^3}+dx\text{ArcTan}(ax)\log(cx^n)+\frac{1}{3}ex^3\text{ArcTan}(ax)\log(cx^n)-dnx\text{ArcTan}(ax)-\frac{1}{9}enx^3\text{ArcTan}(ax)-\frac{ex^2\log(cx^n)}{6a}+\frac{5enx^2}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

```
[Out] (5*e*n*x^2)/(36*a) - d*n*x*ArcTan[a*x] - (e*n*x^3*ArcTan[a*x])/9 - (e*x^2*L
og[c*x^n])/(6*a) + d*x*ArcTan[a*x]*Log[c*x^n] + (e*x^3*ArcTan[a*x]*Log[c*x^
n])/3 + (d*n*Log[1 + a^2*x^2])/(2*a) - (e*n*Log[1 + a^2*x^2])/(18*a^3) - ((
3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) - ((3*a^2*d - e)*n*PolyLo
g[2, -(a^2*x^2)])/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2435

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5032

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTan[c*x], u, x]
- Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right) \tan^{-1}(ax) \log\left(cx^n\right)\, dx &= -\frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan^{-1}(ax) \log\left(cx^n\right) + \frac{1}{3} ex^3 \tan^{-1}(ax) \log\left(cx^n\right) \\ &= \frac{enx^2}{12a} - \frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan^{-1}(ax) \log\left(cx^n\right) + \frac{1}{3} ex^3 \tan^{-1}(ax) \log \\ &= \frac{enx^2}{12a} - dnx \tan^{-1}(ax) - \frac{1}{9} enx^3 \tan^{-1}(ax) - \frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan \\ &= \frac{enx^2}{12a} - dnx \tan^{-1}(ax) - \frac{1}{9} enx^3 \tan^{-1}(ax) - \frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan \\ &= \frac{enx^2}{12a} - dnx \tan^{-1}(ax) - \frac{1}{9} enx^3 \tan^{-1}(ax) - \frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan \\ &= \frac{5enx^2}{36a} - dnx \tan^{-1}(ax) - \frac{1}{9} enx^3 \tan^{-1}(ax) - \frac{ex^2 \log\left(cx^n\right)}{6a} + dx \tan\end{aligned}$$

Mathematica [A]
time = 0.09, size = 165, normalized size = 0.91

$$\frac{5a^2enx^2 - 6a^2ex^2 \log\left(cx^n\right) - 4a^3x \tan^{-1}(ax) \left(n\left(9d+ex^2\right) - 3\left(3d+ex^2\right) \log\left(cx^n\right)\right) + 18a^2dn \log\left(1+a^2x^2\right) - 2en \log\left(1+a^2x^2\right) - 18a^2d \log\left(cx^n\right) \log\left(1+a^2x^2\right) + 6e \log\left(cx^n\right) \log\left(1+a^2x^2\right) + 3\left(-3a^2d+e\right) n\text{Li}_2\left(-a^2x^2\right)}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcTan[a*x]*Log[c*x^n],x]
```

```
[Out] (5*a^2*e*n*x^2 - 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTan[a*x]*(n*(9*d + e*x
^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + 18*a^2*d*n*Log[1 + a^2*x^2] - 2*e*n*Log
[1 + a^2*x^2] - 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2] + 6*e*Log[c*x^n]*Log[1
 + a^2*x^2] + 3*(-3*a^2*d + e)*n*PolyLog[2, -(a^2*x^2)])/(36*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.61, size = 78943, normalized size = 433.75

method	result	size
risch	Expression too large to display	2700
default	Expression too large to display	78943

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arctan(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*x^2*e*log(c) - 3*a^3*integrate(2*(x^2*e + d)*arctan(a*x)*log(x^n)
, x) - 2*(a^3*x^3*e*log(c) + 3*a^3*d*x*log(c))*arctan(a*x) + (3*a^2*d*log(c
) - e*log(c))*log(a^2*x^2 + 1))/a^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arctan(a*x)*log(c*x^n), x)
```

Sympy [A]
time = 54.39, size = 221, normalized size = 1.21

$$-dn\left(\begin{cases} 0 & \text{for}\: a = 0 \\ \begin{cases} x \operatorname{atan}{\left(ax \right)} - \frac{\log{\left(a^{2}x^{2}+1 \right)}}{2a} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases} + \frac{\operatorname{Li}_{2}\left(a^{2}x^{2}e^{i\pi}\right)}{4a} & \text{otherwise} \end{cases}\right) + d\left(\begin{cases} 0 & \text{for}\: a = 0 \\ x \operatorname{atan}{\left(ax \right)} - \frac{\log{\left(a^{2}x^{2}+1 \right)}}{2a} & \text{otherwise} \end{cases}\right)\log{\left(cx^{n} \right)} - \frac{enx^{3}\operatorname{atan}{\left(ax \right)}}{9} + \frac{ex^{3}\log{\left(cx^{n} \right)}\operatorname{atan}{\left(ax \right)}}{3} + \frac{5enx^{2}}{36a} - \frac{en\left(\begin{cases} \frac{x^{2}}{2} & \text{for}\: a = 0 \\ -\frac{\operatorname{Li}_{2}\left(a^{2}x^{2}e^{i\pi}\right)}{2a^{2}} & \text{otherwise} \end{cases}\right)}{6a} - \frac{en\left(\begin{cases} x^{2} & \text{for}\: a^{2} = 0 \\ \frac{\log{\left(a^{2}x^{2}+1 \right)}}{a^{2}} & \text{otherwise} \end{cases}\right)}{18a} - \frac{ex^{2}\log{\left(cx^{n} \right)}}{6a} + \frac{e\left(\begin{cases} x^{2} & \text{for}\: a^{2} = 0 \\ \frac{\log{\left(a^{2}x^{2}+1 \right)}}{a^{2}} & \text{otherwise} \end{cases}\right)\log{\left(cx^{n} \right)}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*atan(a*x)*ln(c*x**n),x)
```

```
[Out] -d*n*Piecewise((0, Eq(a, 0)), (Piecewise((x*atan(a*x) - log(a**2*x**2 + 1)/
(2*a), Ne(a, 0)), (0, True)) + polylog(2, a**2*x**2*exp_polar(I*pi))/(4*a),
 True)) + d*Piecewise((0, Eq(a, 0)), (x*atan(a*x) - log(a**2*x**2 + 1)/(2*a
), True))*log(c*x**n) - e*n*x**3*atan(a*x)/9 + e*x**3*log(c*x**n)*atan(a*x)
/3 + 5*e*n*x**2/(36*a) - e*n*Piecewise((x**2/2, Eq(a, 0)), (-polylog(2, a**
2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) - e*n*Piecewise((x**2, Eq(a*
*2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) - e*x**2*log(c*x**n)/(6*a)
 + e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))*log(c*
x**n)/(6*a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctan(a*x)*log(c*x^n),x, algorithm="giac")
[Out] integrate((x^2*e + d)*arctan(a*x)*log(c*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{atan}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*atan(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*atan(a*x)*(d + e*x^2), x)
```

3.189 $\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=182

$$-\frac{5enx^2}{36a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\cot^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cot^{-1}(ax)\log(cx^n)-$$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arccot(a*x)-1/9*e*n*x^3*arccot(a*x)+1/6*e*x^2*ln(c*x^
n)/a+d*x*arccot(a*x)*ln(c*x^n)+1/3*e*x^3*arccot(a*x)*ln(c*x^n)-1/2*d*n*ln(a
^2*x^2+1)/a+1/18*e*n*ln(a^2*x^2+1)/a^3+1/6*(3*a^2*d-e)*ln(c*x^n)*ln(a^2*x^2
+1)/a^3+1/12*(3*a^2*d-e)*n*polylog(2,-a^2*x^2)/a^3
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5033, 1607, 455, 45, 2435, 4931, 266, 4947, 272, 2438}

$$\frac{n(3a^2d-e)\,\text{PolyLog}(2,-a^2x^2)}{12a^3}-\frac{dn\log(a^2x^2+1)}{2a}+\frac{(3a^2d-e)\log(a^2x^2+1)\log(cx^n)}{6a^3}+\frac{en\log(a^2x^2+1)}{18a^3}+dx\cot^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cot^{-1}(ax)\log(cx^n)+\frac{ex^2\log(cx^n)}{6a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)-\frac{5enx^2}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcCot[a*x] - (e*n*x^3*ArcCot[a*x])/9 + (e*x^2*
Log[c*x^n])/(6*a) + d*x*ArcCot[a*x]*Log[c*x^n] + (e*x^3*ArcCot[a*x]*Log[c*x
^n])/3 - (d*n*Log[1 + a^2*x^2])/(2*a) + (e*n*Log[1 + a^2*x^2])/(18*a^3) + (
(3*a^2*d - e)*Log[c*x^n]*Log[1 + a^2*x^2])/(6*a^3) + ((3*a^2*d - e)*n*PolyL
og[2, -(a^2*x^2)])/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2435

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)\cot^{-1}(ax)\log(cx^n)\,dx &= \frac{ex^2\log(cx^n)}{6a}+dx\cot^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cot^{-1}(ax)\log(cx^n)+\frac{(3\\ &= -\frac{enx^2}{12a}+\frac{ex^2\log(cx^n)}{6a}+dx\cot^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cot^{-1}(ax)\log\\ &= -\frac{enx^2}{12a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\cot^-\\ &= -\frac{enx^2}{12a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\cot^-\\ &= -\frac{enx^2}{12a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\cot^-\\ &= -\frac{5enx^2}{36a}-dnx\cot^{-1}(ax)-\frac{1}{9}enx^3\cot^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\cot\end{aligned}$$

Mathematica [A]
time = 0.09, size = 178, normalized size = 0.98

$$\frac{-5a^2enx^2+36a^2dn\log\left(\frac{1}{a\sqrt{1+\frac{1}{a^2x^2}}\,x}\right)+6a^2ex^2\log(cx^n)-4a^3x\cot^{-1}(ax)\left(n(9d+ex^2)-3(3d+ex^2)\log(cx^n)\right)+2en\log(1+a^2x^2)+18a^2d\log(cx^n)\log(1+a^2x^2)-6e\log(cx^n)\log(1+a^2x^2)+(9a^2dn-3en)\,\mathrm{Li}_2(-a^2x^2)}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCot[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] + 6*a^2*e*x
^2*Log[c*x^n] - 4*a^3*x*ArcCot[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[
c*x^n]) + 2*e*n*Log[1 + a^2*x^2] + 18*a^2*d*Log[c*x^n]*Log[1 + a^2*x^2] - 6
*e*Log[c*x^n]*Log[1 + a^2*x^2] + (9*a^2*d*n - 3*e*n)*PolyLog[2, -(a^2*x^2)]
)/(36*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.17, size = 152337, normalized size = 837.02

method	result	size
risch	Expression too large to display	3017
default	Expression too large to display	152337

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccot(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/36*(69984*a^4*n*e*integrate(1/11664*x^4*log(x)/(a^2*x^3 + x), x) + 209952
*a^4*d*n*integrate(1/11664*x^2*log(x)/(a^2*x^3 + x), x) + 1944*a^4*e*integr
ate(1/216*(2*a*x^4*arctan2(1, a*x) + x^3*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x
)*log(c) + 1944*a^4*d*integrate(1/216*(2*a*x^2*arctan2(1, a*x) + x*log(a^2*
x^2 + 1))/(a^2*x^2 + 1), x)*log(c) + 1944*a^4*e*integrate(1/216*(2*a*x^4*ar
ctan2(1, a*x) + x^3*log(a^2*x^2 + 1))*log(x^n)/(a^2*x^2 + 1), x) + 1944*a^4
*d*integrate(1/216*(2*a*x^2*arctan2(1, a*x) + x*log(a^2*x^2 + 1))*log(x^n)/
(a^2*x^2 + 1), x) - 9*(216*a*integrate(1/216*x*log(a^2*x^2 + 1)/(a^2*x^2 +
1), x) - arctan(a*x)^2/a - 2*arctan(a*x)*arctan(1/(a*x))/a)*a^3*d*log(c) -
1944*a^3*e*integrate(1/216*(a*x^3*log(a^2*x^2 + 1) - 2*x^2*arctan2(1, a*x))
/(a^2*x^2 + 1), x)*log(c) - 1944*a^3*e*integrate(1/216*(a*x^3*log(a^2*x^2 +
 1) - 2*x^2*arctan2(1, a*x))*log(x^n)/(a^2*x^2 + 1), x) - 1944*a^3*d*integr
ate(1/216*(a*x*log(a^2*x^2 + 1) - 2*arctan2(1, a*x))*log(x^n)/(a^2*x^2 + 1)
, x) - 2*(a^3*n*arctan2(1, a*x)*e - 3*a^3*arctan2(1, a*x)*e*log(c))*x^3 - (
a^2*n*e - 3*a^2*e*log(c))*x^2 - 18*(a^3*d*n*arctan2(1, a*x) - a^3*d*arctan2
(1, a*x)*log(c))*x + (9*a^2*d*log(c) - (9*a^2*d - e)*n - 3*e*log(c))*log(a^
2*x^2 + 1) + 6*(a^3*x^3*arctan2(1, a*x)*e + 3*a^3*d*x*arctan2(1, a*x))*log(
x^n))/a^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arccot(a*x)*log(c*x^n), x)
```

Sympy [A]
time = 52.74, size = 231, normalized size = 1.27

$$-dn\left(\begin{cases}\frac{\pi x}{2} & \text{for}\: a = 0\\\begin{cases}x \operatorname{acot}{\left(ax \right)} + \frac{\log{\left(a^{2} x^{2} + 1 \right)}}{2a} & \text{for}\: a \neq 0\\\frac{\pi x}{2} & \text{otherwise}\end{cases} - \frac{\operatorname{Li}_{2}\left(a^{2} x^{2} e^{i\pi}\right)}{4a} & \text{otherwise}\end{cases}\right) + d\left(\begin{cases}\frac{\pi x}{2} & \text{for}\: a = 0\\x \operatorname{acot}{\left(ax \right)} + \frac{\log{\left(a^{2} x^{2} + 1 \right)}}{2a} & \text{otherwise}\end{cases}\right)\log{\left(cx^{n} \right)} - \frac{enx^{3} \operatorname{acot}{\left(ax \right)}}{9} + \frac{ex^{3} \log{\left(cx^{n} \right)} \operatorname{acot}{\left(ax \right)}}{3} - \frac{5enx^{2}}{36a} + \frac{en\left(\begin{cases}\frac{x^{2}}{2} & \text{for}\: a = 0\\- \frac{\operatorname{Li}_{2}\left(a^{2} x^{2} e^{i\pi}\right)}{2a^{2}} & \text{otherwise}\end{cases}\right)}{6a} + \frac{en\left(\begin{cases}x^{2} & \text{for}\: a^{2} = 0\\\frac{\log{\left(a^{2} x^{2} + 1 \right)}}{a^{2}} & \text{otherwise}\end{cases}\right)}{18a} + \frac{ex^{2} \log{\left(cx^{n} \right)}}{6a} - \frac{e\left(\begin{cases}x^{2} & \text{for}\: a^{2} = 0\\\frac{\log{\left(a^{2} x^{2} + 1 \right)}}{a^{2}} & \text{otherwise}\end{cases}\right)\log{\left(cx^{n} \right)}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acot(a*x)*ln(c*x**n),x)
```

```
[Out] -d*n*Piecewise((pi*x/2, Eq(a, 0)), (Piecewise((x*acot(a*x) + log(a**2*x**2
+ 1)/(2*a), Ne(a, 0)), (pi*x/2, True)) - polylog(2, a**2*x**2*exp_polar(I*p
i))/(4*a), True)) + d*Piecewise((pi*x/2, Eq(a, 0)), (x*acot(a*x) + log(a**2
*x**2 + 1)/(2*a), True))*log(c*x**n) - e*n*x**3*acot(a*x)/9 + e*x**3*log(c*
x**n)*acot(a*x)/3 - 5*e*n*x**2/(36*a) + e*n*Piecewise((x**2/2, Eq(a, 0)), (
-polylog(2, a**2*x**2*exp_polar(I*pi))/(2*a**2), True))/(6*a) + e*n*Piecewi
se((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2, True))/(18*a) + e*x**2*lo
g(c*x**n)/(6*a) - e*Piecewise((x**2, Eq(a**2, 0)), (log(a**2*x**2 + 1)/a**2
, True))*log(c*x**n)/(6*a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccot(a*x)*log(c*x^n),x, algorithm="giac")
[Out] integrate((x^2*e + d)*arccot(a*x)*log(c*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{acot}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acot(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*acot(a*x)*(d + e*x^2), x)
```

3.190 $\int (d+ex^2)\sinh^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=244

$$\frac{dn\sqrt{1+a^2x^2}}{a}+\frac{(3a^2d-e)\,n\sqrt{1+a^2x^2}}{3a^3}+\frac{2en(1+a^2x^2)^{3/2}}{27a^3}-dnx\sinh^{-1}(ax)-\frac{1}{9}enx^3\sinh^{-1}(ax)-\frac{(3a^2d-}{}$$

```
[Out] 2/27*e*n*(a^2*x^2+1)^(3/2)/a^3-d*n*x*arcsinh(a*x)-1/9*e*n*x^3*arcsinh(a*x)-
1/3*(3*a^2*d-e)*n*arctanh((a^2*x^2+1)^(1/2))/a^3-1/9*e*n*arctanh((a^2*x^2+1
)^(1/2))/a^3-1/9*e*(a^2*x^2+1)^(3/2)*ln(c*x^n)/a^3+d*x*arcsinh(a*x)*ln(c*x^
n)+1/3*e*x^3*arcsinh(a*x)*ln(c*x^n)+d*n*(a^2*x^2+1)^(1/2)/a+1/3*(3*a^2*d-e)
*n*(a^2*x^2+1)^(1/2)/a^3-1/3*(3*a^2*d-e)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A]
time = 0.15, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.611$, Rules used = {5792, 455, 45, 2434, 272, 52, 65, 214, 5772, 267, 5776}

$$\frac{dn\sqrt{a^2x^2+1}}{a}-\frac{\sqrt{a^2x^2+1}\,(3a^2d-e)\log(cx^n)}{3a^3}-\frac{e(a^2x^2+1)^{3/2}\log(cx^n)}{9a^3}+\frac{n\sqrt{a^2x^2+1}\,(3a^2d-e)}{3a^3}-\frac{n(3a^2d-e)\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)}{3a^3}+\frac{2en(a^2x^2+1)^{3/2}}{27a^3}-\frac{en\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)}{9a^3}+dx\sinh^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\sinh^{-1}(ax)\log(cx^n)-dnx\sinh^{-1}(ax)-\frac{1}{9}enx^3\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[1 + a^2*x^2])/a + ((3*a^2*d - e)*n*Sqrt[1 + a^2*x^2])/(3*a^3) + (
2*e*n*(1 + a^2*x^2)^(3/2))/(27*a^3) - d*n*x*ArcSinh[a*x] - (e*n*x^3*ArcSinh
[a*x])/9 - ((3*a^2*d - e)*n*ArcTanh[Sqrt[1 + a^2*x^2]])/(3*a^3) - (e*n*ArcT
anh[Sqrt[1 + a^2*x^2]])/(9*a^3) - ((3*a^2*d - e)*Sqrt[1 + a^2*x^2]*Log[c*x^
n])/(3*a^3) - (e*(1 + a^2*x^2)^(3/2)*Log[c*x^n])/(9*a^3) + d*x*ArcSinh[a*x]
*Log[c*x^n] + (e*x^3*ArcSinh[a*x]*Log[c*x^n])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5792

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\sinh^{-1}(ax)\log\left(cx^n\right)\,dx = -\frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(cx^n\right)}{3a^3}-\frac{e\left(1+a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\,\mathrm{si}$$

$$= -\frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(cx^n\right)}{3a^3}-\frac{e\left(1+a^2x^2\right)^{3/2}\log\left(cx^n\right)}{9a^3}+dx\,\mathrm{si}$$

$$= -dnx\sinh^{-1}(ax)-\frac{1}{9}enx^3\sinh^{-1}(ax)-\frac{\left(3a^2d-e\right)\sqrt{1+a^2x^2}\,\log\left(\right.}{3a^3}$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a}+\frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3}+\frac{en\left(1+a^2x^2\right)^{3/2}}{27a^3}-dnx$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a}+\frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3}+\frac{en\sqrt{1+a^2x^2}}{9a^3}+en(1$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a}+\frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3}+\frac{2en\left(1+a^2x^2\right)^{3/2}}{27a^3}-dn$$

$$= \frac{dn\sqrt{1+a^2x^2}}{a}+\frac{\left(3a^2d-e\right)n\sqrt{1+a^2x^2}}{3a^3}+\frac{2en\left(1+a^2x^2\right)^{3/2}}{27a^3}-dn$$

Mathematica [A]
time = 0.11, size = 240, normalized size = 0.98

$$\frac{54a^2dn\sqrt{1+a^2x^2}-7en\sqrt{1+a^2x^2}+2a^2enx^2\sqrt{1+a^2x^2}+3\left(9a^2d-2e\right)n\log(x)-27a^2d\sqrt{1+a^2x^2}\,\log\left(cx^n\right)+6e\sqrt{1+a^2x^2}\,\log\left(cx^n\right)-3a^2ex^2\sqrt{1+a^2x^2}\,\log\left(cx^n\right)-3a^3x\sinh^{-1}(ax)\left(n\left(9d+ex^2\right)-3\left(3d+ex^2\right)\log\left(cx^n\right)\right)-27a^2dn\log\left(1+\sqrt{1+a^2x^2}\right)+6en\log\left(1+\sqrt{1+a^2x^2}\right)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSinh[a*x]*Log[c*x^n],x]
```

```
[Out] (54*a^2*d*n*Sqrt[1 + a^2*x^2] - 7*e*n*Sqrt[1 + a^2*x^2] + 2*a^2*e*n*x^2*Sqr
t[1 + a^2*x^2] + 3*(9*a^2*d - 2*e)*n*Log[x] - 27*a^2*d*Sqrt[1 + a^2*x^2]*Lo
g[c*x^n] + 6*e*Sqrt[1 + a^2*x^2]*Log[c*x^n] - 3*a^2*e*x^2*Sqrt[1 + a^2*x^2]
*Log[c*x^n] - 3*a^3*x*ArcSinh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c
*x^n]) - 27*a^2*d*n*Log[1 + Sqrt[1 + a^2*x^2]] + 6*e*n*Log[1 + Sqrt[1 + a^2
*x^2]])/(27*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.94, size = 4077, normalized size = 16.71

method	result	size
default	Expression too large to display	4077

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsinh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/a*(a^2*x^2+1)^(1/2)*csgn(I/(a*x+(a^2*x^2+1)^(1/2)))*Pi*csgn(I/(a*x+(
a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^2*d*n-1/2*I/a*(a^2*x^2+1)
^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^2*
csgn(I*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*d*n-1/2*I/a*(a^2*x^2+1)^(1/2)*Pi*csg
n(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*csgn(I/a*(-1+(a
*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^2*d*n+1/9*I/a^3*(a^2*x^2+
1)^(1/2)*csgn(I/a)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2
+1)^(1/2)))^2*e*n+1/9*I/a^3*(a^2*x^2+1)^(1/2)*csgn(I/(a*x+(a^2*x^2+1)^(1/2)
))*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^2*e*n+
1/6*I*csgn(I/a)*arcsinh(a*x)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*
x+(a^2*x^2+1)^(1/2)))^2*x^3*e*n+1/6*I*arcsinh(a*x)*csgn(I/(a*x+(a^2*x^2+1)^
(1/2)))*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^2
*x^3*e*n+1/6*I*arcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2
*x^2+1)^(1/2))^2))^2*csgn(I*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*x^3*e*n+1/6*I*a
rcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2
))*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^2*x^3*e
*n+1/2*I*csgn(I/a)*arcsinh(a*x)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/
(a*x+(a^2*x^2+1)^(1/2)))^2*x*d*n+1/2*I*arcsinh(a*x)*csgn(I/(a*x+(a^2*x^2+1)
^(1/2)))*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^
2*x*d*n+1/2*I*arcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*
x^2+1)^(1/2))^2))^2*csgn(I*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*x*d*n+1/2*I*arcs
inh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*
csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^2*x*d*n+1/
9*I/a^3*(a^2*x^2+1)^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x
^2+1)^(1/2))^2))^2*csgn(I*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*e*n+1/a*ln(a*x+(a
^2*x^2+1)^(1/2)-1)*d*n-1/a*ln(1+a*x+(a^2*x^2+1)^(1/2))*d*n-2/9/a^3*ln(a*x+(
```

```
a^2*x^2+1)^(1/2)-1)*e*n+2/9/a^3*ln(1+a*x+(a^2*x^2+1)^(1/2))*e*n-7/27/a^3*(a
^2*x^2+1)^(1/2)*e*n-1/9*e*n*x^3*arcsinh(a*x)-1/a*(a^2*x^2+1)^(1/2)*d*(ln(c*
x^n)-n*ln(x))+2/9/a^3*(a^2*x^2+1)^(1/2)*e*(ln(c*x^n)-n*ln(x))+1/3*arcsinh(a
*x)*x^3*e*(ln(c*x^n)-n*ln(x))+arcsinh(a*x)*x*d*(ln(c*x^n)-n*ln(x))-1/9/a^3*
n*(3*arcsinh(a*x)*x^3*a^3*e-(a^2*x^2+1)^(1/2)*x^2*a^2*e+9*arcsinh(a*x)*x*a^
3*d-9*(a^2*x^2+1)^(1/2)*a^2*d+2*(a^2*x^2+1)^(1/2)*e)*ln(a*x+(a^2*x^2+1)^(1/
2))+1/18*I/a*(a^2*x^2+1)^(1/2)*csgn(I/a)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*
(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*
x+(a^2*x^2+1)^(1/2)))*x^2*e*n+1/18*I/a*(a^2*x^2+1)^(1/2)*csgn(I/(a*x+(a^2*x
^2+1)^(1/2)))*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))
^2))*csgn(I*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*x^2*e*n+2/27/a*(a^2*x^2+1)^(1/2
)*x^2*e*n+1/a*(a^2*x^2+1)^(1/2)*ln(2)*d*n-1/a*(a^2*x^2+1)^(1/2)*ln(-1+(a*x+
(a^2*x^2+1)^(1/2))^2)*d*n+1/a*(a^2*x^2+1)^(1/2)*ln(a)*d*n-2/9/a^3*(a^2*x^2+
1)^(1/2)*ln(2)*e*n+2*d*n*(a^2*x^2+1)^(1/2)/a-d*n*x*arcsinh(a*x)+1/9*I/a^3*(
a^2*x^2+1)^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/
2))^2))*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^2*
e*n+1/18*I/a*(a^2*x^2+1)^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(
a^2*x^2+1)^(1/2))^2))^3*x^2*e*n+1/18*I/a*(a^2*x^2+1)^(1/2)*Pi*csgn(I/a*(-1+
(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^3*x^2*e*n-1/2*I/a*(a^2*
x^2+1)^(1/2)*csgn(I/a)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2
*x^2+1)^(1/2)))^2*d*n+1/9/a*(a^2*x^2+1)^(1/2)*ln(a)*x^2*e*n+1/2*I/a*(a^2*x^
2+1)^(1/2)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2
)))^3*d*n-1/9*I/a^3*(a^2*x^2+1)^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1
+(a*x+(a^2*x^2+1)^(1/2))^2))^3*e*n-1/9*I/a^3*(a^2*x^2+1)^(1/2)*Pi*csgn(I/a*
(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^3*e*n-1/6*I*arcsinh
(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^3*x
^3*e*n-1/6*I*arcsinh(a*x)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(
a^2*x^2+1)^(1/2)))^3*x^3*e*n-1/2*I*arcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^
(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^3*x*d*n-1/2*I*arcsinh(a*x)*Pi*csgn(I
/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^3*x*d*n-1/6*I*cs
gn(I/a)*arcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)
^(1/2))^2))*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2))
)*x^3*e*n-1/6*I*arcsinh(a*x)*csgn(I/(a*x+(a^2*x^2+1)^(1/2)))*Pi*csgn(I/(a*x
+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*csgn(I*(-1+(a*x+(a^2*x^
2+1)^(1/2))^2))*x^3*e*n-1/2*I*csgn(I/a)*arcsinh(a*x)*Pi*csgn(I/(a*x+(a^2*x^
2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(
1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))*x*d*n-1/18*I/a*(a^2*x^2+1)^(1/2)*csgn(I/a
)*Pi*csgn(I/a*(-1+(a*x+(a^2*x^2+1)^(1/2))^2)/(a*x+(a^2*x^2+1)^(1/2)))^2*x^2
*e*n-1/18*I/a*(a^2*x^2+1)^(1/2)*csgn(I/(a*x+(a^2*x^2+1)^(1/2)))*Pi*csgn(I/(
a*x+(a^2*x^2+1)^(1/2))*(-1+(a*x+(a^2*x^2+1)^(1/2))^2))^2*x^2*e*n-1/18*I/a*(
a^2*x^2+1)^(1/2)*Pi*csgn(I/(a*x+(a^2*x^2+1)^(1/...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*n*(2*x/a^2 + I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^3) + 1/54*a^2
*n*(2*(a^2*x^3 - 3*x)/a^4 - 3*I*(log(I*a*x + 1) - log(-I*a*x + 1))/a^5)*e -
 3*a^2*n*e*integrate(1/9*x^4*log(x)/(a^2*x^2 + 1), x) - 9*a^2*d*n*integrate
(1/9*x^2*log(x)/(a^2*x^2 + 1), x) - 1/2*a^2*d*(2*x/a^2 + I*(log(I*a*x + 1)
- log(-I*a*x + 1))/a^3)*log(c) - 1/18*a^2*(2*(a^2*x^3 - 3*x)/a^4 - 3*I*(log
(I*a*x + 1) - log(-I*a*x + 1))/a^5)*e*log(c) - 1/9*((n - 3*log(c))*x^3*e +
9*(d*n - d*log(c))*x - 3*(x^3*e + 3*d*x)*log(x^n))*log(a*x + sqrt(a^2*x^2 +
 1)) - integrate(-1/9*(a*(n - 3*log(c))*x^3*e + 9*(d*n - d*log(c))*a*x - 3*
(a*x^3*e + 3*a*d*x)*log(x^n))/(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2)), x)
```

Fricas [A]
time = 0.55, size = 429, normalized size = 1.76

$$-\frac{3\left(9a^3dnx - 9a^3dn + (a^3nx^3 - a^3n)\cosh(1) - 3(3a^3dx - 3a^3d + (a^3x^3 - a^3)\cosh(1) + (a^3x^3 - a^3)\sinh(1))\log(c) - 3(a^3nx^3\cosh(1) + a^3nx^3\sinh(1) + 3a^3dnx)\log(x) + (a^3nx^3 - a^3n)\sinh(1)\right)\log\left(ax + \sqrt{a^2x^2+1}\right) + 3(9a^2dn - 2n\cosh(1) - 2n\sinh(1))\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 3(9a^3dn + a^3n\cosh(1) + a^3n\sinh(1) - 3(3a^3d + a^3\cosh(1) + a^3\sinh(1))\log(c))\log\left(-ax + \sqrt{a^2x^2+1}\right) - 3(9a^2dn - 2n\cosh(1) - 2n\sinh(1))\log\left(-ax + \sqrt{a^2x^2+1} - 1\right) - \left(54a^2dn + (2a^2nx^2 - 7n)\cosh(1) - 3(9a^2d + (a^2x^2 - 2)\cosh(1) + (a^2x^2 - 2)\sinh(1))\log(c) - 3(9a^2dn + (a^2nx^2 - 2n)\cosh(1) + (a^2nx^2 - 2n)\sinh(1))\log(x) + (2a^2nx^2 - 7n)\sinh(1)\right)\sqrt{a^2x^2+1}}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(9*a^3*d*n*x - 9*a^3*d*n + (a^3*n*x^3 - a^3*n)*cosh(1) - 3*(3*a^3*
d*x - 3*a^3*d + (a^3*x^3 - a^3)*cosh(1) + (a^3*x^3 - a^3)*sinh(1))*log(c) -
 3*(a^3*n*x^3*cosh(1) + a^3*n*x^3*sinh(1) + 3*a^3*d*n*x)*log(x) + (a^3*n*x^
3 - a^3*n)*sinh(1))*log(a*x + sqrt(a^2*x^2 + 1)) + 3*(9*a^2*d*n - 2*n*cosh(
1) - 2*n*sinh(1))*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*(9*a^3*d*n + a^3*n*
cosh(1) + a^3*n*sinh(1) - 3*(3*a^3*d + a^3*cosh(1) + a^3*sinh(1))*log(c))*l
og(-a*x + sqrt(a^2*x^2 + 1)) - 3*(9*a^2*d*n - 2*n*cosh(1) - 2*n*sinh(1))*lo
g(-a*x + sqrt(a^2*x^2 + 1) - 1) - (54*a^2*d*n + (2*a^2*n*x^2 - 7*n)*cosh(1)
 - 3*(9*a^2*d + (a^2*x^2 - 2)*cosh(1) + (a^2*x^2 - 2)*sinh(1))*log(c) - 3*(
9*a^2*d*n + (a^2*n*x^2 - 2*n)*cosh(1) + (a^2*n*x^2 - 2*n)*sinh(1))*log(x) +
 (2*a^2*n*x^2 - 7*n)*sinh(1))*sqrt(a^2*x^2 + 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^2\right)\log\left(cx^n\right)\operatorname{asinh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asinh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asinh(a*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asinh}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asinh(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*asinh(a*x)*(d + e*x^2), x)
```

3.191 $\int (d+ex^2)\cosh^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=312

$$\frac{dn\sqrt{-1+ax}\sqrt{1+ax}}{a}+\frac{2en\sqrt{-1+ax}\sqrt{1+ax}}{27a^3}+\frac{(9a^2d+2e)\,n\sqrt{-1+ax}\sqrt{1+ax}}{9a^3}+\frac{enx^2\sqrt{-1+ax}\sqrt{}}{27a}$$

```
[Out] 1/27*e*n*(a*x-1)^(3/2)*(a*x+1)^(3/2)/a^3-d*n*x*arccosh(a*x)-1/9*e*n*x^3*arc
cosh(a*x)-1/9*(9*a^2*d+2*e)*n*arctan((a*x-1)^(1/2)*(a*x+1)^(1/2))/a^3+d*x*a
rccosh(a*x)*ln(c*x^n)+1/3*e*x^3*arccosh(a*x)*ln(c*x^n)+d*n*(a*x-1)^(1/2)*(a
*x+1)^(1/2)/a+2/27*e*n*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3+1/9*(9*a^2*d+2*e)*n*
(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3+1/27*e*n*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-
1/9*(9*a^2*d+2*e)*ln(c*x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3-1/9*e*x^2*ln(c*
x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.14, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5908, 471, 75, 2434, 103, 94, 211, 5879, 5883, 102, 12}

$$\frac{en(ax-1)^{3/2}(ax+1)^{3/2}}{27a^3}+\frac{2en\sqrt{ax-1}\sqrt{ax+1}}{27a^3}-\frac{n\text{ArcTan}\left(\sqrt{ax-1}\sqrt{ax+1}\right)(9a^2d+2e)}{9a^3}-\frac{\sqrt{ax-1}\sqrt{ax+1}(9a^2d+2e)\log(cx^n)}{9a^3}+\frac{n\sqrt{ax-1}\sqrt{ax+1}(9a^2d+2e)}{9a^3}+dx\cosh^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\cosh^{-1}(ax)\log(cx^n)-\frac{ex^2\sqrt{ax-1}\sqrt{ax+1}\log(cx^n)}{9a}+\frac{dn\sqrt{ax-1}\sqrt{ax+1}}{a}-dnx\cosh^{-1}(ax)-\frac{1}{9}enx^3\cosh^{-1}(ax)+\frac{enx^2\sqrt{ax-1}\sqrt{ax+1}}{27a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

```
[Out] (d*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a + (2*e*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
/(27*a^3) + ((9*a^2*d + 2*e)*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) + (e*n
*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (e*n*(-1 + a*x)^(3/2)*(1 + a*x)
^(3/2))/(27*a^3) - d*n*x*ArcCosh[a*x] - (e*n*x^3*ArcCosh[a*x])/9 - ((9*a^2*
d + 2*e)*n*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/(9*a^3) - ((9*a^2*d + 2*e)
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[c*x^n])/(9*a^3) - (e*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]*Log[c*x^n] + (e*x^3*ArcCo
sh[a*x]*Log[c*x^n])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
```

```
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\cosh^{-1}(ax)\log\left(cx^n\right)\,dx = -\frac{(9a^2d+2e)\sqrt{-1+ax}\,\sqrt{1+ax}\,\log\left(cx^n\right)}{9a^3} - \frac{ex^2\sqrt{-1+ax}\,\sqrt{1+}}{9a}$$

$$= -\frac{(9a^2d+2e)\sqrt{-1+ax}\,\sqrt{1+ax}\,\log\left(cx^n\right)}{9a^3} - \frac{ex^2\sqrt{-1+ax}\,\sqrt{1+}}{9a}$$

$$= \frac{(9a^2d+2e)\,n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{en(-1+ax)^{3/2}(1+ax)^{3/2}}{27a^3} - a$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{(9a^2d+2e)\,n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{enx}{}$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{(9a^2d+2e)\,n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3} + \frac{enx}{}$$

$$= \frac{dn\sqrt{-1+ax}\,\sqrt{1+ax}}{a} + \frac{2en\sqrt{-1+ax}\,\sqrt{1+ax}}{27a^3} + \frac{(9a^2d+2e)\,n}{}$$

Mathematica [A]
time = 0.17, size = 145, normalized size = 0.46

$$\frac{3(9a^2d+2e)\,n\tan^{-1}\left(\frac{1}{\sqrt{-1+ax}\,\sqrt{1+ax}}\right)-3a^3x\cosh^{-1}(ax)\,(n(9d+ex^2)-3(3d+ex^2)\log(cx^n))+\sqrt{-1+ax}\,\sqrt{1+ax}\,(n(7e+2a^2(27d+ex^2))-3(2e+a^2(9d+ex^2))\log(cx^n))}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCosh[a*x]*Log[c*x^n],x]
```

```
[Out] (3*(9*a^2*d + 2*e)*n*ArcTan[1/(Sqrt[-1 + a*x]*Sqrt[1 + a*x])] - 3*a^3*x*Arc
Cosh[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) + Sqrt[-1 + a*x]*S
qrt[1 + a*x]*(n*(7*e + 2*a^2*(27*d + e*x^2)) - 3*(2*e + a^2*(9*d + e*x^2))*
Log[c*x^n]))/(27*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.38, size = 4732, normalized size = 15.17

method	result	size
default	Expression too large to display	4732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccosh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*I/a*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*
x+1)^(1/2))^2))*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2)))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*csgn(I/a)*x^2*e*n+1
/6*I*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))^2))^2*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*arccosh(a*x)*
Pi*x^3*e*n+1/2*I*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x
-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(a*x)*Pi*csgn(I/a)*x*d*n-1/9*e*n*x^3*arc
cosh(a*x)-2/9/a^3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e*(ln(c*x^n)-n*ln(x))-1/a*(a*
x+1)^(1/2)*(a*x-1)^(1/2)*d*(ln(c*x^n)-n*ln(x))+I/a*n*ln(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2)-I)*d-1/3*ln(a)*arccosh(a*x)*x^3*e*n-1/3*arccosh(a*x)*ln(2)*x^3
*e*n+1/3*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*x^3*e*n-ln(
a)*arccosh(a*x)*x*d*n-arccosh(a*x)*ln(2)*x*d*n+arccosh(a*x)*ln(1+(a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2))^2)*x*d*n-I/a*n*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)+I
)*d+2/9*I/a^3*n*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)-I)*e-2/9*I/a^3*n*ln(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2)+I)*e-1/18*I/a*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))^2*csgn(I*(1+(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))^2))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*e*n-1/18*I/a*csg
n(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^
2))*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*
x+1)^(1/2)))^2*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*e*n-1/18*I/a*csgn(I/a*(1+
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*P
i*(a*x+1)^(1/2)*(a*x-1)^(1/2)*csgn(I/a)*x^2*e*n+1/6*I*csgn(I/(a*x+(a*x-1)^(
```

```
1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*csgn(I/a*(1+(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arc
cosh(a*x)*Pi*x^3*e*n+1/6*I*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(a*x)*Pi*csgn(I/a)*x^3*e*n+1/2
*I*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))^2))^2*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*arccosh(a*x)*Pi*x*d*n
+1/2*I*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2))^2))^2*csgn(I*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*arccosh(a*x
)*Pi*x*d*n+1/2*I*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))^2))*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^2*arccosh(a*x)*Pi*x*d*n+1/2*I/a*csgn(I/(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))^3*P
i*(a*x+1)^(1/2)*(a*x-1)^(1/2)*d*n+1/2*I/a*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))^3*Pi*(a*x+1)^(1/2)*(a*x-
1)^(1/2)*d*n+1/9*I/a^3*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*
x-1)^(1/2)*(a*x+1)^(1/2))^2))^3*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e*n+1/9*I/a^
3*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2)))^3*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e*n+1/6*I*csgn(I/(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))^2*csgn(I/(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2)))*arccosh(a*x)*Pi*x^3*e*n-1/9/a^3*n*(3*arccosh(
a*x)*x^3*a^3*e-(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2*a^2*e+9*arccosh(a*x)*x*a^3*d
-9*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a^2*d-2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e)*ln(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-d*n*x*arccosh(a*x)+1/a*ln(a)*(a*x+1)^(1/2)*(
a*x-1)^(1/2)*d*n+1/a*ln(2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*d*n-1/a*ln(1+(a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*d*n+2/9/a^3*ln(a)
*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e*n+2/9/a^3*ln(2)*(a*x+1)^(1/2)*(a*x-1)^(1/2)*
e*n-2/9/a^3*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*e*n+1/2*I/a*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))^2))*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*csgn(I/a)
*d*n+1/2*I/a*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2))^2))*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*csgn(I*(1+(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*d*n+1/9*I
/a^3*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))^2))*csgn(I/a*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)/(a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2)))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*csgn(I/a)*e*n+1/9*I/a^3
*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2))^2))*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*csgn(I*(1+(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))^2))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*e*n-1/18*I/a*csgn(I/(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))^2
*csgn(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*Pi*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x
^2*e*n+2/27*e*n*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+1/3*arccosh(a*x)*x^3*e*(l
n(c*x^n)-n*ln(x))+arccosh(a*x)*x*d*(ln(c*x^n)-n*ln(x))+1/18*I/a*csgn(I/(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2))*csgn
(I/(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*csgn(I*(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*d*n + n*e)*(log(a*x + 1)*log(x) + dilog(-a*x))/a^3 - 1/6*(3*a^2*
d*n + n*e)*(log(-a*x + 1)*log(x) + dilog(a*x))/a^3 - 1/18*(9*(d*n - d*log(c
))*a^2 + (n - 3*log(c))*e)*log(a*x + 1)/a^3 + 1/18*(9*(d*n - d*log(c))*a^2
+ (n - 3*log(c))*e)*log(a*x - 1)/a^3 + 1/54*(2*a^3*(2*n - 3*log(c))*x^3*e -
 9*(3*a^2*d*n + n*e)*log(a*x + 1)*log(x) + 9*(3*a^2*d*n + n*e)*log(a*x - 1)
*log(x) + 6*(9*(2*d*n - d*log(c))*a^3 + a*(4*n - 3*log(c))*e)*x - 6*(a^3*(n
 - 3*log(c))*x^3*e + 9*(d*n - d*log(c))*a^3*x - 3*(a^3*x^3*e + 3*a^3*d*x)*l
og(x^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)) - 3*(2*a^3*x^3*e + 6*(3*a^3
*d + a*e)*x - 3*(3*a^2*d + e)*log(a*x + 1) + 3*(3*a^2*d + e)*log(a*x - 1))*
log(x^n))/a^3 + integrate(-1/9*(a*(n - 3*log(c))*x^3*e + 9*(d*n - d*log(c))
*a*x - 3*(a*x^3*e + 3*a*d*x)*log(x^n))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x +
1)*sqrt(a*x - 1) - a*x), x)
```

Fricas [A]
time = 0.49, size = 390, normalized size = 1.25

$$-\frac{6\,(9\,a^2dn+2\,n\cosh\left(1\right)+2\,n\sinh\left(1\right))\arctan\left(-ax+\sqrt{a^2x^2-1}\right)+3\,(9\,a^3dnx-9\,a^3dn+(a^3nx^3-a^3n)\cosh\left(1\right)-3\,(3\,a^3dx-3\,a^3d+(a^3x^3-a^3)\cosh\left(1\right)+(a^3x^3-a^3)\sinh\left(1\right))\log\left(c\right)-3\,(a^3nx^3\cosh\left(1\right)+a^3nx^3\sinh\left(1\right)+3\,a^3dnx)\log\left(x\right)+(a^3nx^3-a^3n)\sinh\left(1\right))\log\left(ax+\sqrt{a^2x^2-1}\right)-3\,(9\,a^3dn+a^3n\cosh\left(1\right)+a^3n\sinh\left(1\right)-3\,(3\,a^3d+a^3\cosh\left(1\right)+a^3\sinh\left(1\right))\log\left(c\right))\log\left(-ax+\sqrt{a^2x^2-1}\right)-(54\,a^2dn+(2\,a^2nx^2+7\,n)\cosh\left(1\right)-3\,(9\,a^2d+(a^2x^2+2)\cosh\left(1\right)+(a^2x^2+2)\sinh\left(1\right))\log\left(c\right)-3\,(9\,a^2dn+(a^2nx^2+2\,n)\cosh\left(1\right)+(a^2nx^2+2\,n)\sinh\left(1\right))\log\left(x\right)+(2\,a^2nx^2+7\,n)\sinh\left(1\right))\sqrt{a^2x^2-1}}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] -1/27*(6*(9*a^2*d*n + 2*n*cosh(1) + 2*n*sinh(1))*arctan(-a*x + sqrt(a^2*x^2
 - 1)) + 3*(9*a^3*d*n*x - 9*a^3*d*n + (a^3*n*x^3 - a^3*n)*cosh(1) - 3*(3*a^
3*d*x - 3*a^3*d + (a^3*x^3 - a^3)*cosh(1) + (a^3*x^3 - a^3)*sinh(1))*log(c)
 - 3*(a^3*n*x^3*cosh(1) + a^3*n*x^3*sinh(1) + 3*a^3*d*n*x)*log(x) + (a^3*n*
x^3 - a^3*n)*sinh(1))*log(a*x + sqrt(a^2*x^2 - 1)) - 3*(9*a^3*d*n + a^3*n*c
osh(1) + a^3*n*sinh(1) - 3*(3*a^3*d + a^3*cosh(1) + a^3*sinh(1))*log(c))*lo
g(-a*x + sqrt(a^2*x^2 - 1)) - (54*a^2*d*n + (2*a^2*n*x^2 + 7*n)*cosh(1) - 3
*(9*a^2*d + (a^2*x^2 + 2)*cosh(1) + (a^2*x^2 + 2)*sinh(1))*log(c) - 3*(9*a^
2*d*n + (a^2*n*x^2 + 2*n)*cosh(1) + (a^2*n*x^2 + 2*n)*sinh(1))*log(x) + (2*
a^2*n*x^2 + 7*n)*sinh(1))*sqrt(a^2*x^2 - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^{2}\right) \log \left(cx^{n}\right) \operatorname{acosh}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acosh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acosh(a*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{acosh}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acosh(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acosh(a*x)*(d + e*x^2), x)
```

3.192 $\int (d+ex^2) \tanh^{-1}(ax) \log(cx^n) \, dx$

Optimal. Leaf size=180

$$-\frac{5enx^2}{36a}-dnx\tanh^{-1}(ax)-\frac{1}{9}enx^3\tanh^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\tanh^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\tanh^{-1}(ax)\log$$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arctanh(a*x)-1/9*e*n*x^3*arctanh(a*x)+1/6*e*x^2*ln(c*
x^n)/a+d*x*arctanh(a*x)*ln(c*x^n)+1/3*e*x^3*arctanh(a*x)*ln(c*x^n)-1/2*d*n*
ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*ln(-
a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Rubi [A]
time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6123, 1607, 455, 45, 2435, 6021, 266, 6037, 272, 2438}

$$\frac{n(3a^2d+e)\,\text{PolyLog}(2,a^2x^2)}{12a^3}-\frac{dn\log(1-a^2x^2)}{2a}+\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3}-\frac{en\log(1-a^2x^2)}{18a^3}+dx\tanh^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\tanh^{-1}(ax)\log(cx^n)+\frac{ex^2\log(cx^n)}{6a}-dnx\tanh^{-1}(ax)-\frac{1}{9}enx^3\tanh^{-1}(ax)-\frac{5enx^2}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcTanh[a*x] - (e*n*x^3*ArcTanh[a*x])/9 + (e*x^
2*Log[c*x^n])/(6*a) + d*x*ArcTanh[a*x]*Log[c*x^n] + (e*x^3*ArcTanh[a*x]*Log
[c*x^n])/3 - (d*n*Log[1 - a^2*x^2])/(2*a) - (e*n*Log[1 - a^2*x^2])/(18*a^3)
 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) + ((3*a^2*d + e)*n*P
olyLog[2, a^2*x^2])/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2435

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6123

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (d+ex^2)\tanh^{-1}(ax)\log(cx^n)\,dx &= \frac{ex^2\log(cx^n)}{6a} + dx\tanh^{-1}(ax)\log(cx^n) + \frac{1}{3}ex^3\tanh^{-1}(ax)\log(cx^n \\
&= -\frac{enx^2}{12a} + \frac{ex^2\log(cx^n)}{6a} + dx\tanh^{-1}(ax)\log(cx^n) + \frac{1}{3}ex^3\tanh^{-1}(a \\
&= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log(cx^n)}{6a} + d \\
&= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log(cx^n)}{6a} + d \\
&= -\frac{enx^2}{12a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log(cx^n)}{6a} + d \\
&= -\frac{5enx^2}{36a} - dnx\tanh^{-1}(ax) - \frac{1}{9}enx^3\tanh^{-1}(ax) + \frac{ex^2\log(cx^n)}{6a} +
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 167, normalized size = 0.93

$$\frac{-5a^2enx^2+6a^2ex^2\log(cx^n)-4a^3x\tanh^{-1}(ax)\left(n(9d+ex^2)-3(3d+ex^2)\log(cx^n)\right)-18a^2dn\log(1-a^2x^2)+18a^2d\log(cx^n)\log(1-a^2x^2)+6e\log(cx^n)\log(1-a^2x^2)-2en\log(-1+a^2x^2)+3(3a^2d+e)n\mathrm{Li}_2(a^2x^2)}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcTanh[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 6*a^2*e*x^2*Log[c*x^n] - 4*a^3*x*ArcTanh[a*x]*(n*(9*d + e
*x^2) - 3*(3*d + e*x^2)*Log[c*x^n]) - 18*a^2*d*n*Log[1 - a^2*x^2] + 18*a^2*
d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 - a^2*x^2] - 2*e*n*Log
[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2])/(36*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.38, size = 90894, normalized size = 504.97

method	result	size
risch	Expression too large to display	1939
default	Expression too large to display	90894

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arctanh(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 364, normalized size = 2.02

$$-\frac{1}{36}n\left(\frac{18\,(i\,\pi d-2\,d)\log\left(x\right)}{a}+\frac{6\,(3\,a^{2}d+e)(\log\left(ax-1\right)\log\left(ax\right)+\mathrm{Li}_{2}(-ax+1))}{a^{3}}+\frac{6\,(3\,a^{2}d+e)(\log\left(ax+1\right)\log\left(-ax\right)+\mathrm{Li}_{2}(ax+1))}{a^{3}}+\frac{2\,(9\,a^{2}d+e)\log\left(ax+1\right)}{a^{3}}+\frac{-2i\,\pi a^{3}x^{3}e-18i\,\pi a^{3}dx+5\,a^{2}x^{2}e+2\,(a^{3}x^{3}e+9\,a^{3}dx)\log\left(ax+1\right)-2\,(a^{3}x^{3}e+9\,a^{3}dx-9\,a^{2}d-e)\log\left(ax-1\right)}{a^{3}}\right)+\frac{1}{36}\left(\left(6\,x^{3}\log\left(ax+1\right)-a\left(\frac{2\,a^{2}x^{3}-3\,ax^{2}+6\,x}{a^{3}}-\frac{6\,\log\left(ax+1\right)}{a^{4}}\right)\right)e-\left(6\,x^{3}\log\left(-ax+1\right)-a\left(\frac{2\,a^{2}x^{3}+3\,ax^{2}+6\,x}{a^{3}}+\frac{6\,\log\left(ax-1\right)}{a^{4}}\right)\right)e-\frac{18\,(ax-(ax+1)\log\left(ax+1\right)+1)d}{a}+\frac{18\,(ax-(ax-1)\log\left(-ax+1\right)-1)d}{a}\right)\log\left(cx^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/36*n*(18*(I*pi*d - 2*d)*log(x)/a + 6*(3*a^2*d + e)*(log(a*x - 1)*log(a*x
) + dilog(-a*x + 1))/a^3 + 6*(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilog(
a*x + 1))/a^3 + 2*(9*a^2*d + e)*log(a*x + 1)/a^3 + (-2*I*pi*a^3*x^3*e - 18*
I*pi*a^3*d*x + 5*a^2*x^2*e + 2*(a^3*x^3*e + 9*a^3*d*x)*log(a*x + 1) - 2*(a^
3*x^3*e + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/36*((6*x^3*log(a*
x + 1) - a*((2*a^2*x^3 - 3*a*x^2 + 6*x)/a^3 - 6*log(a*x + 1)/a^4))*e - (6*x
^3*log(-a*x + 1) - a*((2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 + 6*log(a*x - 1)/a^4)
)*e - 18*(a*x - (a*x + 1)*log(a*x + 1) + 1)*d/a + 18*(a*x - (a*x - 1)*log(-
a*x + 1) - 1)*d/a)*log(c*x^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arctanh(a*x)*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d+ex^{2}\right)\log\left(cx^{n}\right)\operatorname{atanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*atanh(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*atanh(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arctanh(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)*arctanh(a*x)*log(c*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{atanh}(a\,x)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*atanh(a*x)*(d + e*x^2),x)
[Out] int(log(c*x^n)*atanh(a*x)*(d + e*x^2), x)
```

3.193 $\int (d+ex^2)\coth^{-1}(ax)\log(cx^n)\,dx$

Optimal. Leaf size=180

$$-\frac{5enx^2}{36a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)+\frac{ex^2\log(cx^n)}{6a}+dx\coth^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\coth^{-1}(ax)\log($$

```
[Out] -5/36*e*n*x^2/a-d*n*x*arccoth(a*x)-1/9*e*n*x^3*arccoth(a*x)+1/6*e*x^2*ln(c*
x^n)/a+d*x*arccoth(a*x)*ln(c*x^n)+1/3*e*x^3*arccoth(a*x)*ln(c*x^n)-1/2*d*n*
ln(-a^2*x^2+1)/a-1/18*e*n*ln(-a^2*x^2+1)/a^3+1/6*(3*a^2*d+e)*ln(c*x^n)*ln(-
a^2*x^2+1)/a^3+1/12*(3*a^2*d+e)*n*polylog(2,a^2*x^2)/a^3
```

Rubi [A]

time = 0.10, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6124, 1607, 455, 45, 2435, 6022, 266, 6038, 272, 2438}

$$\frac{n(3a^2d+e)\,\text{PolyLog}(2,a^2x^2)}{12a^3}-\frac{dn\log(1-a^2x^2)}{2a}+\frac{(3a^2d+e)\log(1-a^2x^2)\log(cx^n)}{6a^3}-\frac{en\log(1-a^2x^2)}{18a^3}+dx\coth^{-1}(ax)\log(cx^n)+\frac{1}{3}ex^3\coth^{-1}(ax)\log(cx^n)+\frac{ex^2\log(cx^n)}{6a}-dnx\coth^{-1}(ax)-\frac{1}{9}enx^3\coth^{-1}(ax)-\frac{5enx^2}{36a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*e*n*x^2)/(36*a) - d*n*x*ArcCoth[a*x] - (e*n*x^3*ArcCoth[a*x])/9 + (e*x^
2*Log[c*x^n])/(6*a) + d*x*ArcCoth[a*x]*Log[c*x^n] + (e*x^3*ArcCoth[a*x]*Log
[c*x^n])/3 - (d*n*Log[1 - a^2*x^2])/(2*a) - (e*n*Log[1 - a^2*x^2])/(18*a^3)
 + ((3*a^2*d + e)*Log[c*x^n]*Log[1 - a^2*x^2])/(6*a^3) + ((3*a^2*d + e)*n*P
olyLog[2, a^2*x^2])/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2435

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))], x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)], x]}, Dist[a + b*L
og[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[{a, b, c
, d, e, f, n}, x] && PolynomialQ[Px, x] && MemberQ[{ArcTan, ArcCot, ArcTanh
, ArcCoth}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\coth^{-1}(ax)\log\left(cx^n\right)\,dx = \frac{ex^2\log\left(cx^n\right)}{6a} + dx\coth^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\coth^{-1}(ax)\log\left(cx^n\right)$$

$$= -\frac{enx^2}{12a} + \frac{ex^2\log\left(cx^n\right)}{6a} + dx\coth^{-1}(ax)\log\left(cx^n\right) + \frac{1}{3}ex^3\coth^{-1}(ax)$$

$$= -\frac{enx^2}{12a} - dnx\coth^{-1}(ax) - \frac{1}{9}enx^3\coth^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx$$

$$= -\frac{enx^2}{12a} - dnx\coth^{-1}(ax) - \frac{1}{9}enx^3\coth^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx$$

$$= -\frac{enx^2}{12a} - dnx\coth^{-1}(ax) - \frac{1}{9}enx^3\coth^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx$$

$$= -\frac{5enx^2}{36a} - dnx\coth^{-1}(ax) - \frac{1}{9}enx^3\coth^{-1}(ax) + \frac{ex^2\log\left(cx^n\right)}{6a} + dx$$

Mathematica [A]
time = 0.10, size = 178, normalized size = 0.99

$$\frac{-5a^2enx^2 + 36a^2dn\log\left(\frac{1}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\right) + 6a^2ex^2\log\left(cx^n\right) - 4a^3x\coth^{-1}(ax)\left(n(9d+ex^2) - 3(3d+ex^2)\log\left(cx^n\right)\right) + 18a^2d\log\left(cx^n\right)\log\left(1-a^2x^2\right) + 6e\log\left(cx^n\right)\log\left(1-a^2x^2\right) - 2en\log\left(-1+a^2x^2\right) + 3(3a^2d+e)\,n\mathrm{Li}_2(a^2x^2)}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCoth[a*x]*Log[c*x^n],x]
```

```
[Out] (-5*a^2*e*n*x^2 + 36*a^2*d*n*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] + 6*a^2*e*x
^2*Log[c*x^n] - 4*a^3*x*ArcCoth[a*x]*(n*(9*d + e*x^2) - 3*(3*d + e*x^2)*Log
[c*x^n]) + 18*a^2*d*Log[c*x^n]*Log[1 - a^2*x^2] + 6*e*Log[c*x^n]*Log[1 - a^
2*x^2] - 2*e*n*Log[-1 + a^2*x^2] + 3*(3*a^2*d + e)*n*PolyLog[2, a^2*x^2])/(
36*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 29.14, size = 1911, normalized size = 10.62

method	result	size
risch	Expression too large to display	1911

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccoth(a*x)*ln(c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 11/18*e/a^3*n*ln(x)+1/12*I*Pi*csgn(I*c*x^n)^3*e*ln(a*x-1)*x^3-1/12*I*Pi*csg
n(I*c*x^n)^3*e/a^3*ln(a*x-1)+1/12*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e/a*x^2-1/
```

```
12*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*e*ln(a*x-1)*x^3+1/12*I*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2*e/a^3*ln(a*x-1)-1/4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d*ln(a*x-
1)*x+1/4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d/a*ln(a*x-1)+1/4*I*Pi*csgn(I*x^n)*
csgn(I*c*x^n)^2*d*ln(a*x+1)*x+1/4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*d/a*ln(a
*x+1)+1/12*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*e*ln(a*x+1)*x^3+1/12*I*Pi*csgn(
I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e*ln(a*x-1)*x^3-1/2*d*n/a*ln(a*x+1)-1/18*e*n
/a^3*ln(a*x-1)-1/6*e*n/a^3*dilog(a*x)-1/18*e*n*x^3*ln(a*x+1)-1/18*e*n/a^3*l
n(a*x+1)+1/6*e*n/a^3*dilog(a*x+1)+1/2*ln(c)*d*ln(a*x+1)*x+1/2*ln(c)*d/a*ln(
a*x+1)+1/6*ln(c)*e*ln(a*x+1)*x^3+1/6*ln(c)*e/a^3*ln(a*x+1)+1/2*I*Pi*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)*d/a-5/36*e*n*x^2/a-11/18*ln(c)*e/a^3+1/12*I*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2*e/a*x^2-1/2*d*n*dilog(a*x)/a-1/2*d*n/a*ln(a*x-
1)-1/2*d*n*x*ln(a*x+1)+1/2*d*n*dilog(a*x+1)/a-1/12*I*Pi*csgn(I*c*x^n)^3*e/a
*x^2-1/2*d*n*ln(a*x-1)*ln(x)*x-1/4*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
*d*ln(a*x+1)*x-1/4*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*d/a*ln(a*x+1)+1
/2*I*Pi*csgn(I*c*x^n)^3*d/a-1/6*(ln(x^n)-n*ln(x))*e*ln(a*x-1)*x^3+1/6*(ln(x
^n)-n*ln(x))*e/a^3*ln(a*x-1)-1/2*(ln(x^n)-n*ln(x))*d*ln(a*x-1)*x+1/2*(ln(x^
n)-n*ln(x))*d/a*ln(a*x-1)-1/6*ln(c)*e*ln(a*x-1)*x^3+1/6*ln(c)*e/a^3*ln(a*x-
1)+1/6*(ln(x^n)-n*ln(x))*e*ln(a*x+1)*x^3+1/6*(ln(x^n)-n*ln(x))*e/a^3*ln(a*x
+1)+1/6*e*n/a^3*ln(a*x+1)*ln(x)+1/2*d*n*x*ln(a*x+1)*ln(x)+1/2*d*n*ln(x)*ln(
a*x+1)/a-1/4*I*Pi*csgn(I*c*x^n)^3*d/a*ln(a*x+1)-1/12*I*Pi*csgn(I*c*x^n)^3*e
*ln(a*x+1)*x^3-1/12*I*Pi*csgn(I*c*x^n)^3*e/a^3*ln(a*x+1)-ln(c)*d/a+11/36*I*
Pi*csgn(I*c*x^n)^3*e/a^3-d/a*ln(x^n)-1/12*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)*e*ln(a*x+1)*x^3-1/12*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e/a^3
*ln(a*x+1)-1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*d/a+1/6*e*n/a*x^2*ln(x)-1/4
*I*Pi*csgn(I*c*x^n)^3*d*ln(a*x+1)*x-1/12*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e*l
n(a*x-1)*x^3+1/12*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*e/a^3*ln(a*x+1)+11/36*I*
Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e/a^3+1/4*I*Pi*csgn(I*c*x^n)^3*d*ln(
a*x-1)*x-1/4*I*Pi*csgn(I*c*x^n)^3*d/a*ln(a*x-1)-1/6*e*n*ln(a*x-1)*ln(x)*x^3
-1/4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*d*ln(a*x-1)*x+1/4*I*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2*d/a*ln(a*x-1)-1/2*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d/a+1/2*d*n*
x*ln(a*x-1)+1/18*e*n*x^3*ln(a*x-1)-1/2*ln(c)*d*ln(a*x-1)*x+1/2*ln(c)*d/a*ln
(a*x-1)-11/18*e/a^3*ln(x^n)-11/36*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e/a^3-11/3
6*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2*e/a^3+1/6*e*n*ln(a*x+1)*ln(x)*x^3+1/12*I
*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e/a^3*ln(a*x-1)+1/6*e*n/a^3*ln(-a*x+1)*ln(x)-
1/6*e*n/a^3*ln(-a*x+1)*ln(a*x)+1/2*d*n/a*ln(-a*x+1)*ln(x)-1/2*d*n/a*ln(-a*x
+1)*ln(a*x)+1/6*(ln(x^n)-n*ln(x))*e/a*x^2+1/2*(ln(x^n)-n*ln(x))*d*ln(a*x+1)
*x+1/2*(ln(x^n)-n*ln(x))*d/a*ln(a*x+1)-1/12*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)*e/a*x^2-1/12*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*e/a^3*ln(a*
x-1)+d/a*n*ln(x)+1/6*ln(c)*e/a*x^2+1/4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d*ln(
a*x+1)*x+1/4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*d/a*ln(a*x+1)+1/4*I*Pi*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)*d*ln(a*x-1)*x-1/4*I*Pi*csgn(I*c)*csgn(I*x^n)*cs
gn(I*c*x^n)*d/a*ln(a*x-1)+1/12*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e*ln(a*x+1)*x
^3+1/12*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*e/a^3*ln(a*x+1)
```

Maxima [A]

time = 0.37, size = 328, normalized size = 1.82

$$-\frac{1}{36}n\left(\frac{6\,(3\,a^2d+e)(\log\left(ax-1\right)\log\left(ax\right)+\mathrm{Li}_2(-ax+1))}{a^3}+\frac{6\,(3\,a^2d+e)(\log\left(ax+1\right)\log\left(-ax\right)+\mathrm{Li}_2(ax+1))}{a^3}+\frac{2\,(9\,a^2d+e)\log\left(ax+1\right)}{a^3}+\frac{5\,a^2x^2e+2\,(a^3x^3e+9\,a^3dx)\log\left(ax+1\right)-2\,(a^3x^3e+9\,a^3dx-9\,a^2d-e)\log\left(ax-1\right)}{a^3}\right)+\frac{1}{12}\left(6\left(x\log\left(\frac{1}{ax}+1\right)+\frac{\log\left(ax+1\right)}{a}\right)d-6\left(x\log\left(-\frac{1}{ax}+1\right)-\frac{\log\left(ax-1\right)}{a}\right)d+\left(2\,x^3\log\left(\frac{1}{ax}+1\right)+\frac{\frac{ax^2-2\,x}{a}+\frac{2\,\log(ax+1)}{a^2}}{a}\right)e-\left(2\,x^3\log\left(-\frac{1}{ax}+1\right)-\frac{\frac{ax^2+2\,x}{a}+\frac{2\,\log(ax-1)}{a^2}}{a}\right)e\right)\log\left(cx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/36*n*(6*(3*a^2*d + e)*(log(a*x - 1)*log(a*x) + dilog(-a*x + 1))/a^3 + 6*
(3*a^2*d + e)*(log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a^3 + 2*(9*a^2*d +
e)*log(a*x + 1)/a^3 + (5*a^2*x^2*e + 2*(a^3*x^3*e + 9*a^3*d*x)*log(a*x + 1)
 - 2*(a^3*x^3*e + 9*a^3*d*x - 9*a^2*d - e)*log(a*x - 1))/a^3) + 1/12*(6*(x*
log(1/(a*x) + 1) + log(a*x + 1)/a)*d - 6*(x*log(-1/(a*x) + 1) - log(a*x - 1
)/a)*d + (2*x^3*log(1/(a*x) + 1) + ((a*x^2 - 2*x)/a + 2*log(a*x + 1)/a^2)/a
)*e - (2*x^3*log(-1/(a*x) + 1) - ((a*x^2 + 2*x)/a + 2*log(a*x - 1)/a^2)/a)*
e)*log(c*x^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arccoth(a*x)*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^{2}\right) \log \left(cx^{n}\right) \operatorname{acoth}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acoth(a*x)*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acoth(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccoth(a*x)*log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)*arccoth(a*x)*log(c*x^n), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,x^n\right)\,\mathrm{acoth}(a\,x)\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acoth(a*x)*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acoth(a*x)*(d + e*x^2), x)
```

3.194 $\int (d+ex^2)\sin^{-1}(ax)^2 \log(cx^n)\,dx$

Optimal. Leaf size=482

$$2dnx+\frac{2enx}{27a^2}+\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3-\frac{2dn\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{a}-\frac{4en\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{27a^3}-\frac{2(9a^2d+2}{}$$

```
[Out] 2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3+2/27*e*n*(-a^2*x^
2+1)^(3/2)*arcsin(a*x)/a^3-d*n*x*arcsin(a*x)^2-1/9*e*n*x^3*arcsin(a*x)^2+4/
9*(9*a^2*d+2*e)*n*arcsin(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))/a^3-2*d*x*l
n(c*x^n)-4/9*e*x*ln(c*x^n)/a^2-2/27*e*x^3*ln(c*x^n)+d*x*arcsin(a*x)^2*ln(c*
x^n)+1/3*e*x^3*arcsin(a*x)^2*ln(c*x^n)-2/9*I*(9*a^2*d+2*e)*n*polylog(2,-I*a
*x-(-a^2*x^2+1)^(1/2))/a^3+2/9*I*(9*a^2*d+2*e)*n*polylog(2,I*a*x+(-a^2*x^2+
1)^(1/2))/a^3-2*d*n*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a-4/27*e*n*arcsin(a*x)*(
-a^2*x^2+1)^(1/2)/a^3-2/9*(9*a^2*d+2*e)*n*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a^
3-2/27*e*n*x^2*arcsin(a*x)*(-a^2*x^2+1)^(1/2)/a+2*d*arcsin(a*x)*ln(c*x^n)*(
-a^2*x^2+1)^(1/2)/a+4/9*e*arcsin(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a^3+2/9*
e*x^2*arcsin(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.45, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4757, 4715, 4767, 8, 4723, 4795, 30, 2434, 6, 4783, 4803, 4268, 2317, 2438}

$$-\frac{2in(9a^2d+2e)\,\text{PolyLog}\left(2,-e^{i\text{ArcSin}(ax)}\right)}{9a^3}+\frac{2in(9a^2d+2e)\,\text{PolyLog}\left(2,e^{i\text{ArcSin}(ax)}\right)}{9a^3}+\frac{2d\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)\log(cx^n)}{a}+\frac{2ex^2\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)\log(cx^n)}{9a}-\frac{2dn\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)}{a}-\frac{2enx^2\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)}{27a}-\frac{4ex\log(cx^n)}{9a^2}+\frac{4}{9}nx\left(\frac{2e}{a^2}+9d\right)+\frac{2enx}{27a^2}+\frac{4e\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)\log(cx^n)}{9a^3}-\frac{2n\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)(9a^2d+2e)}{9a^3}+\frac{4n\text{ArcSin}(ax)(9a^2d+2e)\tanh^{-1}\left(e^{i\text{ArcSin}(ax)}\right)}{9a^3}+\frac{2en(1-a^2x^2)^{3/2}\,\text{ArcSin}(ax)}{27a^3}-\frac{4en\sqrt{1-a^2x^2}\,\text{ArcSin}(ax)}{27a^3}+dx\text{ArcSin}(ax)^2\log(cx^n)+\frac{1}{3}ex^3\text{ArcSin}(ax)^2\log(cx^n)-dnx\text{ArcSin}(ax)^2-\frac{1}{9}enx^3\text{ArcSin}(ax)^2-2dx\log(cx^n)-\frac{2}{27}ex^3\log(cx^n)+2dnx+\frac{2}{27}enx^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (2*e*n*x)/(27*a^2) + (4*(9*d + (2*e)/a^2)*n*x)/9 + (2*e*n*x^3)/27
 - (2*d*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a - (4*e*n*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(27*a^3) - (2*(9*a^2*d + 2*e)*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(9*a
^3) - (2*e*n*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(27*a) + (2*e*n*(1 - a^2*x^
2)^(3/2)*ArcSin[a*x])/(27*a^3) - d*n*x*ArcSin[a*x]^2 - (e*n*x^3*ArcSin[a*x]
^2)/9 + (4*(9*a^2*d + 2*e)*n*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])])/(9*a^3
) - 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])/27
 + (2*d*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 - a^2*x^2
]*ArcSin[a*x]*Log[c*x^n])/(9*a^3) + (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*
Log[c*x^n])/(9*a) + d*x*ArcSin[a*x]^2*Log[c*x^n] + (e*x^3*ArcSin[a*x]^2*Log
[c*x^n])/3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, -E^(I*ArcSin[a*x])])/a
^3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, E^(I*ArcSin[a*x])])/a^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4783

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4795

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4803

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]], Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\sin^{-1}(ax)^2\log\left(cx^n\right)\,dx = -2dx\log\left(cx^n\right) - \frac{4ex\log\left(cx^n\right)}{9a^2} - \frac{2}{27}ex^3\log\left(cx^n\right) + \frac{2d\sqrt{1-a^2x^2}\,\text{si}}{}$$

$$= -2dx\log\left(cx^n\right) - \frac{4ex\log\left(cx^n\right)}{9a^2} - \frac{2}{27}ex^3\log\left(cx^n\right) + \frac{2d\sqrt{1-a^2x^2}\,\text{si}}{}$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{2}{81}enx^3 - 2dx\log\left(cx^n\right) - \frac{4ex\log\left(cx^n\right)}{9a^2} - \frac{2}{27}ex^3$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{2}{81}enx^3 - \frac{2(9a^2d+2e)\,n\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{9a^3} +$$

$$= -\frac{2enx}{27a^2} + \frac{2(9a^2d+2e)\,nx}{9a^2} + \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{4}{81}enx^3 - \frac{2dn\sqrt{1-}}{}$$

$$= 2dnx - \frac{2enx}{27a^2} + \frac{2(9a^2d+2e)\,nx}{9a^2} + \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{2}{27}enx^3 - \frac{2d}{}$$

$$= 2dnx + \frac{2enx}{27a^2} + \frac{2(9a^2d+2e)\,nx}{9a^2} + \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{2}{27}enx^3 - \frac{2d}{}$$

$$= 2dnx + \frac{2enx}{27a^2} + \frac{2(9a^2d+2e)\,nx}{9a^2} + \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx + \frac{2}{27}enx^3 - \frac{2d}{}$$

Mathematica [**A**]
time = 0.60, size = 456, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSin[a*x]^2*Log[c*x^n],x]
```

```
[Out] (162*a^3*d*n*x + 26*a*e*n*x + 2*a^3*e*n*x^3 - 108*a^2*d*n*Sqrt[1 - a^2*x^2]
*ArcSin[a*x] - 14*e*n*Sqrt[1 - a^2*x^2]*ArcSin[a*x] - 4*a^2*e*n*x^2*Sqrt[1
- a^2*x^2]*ArcSin[a*x] - 27*a^3*d*n*x*ArcSin[a*x]^2 - 3*a^3*e*n*x^3*ArcSin[
a*x]^2 - 54*a^2*d*n*ArcSin[a*x]*Log[1 - E^(I*ArcSin[a*x])] - 12*e*n*ArcSin[
a*x]*Log[1 - E^(I*ArcSin[a*x])] + 54*a^2*d*n*ArcSin[a*x]*Log[1 + E^(I*ArcSi
n[a*x])] + 12*e*n*ArcSin[a*x]*Log[1 + E^(I*ArcSin[a*x])] - 54*a^3*d*x*Log[c
*x^n] - 12*a*e*x*Log[c*x^n] - 2*a^3*e*x^3*Log[c*x^n] + 54*a^2*d*Sqrt[1 - a^
2*x^2]*ArcSin[a*x]*Log[c*x^n] + 12*e*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^
n] + 6*a^2*e*x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]*Log[c*x^n] + 27*a^3*d*x*ArcS
in[a*x]^2*Log[c*x^n] + 9*a^3*e*x^3*ArcSin[a*x]^2*Log[c*x^n] - (6*I)*(9*a^2*
```

```
d + 2*e)*n*PolyLog[2, -E^(I*ArcSin[a*x])] + (6*I)*(9*a^2*d + 2*e)*n*PolyLog
[2, E^(I*ArcSin[a*x])])/(27*a^3)
```

Maple [F]
time = 3.98, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \arcsin \left(a x\right)^{2} \ln \left(c\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arcsin(a*x)^2*ln(c*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*(x^3*e + 3*d*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2*log(x^n) -
 1/9*((n*e - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(a*x, sqrt(a*x
+ 1)*sqrt(-a*x + 1))^2 + integrate(2/9*(3*(a*x^3*e + 3*a*d*x)*arctan2(a*x,
sqrt(a*x + 1)*sqrt(-a*x + 1))*log(x^n) - ((a*n*e - 3*a*e*log(c))*x^3 + 9*(a
*d*n - a*d*log(c))*x)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)))*sqrt(a*x
+ 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arcsin(a*x)^2*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{asin}^{2}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asin(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asin(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsin(a*x)^2*log(c*x^n),x, algorithm="giac")
[Out] integrate((x^2*e + d)*arcsin(a*x)^2*log(c*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asin}(a\,x)^2\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asin(a*x)^2*(d + e*x^2),x)
[Out] int(log(c*x^n)*asin(a*x)^2*(d + e*x^2), x)
```

3.195 $\int (d+ex^2)\cos^{-1}(ax)^2\log(cx^n)\,dx$

Optimal. Leaf size=490

$$2dnx+\frac{2enx}{27a^2}+\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2dn\sqrt{1-a^2x^2}\cos^{-1}(ax)}{a}+\frac{4en\sqrt{1-a^2x^2}\cos^{-1}(ax)}{27a^3}+\frac{2(9a^2d+2$$

```
[Out] 2*d*n*x+2/27*e*n*x/a^2+4/9*(9*d+2*e/a^2)*n*x+2/27*e*n*x^3-2/27*e*n*(-a^2*x^
2+1)^(3/2)*arccos(a*x)/a^3-d*n*x*arccos(a*x)^2-1/9*e*n*x^3*arccos(a*x)^2+2/
9*I*(9*a^2*d+2*e)*n*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a^3-2*d*x*ln(c*
x^n)-4/9*e*x*ln(c*x^n)/a^2-2/27*e*x^3*ln(c*x^n)+d*x*arccos(a*x)^2*ln(c*x^n)
+1/3*e*x^3*arccos(a*x)^2*ln(c*x^n)-2/9*I*(9*a^2*d+2*e)*n*polylog(2,-I*(a*x+
I*(-a^2*x^2+1)^(1/2)))/a^3+4/9*I*(9*a^2*d+2*e)*n*arccos(a*x)*arctan(a*x+I*(
-a^2*x^2+1)^(1/2))/a^3+2*d*n*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a+4/27*e*n*arcc
os(a*x)*(-a^2*x^2+1)^(1/2)/a^3+2/9*(9*a^2*d+2*e)*n*arccos(a*x)*(-a^2*x^2+1)
^(1/2)/a^3+2/27*e*n*x^2*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a-2*d*arccos(a*x)*ln
(c*x^n)*(-a^2*x^2+1)^(1/2)/a-4/9*e*arccos(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)
/a^3-2/9*e*x^2*arccos(a*x)*ln(c*x^n)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.47, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4758, 4716, 4768, 8, 4724, 4796, 30, 2434, 6, 4784, 4804, 4266, 2317, 2438}

$$-\frac{2in(9a^2d+2e)\,\text{PolyLog}\left(2,-ie^{i\text{ArcCos}(ax)}\right)}{9a^3}+\frac{2in(9a^2d+2e)\,\text{PolyLog}\left(2,ie^{i\text{ArcCos}(ax)}\right)}{9a^3}-\frac{2d\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)\log(cx^n)}{a}-\frac{2ex^2\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)\log(cx^n)}{9a}+\frac{2dn\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)}{a}+\frac{2enx^2\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)}{27a}-\frac{4ex\log(cx^n)}{9a^2}+\frac{4}{9}nx\left(\frac{2e}{a^2}+9d\right)+\frac{2enx}{27a^2}+\frac{4in\text{ArcCos}(ax)(9a^2d+2e)\,\text{ArcTan}\left(e^{i\text{ArcCos}(ax)}\right)}{9a^3}-\frac{4e\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)\log(cx^n)}{9a^3}+\frac{2n\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)(9a^2d+2e)}{9a^3}-\frac{2en(1-a^2x^2)^{3/2}\,\text{ArcCos}(ax)}{27a^3}+\frac{4en\sqrt{1-a^2x^2}\,\text{ArcCos}(ax)}{27a^3}+dx\text{ArcCos}(ax)^2\log(cx^n)+\frac{1}{3}ex^3\text{ArcCos}(ax)^2\log(cx^n)-dnx\text{ArcCos}(ax)^2-\frac{1}{9}enx^3\text{ArcCos}(ax)^2-2dx\log(cx^n)-\frac{2}{27}ex^3\log(cx^n)+2dnx+\frac{2}{27}enx^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (2*e*n*x)/(27*a^2) + (4*(9*d + (2*e)/a^2)*n*x)/9 + (2*e*n*x^3)/27
 + (2*d*n*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + (4*e*n*Sqrt[1 - a^2*x^2]*ArcCo
s[a*x])/(27*a^3) + (2*(9*a^2*d + 2*e)*n*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a
^3) + (2*e*n*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) - (2*e*n*(1 - a^2*x^
2)^(3/2)*ArcCos[a*x])/(27*a^3) - d*n*x*ArcCos[a*x]^2 - (e*n*x^3*ArcCos[a*x]
^2)/9 + (((4*I)/9)*(9*a^2*d + 2*e)*n*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])])
/a^3 - 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) - (2*e*x^3*Log[c*x^n])
/27 - (2*d*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Log[c*x^n])/a - (4*e*Sqrt[1 - a^2*
x^2]*ArcCos[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]*Log[c*x^n])/(9*a) + d*x*ArcCos[a*x]^2*Log[c*x^n] + (e*x^3*ArcCos[a*x]^2*
Log[c*x^n])/3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, (-I)*E^(I*ArcCos[a*
x])])/a^3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, I*E^(I*ArcCos[a*x])])/a
^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\cos^{-1}(ax)^2\log\left(cx^n\right)\,dx = -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1-a^2x^2}\,\mathrm{c}\cdots}{}$$

$$= -2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\log\left(cx^n\right)-\frac{2d\sqrt{1-a^2x^2}\,\mathrm{c}\cdots}{}$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3-2dx\log\left(cx^n\right)-\frac{4ex\log\left(cx^n\right)}{9a^2}-\frac{2}{27}ex^3\cdots$$

$$= \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{81}enx^3+\frac{2(9a^2d+2e)\,n\sqrt{1-a^2x^2}\,\cos^{-1}(ax)}{9a^3}\cdots$$

$$= -\frac{2enx}{27a^2}+\frac{2(9a^2d+2e)\,nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{4}{81}enx^3+\frac{2dn\sqrt{1-\cdots}}{}$$

$$= 2dnx-\frac{2enx}{27a^2}+\frac{2(9a^2d+2e)\,nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2d\cdots}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2(9a^2d+2e)\,nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2d\cdots}{}$$

$$= 2dnx+\frac{2enx}{27a^2}+\frac{2(9a^2d+2e)\,nx}{9a^2}+\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx+\frac{2}{27}enx^3+\frac{2d\cdots}{}$$

Mathematica [A]
time = 0.58, size = 564, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcCos[a*x]^2*Log[c*x^n],x]
```

```
[Out] 2*d*n*x + (4*e*n*x)/(9*a^2) + (2*e*n*x^3)/81 + (e*n*(-9*a*x - 12*(1 - a^2*x
^2)^(3/2)*ArcCos[a*x] + Cos[3*ArcCos[a*x]]))/(162*a^3) + (d*n*(-2*a*x - 2*S
qrt[1 - a^2*x^2]*ArcCos[a*x] + a*x*ArcCos[a*x]^2)*Log[x])/a + (e*n*(-12*a*x
 - 2*a^3*x^3 - 12*Sqrt[1 - a^2*x^2]*ArcCos[a*x] - 6*a^2*x^2*Sqrt[1 - a^2*x^
2]*ArcCos[a*x] + 9*a^3*x^3*ArcCos[a*x]^2)*Log[x])/(27*a^3) + (d*(-2*Sqrt[1
- a^2*x^2]*ArcCos[a*x] + a*x*(-2 + ArcCos[a*x]^2))*(-n - n*Log[x] + Log[c*x
```

```
^n]))/a + (2*d*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*Log[1 -
 I*E^(I*ArcCos[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] - I*PolyLo
g[2, (-I)*E^(I*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/a + (4*e
*n*(a*x + Sqrt[1 - a^2*x^2]*ArcCos[a*x] - ArcCos[a*x]*Log[1 - I*E^(I*ArcCos
[a*x])] + ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] - I*PolyLog[2, (-I)*E^(I
*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/(9*a^3) + (e*(-n + 3*(
-(n*Log[x]) + Log[c*x^n]))*(27*a*x*(-2 + ArcCos[a*x]^2) - (2 - 9*ArcCos[a*x
]^2)*Cos[3*ArcCos[a*x]] - 6*ArcCos[a*x]*(9*Sqrt[1 - a^2*x^2] + Sin[3*ArcCos
[a*x]])))/(324*a^3)
```

Maple [**F**]
time = 2.94, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d \right) \arccos \left(a x \right)^{2} \ln \left(c\,x^{n}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
[Out] int((e*x^2+d)*arccos(a*x)^2*ln(c*x^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="maxima")
[Out] 1/3*(x^3*e + 3*d*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*log(x^n) -
 1/9*((n*e - 3*e*log(c))*x^3 + 9*(d*n - d*log(c))*x)*arctan2(sqrt(a*x + 1)*
sqrt(-a*x + 1), a*x)^2 - integrate(2/9*(3*(a*x^3*e + 3*a*d*x)*arctan2(sqrt(
a*x + 1)*sqrt(-a*x + 1), a*x)*log(x^n) - ((a*n*e - 3*a*e*log(c))*x^3 + 9*(a
*d*n - a*d*log(c))*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))*sqrt(a*x
+ 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="fricas")
[Out] integral((x^2*e + d)*arccos(a*x)^2*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^{2}\right)\log\left(cx^{n}\right)\operatorname{acos}^{2}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acos(a*x)**2*ln(c*x**n),x)
[Out] Integral((d + e*x**2)*log(c*x**n)*acos(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccos(a*x)^2*log(c*x^n),x, algorithm="giac")
[Out] integrate((x^2*e + d)*arccos(a*x)^2*log(c*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^{n}\right)\,{\mathrm{acos}\left(a\,x\right)}^{2}\,\left(e\,x^{2}+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acos(a*x)^2*(d + e*x^2),x)
[Out] int(log(c*x^n)*acos(a*x)^2*(d + e*x^2), x)
```

3.196 $\int (d+ex^2)\sinh^{-1}(ax)^2\log(cx^n)\,dx$

Optimal. Leaf size=458

$$-2dnx+\frac{2enx}{27a^2}-\frac{4}{9}\left(9d-\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+\frac{2(9a^2d-2e)\,n\sqrt{1+a^2x^2}\,\sinh^{-1}(a}{9a^3}$$

```
[Out] -2*d*n*x+2/27*e*n*x/a^2-4/9*(9*d-2*e/a^2)*n*x-2/27*e*n*x^3+2/27*e*n*(a^2*x^
2+1)^(3/2)*arcsinh(a*x)/a^3-d*n*x*arcsinh(a*x)^2-1/9*e*n*x^3*arcsinh(a*x)^2
-4/9*(9*a^2*d-2*e)*n*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))/a^3+2*d*x*
ln(c*x^n)-4/9*e*x*ln(c*x^n)/a^2+2/27*e*x^3*ln(c*x^n)+d*x*arcsinh(a*x)^2*ln(
c*x^n)+1/3*e*x^3*arcsinh(a*x)^2*ln(c*x^n)-2/9*(9*a^2*d-2*e)*n*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))/a^3+2/9*(9*a^2*d-2*e)*n*polylog(2,a*x+(a^2*x^2+1)^(1/2
))/a^3+2*d*n*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a+2/9*(9*a^2*d-2*e)*n*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)/a^3-4/27*e*n*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3+2/27*
e*n*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-2*d*arcsinh(a*x)*ln(c*x^n)*(a^2*x^
2+1)^(1/2)/a+4/9*e*arcsinh(a*x)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a^3-2/9*e*x^2*a
rcsinh(a*x)*ln(c*x^n)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.45, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5793, 5772, 5798, 8, 5776, 5812, 30, 2434, 6, 5806, 5816, 4267, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x + (2*e*n*x)/(27*a^2) - (4*(9*d - (2*e)/a^2)*n*x)/9 - (2*e*n*x^3)/2
7 + (2*d*n*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + (2*(9*a^2*d - 2*e)*n*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x])/(9*a^3) - (4*e*n*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
27*a^3) + (2*e*n*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) + (2*e*n*(1 + a
^2*x^2)^(3/2)*ArcSinh[a*x])/(27*a^3) - d*n*x*ArcSinh[a*x]^2 - (e*n*x^3*ArcS
inh[a*x]^2)/9 - (4*(9*a^2*d - 2*e)*n*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/
(9*a^3) + 2*d*x*Log[c*x^n] - (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^
n])/27 - (2*d*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*Log[c*x^n])/a + (4*e*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]*Log[c*x^n])/(9*a) + d*x*ArcSinh[a*x]^2*Log[c*x^n] + (e*x^3*ArcSinh
[a*x]^2*Log[c*x^n])/3 - (2*(9*a^2*d - 2*e)*n*PolyLog[2, -E^ArcSinh[a*x]])/(
9*a^3) + (2*(9*a^2*d - 2*e)*n*PolyLog[2, E^ArcSinh[a*x]])/(9*a^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int (d+ex^2)\sinh^{-1}(ax)^2\log(cx^n)\,dx = 2dx\log(cx^n) - \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}ex^3\log(cx^n) - \frac{2d\sqrt{1+a^2x^2}\,\text{si}}{}$$

$$= 2dx\log(cx^n) - \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}ex^3\log(cx^n) - \frac{2d\sqrt{1+a^2x^2}\,\text{si}}{}$$

$$= -\frac{2}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{2}{81}enx^3 + 2dx\log(cx^n) - \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}$$

$$= -\frac{2}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{2}{81}enx^3 + \frac{2\left(9d - \frac{2e}{a^2}\right)n\sqrt{1+a^2x^2}\,\sinh^{-1}(ax}{9a}$$

$$= -\frac{2enx}{27a^2} - \frac{4}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{4}{81}enx^3 + \frac{2dn\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}$$

$$= -2dnx - \frac{2enx}{27a^2} - \frac{4}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 + \frac{2dn\sqrt{1+a^2x^2}\,\text{sin}}{a}$$

$$= -2dnx + \frac{2enx}{27a^2} - \frac{4}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 + \frac{2dn\sqrt{1+a^2x^2}\,\text{sin}}{a}$$

$$= -2dnx + \frac{2enx}{27a^2} - \frac{4}{9}\left(9d - \frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 + \frac{2dn\sqrt{1+a^2x^2}\,\text{sin}}{a}$$

Mathematica [A]
time = 0.51, size = 516, normalized size = 1.13

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*ArcSinh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x + (4*e*n*x)/(9*a^2) - (2*e*n*x^3)/81 + (2*e*n*(-1/3*(a*x) - (a^3*x
^3)/9 + ((1 + a^2*x^2)^(3/2)*ArcSinh[a*x])/3))/(9*a^3) + (d*n*(2*a*x - 2*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*ArcSinh[a*x]^2)*Log[x])/a + (e*n*(-12*a*
x + 2*a^3*x^3 + 12*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 6*a^2*x^2*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x] + 9*a^3*x^3*ArcSinh[a*x]^2)*Log[x])/(27*a^3) + (d*(-2*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(2 + ArcSinh[a*x]^2))*(-n - n*Log[x] + Lo
g[c*x^n]))/a + (e*(27*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + a*x*(-26 - 9*ArcSinh
[a*x]^2 + (2 + 9*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]]) - 3*ArcSinh[a*x]*Cos
h[3*ArcSinh[a*x]])*(-n + 3*(-(n*Log[x]) + Log[c*x^n])))/(162*a^3) + (2*d*n*
(-(a*x) + Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E^(-ArcSinh
[a*x])] - ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -E^(-ArcSinh
```

```
[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]))/a - (4*e*n*(-(a*x) + Sqrt[1 + a^2
*x^2]*ArcSinh[a*x] + ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]
*Log[1 + E^(-ArcSinh[a*x])] + PolyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E
^(-ArcSinh[a*x])]))/(9*a^3)
```

Maple [F]
time = 1.64, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \operatorname{arcsinh}\left(a x\right)^{2} \ln \left(c\,x^{n}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arcsinh(a*x)^2*ln(c*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*((n - 3*log(c))*x^3*e + 9*(d*n - d*log(c))*x - 3*(x^3*e + 3*d*x)*log(x
^n))*log(a*x + sqrt(a^2*x^2 + 1))^2 - integrate(-2/9*(a^3*(n - 3*log(c))*x^
5*e + (9*(d*n - d*log(c))*a^3 + a*(n - 3*log(c))*e)*x^3 + 9*(d*n - d*log(c)
)*a*x - 3*(a^3*x^5*e + (3*a^3*d + a*e)*x^3 + 3*a*d*x)*log(x^n) + (a^2*(n -
3*log(c))*x^4*e + 9*(d*n - d*log(c))*a^2*x^2 - 3*(a^2*x^4*e + 3*a^2*d*x^2)*
log(x^n))*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^3 + a*x +
(a^2*x^2 + 1)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arcsinh(a*x)^2*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x^{2}\right) \log{\left(c x^{n} \right)} \operatorname{asinh}^{2}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*asinh(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*asinh(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arcsinh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,\mathrm{asinh}(a\,x)^2\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*asinh(a*x)^2*(d + e*x^2), x)
```

3.197 $\int \left(d+ex^2\right) \cosh^{-1}(ax)^2 \log\left(cx^n\right)\, dx$

Optimal. Leaf size=508

$$-2dnx-\frac{2enx}{27a^2}-\frac{4}{9}\left(9d+\frac{2e}{a^2}\right)nx-\frac{2}{27}enx^3+\frac{2dn\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a}+\frac{4en\sqrt{-1+ax}\,\sqrt{1+ax}\,}{27a^3}$$

```
[Out] -2*d*n*x-2/27*e*n*x/a^2-4/9*(9*d+2*e/a^2)*n*x-2/27*e*n*x^3+2/27*e*n*(a*x-1)
^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a^3-d*n*x*arccosh(a*x)^2-1/9*e*n*x^3*arcc
osh(a*x)^2-4/9*(9*a^2*d+2*e)*n*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a^3+2*d*x*ln(c*x^n)+4/9*e*x*ln(c*x^n)/a^2+2/27*e*x^3*ln(c*x^n)+d*x
*arccosh(a*x)^2*ln(c*x^n)+1/3*e*x^3*arccosh(a*x)^2*ln(c*x^n)-2/9*I*(9*a^2*d
+2*e)*n*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/a^3+2/9*I*(9*a^2*d+2
*e)*n*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))/a^3+2*d*n*arccosh(a*x
)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+4/27*e*n*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)
^(1/2)/a^3+2/9*(9*a^2*d+2*e)*n*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3
+2/27*e*n*x^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-2*d*arccosh(a*x)*l
n(c*x^n)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a-4/9*e*arccosh(a*x)*ln(c*x^n)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)/a^3-2/9*e*x^2*arccosh(a*x)*ln(c*x^n)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A]

time = 0.98, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5909, 5879, 5915, 8, 5883, 5939, 30, 2434, 6, 5927, 5947, 4265, 2317, 2438, 41}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

```
[Out] -2*d*n*x - (2*e*n*x)/(27*a^2) - (4*(9*d + (2*e)/a^2)*n*x)/9 - (2*e*n*x^3)/2
7 + (2*d*n*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/a + (4*e*n*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(27*a^3) + (2*(9*a^2*d + 2*e)*n*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(9*a^3) + (2*e*n*x^2*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x])/(27*a) + (2*e*n*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcC
osh[a*x])/(27*a^3) - d*n*x*ArcCosh[a*x]^2 - (e*n*x^3*ArcCosh[a*x]^2)/9 - (4
*(9*a^2*d + 2*e)*n*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/(9*a^3) + 2*d*x*Log
[c*x^n] + (4*e*x*Log[c*x^n])/(9*a^2) + (2*e*x^3*Log[c*x^n])/27 - (2*d*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/a - (4*e*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a^3) - (2*e*x^2*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x]*Log[c*x^n])/(9*a) + d*x*ArcCosh[a*x]^2*Log[c*x^n] +
(e*x^3*ArcCosh[a*x]^2*Log[c*x^n])/3 + (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[
```

```
2, (-I)*E^ArcCosh[a*x]])/a^3 - (((2*I)/9)*(9*a^2*d + 2*e)*n*PolyLog[2, I*E^
ArcCosh[a*x]])/a^3
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
```

```
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rubi steps

$$\int (d+ex^2)\cosh^{-1}(ax)^2\log(cx^n)\, dx = 2dx\log(cx^n) + \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}ex^3\log(cx^n) - \frac{2d\sqrt{-1+ax}\,\sqrt{1}}{}$$

$$= 2dx\log(cx^n) + \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}ex^3\log(cx^n) - \frac{2d\sqrt{-1+ax}\,\sqrt{1}}{}$$

$$= -\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{2}{81}enx^3 + 2dx\log(cx^n) + \frac{4ex\log(cx^n)}{9a^2} + \frac{2}{27}e$$

$$= -\frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{2}{81}enx^3 + \frac{2(9a^2d+2e)\,n\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3}$$

$$= \frac{2enx}{27a^2} - \frac{2(9a^2d+2e)\,nx}{9a^2} - \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{4}{81}enx^3 + \frac{2dn\sqrt{-1}}{}$$

$$= -2dnx + \frac{2enx}{27a^2} - \frac{2(9a^2d+2e)\,nx}{9a^2} - \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 +$$

$$= -2dnx - \frac{2enx}{27a^2} - \frac{2(9a^2d+2e)\,nx}{9a^2} - \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 +$$

$$= -2dnx - \frac{2enx}{27a^2} - \frac{2(9a^2d+2e)\,nx}{9a^2} - \frac{2}{9}\left(9d+\frac{2e}{a^2}\right)nx - \frac{2}{27}enx^3 +$$

Mathematica [A]
time = 2.48, size = 619, normalized size = 1.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*ArcCosh[a*x]^2*Log[c*x^n],x]
```

```
[Out] (-648*a^3*d*n*x - 144*a*e*n*x - 8*a^3*e*n*x^3 + 2*e*n*(9*a*x + 12*((-1 + a*
x)/(1 + a*x))^(3/2)*(1 + a*x)^3*ArcCosh[a*x] - Cosh[3*ArcCosh[a*x]]) + 324*
a^2*d*n*(2*a*x - 2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + a*x*ArcCosh[
a*x]^2)*Log[x] + 12*e*n*(2*a*x*(6 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*(2 + a^2*x^2)*ArcCosh[a*x] + 9*a^3*x^3*ArcCosh[a*x]^2)*Log[x] + 324*a^2*
d*(2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x] - a*x*(2 + ArcCosh[a
*x]^2))*(n + n*Log[x] - Log[c*x^n]) + 648*a^2*d*n*(-(a*x) + Sqrt[(-1 + a*x)
/(1 + a*x)]*ArcCosh[a*x] + a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + I*
ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] - I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] + I*PolyLog[2, (-I)/E^ArcCosh[a*x]] - I*PolyLog[2, I/E^ArcCosh[a*x]]
```

```
) + 144*e*n*(-(a*x) + Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + a*x*Sqrt[(-
1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + I*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]]
 - I*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + I*PolyLog[2, (-I)/E^ArcCosh[a
*x]] - I*PolyLog[2, I/E^ArcCosh[a*x]]) - e*(n + 3*n*Log[x] - 3*Log[c*x^n])*
(27*a*x*(2 + ArcCosh[a*x]^2) + (2 + 9*ArcCosh[a*x]^2)*Cosh[3*ArcCosh[a*x]]
- 6*ArcCosh[a*x]*(9*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + Sinh[3*ArcCosh[a
*x]])))/(324*a^3)
```

Maple [**F**]
time = 2.07, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right) \operatorname{arccosh}\left(a x\right)^{2} \ln \left(c\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

```
[Out] int((e*x^2+d)*arccosh(a*x)^2*ln(c*x^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*((n - 3*log(c))*x^3*e + 9*(d*n - d*log(c))*x - 3*(x^3*e + 3*d*x)*log(x
^n))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2 - integrate(-2/9*(a^3*(n - 3*
log(c))*x^5*e + (9*(d*n - d*log(c))*a^3 - a*(n - 3*log(c))*e)*x^3 - 9*(d*n
- d*log(c))*a*x + (a^2*(n - 3*log(c))*x^4*e + 9*(d*n - d*log(c))*a^2*x^2 -
3*(a^2*x^4*e + 3*a^2*d*x^2)*log(x^n))*sqrt(a*x + 1)*sqrt(a*x - 1) - 3*(a^3*
x^5*e + (3*a^3*d - a*e)*x^3 - 3*a*d*x)*log(x^n))*log(a*x + sqrt(a*x + 1)*sq
rt(a*x - 1))/(a^3*x^3 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)*arccosh(a*x)^2*log(c*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^2\right) \log \left(cx^n\right) \operatorname{acosh}^2 \left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*acosh(a*x)**2*ln(c*x**n),x)
```

```
[Out] Integral((d + e*x**2)*log(c*x**n)*acosh(a*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*arccosh(a*x)^2*log(c*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,x^n\right)\,{\mathrm{acosh}\left(a\,x\right)}^2\,\left(e\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2),x)
```

```
[Out] int(log(c*x^n)*acosh(a*x)^2*(d + e*x^2), x)
```

3.198 $\int \frac{(a+b\log(cx^n))^p \mathbf{Li}_k(ex^q)}{x}\, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\log(cx^n))^p \operatorname{Li}_k(ex^q)}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\log(cx^n))^p \operatorname{PolyLog}(k,ex^q)}{x}\, dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
[Out] Defer[Int][((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x, x]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^p \operatorname{Li}_k(ex^q)}{x}\, dx = \int \frac{(a+b\log(cx^n))^p \operatorname{Li}_k(ex^q)}{x}\, dx$$

Mathematica [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))^p \operatorname{Li}_k(ex^q)}{x}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x,x]
[Out] Integrate[((a + b*Log[c*x^n])^p*PolyLog[k, e*x^q])/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^p \operatorname{polylog}(k,e\,x^q)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^p*polylog(k,e*x^q)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*polylog(k, x^q*e)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)^p*polylog(k, x^q*e)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log{(cx^{n})})^{p} \operatorname{Li}_{k}(ex^{q})}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**p*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**p*polylog(k, e*x**q)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*polylog(k, x^q*e)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\text{polylog}(k, e\,x^q)\ (a + b\ \ln\left(c\,x^n\right))^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x,x)
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^p)/x, x)
```

3.199 $\int \frac{(a+b\log(cx^n))^3\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=104

$$\frac{(a+b\log(cx^n))^3\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{3bn(a+b\log(cx^n))^2\operatorname{Li}_{2+k}(ex^q)}{q^2}+\frac{6b^2n^2(a+b\log(cx^n))\operatorname{Li}_{3+k}(ex^q)}{q^3}-\frac{6b^3n^3\operatorname{Li}_{4+k}(ex^q)}{q^4}$$

```
[Out] (a+b*ln(c*x^n))^3*polylog(1+k,e*x^q)/q-3*b*n*(a+b*ln(c*x^n))^2*polylog(2+k,
e*x^q)/q^2+6*b^2*n^2*(a+b*ln(c*x^n))*polylog(3+k,e*x^q)/q^3-6*b^3*n^3*polyl
og(4+k,e*x^q)/q^4
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2430, 6724}

$$\frac{6b^2n^2\text{PolyLog}(k+3,ex^q)\,(a+b\log(cx^n))}{q^3}-\frac{3bn\text{PolyLog}(k+2,ex^q)\,(a+b\log(cx^n))^2}{q^2}+\frac{\text{PolyLog}(k+1,ex^q)\,(a+b\log(cx^n))^3}{q}-\frac{6b^3n^3\text{PolyLog}(k+4,ex^q)}{q^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q])/q - (3*b*n*(a + b*Log[c*x^n])^
2*PolyLog[2 + k, e*x^q])/q^2 + (6*b^2*n^2*(a + b*Log[c*x^n])*PolyLog[3 + k,
 e*x^q])/q^3 - (6*b^3*n^3*PolyLog[4 + k, e*x^q])/q^4
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))^3 \operatorname{Li}_k(ex^q)}{x} dx = \frac{(a+b\log(cx^n))^3 \operatorname{Li}_{1+k}(ex^q)}{q} - \frac{(3bn)\int \frac{(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)}{x} dx}{q}$$
$$= \frac{(a+b\log(cx^n))^3 \operatorname{Li}_{1+k}(ex^q)}{q} - \frac{3bn(a+b\log(cx^n))^2 \operatorname{Li}_{2+k}(ex^q)}{q^2} + \frac{(6b^2n$$
$$= \frac{(a+b\log(cx^n))^3 \operatorname{Li}_{1+k}(ex^q)}{q} - \frac{3bn(a+b\log(cx^n))^2 \operatorname{Li}_{2+k}(ex^q)}{q^2} + \frac{6b^2n^2$$
$$= \frac{(a+b\log(cx^n))^3 \operatorname{Li}_{1+k}(ex^q)}{q} - \frac{3bn(a+b\log(cx^n))^2 \operatorname{Li}_{2+k}(ex^q)}{q^2} + \frac{6b^2n^2$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.95

$$\frac{q^3(a+b\log(cx^n))^3 \operatorname{Li}_{1+k}(ex^q) - 3bn\left(q^2(a+b\log(cx^n))^2 \operatorname{Li}_{2+k}(ex^q) + 2bn(-q(a+b\log(cx^n)) \operatorname{Li}_{3+k}(ex^q) + bn\operatorname{Li}_{4+k}(ex^q))\right)}{q^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^3*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (q^3*(a + b*Log[c*x^n])^3*PolyLog[1 + k, e*x^q] - 3*b*n*(q^2*(a + b*Log[c*x
^n])^2*PolyLog[2 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*x^n])*PolyLog[3 + k,
 e*x^q]) + b*n*PolyLog[4 + k, e*x^q])))/q^4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^3 \operatorname{polylog}(k, e\,x^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3*polylog(k,e*x^q)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^3*polylog(k,e*x^q)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*polylog(k, x^q*e)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*x^n)^3 + 3*a*b^2*log(c*x^n)^2 + 3*a^2*b*log(c*x^n) + a^
3)*polylog(k, x^q*e)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log(cx^n))^3 \operatorname{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**3*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**3*polylog(k, e*x**q)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^3*polylog(k, x^q*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(k, e\,x^q)\,\left(a + b\,\ln\left(c\,x^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x,x)
```

```
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^3)/x, x)
```

3.200 $\int \frac{(a+b\log(cx^n))^2\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=72

$$\frac{(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{2bn(a+b\log(cx^n))\operatorname{Li}_{2+k}(ex^q)}{q^2}+\frac{2b^2n^2\operatorname{Li}_{3+k}(ex^q)}{q^3}$$

```
[Out] (a+b*ln(c*x^n))^2*polylog(1+k,e*x^q)/q-2*b*n*(a+b*ln(c*x^n))*polylog(2+k,e*
x^q)/q^2+2*b^2*n^2*polylog(3+k,e*x^q)/q^3
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2430, 6724}

$$-\frac{2bn\text{PolyLog}(k+2,ex^q)\,(a+b\log(cx^n))}{q^2}+\frac{\text{PolyLog}(k+1,ex^q)\,(a+b\log(cx^n))^2}{q}+\frac{2b^2n^2\text{PolyLog}(k+3,ex^q)}{q^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q])/q - (2*b*n*(a + b*Log[c*x^n])*
PolyLog[2 + k, e*x^q])/q^2 + (2*b^2*n^2*PolyLog[3 + k, e*x^q])/q^3
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))^2\operatorname{Li}_k(ex^q)}{x}\,dx &= \frac{(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{(2bn)\int \frac{(a+b\log(cx^n))\operatorname{Li}_{1+k}(ex^q)}{x}\,dx}{q}\\ &= \frac{(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{2bn(a+b\log(cx^n))\operatorname{Li}_{2+k}(ex^q)}{q^2}+\frac{(2b^2n^2\\ &= \frac{(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)}{q}-\frac{2bn(a+b\log(cx^n))\operatorname{Li}_{2+k}(ex^q)}{q^2}+\frac{2b^2n^2\mathrm{I}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 0.96

$$\frac{q^2(a+b\log(cx^n))^2\operatorname{Li}_{1+k}(ex^q)+2bn(-q(a+b\log(cx^n))\operatorname{Li}_{2+k}(ex^q)+bn\operatorname{Li}_{3+k}(ex^q))}{q^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])^2*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (q^2*(a + b*Log[c*x^n])^2*PolyLog[1 + k, e*x^q] + 2*b*n*(-(q*(a + b*Log[c*x
^n])*PolyLog[2 + k, e*x^q]) + b*n*PolyLog[3 + k, e*x^q]))/q^3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))^2\operatorname{polylog}(k,e\,x^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^2*polylog(k,e*x^q)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))^2*polylog(k,e*x^q)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*polylog(k, x^q*e)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2)*polylog(k, x^q*e)/x, x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))^2\operatorname{Li}_k(ex^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))**2*polylog(k, e*x**q)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^2*polylog(k, x^q*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(k, e\,x^q)\ \left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x,x)
```

```
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n))^2)/x, x)
```

3.201 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_k(ex^q)}{x}\,dx$

Optimal. Leaf size=40

$$\frac{(a+b\log(cx^n))\,\mathrm{Li}_{1+k}(ex^q)}{q}-\frac{bn\mathrm{Li}_{2+k}(ex^q)}{q^2}$$

```
[Out] (a+b*ln(c*x^n))*polylog(1+k,e*x^q)/q-b*n*polylog(2+k,e*x^q)/q^2
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2430, 6724}

$$\frac{\mathrm{PolyLog}(k+1,ex^q)\,(a+b\log(cx^n))}{q}-\frac{bn\mathrm{PolyLog}(k+2,ex^q)}{q^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

```
[Out] ((a + b*Log[c*x^n])*PolyLog[1 + k, e*x^q])/q - (b*n*PolyLog[2 + k, e*x^q])/
q^2
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_k(ex^q)}{x}\,dx &= \frac{(a+b\log(cx^n))\,\mathrm{Li}_{1+k}(ex^q)}{q}-\frac{(bn)\int \frac{\mathrm{Li}_{1+k}(ex^q)}{x}\,dx}{q}\\ &= \frac{(a+b\log(cx^n))\,\mathrm{Li}_{1+k}(ex^q)}{q}-\frac{bn\mathrm{Li}_{2+k}(ex^q)}{q^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.28

$$\frac{a\mathrm{Li}_{1+k}(ex^q)}{q}+\frac{b\log\left(cx^n\right)\mathrm{Li}_{1+k}(ex^q)}{q}-\frac{bn\mathrm{Li}_{2+k}(ex^q)}{q^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[k, e*x^q])/x,x]
```

```
[Out] (a*PolyLog[1 + k, e*x^q])/q + (b*Log[c*x^n]*PolyLog[1 + k, e*x^q])/q - (b*n
*PolyLog[2 + k, e*x^q])/q^2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\ln\left(c\,x^{n}\right)\right)\mathrm{polylog}\left(k,e\,x^{q}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(k,e*x^q)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(k,e*x^q)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(k, x^q*e)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n) + a)*polylog(k, x^q*e)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(cx^{n}\right)\right)\mathrm{Li}_{k}\left(ex^{q}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(k,e*x**q)/x,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(k, e*x**q)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(k,e*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(k, x^q*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{polylog}(k, e\,x^q)\ (a + b\,\ln(c\,x^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((polylog(k, e*x^q)*(a + b*log(c*x^n)))/x, x)
```

3.202 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\text{Li}_k(ex^q)}{x\,(a+b\log{(cx^n)})},x\right)$$

```
[Out] Unintegrable(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{PolyLog}(k,ex^q)}{x\,(a+b\log{(cx^n)})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
[Out] Defer[Int][PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])), x]
```

Rubi steps

$$\int \frac{\text{Li}_k(ex^q)}{x\,(a+b\log{(cx^n)})}\,dx = \int \frac{\text{Li}_k(ex^q)}{x\,(a+b\log{(cx^n)})}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k(ex^q)}{x\,(a+b\log{(cx^n)})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])),x]
[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(k,e\,x^{q}\right)}{x\,(a+b\ln{(c\,x^{n})})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
[Out] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(polylog(k, x^q*e)/(b*x*log(c*x^n) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n)),x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{polylog}(k,e\,x^q)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))), x)
```

3.203 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\text{Li}_k(ex^q)}{bn\,(a+b\log(cx^n))}+\frac{q\text{Int}\left(\frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))},x\right)}{bn}$$

[Out] -polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))+q*Unintegrable(polylog(-1+k,e*x^q)/x/(a+b*ln(c*x^n)),x)/b/n

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{PolyLog}(k,ex^q)}{x\,(a+b\log(cx^n))^2}\,dx$$

Verification is not applicable to the result.

[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]

[Out] -(PolyLog[k, e*x^q]/(b*n*(a + b*Log[c*x^n]))) + (q*Defer[Int][PolyLog[-1 + k, e*x^q]/(x*(a + b*Log[c*x^n])), x])/(b*n)

Rubi steps

$$\int \frac{\text{Li}_k(ex^q)}{x\,(a+b\log(cx^n))^2}\,dx=-\frac{\text{Li}_k(ex^q)}{bn\,(a+b\log(cx^n))}+\frac{q\int\frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))}\,dx}{bn}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k(ex^q)}{x\,(a+b\log(cx^n))^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]

[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]

Maple [**A**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(k, e\,x^{q}\right)}{x\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^2,x)
[Out] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)^2*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(polylog(k, x^q*e)/(b^2*x*log(c*x^n)^2 + 2*a*b*x*log(c*x^n) + a^2*x
), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**2,x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{polylog}(k, e\,x^q)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^2), x)
```

3.204 $\int \frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{q\text{Li}_{-1+k}(ex^q)}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k(ex^q)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{q^2\text{Int}\left(\frac{\text{Li}_{-2+k}(ex^q)}{x(a+b\log(cx^n))},x\right)}{2b^2n^2}$$

```
[Out] -1/2*q*polylog(-1+k,e*x^q)/b^2/n^2/(a+b*ln(c*x^n))-1/2*polylog(k,e*x^q)/b/n
/(a+b*ln(c*x^n))^2+1/2*q^2*Unintegrable(polylog(-2+k,e*x^q)/x/(a+b*ln(c*x^n
)),x)/b^2/n^2
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{PolyLog}(k,ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
```

```
[Out] -1/2*(q*PolyLog[-1 + k, e*x^q])/(b^2*n^2*(a + b*Log[c*x^n])) - PolyLog[k, e
*x^q]/(2*b*n*(a + b*Log[c*x^n])^2) + (q^2*Defer[Int][PolyLog[-2 + k, e*x^q]
/(x*(a + b*Log[c*x^n])), x])/(2*b^2*n^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_k(ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx &= -\frac{\text{Li}_k(ex^q)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{q\int \frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))^2}\,dx}{2bn}\\
&= -\frac{q\text{Li}_{-1+k}(ex^q)}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k(ex^q)}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{q^2\int \frac{\text{Li}_{-2+k}(ex^q)}{x(a+b\log(cx^n))}\,dx}{2b^2n^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_k(ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3),x]
[Out] Integrate[PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^3), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(k, e\,x^{q}\right)}{x\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^3,x)
[Out] int(polylog(k,e*x^q)/x/(a+b*ln(c*x^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)^3*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="fricas")
[Out] integral(polylog(k, x^q*e)/(b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 + 3
*a^2*b*x*log(c*x^n) + a^3*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{k}\left(ex^{q}\right)}{x\left(a+b\log\left(cx^{n}\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x**q)/x/(a+b*ln(c*x**n))**3,x)
[Out] Integral(polylog(k, e*x**q)/(x*(a + b*log(c*x**n))**3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(k,e*x^q)/x/(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(polylog(k, x^q*e)/((b*log(c*x^n) + a)^3*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(k, e\,x^q)}{x\,\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3),x)
[Out] int(polylog(k, e*x^q)/(x*(a + b*log(c*x^n))^3), x)
```

3.205 $\int \frac{\log(x)\mathbf{Li}_n(ax)}{x}\,dx$

Optimal. Leaf size=20

$$\log(x)\text{Li}_{1+n}(ax) - \text{Li}_{2+n}(ax)$$

```
[Out] ln(x)*polylog(1+n,a*x)-polylog(2+n,a*x)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2430, 6724}

$$\log(x)\text{PolyLog}(n+1, ax) - \text{PolyLog}(n+2, ax)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*PolyLog[n, a*x])/x,x]
```

```
[Out] Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(x)\text{Li}_n(ax)}{x}\,dx &= \log(x)\text{Li}_{1+n}(ax) - \int \frac{\text{Li}_{1+n}(ax)}{x}\,dx \\ &= \log(x)\text{Li}_{1+n}(ax) - \text{Li}_{2+n}(ax) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\log(x)\text{Li}_{1+n}(ax) - \text{Li}_{2+n}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*PolyLog[n, a*x])/x,x]
[Out] Log[x]*PolyLog[1 + n, a*x] - PolyLog[2 + n, a*x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)\,\mathrm{polylog}(n, ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*polylog(n,a*x)/x,x)
[Out] int(ln(x)*polylog(n,a*x)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="maxima")
[Out] integrate(log(x)*polylog(n, a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="fricas")
[Out] integral(log(x)*polylog(n, a*x)/x, x)
```

Sympy [A]
time = 1.14, size = 15, normalized size = 0.75

$$\log(x)\,\mathrm{Li}_{n+1}(ax) - \mathrm{Li}_{n+2}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*polylog(n,a*x)/x,x)
[Out] log(x)*polylog(n + 1, a*x) - polylog(n + 2, a*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*polylog(n,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*polylog(n, a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln(x)\ \mathrm{polylog}(n, a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*polylog(n, a*x))/x,x)
```

```
[Out] int((log(x)*polylog(n, a*x))/x, x)
```

3.206 $\int \frac{\log^2(x)\mathbf{Li}_n(ax)}{x} \, dx$

Optimal. Leaf size=33

$$\log^2(x)\mathrm{Li}_{1+n}(ax) - 2\log(x)\mathrm{Li}_{2+n}(ax) + 2\mathrm{Li}_{3+n}(ax)$$

```
[Out] ln(x)^2*polylog(1+n,a*x)-2*ln(x)*polylog(2+n,a*x)+2*polylog(3+n,a*x)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2430, 6724}

$$2\mathrm{PolyLog}(n+3, ax) + \log^2(x)\mathrm{PolyLog}(n+1, ax) - 2\log(x)\mathrm{PolyLog}(n+2, ax)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

```
[Out] Log[x]^2*PolyLog[1 + n, a*x] - 2*Log[x]*PolyLog[2 + n, a*x] + 2*PolyLog[3 +
 n, a*x]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(x)\mathrm{Li}_n(ax)}{x} \, dx &= \log^2(x)\mathrm{Li}_{1+n}(ax) - 2\int \frac{\log(x)\mathrm{Li}_{1+n}(ax)}{x} \, dx \\
&= \log^2(x)\mathrm{Li}_{1+n}(ax) - 2\log(x)\mathrm{Li}_{2+n}(ax) + 2\int \frac{\mathrm{Li}_{2+n}(ax)}{x} \, dx \\
&= \log^2(x)\mathrm{Li}_{1+n}(ax) - 2\log(x)\mathrm{Li}_{2+n}(ax) + 2\mathrm{Li}_{3+n}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$\log^2(x)\text{Li}_{1+n}(ax) - 2\log(x)\text{Li}_{2+n}(ax) + 2\text{Li}_{3+n}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^2*PolyLog[n, a*x])/x,x]
```

```
[Out] Log[x]^2*PolyLog[1 + n, a*x] - 2*Log[x]*PolyLog[2 + n, a*x] + 2*PolyLog[3 +
 n, a*x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)^2 \operatorname{polylog}(n, ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2*polylog(n,a*x)/x,x)
```

```
[Out] int(ln(x)^2*polylog(n,a*x)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(log(x)^2*polylog(n, a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)^2*polylog(n, a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)^2 \operatorname{Li}_n(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2*polylog(n,a*x)/x,x)
```

```
[Out] Integral(log(x)**2*polylog(n, a*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*polylog(n,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)^2*polylog(n, a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)^2 \operatorname{polylog}(n, a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2*polylog(n, a*x))/x,x)
```

```
[Out] int((log(x)^2*polylog(n, a*x))/x, x)
```

3.207 $\int \left(\frac{q\text{Li}_{-1+k}(ex^q)}{bnx(a+b\log(cx^n))}-\frac{\text{Li}_k(ex^q)}{x(a+b\log(cx^n))^2}\right)\,dx$

Optimal. Leaf size=26

$$\frac{\text{Li}_k(ex^q)}{bn\left(a+b\log\left(cx^n\right)\right)}$$

[Out] polylog(k,e*x^q)/b/n/(a+b*ln(c*x^n))

Rubi [A]
time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2431}

$$\frac{\text{PolyLog}(k,ex^q)}{bn\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

[In] Int[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]

[Out] PolyLog[k, e*x^q]/(b*n*(a + b*Log[c*x^n]))

Rule 2431

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k, e*x^q]*((a + b*Log[c*x^n])^(p + 1)/
(b*n*(p + 1))), x] - Dist[q/(b*n*(p + 1)), Int[PolyLog[k - 1, e*x^q]*((a +
b*Log[c*x^n])^(p + 1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && LtQ[
p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{q\text{Li}_{-1+k}(ex^q)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k(ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx &= \frac{q\int \frac{\text{Li}_{-1+k}(ex^q)}{x(a+b\log(cx^n))}\,dx}{bn}-\int \frac{\text{Li}_k(ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\,dx\\ &= \frac{\text{Li}_k(ex^q)}{bn\left(a+b\log\left(cx^n\right)\right)}\end{aligned}$$

Mathematica [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\frac{q\text{Li}_{-1+k}(ex^q)}{bnx\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Li}_k(ex^q)}{x\left(a+b\log\left(cx^n\right)\right)^2}\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k
, e*x^q]/(x*(a + b*Log[c*x^n])^2),x]
```

```
[Out] Integrate[(q*PolyLog[-1 + k, e*x^q])/(b*n*x*(a + b*Log[c*x^n])) - PolyLog[k
, e*x^q]/(x*(a + b*Log[c*x^n])^2), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{q \operatorname{polylog}\left(k-1, e\,x^{q}\right)}{bnx\left(a+b\ln\left(c\,x^{n}\right)\right)}-\frac{\operatorname{polylog}\left(k, e\,x^{q}\right)}{x\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(q*polylog(k-1,e*x^q)/b/n/x/(a+b*ln(c*x^n))-polylog(k,e*x^q)/x/(a+b*ln(c
*x^n))^2,x)
```

```
[Out] int(q*polylog(k-1,e*x^q)/b/n/x/(a+b*ln(c*x^n))-polylog(k,e*x^q)/x/(a+b*ln(c
*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] integrate(q*polylog(k - 1, x^q*e)/((b*log(c*x^n) + a)*b*n*x) - polylog(k, x
^q*e)/((b*log(c*x^n) + a)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*n*polylog(k, x^q*e) - (b*q*log(c*x^n) + a*q)*polylog(k - 1, x^
q*e))/(b^3*n*x*log(c*x^n)^2 + 2*a*b^2*n*x*log(c*x^n) + a^2*b*n*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{aq\operatorname{Li}_{k-1}(ex^{q})}{a^{2}x+2abx\log\left(cx^{n}\right)+b^{2}x\log\left(cx^{n}\right)^{2}}\,dx+\int\left(-\frac{bn\operatorname{Li}_{k}(ex^{q})}{a^{2}x+2abx\log\left(cx^{n}\right)+b^{2}x\log\left(cx^{n}\right)^{2}}\right)dx+\int\frac{bq\log\left(cx^{n}\right)\operatorname{Li}_{k-1}(ex^{q})}{a^{2}x+2abx\log\left(cx^{n}\right)+b^{2}x\log\left(cx^{n}\right)^{2}}\,dx}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x**q)/b/n/x/(a+b*ln(c*x**n))-polylog(k,e*x**q)/x
/(a+b*ln(c*x**n))**2,x)
```

```
[Out] (Integral(a*q*polylog(k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*x
*log(c*x**n)**2), x) + Integral(-b*n*polylog(k, e*x**q)/(a**2*x + 2*a*b*x*l
og(c*x**n) + b**2*x*log(c*x**n)**2), x) + Integral(b*q*log(c*x**n)*polylog(
k - 1, e*x**q)/(a**2*x + 2*a*b*x*log(c*x**n) + b**2*x*log(c*x**n)**2), x))/
(b*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(q*polylog(-1+k,e*x^q)/b/n/x/(a+b*log(c*x^n))-polylog(k,e*x^q)/x/(
a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(q*polylog(k - 1, x^q*e)/((b*log(c*x^n) + a)*b*n*x) - polylog(k, x
^q*e)/((b*log(c*x^n) + a)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{q\,\mathrm{polylog}(k-1,e\,x^q)}{b\,n\,x\,\left(a+b\,\ln\left(c\,x^n\right)\right)}-\frac{\mathrm{polylog}(k,e\,x^q)}{x\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^q
)/(x*(a + b*log(c*x^n))^2),x)
```

```
[Out] int((q*polylog(k - 1, e*x^q))/(b*n*x*(a + b*log(c*x^n))) - polylog(k, e*x^q
)/(x*(a + b*log(c*x^n))^2), x)
```

3.208 $\int x^2(a+b\log(cx^n))\,\mathbf{Li}_2(ex)\,dx$

Optimal. Leaf size=217

$$\frac{5bnx}{27e^2}+\frac{7bnx^2}{108e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}x^3(a+b\log(cx^n))+\frac{2bn\log(1-ex)}{27e^3}-$$

```
[Out] 5/27*b*n*x/e^2+7/108*b*n*x^2/e+1/27*b*n*x^3-1/9*x*(a+b*ln(c*x^n))/e^2-1/18*
x^2*(a+b*ln(c*x^n))/e-1/27*x^3*(a+b*ln(c*x^n))+2/27*b*n*ln(-e*x+1)/e^3-2/27
*b*n*x^3*ln(-e*x+1)-1/9*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3+1/9*x^3*(a+b*ln(c*x^
n))*ln(-e*x+1)-1/9*b*n*polylog(2,e*x)/e^3-1/9*b*n*x^3*polylog(2,e*x)+1/3*x^
3*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Rubi [A]
time = 0.13, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2432, 2442, 45, 2423, 2438}

$$\frac{1}{3}x^3\text{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{bn\text{PolyLog}(2,ex)}{9e^3}-\frac{1}{9}bnx^3\text{PolyLog}(2,ex)-\frac{\log(1-ex)(a+b\log(cx^n))}{9e^3}-\frac{x(a+b\log(cx^n))}{9e^2}+\frac{1}{9}x^3\log(1-ex)(a+b\log(cx^n))-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}x^3(a+b\log(cx^n))+\frac{2bn\log(1-ex)}{27e^3}+\frac{5bnx}{27e^2}-\frac{2}{27}bnx^3\log(1-ex)+\frac{7bnx^2}{108e}+\frac{1}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (5*b*n*x)/(27*e^2) + (7*b*n*x^2)/(108*e) + (b*n*x^3)/27 - (x*(a + b*Log[c*x
^n]))/(9*e^2) - (x^2*(a + b*Log[c*x^n]))/(18*e) - (x^3*(a + b*Log[c*x^n]))/
27 + (2*b*n*Log[1 - e*x])/(27*e^3) - (2*b*n*x^3*Log[1 - e*x])/27 - ((a + b*
Log[c*x^n])*Log[1 - e*x])/(9*e^3) + (x^3*(a + b*Log[c*x^n])*Log[1 - e*x])/9
 - (b*n*PolyLog[2, e*x])/(9*e^3) - (b*n*x^3*PolyLog[2, e*x])/9 + (x^3*(a +
b*Log[c*x^n])*PolyLog[2, e*x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2(a+b\log(cx^n))\,\mathrm{Li}_2(ex)\,dx &= -\frac{1}{9}bnx^3\mathrm{Li}_2(ex)+\frac{1}{3}x^3(a+b\log(cx^n))\,\mathrm{Li}_2(ex)+\frac{1}{3}\int x^2(a+b\log(cx^n))\,\mathrm{l} \\
&= -\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}x^3(a+b\log(cx^n))-\frac{1}{27}bn \\
&= \frac{bnx}{9e^2}+\frac{bnx^2}{36e}+\frac{1}{81}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}x^3 \\
&= \frac{4bnx}{27e^2}+\frac{5bnx^2}{108e}+\frac{2}{81}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27} \\
&= \frac{4bnx}{27e^2}+\frac{5bnx^2}{108e}+\frac{2}{81}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27} \\
&= \frac{5bnx}{27e^2}+\frac{7bnx^2}{108e}+\frac{1}{27}bnx^3-\frac{x(a+b\log(cx^n))}{9e^2}-\frac{x^2(a+b\log(cx^n))}{18e}-\frac{1}{27}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 196, normalized size = 0.90

$$\frac{(a-bn\log(x)+b\log(cx^n))\left(-ex(6+3ex+2e^2x^2)+6(-1+e^3x^3)\log(1-ex)+18e^3x^3\mathrm{Li}_2(ex)\right)}{54e^3}+\frac{bn\left(20ex+7e^2x^2+4e^3x^3+8\log(1-ex)-8e^3x^3\log(1-ex)+2\log(x)\left(-ex(6+3ex+2e^2x^2)+6(-1+e^3x^3)\log(1-ex)\right)+12(-1-e^3x^3+3e^3x^3\log(x))\,\mathrm{Li}_2(ex)\right)}{108e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 +
e^3*x^3)*Log[1 - e*x] + 18*e^3*x^3*PolyLog[2, e*x]))/(54*e^3) + (b*n*(20*e*
x + 7*e^2*x^2 + 4*e^3*x^3 + 8*Log[1 - e*x] - 8*e^3*x^3*Log[1 - e*x] + 2*Log
[x]*(-(e*x*(6 + 3*e*x + 2*e^2*x^2)) + 6*(-1 + e^3*x^3)*Log[1 - e*x]) + 12*(
-1 - e^3*x^3 + 3*e^3*x^3*Log[x])*PolyLog[2, e*x]))/(108*e^3)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\ln(c\,x^{n}))\operatorname{polylog}(2,ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*polylog(2,e*x),x)
```

```
[Out] int(x^2*(a+b*ln(c*x^n))*polylog(2,e*x),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] 1/54*(18*x^3*dilog(x*e)*e^3 - 2*x^3*e^3 - 3*x^2*e^2 - 6*x*e + 6*(x^3*e^3 -
1)*log(-x*e + 1))*a*e^(-3) + 1/54*((6*(3*x^3*e^3*log(x^n) - (n*e^3 - 3*e^3*
log(c))*x^3)*dilog(x*e) - 2*((2*n*e^3 - 3*e^3*log(c))*x^3 - 3*n*log(x))*log
(-x*e + 1) - (2*x^3*e^3 + 3*x^2*e^2 + 6*x*e - 6*(x^3*e^3 - 1)*log(-x*e + 1)
)*log(x^n))*e^(-3) - 54*integrate(-1/54*(6*(n*e^3 - e^3*log(c))*x^3 + n*x^2
*e^2 + 3*n*x*e - 6*n*log(x) - 6*n)/(x*e^3 - e^2), x))*b
```

Fricas [**A**]
time = 0.36, size = 230, normalized size = 1.06

$$\frac{1}{108}\left(4\,(bn-a)x^{3}e^{3}+(7\,bn-6\,a)x^{2}e^{2}+4\,(5\,bn-3\,a)xe-12\,\left((bn-3\,a)x^{3}e^{3}+bn\right)\mathrm{Li}_{2}(xe)-4\,\left((2\,bn-3\,a)x^{3}e^{3}-2\,bn+3\,a\right)\log\left(-xe+1\right)+2\,\left(18\,bx^{3}\mathrm{Li}_{2}(xe)\,e^{3}-2\,bx^{3}e^{3}-3\,bx^{2}e^{2}-6\,bxe+6\,\left(bx^{3}e^{3}-b\right)\log\left(-xe+1\right)\right)\log\left(c\right)+2\,\left(18\,bnx^{3}\mathrm{Li}_{2}(xe)\,e^{3}-2\,bnx^{3}e^{3}-3\,bnx^{2}e^{2}-6\,bnxe+6\,\left(bnx^{3}e^{3}-bn\right)\log\left(-xe+1\right)\right)\log\left(x\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] 1/108*(4*(b*n - a)*x^3*e^3 + (7*b*n - 6*a)*x^2*e^2 + 4*(5*b*n - 3*a)*x*e -
12*((b*n - 3*a)*x^3*e^3 + b*n)*dilog(x*e) - 4*((2*b*n - 3*a)*x^3*e^3 - 2*b*
n + 3*a)*log(-x*e + 1) + 2*(18*b*x^3*dilog(x*e)*e^3 - 2*b*x^3*e^3 - 3*b*x^2
*e^2 - 6*b*x*e + 6*(b*x^3*e^3 - b)*log(-x*e + 1))*log(c) + 2*(18*b*n*x^3*di
```

```
log(x*e)*e^3 - 2*b*n*x^3*e^3 - 3*b*n*x^2*e^2 - 6*b*n*x*e + 6*(b*n*x^3*e^3 -
 b*n)*log(-x*e + 1))*log(x))*e^(-3)
```

Sympy [A]
time = 76.59, size = 250, normalized size = 1.15

$$\begin{cases} -\frac{ax^3 \operatorname{Li}_1(ex)}{9} + \frac{ax^3 \operatorname{Li}_2(ex)}{3} - \frac{ax^3}{27} - \frac{ax^2}{18e} - \frac{ax}{9e^2} + \frac{a \operatorname{Li}_1(ex)}{9e^3} + \frac{2bnx^3 \operatorname{Li}_1(ex)}{27} - \frac{bnx^3 \operatorname{Li}_2(ex)}{9} + \frac{bnx^3}{27} - \frac{bx^3 \log(cx^n) \operatorname{Li}_1(ex)}{9} + \frac{bx^3 \log(cx^n) \operatorname{Li}_2(ex)}{3} - \frac{bx^3 \log(cx^n)}{27} + \frac{7bnx^2}{108e} - \frac{bx^2 \log(cx^n)}{18e} + \frac{5bnx}{27e^2} - \frac{bx \log(cx^n)}{9e^2} - \frac{2bn \operatorname{Li}_1(ex)}{27e^3} - \frac{bn \operatorname{Li}_2(ex)}{9e^3} + \frac{b \log(cx^n) \operatorname{Li}_1(ex)}{9e^3} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Piecewise((-a*x**3*polylog(1, e*x)/9 + a*x**3*polylog(2, e*x)/3 - a*x**3/27
 - a*x**2/(18*e) - a*x/(9*e**2) + a*polylog(1, e*x)/(9*e**3) + 2*b*n*x**3*p
olylog(1, e*x)/27 - b*n*x**3*polylog(2, e*x)/9 + b*n*x**3/27 - b*x**3*log(c
*x**n)*polylog(1, e*x)/9 + b*x**3*log(c*x**n)*polylog(2, e*x)/3 - b*x**3*lo
g(c*x**n)/27 + 7*b*n*x**2/(108*e) - b*x**2*log(c*x**n)/(18*e) + 5*b*n*x/(27
*e**2) - b*x*log(c*x**n)/(9*e**2) - 2*b*n*polylog(1, e*x)/(27*e**3) - b*n*p
olylog(2, e*x)/(9*e**3) + b*log(c*x**n)*polylog(1, e*x)/(9*e**3), Ne(e, 0))
, (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*dilog(x*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{polylog}(2, e\,x)\,\left(a + b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x^2*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

3.209 $\int x(a+b\log(cx^n))\,\mathbf{Li}_2(ex)\,dx$

Optimal. Leaf size=185

$$\frac{bnx}{2e}+\frac{3}{16}bnx^2-\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log(1-ex)}{4e^2}-\frac{1}{4}bnx^2\log(1-ex)-\frac{(a+b\log(cx^n}{4e}$$

```
[Out] 1/2*b*n*x/e+3/16*b*n*x^2-1/4*x*(a+b*ln(c*x^n))/e-1/8*x^2*(a+b*ln(c*x^n))+1/
4*b*n*ln(-e*x+1)/e^2-1/4*b*n*x^2*ln(-e*x+1)-1/4*(a+b*ln(c*x^n))*ln(-e*x+1)/
e^2+1/4*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)-1/4*b*n*polylog(2,e*x)/e^2-1/4*b*n*x
^2*polylog(2,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)
```

Rubi [A]

time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2432, 2442, 45, 2423, 2438}

$$\frac{1}{2}x^2\text{PolyLog}(2,ex)(a+b\log(cx^n))-\frac{bn\text{PolyLog}(2,ex)}{4e^2}-\frac{1}{4}bnx^2\text{PolyLog}(2,ex)-\frac{\log(1-ex)(a+b\log(cx^n))}{4e^2}-\frac{x(a+b\log(cx^n))}{4e}+\frac{1}{4}x^2\log(1-ex)(a+b\log(cx^n))-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log(1-ex)}{4e^2}-\frac{1}{4}bnx^2\log(1-ex)+\frac{bnx}{2e}+\frac{3}{16}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (b*n*x)/(2*e) + (3*b*n*x^2)/16 - (x*(a + b*Log[c*x^n]))/(4*e) - (x^2*(a + b
*Log[c*x^n]))/8 + (b*n*Log[1 - e*x])/(4*e^2) - (b*n*x^2*Log[1 - e*x])/4 - (
(a + b*Log[c*x^n])*Log[1 - e*x])/(4*e^2) + (x^2*(a + b*Log[c*x^n])*Log[1 -
e*x])/4 - (b*n*PolyLog[2, e*x])/(4*e^2) - (b*n*x^2*PolyLog[2, e*x])/4 + (x^
2*(a + b*Log[c*x^n])*PolyLog[2, e*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
```

```
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x(a+b\log(cx^n))\,\mathrm{Li}_2(ex)\,dx &= -\frac{1}{4}bnx^2\mathrm{Li}_2(ex)+\frac{1}{2}x^2(a+b\log(cx^n))\,\mathrm{Li}_2(ex)+\frac{1}{2}\int x(a+b\log(cx^n))\log \\
&= -\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))-\frac{1}{8}bnx^2\log(1-ex)-\frac{(a+b\,l}{} \\
&= \frac{bnx}{4e}+\frac{1}{16}bnx^2-\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))-\frac{1}{8}bnx^2\log(1 \\
&= \frac{3bnx}{8e}+\frac{1}{8}bnx^2-\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log(1-ex}{8e^2} \\
&= \frac{3bnx}{8e}+\frac{1}{8}bnx^2-\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log(1-ex}{8e^2} \\
&= \frac{bnx}{2e}+\frac{3}{16}bnx^2-\frac{x(a+b\log(cx^n))}{4e}-\frac{1}{8}x^2(a+b\log(cx^n))+\frac{bn\log(1-ex}{4e^2}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 168, normalized size = 0.91

$$\frac{(a-bn\log(x)+b\log(cx^n))\left(-ex(2+ex)+2(-1+e^2x^2)\log(1-ex)+4e^2x^2\mathrm{Li}_2(ex)\right)}{8e^2}+\frac{bn\left(8ex+3e^2x^2+4\log(1-ex)-4e^2x^2\log(1-ex)+\log(x)\left(-2ex(2+ex)+4(-1+e^2x^2)\log(1-ex)\right)+\left(-4-4e^2x^2+8e^2x^2\log(x)\right)\mathrm{Li}_2(ex)\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x*(2 + e*x)) + 2*(-1 + e^2*x^2)*Log[1
 - e*x] + 4*e^2*x^2*PolyLog[2, e*x]))/(8*e^2) + (b*n*(8*e*x + 3*e^2*x^2 + 4
*Log[1 - e*x] - 4*e^2*x^2*Log[1 - e*x] + Log[x]*(-2*e*x*(2 + e*x) + 4*(-1 +
 e^2*x^2)*Log[1 - e*x]) + (-4 - 4*e^2*x^2 + 8*e^2*x^2*Log[x])*PolyLog[2, e*
x]))/(16*e^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))\,\mathrm{polylog}(2,ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*polylog(2,e*x),x)
```

```
[Out] int(x*(a+b*ln(c*x^n))*polylog(2,e*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2*dilog(x*e)*e^2 - x^2*e^2 - 2*x*e + 2*(x^2*e^2 - 1)*log(-x*e + 1)
)*a*e^(-2) + 1/8*((2*(2*x^2*e^2*log(x^n) - (n*e^2 - 2*e^2*log(c))*x^2)*dilo
g(x*e) - 2*((n*e^2 - e^2*log(c))*x^2 - n*log(x))*log(-x*e + 1) - (x^2*e^2 +
 2*x*e - 2*(x^2*e^2 - 1)*log(-x*e + 1))*log(x^n))*e^(-2) - 8*integrate(-1/8
*((3*n*e^2 - 2*e^2*log(c))*x^2 + n*x*e - 2*n*log(x) - 2*n)/(x*e^2 - e), x))
*b
```

Fricas [A]
time = 0.37, size = 196, normalized size = 1.06

$$\frac{1}{16}\left((3\,bn-2\,a)x^2e^2+4\,(2\,bn-a)xe-4\,\left((bn-2\,a)x^2e^2+bn\right)\mathrm{Li}_2(xe)-4\,\left((bn-a)x^2e^2-bn+a\right)\log\left(-xe+1\right)+2\,\left(4\,bx^2\mathrm{Li}_2(xe)\,e^2-bx^2e^2-2\,bxe+2\,\left(bx^2e^2-b\right)\log\left(-xe+1\right)\right)\log\left(c\right)+2\,\left(4\,bnx^2\mathrm{Li}_2(xe)\,e^2-bnx^2e^2-2\,bnxe+2\,\left(bnx^2e^2-bn\right)\log\left(-xe+1\right)\right)\log\left(x\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] 1/16*((3*b*n - 2*a)*x^2*e^2 + 4*(2*b*n - a)*x*e - 4*((b*n - 2*a)*x^2*e^2 +
b*n)*dilog(x*e) - 4*((b*n - a)*x^2*e^2 - b*n + a)*log(-x*e + 1) + 2*(4*b*x^
2*dilog(x*e)*e^2 - b*x^2*e^2 - 2*b*x*e + 2*(b*x^2*e^2 - b)*log(-x*e + 1))*l
og(c) + 2*(4*b*n*x^2*dilog(x*e)*e^2 - b*n*x^2*e^2 - 2*b*n*x*e + 2*(b*n*x^2*
e^2 - b*n)*log(-x*e + 1))*log(x))*e^(-2)
```

Sympy [A]
time = 24.94, size = 206, normalized size = 1.11

$$\begin{cases} -\frac{ax^2\operatorname{Li}_1(ex)}{4}+\frac{ax^2\operatorname{Li}_2(ex)}{2}-\frac{ax^2}{8}-\frac{ax}{4e}+\frac{a\operatorname{Li}_1(ex)}{4e^2}+\frac{bnx^2\operatorname{Li}_1(ex)}{4}-\frac{bnx^2\operatorname{Li}_2(ex)}{4}+\frac{3bnx^2}{16}-\frac{bx^2\log(cx^n)\operatorname{Li}_1(ex)}{4}+\frac{bx^2\log(cx^n)\operatorname{Li}_2(ex)}{2}-\frac{bx^2\log(cx^n)}{8}+\frac{bnx}{2e}-\frac{bx\log(cx^n)}{4e}-\frac{bn\operatorname{Li}_1(ex)}{4e^2}-\frac{bn\operatorname{Li}_2(ex)}{4e^2}+\frac{b\log(cx^n)\operatorname{Li}_1(ex)}{4e^2} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Piecewise((-a*x**2*polylog(1, e*x)/4 + a*x**2*polylog(2, e*x)/2 - a*x**2/8
- a*x/(4*e) + a*polylog(1, e*x)/(4*e**2) + b*n*x**2*polylog(1, e*x)/4 - b*n
*x**2*polylog(2, e*x)/4 + 3*b*n*x**2/16 - b*x**2*log(c*x**n)*polylog(1, e*x
)/4 + b*x**2*log(c*x**n)*polylog(2, e*x)/2 - b*x**2*log(c*x**n)/8 + b*n*x/(
2*e) - b*x*log(c*x**n)/(4*e) - b*n*polylog(1, e*x)/(4*e**2) - b*n*polylog(2
, e*x)/(4*e**2) + b*log(c*x**n)*polylog(1, e*x)/(4*e**2), Ne(e, 0)), (0, Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*dilog(x*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, e*x)*(a + b*log(c*x^n)),x)
```

```
[Out] int(x*polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

3.210 $\int (a+b\log(cx^n))\,\mathbf{Li}_2(ex)\,dx$

Optimal. Leaf size=106

$$3bnx-x(a+b\log(cx^n))+\frac{2bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-\frac{bn\text{Li}_2(ex)}{e}-bnx$$

```
[Out] 3*b*n*x-x*(a+b*ln(c*x^n))+2*b*n*(-e*x+1)*ln(-e*x+1)/e-(-e*x+1)*(a+b*ln(c*x^
n))*ln(-e*x+1)/e-b*n*polylog(2,e*x)/e-b*n*x*polylog(2,e*x)+x*(a+b*ln(c*x^n)
)*polylog(2,e*x)
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2428, 2436, 2332, 2417, 2458, 45, 2393, 2352}

$$x\text{PolyLog}(2,ex)(a+b\log(cx^n))-bnx\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{e}-\frac{(1-ex)\log(1-ex)(a+b\log(cx^n))}{e}-x(a+b\log(cx^n))+\frac{2bn(1-ex)\log(1-ex)}{e}+3bnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] 3*b*n*x - x*(a + b*Log[c*x^n]) + (2*b*n*(1 - e*x)*Log[1 - e*x])/e - ((1 - e
*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e - (b*n*PolyLog[2, e*x])/e - b*n*x*Po
lyLog[2, e*x] + x*(a + b*Log[c*x^n])*PolyLog[2, e*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
```

```
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2428

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> Simp[(-b)*n*x*PolyLog[k, e*x^q], x] + (-Dist[q, Int[PolyLog[k -
1, e*x^q]*(a + b*Log[c*x^n]), x], x] + Dist[b*n*q, Int[PolyLog[k - 1, e*x^q
], x], x] + Simp[x*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]), x]) /; FreeQ[{a, b
, c, e, n, q}, x] && IGtQ[k, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\log(cx^n))\operatorname{Li}_2(ex)\,dx &= -bnx\operatorname{Li}_2(ex)+x(a+b\log(cx^n))\operatorname{Li}_2(ex)-(bn)\int \log(1-ex)\,dx+\int (a \\
&= -x(a+b\log(cx^n))-\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-bnx\operatorname{Li}_2(ex) \\
&= 2bnx-x(a+b\log(cx^n))+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log(}{e} \\
&= 2bnx-x(a+b\log(cx^n))+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log(}{e} \\
&= 2bnx-x(a+b\log(cx^n))+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log(}{e} \\
&= 2bnx-x(a+b\log(cx^n))+\frac{bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log(}{e} \\
&= 3bnx-x(a+b\log(cx^n))+\frac{2bn(1-ex)\log(1-ex)}{e}-\frac{(1-ex)(a+b\log}{e}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 113, normalized size = 1.07

$$
(a+b(-n\log(x)+\log(cx^n)))\left(-x+\left(-\frac{1}{e}+x\right)\log(1-ex)+x\operatorname{Li}_2(ex)\right)+\frac{bn(3ex+2\log(1-ex)-2ex\log(1-ex)+\log(x)(-ex+(-1+ex)\log(1-ex))+(-1-ex+ex\log(x))\operatorname{Li}_2(ex))}{e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])*PolyLog[2, e*x],x]
```

```
[Out] (a + b*(-(n*Log[x]) + Log[c*x^n]))*(-x + (-e^(-1) + x)*Log[1 - e*x] + x*Pol
yLog[2, e*x]) + (b*n*(3*e*x + 2*Log[1 - e*x] - 2*e*x*Log[1 - e*x] + Log[x]*
(-(e*x) + (-1 + e*x)*Log[1 - e*x]) + (-1 - e*x + e*x*Log[x])*PolyLog[2, e*x
]))/e
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \left(a+b\ln\left(c\,x^{n}\right)\right)\operatorname{polylog}\left(2,ex\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(2,e*x),x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(2,e*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="maxima")
```

```
[Out] (x*dilog(x*e)*e - x*e + (x*e - 1)*log(-x*e + 1))*a*e^(-1) + (((x*e*log(x^n)
 - (n*e - e*log(c))*x)*dilog(x*e) - ((2*n*e - e*log(c))*x - n*log(x))*log(-
x*e + 1) - (x*e - (x*e - 1)*log(-x*e + 1))*log(x^n))*e^(-1) - integrate(-((
3*n*e - e*log(c))*x - n*log(x) - n)/(x*e - 1), x))*b
```

Fricas [**A**]
time = 0.36, size = 146, normalized size = 1.38

$$((3\,bn-a)xe-((bn-a)xe+bn)\mathrm{Li}_2(xe)-((2\,bn-a)xe-2\,bn+a)\log\left(-xe+1\right)+(bx\mathrm{Li}_2(xe)\,e-bxe+(bxe-b)\log\left(-xe+1\right))\log\left(c\right)+(bnx\mathrm{Li}_2(xe)\,e-bnxe+(bnxe-bn)\log\left(-xe+1\right))\log\left(x\right))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="fricas")
```

```
[Out] ((3*b*n - a)*x*e - ((b*n - a)*x*e + b*n)*dilog(x*e) - ((2*b*n - a)*x*e - 2*
b*n + a)*log(-x*e + 1) + (b*x*dilog(x*e)*e - b*x*e + (b*x*e - b)*log(-x*e +
 1))*log(c) + (b*n*x*dilog(x*e)*e - b*n*x*e + (b*n*x*e - b*n)*log(-x*e + 1)
)*log(x))*e^(-1)
```

Sympy [**A**]
time = 6.96, size = 136, normalized size = 1.28

$$\begin{cases} -ax\,\mathrm{Li}_{1}\left(ex\right) + ax\,\mathrm{Li}_{2}\left(ex\right) - ax + \frac{a\,\mathrm{Li}_{1}\left(ex\right)}{e} + 2bnx\,\mathrm{Li}_{1}\left(ex\right) - bnx\,\mathrm{Li}_{2}\left(ex\right) + 3bnx - bx\log\left(cx^{n}\right)\mathrm{Li}_{1}\left(ex\right) + bx\log\left(cx^{n}\right)\mathrm{Li}_{2}\left(ex\right) - bx\log\left(cx^{n}\right) - \frac{2bn\,\mathrm{Li}_{1}\left(ex\right)}{e} - \frac{bn\,\mathrm{Li}_{2}\left(ex\right)}{e} + \frac{b\log\left(cx^{n}\right)\mathrm{Li}_{1}\left(ex\right)}{e} & \text{for}\: e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x),x)
```

```
[Out] Piecewise((-a*x*polylog(1, e*x) + a*x*polylog(2, e*x) - a*x + a*polylog(1,
e*x)/e + 2*b*n*x*polylog(1, e*x) - b*n*x*polylog(2, e*x) + 3*b*n*x - b*x*lo
g(c*x**n)*polylog(1, e*x) + b*x*log(c*x**n)*polylog(2, e*x) - b*x*log(c*x**
n) - 2*b*n*polylog(1, e*x)/e - b*n*polylog(2, e*x)/e + b*log(c*x**n)*polylo
g(1, e*x)/e, Ne(e, 0)), (0, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*dilog(x*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}(2, e\,x)\,\left(a + b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, e*x)*(a + b*log(c*x^n)),x)
[Out] int(polylog(2, e*x)*(a + b*log(c*x^n)), x)
```

3.211 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_2(ex)}{x}\,dx$

Optimal. Leaf size=26

$$(a+b\log(cx^n))\,\mathrm{Li}_3(ex)-bn\mathrm{Li}_4(ex)$$

[Out] (a+b*ln(c*x^n))*polylog(3,e*x)-b*n*polylog(4,e*x)

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2430, 6724}

$$\mathrm{PolyLog}(3,ex)\,(a+b\log(cx^n))-bn\mathrm{PolyLog}(4,ex)$$

Antiderivative was successfully verified.

[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]

[Out] (a + b*Log[c*x^n])*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_2(ex)}{x}\,dx &= (a+b\log(cx^n))\,\mathrm{Li}_3(ex)-(bn)\int \frac{\mathrm{Li}_3(ex)}{x}\,dx\\ &= (a+b\log(cx^n))\,\mathrm{Li}_3(ex)-bn\mathrm{Li}_4(ex)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.15

$$a\mathrm{Li}_3(ex)+b\log(cx^n)\,\mathrm{Li}_3(ex)-bn\mathrm{Li}_4(ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x,x]
```

```
[Out] a*PolyLog[3, e*x] + b*Log[c*x^n]*PolyLog[3, e*x] - b*n*PolyLog[4, e*x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^{n}))\,\mathrm{polylog}(2,ex)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(2,e*x)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(2,e*x)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x))*dilog(x
*e) + 1/2*integrate((2*b*log(-x*e + 1)*log(x)*log(x^n) - (b*n*log(x)^2 - 2*
(b*log(c) + a)*log(x))*log(-x*e + 1))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*dilog(x*e)*log(c*x^n) + a*dilog(x*e))/x, x)
```

Sympy [A]
time = 9.09, size = 26, normalized size = 1.00

$$a\,\mathrm{Li}_{3}(ex) + b(-n\,\mathrm{Li}_{4}(ex) + \log(cx^{n})\,\mathrm{Li}_{3}(ex))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x,x)
```

```
[Out] a*polylog(3, e*x) + b*(-n*polylog(4, e*x) + log(c*x**n)*polylog(3, e*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*dilog(x*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x, x)
```

3.212 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_2(ex)}{x^2}\,dx$

Optimal. Leaf size=142

$$2ben\log(x)-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-2ben\log(1-ex)+\frac{2bn\log(1-ex)}{x}-e(a+b\log\left(cx^n\right))\,\mathrm{l}$$

```
[Out] 2*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-2*b*e*n*ln(-e*x+1)+
2*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1)/
x-b*e*n*polylog(2,e*x)-b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,e*x)/
x
```

Rubi [A]
time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2432, 2442, 36, 29, 31, 2423, 2338, 2438}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-ben\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{x}+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{1}{2}ben\log^2(x)+2ben\log(x)-2ben\log(1-ex)+\frac{2bn\log(1-ex)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

```
[Out] 2*b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - 2*b*e*n
*Log[1 - e*x] + (2*b*n*Log[1 - e*x])/x - e*(a + b*Log[c*x^n])*Log[1 - e*x]
+ ((a + b*Log[c*x^n])*Log[1 - e*x])/x - b*e*n*PolyLog[2, e*x] - (b*n*PolyLo
g[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{x^2}\,dx = -\frac{bn\operatorname{Li}_2(ex)}{x} - \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{x} - (bn)\int \frac{\log(1-ex)}{x^2}\,dx - \int \frac{(a+$$

$$= e\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{x} - e(a+b\log(cx^n))\log(1-ex) +$$

$$= e\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{x} - e(a+b\log(cx^n))\log(1-ex) +$$

$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) - ben\log(1-ex) + \frac{2}{}$$

$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) - ben\log(1-ex) + \frac{2}{}$$

$$= 2ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) - 2ben\log(1-ex) +$$

Mathematica [A]
time = 0.12, size = 115, normalized size = 0.81

$$\frac{(a-bn\log(x)+b\log(cx^n))(ex\log(x)+(1-ex)\log(1-ex)-\mathrm{Li}_2(ex))}{x}+\frac{bn(ex\log^2(x)-4(-1+ex)\log(1-ex)+\log(x)(4ex+(2-2ex)\log(1-ex))-2(1+ex+\log(x))\mathrm{Li}_2(ex))}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^2,x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(e*x*Log[x] + (1 - e*x)*Log[1 - e*x] - Pol
yLog[2, e*x]))/x + (b*n*(e*x*Log[x]^2 - 4*(-1 + e*x)*Log[1 - e*x] + Log[x]*
(4*e*x + (2 - 2*e*x)*Log[1 - e*x]) - 2*(1 + e*x + Log[x])*PolyLog[2, e*x]))
/(2*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\,\mathrm{polylog}(2,ex)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(2,e*x)/x^2,x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(2,e*x)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="maxima")
```

```
[Out] (e*log(x) - ((x*e - 1)*log(-x*e + 1) + dilog(x*e))/x)*a - b*(((n + log(c) +
 log(x^n))*dilog(x*e) - (n*x*e*log(x) + 2*n + log(c))*log(-x*e + 1) - (x*e*
log(x) - (x*e - 1)*log(-x*e + 1))*log(x^n))/x + integrate((2*n*e + e*log(c)
 + (2*n*x*e^2 - n*e)*log(x))/(x^2*e - x), x))
```

Fricas [A]
time = 0.39, size = 144, normalized size = 1.01

$$\frac{bnxe\log(x)^2-2(bnxe+bn+a)\mathrm{Li}_2(xe)-2((2bn+a)xe-2bn-a)\log(-xe+1)-2(b\mathrm{Li}_2(xe)+(bxe-b)\log(-xe+1))\log(c)+2(bxe\log(c)-bn\mathrm{Li}_2(xe)+(2bn+a)xe-(bnxe-bn)\log(-xe+1))\log(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*x*e*log(x)^2 - 2*(b*n*x*e + b*n + a)*dilog(x*e) - 2*((2*b*n + a)*x
*e - 2*b*n - a)*log(-x*e + 1) - 2*(b*dilog(x*e) + (b*x*e - b)*log(-x*e + 1)
```

```
)*log(c) + 2*(b*x*e*log(c) - b*n*dilog(x*e) + (2*b*n + a)*x*e - (b*n*x*e -
b*n)*log(-x*e + 1))*log(x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log(cx^n))\operatorname{Li}_2(ex)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**2,x)
[Out] Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*dilog(x*e)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, e\,x)\,\left(a + b\,\ln\left(c\,x^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2,x)
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^2, x)
```

3.213 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_2(ex)}{x^3}\,dx$

Optimal. Leaf size=202

$$-\frac{ben}{2x}+\frac{1}{4}be^2n\log(x)-\frac{1}{8}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{4x}+\frac{1}{4}e^2\log(x)\,(a+b\log(cx^n))-\frac{1}{4}be^2n\log(1-ex)+\frac{bn}{}$$

```
[Out] -1/2*b*e*n/x+1/4*b*e^2*n*ln(x)-1/8*b*e^2*n*ln(x)^2-1/4*e*(a+b*ln(c*x^n))/x+
1/4*e^2*ln(x)*(a+b*ln(c*x^n))-1/4*b*e^2*n*ln(-e*x+1)+1/4*b*n*ln(-e*x+1)/x^2
-1/4*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/4*(a+b*ln(c*x^n))*ln(-e*x+1)/x^2-1/4*
b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/2*(a+b*ln(c*x^n))*polyl
og(2,e*x)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2432, 2442, 46, 2423, 2338, 2438}

$$-\frac{\text{PolyLog}(2,ex)\,(a+b\log(cx^n))}{2x^2}-\frac{1}{4}be^2n\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{4x^2}+\frac{1}{4}e^2\log(x)\,(a+b\log(cx^n))-\frac{1}{4}e^2\log(1-ex)\,(a+b\log(cx^n))-\frac{e(a+b\log(cx^n))}{4x}+\frac{\log(1-ex)\,(a+b\log(cx^n))}{4x^2}-\frac{1}{8}be^2n\log^2(x)+\frac{1}{4}be^2n\log(x)-\frac{1}{4}be^2n\log(1-ex)+\frac{bn\log(1-ex)}{4x^2}-\frac{ben}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

```
[Out] -1/2*(b*e*n)/x + (b*e^2*n*Log[x])/4 - (b*e^2*n*Log[x]^2)/8 - (e*(a + b*Log[
c*x^n]))/(4*x) + (e^2*Log[x]*(a + b*Log[c*x^n]))/4 - (b*e^2*n*Log[1 - e*x])
/4 + (b*n*Log[1 - e*x])/(4*x^2) - (e^2*(a + b*Log[c*x^n])*Log[1 - e*x])/4 +
 ((a + b*Log[c*x^n])*Log[1 - e*x])/(4*x^2) - (b*e^2*n*PolyLog[2, e*x])/4 -
(b*n*PolyLog[2, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/(2*x^2
)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
```

```
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_2(ex)}{x^3}\,dx = -\frac{bn\mathrm{Li}_2(ex)}{4x^2} - \frac{(a+b\log(cx^n))\,\mathrm{Li}_2(ex)}{2x^2} - \frac{1}{2}\int \frac{(a+b\log(cx^n))\log(1-ex)}{x^3}$$

$$= -\frac{e(a+b\log(cx^n))}{4x} + \frac{1}{4}e^2\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{8x^2} - \frac{1}{4}e^2(a$$

$$= -\frac{ben}{4x} - \frac{e(a+b\log(cx^n))}{4x} + \frac{1}{4}e^2\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{8x^2}$$

$$= -\frac{3ben}{8x} + \frac{1}{8}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{4x} + \frac{1}{4}e^2\log(x)(a$$

$$= -\frac{3ben}{8x} + \frac{1}{8}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{4x} + \frac{1}{4}e^2\log(x)(a$$

$$= -\frac{ben}{2x} + \frac{1}{4}be^2n\log(x) - \frac{1}{8}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{4x} + \frac{1}{4}e^2\log(x)(a$$

Mathematica [**A**]

time = 0.15, size = 163, normalized size = 0.81

$$\frac{(a-bn\log(x)+b\log(cx^n))(-ex+e^2x^2\log(x)+\log(1-ex)-e^2x^2\log(1-ex)-2\mathrm{Li}_2(ex))}{4x^2}+\frac{bn(-4ex+e^2x^2\log^2(x)+2\log(1-ex)-2e^2x^2\log(1-ex)-2(-1+ex)\log(x)(-ex+(1+ex)\log(1-ex))-2(1+e^2x^2+2\log(x))\mathrm{Li}_2(ex))}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[2, e*x])/x^3,x]
```

```
[Out] ((a - b*n*Log[x] + b*Log[c*x^n])*(-(e*x) + e^2*x^2*Log[x] + Log[1 - e*x] -
e^2*x^2*Log[1 - e*x] - 2*PolyLog[2, e*x]))/(4*x^2) + (b*n*(-4*e*x + e^2*x^2
*Log[x]^2 + 2*Log[1 - e*x] - 2*e^2*x^2*Log[1 - e*x] - 2*(-1 + e*x)*Log[x]*(
-(e*x) + (1 + e*x)*Log[1 - e*x]) - 2*(1 + e^2*x^2 + 2*Log[x])*PolyLog[2, e*
x]))/(8*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\,\mathrm{polylog}(2,ex)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(2,e*x)/x^3,x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(2,e*x)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(e^2*log(x) - (x*e + (x^2*e^2 - 1)*log(-x*e + 1) + 2*dilog(x*e))/x^2)*a
 - 1/4*b*(((n + 2*log(c) + 2*log(x^n))*dilog(x*e) - (n*x^2*e^2*log(x) + n +
 log(c))*log(-x*e + 1) - (x^2*e^2*log(x) - x*e - (x^2*e^2 - 1)*log(-x*e + 1
))*log(x^n))/x^2 + 4*integrate(-1/4*(n*x*e^2 - 2*n*e - e*log(c) - (2*n*x^2*
e^3 - n*x*e^2)*log(x))/(x^3*e - x^2), x))
```

Fricas [A]
time = 0.38, size = 182, normalized size = 0.90

$$\frac{bnx^2e^2\log(x)^2-2(2bn+a)xe-2(bnx^2e^2+bn+2a)\mathrm{Li}_2(xe)-2((bn+a)x^2e^2-bn-a)\log(-xe+1)-2(bxe+2b\mathrm{Li}_2(xe)+(bx^2e^2-b)\log(-xe+1))\log(c)+2(bx^2e^2\log(c)+(bn+a)x^2e^2-bnxe-2bn\mathrm{Li}_2(xe)-(bnx^2e^2-bn)\log(-xe+1))\log(x)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b*n*x^2*e^2*log(x)^2 - 2*(2*b*n + a)*x*e - 2*(b*n*x^2*e^2 + b*n + 2*a)
*dilog(x*e) - 2*((b*n + a)*x^2*e^2 - b*n - a)*log(-x*e + 1) - 2*(b*x*e + 2*
b*dilog(x*e) + (b*x^2*e^2 - b)*log(-x*e + 1))*log(c) + 2*(b*x^2*e^2*log(c)
+ (b*n + a)*x^2*e^2 - b*n*x*e - 2*b*n*dilog(x*e) - (b*n*x^2*e^2 - b*n)*log(
-x*e + 1))*log(x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(2,e*x)/x**3,x)
[Out] Integral((a + b*log(c*x**n))*polylog(2, e*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(2,e*x)/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*dilog(x*e)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2,e\,x)\,\left(a+b\,\ln\left(c\,x^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3,x)
[Out] int((polylog(2, e*x)*(a + b*log(c*x^n)))/x^3, x)
```

3.214 $\int x^2(a+b\log(cx^n))\,\mathbf{Li}_3(ex)\,dx$

Optimal. Leaf size=253

$$-\frac{2bnx}{27e^2}-\frac{bnx^2}{36e}-\frac{4}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+\frac{1}{81}x^3(a+b\log(cx^n))-\frac{bn\log(1-ex)}{27e^3}+$$

```
[Out] -2/27*b*n*x/e^2-1/36*b*n*x^2/e-4/243*b*n*x^3+1/27*x*(a+b*ln(c*x^n))/e^2+1/5
4*x^2*(a+b*ln(c*x^n))/e+1/81*x^3*(a+b*ln(c*x^n))-1/27*b*n*ln(-e*x+1)/e^3+1/
27*b*n*x^3*ln(-e*x+1)+1/27*(a+b*ln(c*x^n))*ln(-e*x+1)/e^3-1/27*x^3*(a+b*ln(
c*x^n))*ln(-e*x+1)+1/27*b*n*polylog(2,e*x)/e^3+2/27*b*n*x^3*polylog(2,e*x)-
1/9*x^3*(a+b*ln(c*x^n))*polylog(2,e*x)-1/9*b*n*x^3*polylog(3,e*x)+1/3*x^3*(
a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A]
time = 0.18, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2432, 2442, 45, 2423, 2438, 6726}

$$-\frac{1}{9}x^3\text{PolyLog}(2,ex)(a+b\log(cx^n))+\frac{1}{3}x^3\text{PolyLog}(3,ex)(a+b\log(cx^n))+\frac{bn\text{PolyLog}(2,ex)}{27e^3}+\frac{2}{27}bnx^3\text{PolyLog}(2,ex)-\frac{1}{9}bnx^3\text{PolyLog}(3,ex)+\frac{\log(1-ex)(a+b\log(cx^n))}{27e^3}+\frac{x(a+b\log(cx^n))}{27e^2}-\frac{1}{27}x^3\log(1-ex)(a+b\log(cx^n))+\frac{x^2(a+b\log(cx^n))}{54e}+\frac{1}{81}x^3(a+b\log(cx^n))-\frac{bn\log(1-ex)}{27e^3}-\frac{2bnx}{27e^2}+\frac{1}{27}bnx^3\log(1-ex)-\frac{bnx^2}{36e}-\frac{4}{243}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] (-2*b*n*x)/(27*e^2) - (b*n*x^2)/(36*e) - (4*b*n*x^3)/243 + (x*(a + b*Log[c*
x^n]))/(27*e^2) + (x^2*(a + b*Log[c*x^n]))/(54*e) + (x^3*(a + b*Log[c*x^n])
)/81 - (b*n*Log[1 - e*x])/(27*e^3) + (b*n*x^3*Log[1 - e*x])/27 + ((a + b*Lo
g[c*x^n])*Log[1 - e*x])/(27*e^3) - (x^3*(a + b*Log[c*x^n])*Log[1 - e*x])/27
 + (b*n*PolyLog[2, e*x])/(27*e^3) + (2*b*n*x^3*PolyLog[2, e*x])/27 - (x^3*(
a + b*Log[c*x^n])*PolyLog[2, e*x])/9 - (b*n*x^3*PolyLog[3, e*x])/9 + (x^3*(
a + b*Log[c*x^n])*PolyLog[3, e*x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x^2(a+b\log(cx^n))\operatorname{Li}_3(ex)\,dx = -\frac{1}{9}bnx^3\operatorname{Li}_3(ex)+\frac{1}{3}x^3(a+b\log(cx^n))\operatorname{Li}_3(ex)-\frac{1}{3}\int x^2(a+b\log(cx^n))$$
$$= \frac{2}{27}bnx^3\operatorname{Li}_2(ex)-\frac{1}{9}x^3(a+b\log(cx^n))\operatorname{Li}_2(ex)-\frac{1}{9}bnx^3\operatorname{Li}_3(ex)+\frac{1}{3}x^3(a$$
$$= \frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+\frac{1}{81}x^3(a+b\log(cx^n))+\frac{(a+}{}$$
$$= -\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+$$
$$= -\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+$$
$$= -\frac{bnx}{27e^2}-\frac{bnx^2}{108e}-\frac{1}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}+$$
$$= -\frac{4bnx}{81e^2}-\frac{5bnx^2}{324e}-\frac{2}{243}bnx^3+\frac{x(a+b\log(cx^n))}{27e^2}+\frac{x^2(a+b\log(cx^n))}{54e}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x^2(a+b\log(cx^n))\operatorname{Li}_3(ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
[Out] Integrate[x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(a+b\ln(c\,x^n))\operatorname{polylog}(3,ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))*polylog(3,e*x),x)
[Out] int(x^2*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

```
[Out] -1/162*(18*x^3*dilog(x*e)*e^3 - 54*x^3*e^3*polylog(3, x*e) - 2*x^3*e^3 - 3*
x^2*e^2 - 6*x*e + 6*(x^3*e^3 - 1)*log(-x*e + 1))*a*e^(-3) + 1/162*((6*((2*n
 - 3*log(c))*x^3*e^3 - 3*x^3*e^3*log(x^n))*dilog(x*e) + 6*((n - log(c))*x^3
*e^3 - n*log(x))*log(-x*e + 1) + (2*x^3*e^3 + 3*x^2*e^2 + 6*x*e - 6*(x^3*e^
3 - 1)*log(-x*e + 1))*log(x^n) - 18*((n - 3*log(c))*x^3*e^3 - 3*x^3*e^3*log
(x^n))*polylog(3, x*e))*e^(-3) + 162*integrate(-1/162*(2*(4*n - 3*log(c))*x
^3*e^3 + n*x^2*e^2 + 3*n*x*e - 6*n*log(x) - 6*n)/(x*e^3 - e^2), x))*b
```

Fricas [A]
time = 0.38, size = 274, normalized size = 1.08

$$-\frac{1}{972}\left(4\left(4\,bn-3\,a\right)x^{3}e^{3}+9\left(3\,bn-2\,a\right)x^{2}e^{2}+36\left(2\,bn-a\right)xe-36\left(\left(2\,bn-3\,a\right)x^{3}e^{3}+bn\right)\mathrm{Li}_2(xe)-36\left(\left(bn-a\right)x^{3}e^{3}-bn+a\right)\log\left(-xe+1\right)+6\left(18\,bx^{3}\mathrm{Li}_2(xe)\,e^{3}-2\,bx^{3}e^{3}-3\,bx^{2}e^{2}-6\,bxe+6\left(bx^{3}e^{3}-b\right)\log\left(-xe+1\right)\right)\log\left(c\right)+6\left(18\,bnx^{3}\mathrm{Li}_2(xe)\,e^{3}-2\,bnx^{3}e^{3}-3\,bnx^{2}e^{2}-6\,bnxe+6\left(bnx^{3}e^{3}-bn\right)\log\left(-xe+1\right)\right)\log\left(x\right)-108\left(3\,bnx^{3}e^{3}\log\left(x\right)+3\,bx^{3}e^{3}\log\left(c\right)-\left(bn-3\,a\right)x^{3}e^{3}\right)\mathrm{polylog}(3,xe)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

```
[Out] -1/972*(4*(4*b*n - 3*a)*x^3*e^3 + 9*(3*b*n - 2*a)*x^2*e^2 + 36*(2*b*n - a)*
x*e - 36*((2*b*n - 3*a)*x^3*e^3 + b*n)*dilog(x*e) - 36*((b*n - a)*x^3*e^3 -
 b*n + a)*log(-x*e + 1) + 6*(18*b*x^3*dilog(x*e)*e^3 - 2*b*x^3*e^3 - 3*b*x^
2*e^2 - 6*b*x*e + 6*(b*x^3*e^3 - b)*log(-x*e + 1))*log(c) + 6*(18*b*n*x^3*d
ilog(x*e)*e^3 - 2*b*n*x^3*e^3 - 3*b*n*x^2*e^2 - 6*b*n*x*e + 6*(b*n*x^3*e^3
- b*n)*log(-x*e + 1))*log(x) - 108*(3*b*n*x^3*e^3*log(x) + 3*b*x^3*e^3*log(
c) - (b*n - 3*a)*x^3*e^3)*polylog(3, x*e))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(a+b\log{(cx^{n})})\operatorname{Li}_{3}{(ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral(x**2*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x^2*polylog(3, x*e), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, e*x)*(a + b*log(c*x^n)),x)
[Out] \text{Hanged}
```

3.215 $\int x(a+b\log(cx^n))\,\mathbf{Li}_3(ex)\,dx$

Optimal. Leaf size=221

$$-\frac{5bnx}{16e}-\frac{1}{8}bnx^2+\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))-\frac{3bn\log(1-ex)}{16e^2}+\frac{3}{16}bnx^2\log(1-ex)+\frac{(a+b\log(\cdots$$

```
[Out] -5/16*b*n*x/e-1/8*b*n*x^2+1/8*x*(a+b*ln(c*x^n))/e+1/16*x^2*(a+b*ln(c*x^n))-
3/16*b*n*ln(-e*x+1)/e^2+3/16*b*n*x^2*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e*x
+1)/e^2-1/8*x^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*b*n*polylog(2,e*x)/e^2+1/4*b
*n*x^2*polylog(2,e*x)-1/4*x^2*(a+b*ln(c*x^n))*polylog(2,e*x)-1/4*b*n*x^2*po
lylog(3,e*x)+1/2*x^2*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A]
time = 0.13, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2432, 2442, 45, 2423, 2438, 6726}

$$-\frac{1}{4}x^2\text{PolyLog}(2,ex)(a+b\log(cx^n))+\frac{1}{2}x^2\text{PolyLog}(3,ex)(a+b\log(cx^n))+\frac{bn\text{PolyLog}(2,ex)}{8e^2}+\frac{1}{4}bnx^2\text{PolyLog}(2,ex)-\frac{1}{4}bnx^2\text{PolyLog}(3,ex)+\frac{\log(1-ex)(a+b\log(cx^n))}{8e^2}+\frac{x(a+b\log(cx^n))}{8e}-\frac{1}{8}x^2\log(1-ex)(a+b\log(cx^n))+\frac{1}{16}x^2(a+b\log(cx^n))-\frac{3bn\log(1-ex)}{16e^2}+\frac{3}{16}bnx^2\log(1-ex)-\frac{5bnx}{16e}-\frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] (-5*b*n*x)/(16*e) - (b*n*x^2)/8 + (x*(a + b*Log[c*x^n]))/(8*e) + (x^2*(a +
b*Log[c*x^n]))/16 - (3*b*n*Log[1 - e*x])/(16*e^2) + (3*b*n*x^2*Log[1 - e*x]
)/16 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(8*e^2) - (x^2*(a + b*Log[c*x^n])*
Log[1 - e*x])/8 + (b*n*PolyLog[2, e*x])/(8*e^2) + (b*n*x^2*PolyLog[2, e*x])
/4 - (x^2*(a + b*Log[c*x^n])*PolyLog[2, e*x])/4 - (b*n*x^2*PolyLog[3, e*x])
/4 + (x^2*(a + b*Log[c*x^n])*PolyLog[3, e*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x(a+b\log(cx^n))\operatorname{Li}_3(ex)\,dx = -\frac{1}{4}bnx^2\operatorname{Li}_3(ex)+\frac{1}{2}x^2(a+b\log(cx^n))\operatorname{Li}_3(ex)-\frac{1}{2}\int x(a+b\log(cx^n))\operatorname{Li}_2$$
$$=\frac{1}{4}bnx^2\operatorname{Li}_2(ex)-\frac{1}{4}x^2(a+b\log(cx^n))\operatorname{Li}_2(ex)-\frac{1}{4}bnx^2\operatorname{Li}_3(ex)+\frac{1}{2}x^2(a+b$$
$$=\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))+\frac{(a+b\log(cx^n))\log(1-ex)}{8e^2}$$
$$=-\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))+\frac{(a+b\log}{}$$
$$=-\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))+\frac{1}{16}bnx^2\log$$
$$=-\frac{bnx}{8e}-\frac{1}{32}bnx^2+\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))+\frac{1}{16}bnx^2\log$$
$$=-\frac{3bnx}{16e}-\frac{1}{16}bnx^2+\frac{x(a+b\log(cx^n))}{8e}+\frac{1}{16}x^2(a+b\log(cx^n))-\frac{bn\log(1}{16e}$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int x(a+b\log(cx^n))\operatorname{Li}_3(ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
[Out] Integrate[x*(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\,x^n))\operatorname{polylog}(3,ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*x^n))*polylog(3,e*x),x)
[Out] int(x*(a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

```
[Out] -1/16*(4*x^2*dilog(x*e)*e^2 - 8*x^2*e^2*polylog(3, x*e) - x^2*e^2 - 2*x*e +
 2*(x^2*e^2 - 1)*log(-x*e + 1))*a*e^(-2) + 1/16*((4*((n - log(c))*x^2*e^2 -
 x^2*e^2*log(x^n))*dilog(x*e) + ((3*n - 2*log(c))*x^2*e^2 - 2*n*log(x))*log
(-x*e + 1) + (x^2*e^2 + 2*x*e - 2*(x^2*e^2 - 1)*log(-x*e + 1))*log(x^n) - 4
*((n - 2*log(c))*x^2*e^2 - 2*x^2*e^2*log(x^n))*polylog(3, x*e))*e^(-2) + 16
*integrate(-1/16*(2*(2*n - log(c))*x^2*e^2 + n*x*e - 2*n*log(x) - 2*n)/(x*e
^2 - e), x))*b
```

Fricas [A]
time = 0.36, size = 241, normalized size = 1.09

$$-\frac{1}{16}\left((2\,bn-a)x^2e^2+(5\,bn-2\,a)xe-2\left(2\,(bn-a)x^2e^2+bn\right)\mathrm{Li}_2(xe)-\left((3\,bn-2\,a)x^2e^2-3\,bn+2\,a\right)\log\left(-xe+1\right)+\left(4\,bx^2\mathrm{Li}_2(xe)\,e^2-bx^2e^2-2\,bxe+2\left(bx^2e^2-b\right)\log\left(-xe+1\right)\right)\log\left(c\right)+\left(4\,bnx^2\mathrm{Li}_2(xe)\,e^2-bnx^2e^2-2\,bnxe+2\left(bnx^2e^2-bn\right)\log\left(-xe+1\right)\right)\log\left(x\right)-4\left(2\,bnx^2e^2\log\left(x\right)+2\,bx^2e^2\log\left(c\right)-(bn-2\,a)x^2e^2\right)\mathrm{polylog}(3,xe)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

```
[Out] -1/16*((2*b*n - a)*x^2*e^2 + (5*b*n - 2*a)*x*e - 2*(2*(b*n - a)*x^2*e^2 + b
*n)*dilog(x*e) - ((3*b*n - 2*a)*x^2*e^2 - 3*b*n + 2*a)*log(-x*e + 1) + (4*b
*x^2*dilog(x*e)*e^2 - b*x^2*e^2 - 2*b*x*e + 2*(b*x^2*e^2 - b)*log(-x*e + 1)
)*log(c) + (4*b*n*x^2*dilog(x*e)*e^2 - b*n*x^2*e^2 - 2*b*n*x*e + 2*(b*n*x^2
*e^2 - b*n)*log(-x*e + 1))*log(x) - 4*(2*b*n*x^2*e^2*log(x) + 2*b*x^2*e^2*l
og(c) - (b*n - 2*a)*x^2*e^2)*polylog(3, x*e))*e^(-2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\log\left(cx^n\right))\,\mathrm{Li}_3\left(ex\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral(x*(a + b*log(c*x**n))*polylog(3, e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*x*polylog(3, x*e), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, e*x)*(a + b*log(c*x^n)),x)
[Out] \text{Hanged}
```

3.216 $\int (a + b\log(cx^n))\,\mathbf{Li}_3(ex)\,dx$

Optimal. Leaf size=131

$$-4bnx+x(a+b\log(cx^n))-\frac{3bn(1-ex)\log(1-ex)}{e}+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}+\frac{bn\text{Li}_2(ex)}{e}+2b$$

```
[Out] -4*b*n*x+x*(a+b*ln(c*x^n))-3*b*n*(-e*x+1)*ln(-e*x+1)/e+(-e*x+1)*(a+b*ln(c*x
^n))*ln(-e*x+1)/e+b*n*polylog(2,e*x)/e+2*b*n*x*polylog(2,e*x)-x*(a+b*ln(c*x
^n))*polylog(2,e*x)-b*n*x*polylog(3,e*x)+x*(a+b*ln(c*x^n))*polylog(3,e*x)
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2428, 2436, 2332, 2417, 2458, 45, 2393, 2352, 6721}

$$-x\text{PolyLog}(2,ex)(a+b\log(cx^n))+x\text{PolyLog}(3,ex)(a+b\log(cx^n))+2bnx\text{PolyLog}(2,ex)-bnx\text{PolyLog}(3,ex)+\frac{bn\text{PolyLog}(2,ex)}{e}+\frac{(1-ex)\log(1-ex)(a+b\log(cx^n))}{e}+x(a+b\log(cx^n))-\frac{3bn(1-ex)\log(1-ex)}{e}-4bnx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
```

```
[Out] -4*b*n*x + x*(a + b*Log[c*x^n]) - (3*b*n*(1 - e*x)*Log[1 - e*x])/e + ((1 -
e*x)*(a + b*Log[c*x^n])*Log[1 - e*x])/e + (b*n*PolyLog[2, e*x])/e + 2*b*n*x
*PolyLog[2, e*x] - x*(a + b*Log[c*x^n])*PolyLog[2, e*x] - b*n*x*PolyLog[3,
e*x] + x*(a + b*Log[c*x^n])*PolyLog[3, e*x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
```

```
(f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2417

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))^(p_.), x_Symbol] :> With[{u = IntHide[Log[d*(e + f*x^m)^r], x]},
Dist[(a + b*Log[c*x^n])^p, u, x] - Dist[b*n*p, Int[Dist[(a + b*Log[c*x^n])^
(p - 1)/x, u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[
p, 0] && RationalQ[m] && (EqQ[p, 1] || (FractionQ[m] && IntegerQ[1/m]) || (
EqQ[r, 1] && EqQ[m, 1] && EqQ[d*e, 1]))
```

Rule 2428

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*PolyLog[k_, (e_.)*(x_)^(q_.)], x_
Symbol] :> Simp[(-b)*n*x*PolyLog[k, e*x^q], x] + (-Dist[q, Int[PolyLog[k -
1, e*x^q]*(a + b*Log[c*x^n]), x], x] + Dist[b*n*q, Int[PolyLog[k - 1, e*x^q
], x], x] + Simp[x*PolyLog[k, e*x^q]*(a + b*Log[c*x^n]), x]) /; FreeQ[{a, b
, c, e, n, q}, x] && IGtQ[k, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$\int (a+b\log(cx^n))\,\mathrm{Li}_3(ex)\,dx = -bnx\mathrm{Li}_3(ex)+x(a+b\log(cx^n))\,\mathrm{Li}_3(ex)+(bn)\int \mathrm{Li}_2(ex)\,dx-\int (a+b\log$$

$$= 2bnx\mathrm{Li}_2(ex)-x(a+b\log(cx^n))\,\mathrm{Li}_2(ex)-bnx\mathrm{Li}_3(ex)+x(a+b\log(cx^n))$$

$$= x(a+b\log(cx^n))+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}+2bnx\mathrm{Li}_2(ex)-$$

$$= -bnx+x(a+b\log(cx^n))+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-2\Big(bn$$

$$= -bnx+x(a+b\log(cx^n))+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-2\Big(bn$$

$$= -bnx+x(a+b\log(cx^n))+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-2\Big(bn$$

$$= -bnx+x(a+b\log(cx^n))+\frac{(1-ex)(a+b\log(cx^n))\log(1-ex)}{e}-2\Big(bn$$

$$= -2bnx+x(a+b\log(cx^n))-\frac{bn(1-ex)\log(1-ex)}{e}+\frac{(1-ex)(a+b\log$$

Mathematica [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b\log(cx^n))\,\mathrm{Li}_3(ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x],x]
[Out] Integrate[(a + b*Log[c*x^n])*PolyLog[3, e*x], x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (a+b\ln(c\,x^n))\,\mathrm{polylog}(3,ex)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(3,e*x),x)
[Out] int((a+b*ln(c*x^n))*polylog(3,e*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="maxima")
```

```
[Out] -(x*dilog(x*e)*e - x*e*polylog(3, x*e) - x*e + (x*e - 1)*log(-x*e + 1))*a*e
^(-1) + ((((2*n - log(c))*x*e - x*e*log(x^n))*dilog(x*e) + ((3*n - log(c))*
x*e - n*log(x))*log(-x*e + 1) + (x*e - (x*e - 1)*log(-x*e + 1))*log(x^n) -
((n - log(c))*x*e - x*e*log(x^n))*polylog(3, x*e))*e^(-1) + integrate(-((4*
n - log(c))*x*e - n*log(x) - n)/(x*e - 1), x))*b
```

Fricas [A]
time = 0.37, size = 184, normalized size = 1.40

$$-((4bn-a)xe-((2bn-a)xe+bn)\mathrm{Li}_2(xe)-((3bn-a)xe-3bn+a)\log(-xe+1)+(bx\mathrm{Li}_2(xe)e-bxe+(bxe-b)\log(-xe+1))\log(c)+(bnx\mathrm{Li}_2(xe)e-bnxe+(bnxe-bn)\log(-xe+1))\log(x)-(bnxe\log(x)+bxe\log(c)-(bn-a)xe)\mathrm{polylog}(3,xe))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="fricas")
```

```
[Out] -((4*b*n - a)*x*e - ((2*b*n - a)*x*e + b*n)*dilog(x*e) - ((3*b*n - a)*x*e -
 3*b*n + a)*log(-x*e + 1) + (b*x*dilog(x*e)*e - b*x*e + (b*x*e - b)*log(-x*
e + 1))*log(c) + (b*n*x*dilog(x*e)*e - b*n*x*e + (b*n*x*e - b*n)*log(-x*e +
 1))*log(x) - (b*n*x*e*log(x) + b*x*e*log(c) - (b*n - a)*x*e)*polylog(3, x*
e))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\log(cx^n))\,\mathrm{Li}_3(ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x),x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(3, x*e), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, e*x)*(a + b*log(c*x^n)),x)
[Out] \text{Hanged}
```

3.217 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_3(ex)}{x}\,dx$

Optimal. Leaf size=26

$$(a + b\log(cx^n))\,\mathrm{Li}_4(ex) - bn\mathrm{Li}_5(ex)$$

```
[Out] (a+b*ln(c*x^n))*polylog(4,e*x)-b*n*polylog(5,e*x)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2430, 6724}

$$\mathrm{PolyLog}(4, ex)\,(a + b\log(cx^n)) - bn\mathrm{PolyLog}(5, ex)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

```
[Out] (a + b*Log[c*x^n])*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a + b\log(cx^n))\,\mathrm{Li}_3(ex)}{x}\,dx &= (a + b\log(cx^n))\,\mathrm{Li}_4(ex) - (bn)\int \frac{\mathrm{Li}_4(ex)}{x}\,dx\\ &= (a + b\log(cx^n))\,\mathrm{Li}_4(ex) - bn\mathrm{Li}_5(ex)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.15

$$a\mathrm{Li}_4(ex) + b\log(cx^n)\,\mathrm{Li}_4(ex) - bn\mathrm{Li}_5(ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x,x]
```

```
[Out] a*PolyLog[4, e*x] + b*Log[c*x^n]*PolyLog[4, e*x] - b*n*PolyLog[5, e*x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a +b \ln \left(c \,x^{n}\right)\right) \operatorname{polylog}\left(3, e x \right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(3,e*x)/x,x)
```

```
[Out] int((a+b*ln(c*x^n))*polylog(3,e*x)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*n*log(x)^3 - 3*b*log(x)^2*log(x^n) - 3*(b*log(c) + a)*log(x)^2)*di
log(x*e) - 1/2*(b*n*log(x)^2 - 2*b*log(x)*log(x^n) - 2*(b*log(c) + a)*log(x
))*polylog(3, x*e) - 1/6*integrate((3*b*log(-x*e + 1)*log(x)^2*log(x^n) - (
2*b*n*log(x)^3 - 3*(b*log(c) + a)*log(x)^2)*log(-x*e + 1))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^n)*polylog(3, x*e) + a*polylog(3, x*e))/x, x)
```

Sympy [**A**]
time = 5.11, size = 26, normalized size = 1.00

$$a \operatorname{Li}_{4}\left(e x\right) + b \left(- n \operatorname{Li}_{5}\left(e x\right) + \log{\left(c x^{n} \right)} \operatorname{Li}_{4}\left(e x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x,x)
```

```
[Out] a*polylog(4, e*x) + b*(-n*polylog(5, e*x) + log(c*x**n)*polylog(4, e*x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*polylog(3, x*e)/x, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.04

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x,x)
[Out] \text{Hanged}
```

3.218 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_3(ex)}{x^2}\,dx$

Optimal. Leaf size=174

$$3ben\log(x)-\frac{1}{2}ben\log^2(x)+e\log(x)\left(a+b\log\left(cx^n\right)\right)-3ben\log(1-ex)+\frac{3bn\log(1-ex)}{x}-e(a+b\log\left(cx^n\right))\,\mathrm{l}$$

```
[Out] 3*b*e*n*ln(x)-1/2*b*e*n*ln(x)^2+e*ln(x)*(a+b*ln(c*x^n))-3*b*e*n*ln(-e*x+1)+
3*b*n*ln(-e*x+1)/x-e*(a+b*ln(c*x^n))*ln(-e*x+1)+(a+b*ln(c*x^n))*ln(-e*x+1)/
x-b*e*n*polylog(2,e*x)-2*b*n*polylog(2,e*x)/x-(a+b*ln(c*x^n))*polylog(2,e*x
)/x-b*n*polylog(3,e*x)/x-(a+b*ln(c*x^n))*polylog(3,e*x)/x
```

Rubi [A]
time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2432, 2442, 36, 29, 31, 2423, 2338, 2438, 6726}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{\text{PolyLog}(3,ex)\left(a+b\log\left(cx^n\right)\right)}{x}-ben\text{PolyLog}(2,ex)-\frac{2bn\text{PolyLog}(2,ex)}{x}-\frac{bn\text{PolyLog}(3,ex)}{x}+e\log(x)\left(a+b\log\left(cx^n\right)\right)-e\log(1-ex)\left(a+b\log\left(cx^n\right)\right)+\frac{\log(1-ex)\left(a+b\log\left(cx^n\right)\right)}{x}-\frac{1}{2}ben\log^2(x)+3ben\log(x)-3ben\log(1-ex)+\frac{3bn\log(1-ex)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
```

```
[Out] 3*b*e*n*Log[x] - (b*e*n*Log[x]^2)/2 + e*Log[x]*(a + b*Log[c*x^n]) - 3*b*e*n
*Log[1 - e*x] + (3*b*n*Log[1 - e*x])/x - e*(a + b*Log[c*x^n])*Log[1 - e*x]
+ ((a + b*Log[c*x^n])*Log[1 - e*x])/x - b*e*n*PolyLog[2, e*x] - (2*b*n*Poly
Log[2, e*x])/x - ((a + b*Log[c*x^n])*PolyLog[2, e*x])/x - (b*n*PolyLog[3, e
*x])/x - ((a + b*Log[c*x^n])*PolyLog[3, e*x])/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{x^2}\,dx = -\frac{bn\operatorname{Li}_3(ex)}{x} - \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{x} + (bn)\int \frac{\operatorname{Li}_2(ex)}{x^2}\,dx + \int \frac{(a+b\log}{}$$
$$= -\frac{2bn\operatorname{Li}_2(ex)}{x} - \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{x} - \frac{bn\operatorname{Li}_3(ex)}{x} - \frac{(a+b\log(cx^n))\,\mathrm{I}}{x}$$
$$= e\log(x)(a+b\log(cx^n)) - e(a+b\log(cx^n))\log(1-ex) + \frac{(a+b\log(cx^n}{x}$$
$$= e\log(x)(a+b\log(cx^n)) - e(a+b\log(cx^n))\log(1-ex) + \frac{(a+b\log(cx^n}{x}$$
$$= -\frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{x} - e(a+b\log($$
$$= -\frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) + \frac{bn\log(1-ex)}{x} - e(a+b\log($$
$$= ben\log(x) - \frac{1}{2}ben\log^2(x) + e\log(x)(a+b\log(cx^n)) - ben\log(1-ex) +$$

Mathematica [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2,x]
[Out] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^2, x]
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\operatorname{polylog}(3,ex)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(3,e*x)/x^2,x)
[Out] int((a+b*ln(c*x^n))*polylog(3,e*x)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="maxima")
```

```
[Out] (e*log(x) - ((x*e - 1)*log(-x*e + 1) + dilog(x*e) + polylog(3, x*e))/x)*a -
 b*(((2*n + log(c) + log(x^n))*dilog(x*e) - (n*x*e*log(x) + 3*n + log(c))*l
og(-x*e + 1) - (x*e*log(x) - (x*e - 1)*log(-x*e + 1))*log(x^n) + (n + log(c
) + log(x^n))*polylog(3, x*e))/x + integrate(((3*n + log(c))*e + (2*n*x*e^2
 - n*e)*log(x))/(x^2*e - x), x))
```

Fricas [**A**]
time = 0.41, size = 167, normalized size = 0.96

$$\frac{bnxe\log\left(x\right)^{2}-2\left(bnxe+2\,bn+a\right)\mathrm{Li}_{2}(xe)-2\left(\left(3\,bn+a\right)xe-3\,bn-a\right)\log\left(-xe+1\right)-2\left(b\mathrm{Li}_{2}(xe)+\left(bxe-b\right)\log\left(-xe+1\right)\right)\log\left(c\right)+2\left(bxe\log\left(c\right)-bn\mathrm{Li}_{2}(xe)+\left(3\,bn+a\right)xe-\left(bnxe-bn\right)\log\left(-xe+1\right)\right)\log\left(x\right)-2\left(bn\log\left(x\right)+bn+b\log\left(c\right)+a\right)\mathrm{polylog}(3,xe)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*x*e*log(x)^2 - 2*(b*n*x*e + 2*b*n + a)*dilog(x*e) - 2*((3*b*n + a)
*x*e - 3*b*n - a)*log(-x*e + 1) - 2*(b*dilog(x*e) + (b*x*e - b)*log(-x*e +
1))*log(c) + 2*(b*x*e*log(c) - b*n*dilog(x*e) + (3*b*n + a)*x*e - (b*n*x*e
- b*n)*log(-x*e + 1))*log(x) - 2*(b*n*log(x) + b*n + b*log(c) + a)*polylog(
3, x*e))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(cx^{n}\right)\right)\mathrm{Li}_{3}\left(ex\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(3, x*e)/x^2, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^2,x)
[Out] \text{Hanged}
```

3.219 $\int \frac{(a+b\log(cx^n))\mathbf{Li}_3(ex)}{x^3}\,dx$

Optimal. Leaf size=238

$$-\frac{5ben}{16x}+\frac{3}{16}be^2n\log(x)-\frac{1}{16}be^2n\log^2(x)-\frac{e(a+b\log(cx^n))}{8x}+\frac{1}{8}e^2\log(x)\left(a+b\log(cx^n)\right)-\frac{3}{16}be^2n\log(1-ex)+$$

```
[Out] -5/16*b*e*n/x+3/16*b*e^2*n*ln(x)-1/16*b*e^2*n*ln(x)^2-1/8*e*(a+b*ln(c*x^n))
/x+1/8*e^2*ln(x)*(a+b*ln(c*x^n))-3/16*b*e^2*n*ln(-e*x+1)+3/16*b*n*ln(-e*x+1
)/x^2-1/8*e^2*(a+b*ln(c*x^n))*ln(-e*x+1)+1/8*(a+b*ln(c*x^n))*ln(-e*x+1)/x^2
-1/8*b*e^2*n*polylog(2,e*x)-1/4*b*n*polylog(2,e*x)/x^2-1/4*(a+b*ln(c*x^n))*
polylog(2,e*x)/x^2-1/4*b*n*polylog(3,e*x)/x^2-1/2*(a+b*ln(c*x^n))*polylog(3
,e*x)/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2432, 2442, 46, 2423, 2338, 2438, 6726}

$$-\frac{\text{PolyLog}(2,ex)\left(a+b\log(cx^n)\right)}{4x^2}-\frac{\text{PolyLog}(3,ex)\left(a+b\log(cx^n)\right)}{2x^2}-\frac{1}{8}be^2n\text{PolyLog}(2,ex)-\frac{bn\text{PolyLog}(2,ex)}{4x^2}-\frac{bn\text{PolyLog}(3,ex)}{4x^2}+\frac{1}{8}e^2\log(x)\left(a+b\log(cx^n)\right)-\frac{1}{8}e^2\log(1-ex)\left(a+b\log(cx^n)\right)-\frac{e(a+b\log(cx^n))}{8x}+\frac{\log(1-ex)\left(a+b\log(cx^n)\right)}{8x^2}-\frac{1}{16}be^2n\log^2(x)+\frac{3}{16}be^2n\log(x)-\frac{3}{16}be^2n\log(1-ex)+\frac{3bn\log(1-ex)}{16x^2}-\frac{5ben}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
```

```
[Out] (-5*b*e*n)/(16*x) + (3*b*e^2*n*Log[x])/16 - (b*e^2*n*Log[x]^2)/16 - (e*(a +
 b*Log[c*x^n]))/(8*x) + (e^2*Log[x]*(a + b*Log[c*x^n]))/8 - (3*b*e^2*n*Log[
1 - e*x])/16 + (3*b*n*Log[1 - e*x])/(16*x^2) - (e^2*(a + b*Log[c*x^n])*Log[
1 - e*x])/8 + ((a + b*Log[c*x^n])*Log[1 - e*x])/(8*x^2) - (b*e^2*n*PolyLog[
2, e*x])/8 - (b*n*PolyLog[2, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLog[2,
 e*x])/(4*x^2) - (b*n*PolyLog[3, e*x])/(4*x^2) - ((a + b*Log[c*x^n])*PolyLo
g[3, e*x])/(2*x^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2423

```
Int[Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.
)]*(b_.))*((g_.)*(x_))^(q_.), x_Symbol] :> With[{u = IntHide[(g*x)^q*Log[d*
(e + f*x^m)^r], x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x,
 u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, r, m, n, q}, x] && (IntegerQ
[(q + 1)/m] || (RationalQ[m] && RationalQ[q])) && NeQ[q, -1]
```

Rule 2432

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.)*PolyLog[k_, (e
_.)*(x_)^(q_.)], x_Symbol] :> Simp[(-b)*n*(d*x)^(m + 1)*(PolyLog[k, e*x^q]/
(d*(m + 1)^2)), x] + (-Dist[q/(m + 1), Int[(d*x)^m*PolyLog[k - 1, e*x^q]*(a
 + b*Log[c*x^n]), x], x] + Dist[b*n*(q/(m + 1)^2), Int[(d*x)^m*PolyLog[k -
1, e*x^q], x], x] + Simp[(d*x)^(m + 1)*PolyLog[k, e*x^q]*((a + b*Log[c*x^n]
)/(d*(m + 1))), x]) /; FreeQ[{a, b, c, d, e, m, n, q}, x] && IGtQ[k, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{x^3}\,dx = -\frac{bn\operatorname{Li}_3(ex)}{4x^2} - \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{2x^2} + \frac{1}{2}\int \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{x^3}\,dx$$
$$= -\frac{bn\operatorname{Li}_2(ex)}{4x^2} - \frac{(a+b\log(cx^n))\operatorname{Li}_2(ex)}{4x^2} - \frac{bn\operatorname{Li}_3(ex)}{4x^2} - \frac{(a+b\log(cx^n))\operatorname{Li}_3}{2x^2}$$
$$= -\frac{e(a+b\log(cx^n))}{8x} + \frac{1}{8}e^2\log(x)(a+b\log(cx^n)) - \frac{1}{8}e^2(a+b\log(cx^n))\log$$
$$= -\frac{ben}{8x} - \frac{e(a+b\log(cx^n))}{8x} + \frac{1}{8}e^2\log(x)(a+b\log(cx^n)) - \frac{1}{8}e^2(a+b\log(cx$$
$$= -\frac{ben}{8x} - \frac{1}{16}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{8x} + \frac{1}{8}e^2\log(x)(a+b\log(cx^n))$$
$$= -\frac{ben}{8x} - \frac{1}{16}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{8x} + \frac{1}{8}e^2\log(x)(a+b\log(cx^n))$$
$$= -\frac{3ben}{16x} + \frac{1}{16}be^2n\log(x) - \frac{1}{16}be^2n\log^2(x) - \frac{e(a+b\log(cx^n))}{8x} + \frac{1}{8}e^2\log(x)$$

Mathematica [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))\operatorname{Li}_3(ex)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3,x]
[Out] Integrate[((a + b*Log[c*x^n])*PolyLog[3, e*x])/x^3, x]
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c\,x^n))\operatorname{polylog}(3,ex)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))*polylog(3,e*x)/x^3,x)
[Out] int((a+b*ln(c*x^n))*polylog(3,e*x)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(e^2*log(x) - (x*e + (x^2*e^2 - 1)*log(-x*e + 1) + 2*dilog(x*e) + 4*pol
ylog(3, x*e))/x^2)*a - 1/16*b*((4*(n + log(c) + log(x^n))*dilog(x*e) - (2*n
*x^2*e^2*log(x) + 3*n + 2*log(c))*log(-x*e + 1) - 2*(x^2*e^2*log(x) - x*e -
 (x^2*e^2 - 1)*log(-x*e + 1))*log(x^n) + 4*(n + 2*log(c) + 2*log(x^n))*poly
log(3, x*e))/x^2 + 16*integrate(-1/16*(2*n*x*e^2 - (5*n + 2*log(c))*e - 2*(
2*n*x^2*e^3 - n*x*e^2)*log(x))/(x^3*e - x^2), x))
```

Fricas [A]
time = 0.36, size = 217, normalized size = 0.91

$$\frac{bnx^2e^2\log(x)^2-(5\,bn+2\,a)xe-2\,(bnx^2e^2+2\,bn+2\,a)\mathrm{Li}_2(xe)-((3\,bn+2\,a)x^2e^2-3\,bn-2\,a)\log(-xe+1)-2\,(bxe+2\,b\mathrm{Li}_2(xe)+(bx^2e^2-b)\log(-xe+1))\log(c)+(2\,bx^2e^2\log(c)+(3\,bn+2\,a)x^2e^2-2\,bnxe-4\,bn\mathrm{Li}_2(xe)-2\,(bnx^2e^2-bn)\log(-xe+1))\log(x)-4\,(2\,bn\log(x)+bn+2\,b\log(c)+2\,a)\mathrm{polylog}(3,xe)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(b*n*x^2*e^2*log(x)^2 - (5*b*n + 2*a)*x*e - 2*(b*n*x^2*e^2 + 2*b*n + 2
*a)*dilog(x*e) - ((3*b*n + 2*a)*x^2*e^2 - 3*b*n - 2*a)*log(-x*e + 1) - 2*(b
*x*e + 2*b*dilog(x*e) + (b*x^2*e^2 - b)*log(-x*e + 1))*log(c) + (2*b*x^2*e^
2*log(c) + (3*b*n + 2*a)*x^2*e^2 - 2*b*n*x*e - 4*b*n*dilog(x*e) - 2*(b*n*x^
2*e^2 - b*n)*log(-x*e + 1))*log(x) - 4*(2*b*n*log(x) + b*n + 2*b*log(c) + 2
*a)*polylog(3, x*e))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\log(cx^n))\,\mathrm{Li}_3(ex)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))*polylog(3,e*x)/x**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))*polylog(3, e*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))*polylog(3,e*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*polylog(3, x*e)/x^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(3, e*x)*(a + b*log(c*x^n)))/x^3,x)
[Out] \text{Hanged}
```

3.220 $\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx$

Optimal. Leaf size=30

$$-\text{Int}\left((dx)^m (a+b\log(cx^n))\log(1-ex^q),x\right)$$

```
[Out] -Unintegrable((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx$$

Verification is not applicable to the result.

```
[In] Int[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

```
[Out] -Defer[Int][(d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q], x]
```

Rubi steps

$$\int -(dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx = -\int (dx)^m (a+b\log(cx^n))\log(1-ex^q)\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(30) = 60.
time = 0.14, size = 266, normalized size = 8.87

$$-\frac{x(dx)^m\left(-aq-amq+2bnq-bnq\,{}_3F_2\left(1,\frac{1}{q}+\frac{m}{q},\frac{1}{q}+\frac{m}{q};1+\frac{1}{q}+\frac{m}{q},1+\frac{1}{q}+\frac{m}{q};ex^q\right)-bq\log(cx^n)-bmq\log(cx^n)+q\,{}_2F_1\left(1,\frac{1+m}{q};\frac{1+m+q}{q};ex^q\right)(a+am-bn+b(1+m)\log(cx^n))+a\log(1-ex^q)+2am\log(1-ex^q)+am^2\log(1-ex^q)-bn\log(1-ex^q)-bmn\log(1-ex^q)+b\log(cx^n)\log(1-ex^q)+2bm\log(cx^n)\log(1-ex^q)+bm^2\log(cx^n)\log(1-ex^q)\right)}{(1+m)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

```
[Out] -((x*(d*x)^m*(-(a*q) - a*m*q + 2*b*n*q - b*n*q*HypergeometricPFQ[{1, q^(-1)
 + m/q, q^(-1) + m/q}, {1 + q^(-1) + m/q, 1 + q^(-1) + m/q}, e*x^q] - b*q*L
og[c*x^n] - b*m*q*Log[c*x^n] + q*Hypergeometric2F1[1, (1 + m)/q, (1 + m + q
)/q, e*x^q]*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]) + a*Log[1 - e*x^q] + 2*a
*m*Log[1 - e*x^q] + a*m^2*Log[1 - e*x^q] - b*n*Log[1 - e*x^q] - b*m*n*Log[1
 - e*x^q] + b*Log[c*x^n]*Log[1 - e*x^q] + 2*b*m*Log[c*x^n]*Log[1 - e*x^q] +
 b*m^2*Log[c*x^n]*Log[1 - e*x^q]))/(1 + m)^3)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(29) = 58$.
time = 0.40, size = 844, normalized size = 28.13

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-e)^{-\frac{m}{q}-\frac{1}{q}} a \left(\frac{q\,x^{1+m}(-e)^{\frac{m}{q}+\frac{1}{q}} \ln(1-e\,x^q)}{1+m}-\frac{q\,x^{1+m+q} e(-e)^{\frac{m}{q}+\frac{1}{q}}(-q-m-1)\Phi\left(e\,x^q,1,\frac{1+m+q}{q}\right)}{(1+m+q)(1+m)}\right)}{q} - \frac{(dx)^m x^{-m}(-e)^{-\frac{m}{q}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*ln(1-e*
x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(
1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q*x^(1+m)*(-e)^(m/q+1/
q)/(1+m)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*Le
rchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m/q-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(
q*x^m*(-e)^(m/q+1/q)/(1+m)*ln(1-e*x^q)-q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*(
-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b
*n/q*(q*x^m*(-e)^(m/q+1/q)*ln(x)/(1+m)*ln(1-e*x^q)+x^m*(-e)^(m/q+1/q)*ln(-e
)/(1+m)*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)+q/(1+m+q)^2*x^
(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)/q)-q/(1+m+q)
*x^(q+m)*e*(-e)^(m/q+1/q)*ln(x)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)/q)-
1/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*ln(-e)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,1,(
1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)*LerchPhi(e*x^q,1,(1+m+q)
/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)^2*LerchPhi(e*x^q,1,(1
+m+q)/q)+1/(1+m+q)*x^(q+m)*e*(-e)^(m/q+1/q)*(-q-m-1)/(1+m)*LerchPhi(e*x^q,2
,(1+m+q)/q)))*x
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="maxima")
```

```
[Out] -(b*d^m*(m + 1)*x*x^m*log(x^n) + (a*d^m*(m + 1) + (d^m*(m + 1)*log(c) - d^m
*n)*b)*x*x^m)*log(-e^(q*log(x) + 1) + 1)/(m^2 + 2*m + 1) + integrate(-((m*q
 + q)*b*d^m*e^(m*log(x) + q*log(x) + 1)*log(x^n) + ((m*q + q)*a*d^m - (d^m*
n*q - (m*q + q)*d^m*log(c))*b)*e^(m*log(x) + q*log(x) + 1))/(m^2 - (m^2 + 2
*m + 1)*e^(q*log(x) + 1) + 2*m + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="fricas")
[Out] integral(-(d*x)^m*b*log(c*x^n)*log(-x^q*e + 1) - (d*x)^m*a*log(-x^q*e + 1),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)**m*(a+b*ln(c*x**n))*ln(1-e*x**q),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x)^m*(a+b*log(c*x^n))*log(1-e*x^q),x, algorithm="giac")
[Out] integrate(-(b*log(c*x^n) + a)*(d*x)^m*log(-x^q*e + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\ln\left(1-e\,x^q\right)\,{\left(d\,x\right)}^m\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)),x)
[Out] int(-log(1 - e*x^q)*(d*x)^m*(a + b*log(c*x^n)), x)
```

3.221 $\int (dx)^m (a+b\log(cx^n))\,\mathbf{Li}_2(ex^q)\,dx$

Optimal. Leaf size=178

$$-\frac{benq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^3(1+m+q)}-\frac{bnq(dx)^{1+m}\log(1-ex^q)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^2}+\frac{(dx)^{1+m}(a-}{}$$

```
[Out] -b*e*n*q^2*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/(1
+m)^3/(1+m+q)-b*n*q*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^3-b*n*(d*x)^(1+m)*polyl
og(2,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e*x^q)/d/(1+m)+
q*Unintegrable((d*x)^m*(a+b*ln(c*x^n))*ln(1-e*x^q),x)/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m (a+b\log(cx^n))\,\mathrm{PolyLog}(2,ex^q)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

```
[Out] -((b*e*n*q^2*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m +
 2*q)/q, e*x^q])/((1 + m)^3*(1 + m + q))) - (b*n*q*(d*x)^(1 + m)*Log[1 - e*
x^q])/(d*(1 + m)^3) - (b*n*(d*x)^(1 + m)*PolyLog[2, e*x^q])/(d*(1 + m)^2) +
 ((d*x)^(1 + m)*(a + b*Log[c*x^n])*PolyLog[2, e*x^q])/(d*(1 + m)) + (q*Defe
r[Int][(d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q], x])/(1 + m)
```

Rubi steps

$$\begin{aligned}\int (dx)^m (a+b\log(cx^n))\,\mathrm{Li}_2(ex^q)\,dx &= -\frac{bn(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^2}+\frac{(dx)^{1+m}(a+b\log(cx^n))\,\mathrm{Li}_2(ex^q)}{d(1+m)}+\frac{q\int(dx)}{}\\ &= -\frac{bnq(dx)^{1+m}\log(1-ex^q)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^2}+\frac{(dx)^{1+m}(a+b\,\mathrm{l}}{d(1}\\ &= -\frac{bnq(dx)^{1+m}\log(1-ex^q)}{d(1+m)^3}-\frac{bn(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^2}+\frac{(dx)^{1+m}(a+b\,\mathrm{l}}{d(1}\\ &= -\frac{benq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^3(1+m+q)}-\frac{bnq(dx)^{1+m}\log(1-}{d(1+m)^3}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\log \left(cx^n\right)\right) \text{Li}_2(ex^q) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q],x]
```

```
[Out] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[2, e*x^q], x]
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(179) = 358$.
time = 0.21, size = 867, normalized size = 4.87

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-e)^{-\frac{m}{q}-\frac{1}{q}} a \left(-\frac{q^2 x^{1+m} (-e)^{\frac{m}{q}+\frac{1}{q}} \ln(1-e\,x^q)}{(1+m)^2}-\frac{q\,x^{1+m} (-e)^{\frac{m}{q}+\frac{1}{q}} \operatorname{polylog}(2,e\,x^q)}{1+m}-\frac{q^2 x^{1+m+q} e (-e)^{\frac{m}{q}+\frac{1}{q}} \Phi\left(e\,x^q,1,\frac{1+m+}{q}\right.}{(1+m)^2}\right.}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*ln(c*x^n))*polylog(2,e*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(-q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*ln
(1-e*x^q)-q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*polylog(2,e*x^q)-q^2*x^(1+m+q)*e*(
-e)^(m/q+1/q)/(1+m)^2*LerchPhi(e*x^q,1,(1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(-m/
q-1/q)*b*ln(c)/q*(-q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)-q*x^(1+m)
*(-e)^(m/q+1/q)/(1+m)*polylog(2,e*x^q)-q^2*x^(1+m+q)*e*(-e)^(m/q+1/q)/(1+m)
^2*LerchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m/q-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*
b*n*(-q^2*x^m*(-e)^(m/q+1/q)/(1+m)^2*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)/(1+m)
*polylog(2,e*x^q)-q^2*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^2*LerchPhi(e*x^q,1,(1+
m+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(-q^2*x^m*(-e)^(m/q+1/q)*ln(x
)/(1+m)^2*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^2*ln(1-e*x^q)+2*q^2
*x^m*(-e)^(m/q+1/q)/(1+m)^3*ln(1-e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(x)/(1+m)*po
lylog(2,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)*polylog(2,e*x^q)+q*x^m*(-e)^
(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q^2*x^(q+m)*e*(-e)^(m/q+1/q)*ln(x)/(1+m)
^2*LerchPhi(e*x^q,1,(1+m+q)/q)-q*x^(q+m)*e*(-e)^(m/q+1/q)*ln(-e)/(1+m)^2*Le
rchPhi(e*x^q,1,(1+m+q)/q)+2*q^2*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e
*x^q,1,(1+m+q)/q)+q*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^2*LerchPhi(e*x^q,2,(1+m+
q)/q)))*x
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="maxima")
```

```
[Out] (((b*d^m*m^2 + 2*b*d^m*m + b*d^m)*x*x^m*log(x^n) + ((b*log(c) + a)*d^m*m^2
+ 2*(b*log(c) + a)*d^m*m + (b*log(c) + a)*d^m - (b*d^m*m + b*d^m)*n)*x*x^m)
*dilog(e^(q*log(x) + 1)) + ((b*d^m*m + b*d^m)*q*x*x^m*log(x^n) + ((b*log(c)
 + a)*d^m*m - 2*b*d^m*n + (b*log(c) + a)*d^m)*q*x*x^m)*log(-e^(q*log(x) + 1
) + 1))/(m^3 + 3*m^2 + 3*m + 1) - integrate(-((b*d^m*m*e + b*d^m*e)*q^2*e^(
m*log(x) + q*log(x))*log(x^n) - (2*b*d^m*n*e - (b*e*log(c) + a*e)*d^m*m - (
b*e*log(c) + a*e)*d^m)*q^2*e^(m*log(x) + q*log(x)))/(m^3 + 3*m^2 - (m^3*e +
 3*m^2*e + 3*m*e + e)*x^q + 3*m + 1), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b*dilog(x^q*e)*log(c*x^n) + (d*x)^m*a*dilog(x^q*e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(2,e*x**q),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(2,e*x^q),x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)*(d*x)^m*dilog(x^q*e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(2,e\,x^q)\,\left(a+b\,\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)),x)
```

```
[Out] int((d*x)^m*polylog(2, e*x^q)*(a + b*log(c*x^n)), x)
```

3.222 $\int (dx)^m (a + b\log(cx^n))\,\mathbf{Li}_3(ex^q)\,dx$

Optimal. Leaf size=245

$$\frac{2benq^3x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^4(1+m+q)}+\frac{2bnq^2(dx)^{1+m}\log(1-ex^q)}{d(1+m)^4}+\frac{2bnq(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^3}-\frac{q(dx)^{1\cdots}}{}$$

```
[Out] 2*b*e*n*q^3*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],e*x^q)/(
1+m)^4/(1+m+q)+2*b*n*q^2*(d*x)^(1+m)*ln(1-e*x^q)/d/(1+m)^4+2*b*n*q*(d*x)^(1
+m)*polylog(2,e*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*(a+b*ln(c*x^n))*polylog(2,e*x^
q)/d/(1+m)^2-b*n*(d*x)^(1+m)*polylog(3,e*x^q)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln
(c*x^n))*polylog(3,e*x^q)/d/(1+m)-q^2*Unintegrable((d*x)^m*(a+b*ln(c*x^n))*
ln(1-e*x^q),x)/(1+m)^2
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m (a+b\log(cx^n))\,\mathrm{PolyLog}(3,ex^q)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

```
[Out] (2*b*e*n*q^3*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m +
 2*q)/q, e*x^q])/((1 + m)^4*(1 + m + q)) + (2*b*n*q^2*(d*x)^(1 + m)*Log[1 -
 e*x^q])/(d*(1 + m)^4) + (2*b*n*q*(d*x)^(1 + m)*PolyLog[2, e*x^q])/(d*(1 +
m)^3) - (q*(d*x)^(1 + m)*(a + b*Log[c*x^n])*PolyLog[2, e*x^q])/(d*(1 + m)^2
) - (b*n*(d*x)^(1 + m)*PolyLog[3, e*x^q])/(d*(1 + m)^2) + ((d*x)^(1 + m)*(a
 + b*Log[c*x^n])*PolyLog[3, e*x^q])/(d*(1 + m)) - (q^2*Defer[Int][(d*x)^m*(
a + b*Log[c*x^n])*Log[1 - e*x^q], x])/(1 + m)^2
```

Rubi steps

$$\int (dx)^m (a+b\log(cx^n))\,\mathrm{Li}_3(ex^q)\,dx = -\frac{bn(dx)^{1+m}\mathrm{Li}_3(ex^q)}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))\,\mathrm{Li}_3(ex^q)}{d(1+m)} - \frac{q\int (dx)}{}$$

$$= \frac{2bnq(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}(a+b\log(cx^n))\,\mathrm{Li}_2(ex^q)}{d(1+m)^2} - \frac{bn(dx}{d}$$

$$= \frac{2bnq(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}(a+b\log(cx^n))\,\mathrm{Li}_2(ex^q)}{d(1+m)^2} - \frac{bn(dx}{d}$$

$$= \frac{2bnq(dx)^{1+m}\mathrm{Li}_2(ex^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}(a+b\log(cx^n))\,\mathrm{Li}_2(ex^q)}{d(1+m)^2} - \frac{bn(dx}{d}$$

$$= 2\left(\frac{benq^3x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ex^q\right)}{(1+m)^4(1+m+q)} + \frac{bnq^2(dx)^{1+m}\log(}{d(1+m)^4}\right.$$

Mathematica [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\log(cx^n))\,\mathrm{Li}_3(ex^q)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q],x]
```

```
[Out] Integrate[(d*x)^m*(a + b*Log[c*x^n])*PolyLog[3, e*x^q], x]
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(246) = 492.
time = 0.86, size = 1065, normalized size = 4.35

method	result	size
meijerg	Expression too large to display	1065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*ln(c*x^n))*polylog(3,e*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q^3*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^3*ln(
1-e*x^q)+q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q*x^(1+m)*(-e)
^(m/q+1/q)/(1+m)*polylog(3,e*x^q)+q^3*x^(1+m+q)*e*(-e)^(m/q+1/q)/(1+m)^3*Le
rchPhi(e*x^q,1,(1+m+q)/q))-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q^3*x^
(1+m)*(-e)^(m/q+1/q)/(1+m)^3*ln(1-e*x^q)+q^2*x^(1+m)*(-e)^(m/q+1/q)/(1+m)^2
*polylog(2,e*x^q)-q*x^(1+m)*(-e)^(m/q+1/q)/(1+m)*polylog(3,e*x^q)+q^3*x^(1+
m+q)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q))+((-e)^(-m/q-1/q)
```

```
/q^2*ln(-e)*(d*x)^m*x^(-m)*b*n*(q^3*x^m*(-e)^(m/q+1/q)/(1+m)^3*ln(1-e*x^q)+
q^2*x^m*(-e)^(m/q+1/q)/(1+m)^2*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)/(1+m)*
polylog(3,e*x^q)+q^3*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e*x^q,1,(1+m
+q)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(q^3*x^m*(-e)^(m/q+1/q)*ln(x)/
(1+m)^3*ln(1-e*x^q)+q^2*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^3*ln(1-e*x^q)-3*q^3
*x^m*(-e)^(m/q+1/q)/(1+m)^4*ln(1-e*x^q)+q^2*x^m*(-e)^(m/q+1/q)*ln(x)/(1+m)^
2*polylog(2,e*x^q)+q*x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)^2*polylog(2,e*x^q)-2*q
^2*x^m*(-e)^(m/q+1/q)/(1+m)^3*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(x)/(
1+m)*polylog(3,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(1+m)*polylog(3,e*x^q)+q*x^
m*(-e)^(m/q+1/q)/(1+m)^2*polylog(3,e*x^q)+q^3*x^(q+m)*e*(-e)^(m/q+1/q)*ln(x
)/(1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q)+q^2*x^(q+m)*e*(-e)^(m/q+1/q)*ln(-e)/(
1+m)^3*LerchPhi(e*x^q,1,(1+m+q)/q)-3*q^3*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^4*L
erchPhi(e*x^q,1,(1+m+q)/q)-q^2*x^(q+m)*e*(-e)^(m/q+1/q)/(1+m)^3*LerchPhi(e*
x^q,2,(1+m+q)/q)))*x
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="maxima")
```

```
[Out] -((((m^2*q + 2*m*q + q)*b*d^m*x*x^m*log(x^n) + ((m^2*q + 2*m*q + q)*a*d^m +
((m^2*q + 2*m*q + q)*d^m*log(c) - 2*(m*n*q + n*q)*d^m)*b)*x*x^m)*dilog(e^(q
*log(x) + 1)) + ((m*q^2 + q^2)*b*d^m*x*x^m*log(x^n) + ((m*q^2 + q^2)*a*d^m
- (3*d^m*n*q^2 - (m*q^2 + q^2)*d^m*log(c))*b)*x*x^m)*log(-e^(q*log(x) + 1)
+ 1) - ((m^3 + 3*m^2 + 3*m + 1)*b*d^m*x*x^m*log(x^n) + ((m^3 + 3*m^2 + 3*m
+ 1)*a*d^m + ((m^3 + 3*m^2 + 3*m + 1)*d^m*log(c) - (m^2*n + 2*m*n + n)*d^m)
*b)*x*x^m)*polylog(3, e^(q*log(x) + 1)))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) +
integrate(-((m*q^3 + q^3)*b*d^m*e^(m*log(x) + q*log(x) + 1)*log(x^n) + ((m*
q^3 + q^3)*a*d^m - (3*d^m*n*q^3 - (m*q^3 + q^3)*d^m*log(c))*b)*e^(m*log(x)
+ q*log(x) + 1))/(m^4 + 4*m^3 + 6*m^2 - (m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*e^(
q*log(x) + 1) + 4*m + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="fricas")
```

```
[Out] integral(((d*x)^m*b*log(c*x^n) + (d*x)^m*a)*polylog(3, x^q*e), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\log(cx^n))\,\mathrm{Li}_3(ex^q)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))*polylog(3,e*x**q),x)
[Out] Integral((d*x)**m*(a + b*log(c*x**n))*polylog(3, e*x**q), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))*polylog(3,e*x^q),x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)*(d*x)^m*polylog(3, x^q*e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\mathrm{polylog}(3,e\,x^q)\,\left(a+b\,\ln\left(c\,x^n\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)),x)
[Out] int((d*x)^m*polylog(3, e*x^q)*(a + b*log(c*x^n)), x)
```

3.223 $\int x^2 \log \left(c(bx^n)^p\right)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{9}npx^3+\frac{1}{3}x^3\log\left(c(bx^n)^p\right)$$

```
[Out] -1/9*n*p*x^3+1/3*x^3*ln(c*(b*x^n)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2341, 2495}

$$\frac{1}{3}x^3\log\left(c(bx^n)^p\right)-\frac{1}{9}npx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(b*x^n)^p],x]
```

```
[Out] -1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p])/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c(bx^n)^p\right)\,dx&=\text{Subst}\left(\int x^2\log\left(b^pcx^{np}\right)\,dx,b^pcx^{np},c(bx^n)^p\right)\\&=-\frac{1}{9}npx^3+\frac{1}{3}x^3\log\left(c(bx^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{1}{9}npx^3+\frac{1}{3}x^3\log\left(c(bx^n)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(b*x^n)^p],x]
```

```
[Out] -1/9*(n*p*x^3) + (x^3*Log[c*(b*x^n)^p])/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2} \ln \left(c (b\,x^{n})^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^n)^p),x)
```

```
[Out] int(x^2*ln(c*(b*x^n)^p),x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.85

$$-\frac{1}{9}\,npx^{3} + \frac{1}{3}\,x^{3} \log \left((bx^{n})^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*n*p*x^3 + 1/3*x^3*log((b*x^n)^p*c)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 1.19

$$\frac{1}{3}\,npx^{3} \log \left(x\right) - \frac{1}{9}\,npx^{3} + \frac{1}{3}\,px^{3} \log \left(b\right) + \frac{1}{3}\,x^{3} \log \left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

Sympy [A]
time = 0.41, size = 22, normalized size = 0.81

$$-\frac{npx^{3}}{9} + \frac{x^{3} \log \left(c (bx^{n})^{p}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**n)**p),x)
```

```
[Out] -n*p*x**3/9 + x**3*log(c*(b*x**n)**p)/3
```

Giac [A]
time = 3.53, size = 32, normalized size = 1.19

$$\frac{1}{3}\,npx^3\log\left(x\right)-\frac{1}{9}\,npx^3+\frac{1}{3}\,px^3\log\left(b\right)+\frac{1}{3}\,x^3\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p),x, algorithm="giac")
```

```
[Out] 1/3*n*p*x^3*log(x) - 1/9*n*p*x^3 + 1/3*p*x^3*log(b) + 1/3*x^3*log(c)
```

Mupad [B]
time = 3.84, size = 23, normalized size = 0.85

$$\frac{x^3\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{3}-\frac{n\,p\,x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(b*x^n)^p),x)
```

```
[Out] (x^3*log(c*(b*x^n)^p))/3 - (n*p*x^3)/9
```

3.224 $\int x \log \left(c(bx^n)^p\right) \, dx$

Optimal. Leaf size=27

$$-\frac{1}{4}npx^2+\frac{1}{2}x^2\log \left(c(bx^n)^p\right)$$

```
[Out] -1/4*n*p*x^2+1/2*x^2*ln(c*(b*x^n)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2341, 2495}

$$\frac{1}{2}x^2\log \left(c(bx^n)^p\right)-\frac{1}{4}npx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(b*x^n)^p],x]
```

```
[Out] -1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int x\log \left(c(bx^n)^p\right)\,dx&=\text{Subst}\left(\int x\log \left(b^pcx^{np}\right)\,dx,b^pcx^{np},c(bx^n)^p\right)\\&=-\frac{1}{4}npx^2+\frac{1}{2}x^2\log \left(c(bx^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{1}{4}npx^2+\frac{1}{2}x^2\log \left(c(bx^n)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(b*x^n)^p],x]
[Out] -1/4*(n*p*x^2) + (x^2*Log[c*(b*x^n)^p])/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x \ln \left(c(b\,x^{n})^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^n)^p),x)
[Out] int(x*ln(c*(b*x^n)^p),x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.85

$$-\frac{1}{4}\,npx^{2}+\frac{1}{2}\,x^{2}\log\left(\left(bx^{n}\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="maxima")
[Out] -1/4*n*p*x^2 + 1/2*x^2*log((b*x^n)^p*c)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 1.19

$$\frac{1}{2}\,npx^{2}\log\left(x\right)-\frac{1}{4}\,npx^{2}+\frac{1}{2}\,px^{2}\log\left(b\right)+\frac{1}{2}\,x^{2}\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="fricas")
[Out] 1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

Sympy [A]
time = 0.22, size = 22, normalized size = 0.81

$$-\frac{npx^{2}}{4}+\frac{x^{2}\log\left(c(bx^{n})^{p}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**n)**p),x)
[Out] -n*p*x**2/4 + x**2*log(c*(b*x**n)**p)/2
```

Giac [A]
time = 4.57, size = 32, normalized size = 1.19

$$\frac{1}{2}npx^2\log(x) - \frac{1}{4}npx^2 + \frac{1}{2}px^2\log(b) + \frac{1}{2}x^2\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p),x, algorithm="giac")
```

```
[Out] 1/2*n*p*x^2*log(x) - 1/4*n*p*x^2 + 1/2*p*x^2*log(b) + 1/2*x^2*log(c)
```

Mupad [B]
time = 3.80, size = 23, normalized size = 0.85

$$\frac{x^2\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2}-\frac{n\,p\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(b*x^n)^p),x)
```

```
[Out] (x^2*log(c*(b*x^n)^p))/2 - (n*p*x^2)/4
```

3.225 $\int \log(c(bx^n)^p)\,dx$

Optimal. Leaf size=18

$$-npx + x\log(c(bx^n)^p)$$

```
[Out] -n*p*x+x*ln(c*(b*x^n)^p)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2332, 2495}

$$x\log(c(bx^n)^p) - npx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p],x]
```

```
[Out] -(n*p*x) + x*Log[c*(b*x^n)^p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \log(c(bx^n)^p)\,dx &= \text{Subst}\left(\int \log(b^p c x^{np})\,dx, b^p c x^{np}, c(bx^n)^p\right)\\ &= -npx + x\log(c(bx^n)^p)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-npx + x\log(c(bx^n)^p)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p],x]
[Out] -(n*p*x) + x*Log[c*(b*x^n)^p]
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
default	$-npx+x\ln\left(c(b\,x^{n})^{p}\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p),x,method=_RETURNVERBOSE)
[Out] -n*p*x+x*ln(c*(b*x^n)^p)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 1.00

$$-npx+x\log\left((bx^{n})^{p}\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="maxima")
[Out] -n*p*x + x*log((b*x^n)^p*c)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 1.17

$$npx\log\left(x\right)-npx+px\log\left(b\right)+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="fricas")
[Out] n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

Sympy [A]
time = 0.12, size = 15, normalized size = 0.83

$$-npx+x\log\left(c(bx^{n})^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p),x)
[Out] -n*p*x + x*log(c*(b*x**n)**p)
```

Giac [A]
time = 5.82, size = 21, normalized size = 1.17

$$npx\log\left(x\right)-npx+px\log\left(b\right)+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p),x, algorithm="giac")
```

```
[Out] n*p*x*log(x) - n*p*x + p*x*log(b) + x*log(c)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.94

$$x\left(\ln\left(c\left(b\,x^n\right)^p\right)-n\,p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p),x)
```

```
[Out] x*(log(c*(b*x^n)^p) - n*p)
```

3.226 $\int \frac{\log(c(bx^n)^p)}{x} \, dx$

Optimal. Leaf size=22

$$\frac{\log^2\left(c(bx^n)^p\right)}{2np}$$

```
[Out] 1/2*ln(c*(b*x^n)^p)^2/n/p
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2338, 2495}

$$\frac{\log^2\left(c(bx^n)^p\right)}{2np}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x,x]
```

```
[Out] Log[c*(b*x^n)^p]^2/(2*n*p)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(c(bx^n)^p\right)}{x} \, dx &= \text{Subst}\left(\int \frac{\log\left(b^p c x^{np}\right)}{x} \, dx, b^p c x^{np}, c(bx^n)^p\right) \\ &= \frac{\log^2\left(c(bx^n)^p\right)}{2np} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log^2\left(c(bx^n)^p\right)}{2np}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x,x]
[Out] Log[c*(b*x^n)^p]^2/(2*n*p)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln(c(b\,x^n)^p)^2}{2pn}$	21
default	$\frac{\ln(c(b\,x^n)^p)^2}{2pn}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(c*(b*x^n)^p)^2/p/n
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$\frac{\log\left((bx^n)^p\,c\right)^2}{2\,np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="maxima")
[Out] 1/2*log((b*x^n)^p*c)^2/(n*p)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.86

$$\frac{1}{2}\,np\log\left(x\right)^2+\left(p\log\left(b\right)+\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="fricas")
[Out] 1/2*n*p*log(x)^2 + (p*log(b) + log(c))*log(x)
```

Sympy [A]
time = 1.00, size = 37, normalized size = 1.68

$$-\begin{cases} -\log\left(x\right)\log\left(b^p c\right) & \text{for}\; n=0 \\ -\log\left(c\right)\log\left(x\right) & \text{for}\; p=0 \\ -\frac{\log\left(c(bx^n)^p\right)^2}{2np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x,x)
```

```
[Out] -Piecewise((-log(x)*log(b**p*c), Eq(n, 0)), (-log(c)*log(x), Eq(p, 0)), (-l
og(c*(b*x**n)**p)**2/(2*n*p), True))
```

Giac [A]
time = 3.76, size = 20, normalized size = 0.91

$$\frac{1}{2}\,np\log\left(x\right)^{2}+p\log\left(b\right)\log\left(x\right)+\log\left(c\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*p*log(x)^2 + p*log(b)*log(x) + log(c)*log(x)
```

Mupad [B]
time = 3.80, size = 20, normalized size = 0.91

$$\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)^2}{2\,n\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x,x)
```

```
[Out] log(c*(b*x^n)^p)^2/(2*n*p)
```

3.227 $\int \frac{\log(c(bx^n)^p)}{x^2} dx$

Optimal. Leaf size=23

$$-\frac{np}{x} - \frac{\log\left(c(bx^n)^p\right)}{x}$$

```
[Out] -n*p/x-ln(c*(b*x^n)^p)/x
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2341, 2495}

$$-\frac{\log\left(c(bx^n)^p\right)}{x} - \frac{np}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^2,x]
[Out] -((n*p)/x) - Log[c*(b*x^n)^p]/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned} \int \frac{\log\left(c(bx^n)^p\right)}{x^2} dx &= \text{Subst}\left(\int \frac{\log\left(b^p cx^{np}\right)}{x^2} dx, b^p cx^{np}, c(bx^n)^p\right) \\ &= -\frac{np}{x} - \frac{\log\left(c(bx^n)^p\right)}{x} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{np}{x} - \frac{\log\left(c(bx^n)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^2,x]
```

```
[Out] -((n*p)/x) - Log[c*(b*x^n)^p]/x
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^n)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^2,x)
```

```
[Out] int(ln(c*(b*x^n)^p)/x^2,x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.00

$$-\frac{np}{x}-\frac{\log\left((bx^n)^p\,c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -n*p/x - log((b*x^n)^p*c)/x
```

Fricas [A]
time = 0.34, size = 20, normalized size = 0.87

$$-\frac{np\log\left(x\right)+np+p\log\left(b\right)+\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="fricas")
```

```
[Out] -(n*p*log(x) + n*p + p*log(b) + log(c))/x
```

Sympy [A]
time = 0.22, size = 17, normalized size = 0.74

$$-\frac{np}{x}-\frac{\log\left(c(bx^n)^p\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**2,x)
```

```
[Out] -n*p/x - log(c*(b*x**n)**p)/x
```

Giac [A]
time = 6.39, size = 25, normalized size = 1.09

$$-\frac{np\log{(x)}}{x} - \frac{np + p\log{(b)} + \log{(c)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^2,x, algorithm="giac")
[Out] -n*p*log(x)/x - (n*p + p*log(b) + log(c))/x
```

Mupad [B]
time = 3.88, size = 19, normalized size = 0.83

$$-\frac{\ln{(c\,(b\,x^n)^p)} + n\,p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^2,x)
[Out] -(log(c*(b*x^n)^p) + n*p)/x
```

3.228 $\int \frac{\log(c(bx^n)^p)}{x^3} \, dx$

Optimal. Leaf size=27

$$-\frac{np}{4x^2} - \frac{\log(c(bx^n)^p)}{2x^2}$$

```
[Out] -1/4*n*p/x^2-1/2*ln(c*(b*x^n)^p)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2341, 2495}

$$-\frac{\log(c(bx^n)^p)}{2x^2} - \frac{np}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^3,x]
```

```
[Out] -1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(c(bx^n)^p)}{x^3} \, dx &= \text{Subst}\left(\int \frac{\log(b^p cx^{np})}{x^3} \, dx, b^p cx^{np}, c(bx^n)^p\right)\\ &= -\frac{np}{4x^2} - \frac{\log(c(bx^n)^p)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{np}{4x^2} - \frac{\log(c(bx^n)^p)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^3,x]
```

```
[Out] -1/4*(n*p)/x^2 - Log[c*(b*x^n)^p]/(2*x^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\, x^{n})^{p}\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^3,x)
```

```
[Out] int(ln(c*(b*x^n)^p)/x^3,x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.85

$$-\frac{np}{4\, x^{2}} - \frac{\log\left(\left(bx^{n}\right)^{p} c\right)}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*n*p/x^2 - 1/2*log((b*x^n)^p*c)/x^2
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.89

$$-\frac{2\, np\log\left(x\right) + np + 2\, p\log\left(b\right) + 2\, \log\left(c\right)}{4\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*n*p*log(x) + n*p + 2*p*log(b) + 2*log(c))/x^2
```

Sympy [A]
time = 0.49, size = 24, normalized size = 0.89

$$-\frac{np}{4x^{2}} - \frac{\log\left(c(bx^{n})^{p}\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**3,x)
```

```
[Out] -n*p/(4*x**2) - log(c*(b*x**n)**p)/(2*x**2)
```

Giac [A]
time = 5.65, size = 28, normalized size = 1.04

$$-\frac{np\log\left(x\right)}{2\,x^2}-\frac{np+2\,p\log\left(b\right)+2\,\log\left(c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^3,x, algorithm="giac")
[Out] -1/2*n*p*log(x)/x^2 - 1/4*(n*p + 2*p*log(b) + 2*log(c))/x^2
```

Mupad [B]
time = 3.84, size = 23, normalized size = 0.85

$$-\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2\,x^2}-\frac{n\,p}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^3,x)
[Out] - log(c*(b*x^n)^p)/(2*x^2) - (n*p)/(4*x^2)
```

3.229 $\int \frac{\log(c(bx^n)^p)}{x^4}\,dx$

Optimal. Leaf size=27

$$-\frac{np}{9x^3}-\frac{\log\left(c(bx^n)^p\right)}{3x^3}$$

```
[Out] -1/9*n*p/x^3-1/3*ln(c*(b*x^n)^p)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2341, 2495}

$$-\frac{\log\left(c(bx^n)^p\right)}{3x^3}-\frac{np}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]/x^4,x]
```

```
[Out] -1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3*x^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(bx^n)^p\right)}{x^4}\,dx &= \text{Subst}\left(\int \frac{\log\left(b^pcx^{np}\right)}{x^4}\,dx, b^pcx^{np}, c(bx^n)^p\right)\\ &= -\frac{np}{9x^3}-\frac{\log\left(c(bx^n)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{np}{9x^3}-\frac{\log\left(c(bx^n)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]/x^4,x]
```

```
[Out] -1/9*(n*p)/x^3 - Log[c*(b*x^n)^p]/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^{n})^{p}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)/x^4,x)
```

```
[Out] int(ln(c*(b*x^n)^p)/x^4,x)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.85

$$-\frac{np}{9\,x^{3}} - \frac{\log\left((bx^{n})^{p}\,c\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*n*p/x^3 - 1/3*log((b*x^n)^p*c)/x^3
```

Fricas [A]
time = 0.36, size = 24, normalized size = 0.89

$$-\frac{3\,np\log\left(x\right) + np + 3\,p\log\left(b\right) + 3\,\log\left(c\right)}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*n*p*log(x) + n*p + 3*p*log(b) + 3*log(c))/x^3
```

Sympy [A]
time = 0.89, size = 24, normalized size = 0.89

$$-\frac{np}{9x^{3}} - \frac{\log\left(c(bx^{n})^{p}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)/x**4,x)
```

```
[Out] -n*p/(9*x**3) - log(c*(b*x**n)**p)/(3*x**3)
```

Giac [A]
time = 3.02, size = 28, normalized size = 1.04

$$-\frac{np\log\left(x\right)}{3\,x^{3}}-\frac{np+3\,p\log\left(b\right)+3\,\log\left(c\right)}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*n*p*log(x)/x^3 - 1/9*(n*p + 3*p*log(b) + 3*log(c))/x^3
```

Mupad [B]
time = 3.87, size = 23, normalized size = 0.85

$$-\frac{\ln\left(c\,{\left(b\,x^{n}\right)}^{p}\right)}{3\,x^{3}}-\frac{n\,p}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)/x^4,x)
```

```
[Out] - log(c*(b*x^n)^p)/(3*x^3) - (n*p)/(9*x^3)
```

3.230 $\int x^2 \log^2 \left(c(bx^n)^p\right) dx$

Optimal. Leaf size=52

$$\frac{2}{27}n^2p^2x^3 - \frac{2}{9}npx^3 \log \left(c(bx^n)^p\right) + \frac{1}{3}x^3 \log^2 \left(c(bx^n)^p\right)$$

```
[Out] 2/27*n^2*p^2*x^3-2/9*n*p*x^3*ln(c*(b*x^n)^p)+1/3*x^3*ln(c*(b*x^n)^p)^2
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2342, 2341, 2495}

$$\frac{1}{3}x^3 \log^2 \left(c(bx^n)^p\right) - \frac{2}{9}npx^3 \log \left(c(bx^n)^p\right) + \frac{2}{27}n^2p^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*Log[c*(b*x^n)^p])/9 + (x^3*Log[c*(b*x^n)^p]
^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int x^2 \log^2\left(c(bx^n)^p\right) dx = \text{Subst}\left(\int x^2 \log^2\left(b^p c x^{np}\right) dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= \frac{1}{3}x^3 \log^2\left(c(bx^n)^p\right) - \text{Subst}\left(\frac{1}{3}(2np)\int x^2 \log\left(b^p c x^{np}\right) dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= \frac{2}{27}n^2p^2x^3 - \frac{2}{9}npx^3 \log\left(c(bx^n)^p\right) + \frac{1}{3}x^3 \log^2\left(c(bx^n)^p\right)$$

Mathematica [A]
time = 0.00, size = 52, normalized size = 1.00

$$\frac{2}{27}n^2p^2x^3 - \frac{2}{9}npx^3 \log\left(c(bx^n)^p\right) + \frac{1}{3}x^3 \log^2\left(c(bx^n)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(b*x^n)^p]^2,x]
[Out] (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*Log[c*(b*x^n)^p])/9 + (x^3*Log[c*(b*x^n)^p]
^2)/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(c(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^n)^p)^2,x)
[Out] int(x^2*ln(c*(b*x^n)^p)^2,x)
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.88

$$\frac{2}{27}\,n^2p^2x^3 - \frac{2}{9}\,npx^3 \log\left((bx^n)^p\,c\right) + \frac{1}{3}\,x^3 \log\left((bx^n)^p\,c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
[Out] 2/27*n^2*p^2*x^3 - 2/9*n*p*x^3*log((b*x^n)^p*c) + 1/3*x^3*log((b*x^n)^p*c)^
2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.
time = 0.34, size = 113, normalized size = 2.17

$$\frac{1}{3}\,n^2p^2x^3\log\left(x\right)^2 + \frac{2}{27}\,n^2p^2x^3 - \frac{2}{9}\,np^2x^3\log\left(b\right) + \frac{1}{3}\,p^2x^3\log\left(b\right)^2 + \frac{1}{3}\,x^3\log\left(c\right)^2 - \frac{2}{9}\,\left(npx^3 - 3\,px^3\log\left(b\right)\right)\log\left(c\right) - \frac{2}{9}\,\left(n^2p^2x^3 - 3\,np^2x^3\log\left(b\right) - 3\,npx^3\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*n^2*p^2*x^3*log(x)^2 + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p^
2*x^3*log(b)^2 + 1/3*x^3*log(c)^2 - 2/9*(n*p*x^3 - 3*p*x^3*log(b))*log(c) -
 2/9*(n^2*p^2*x^3 - 3*n*p^2*x^3*log(b) - 3*n*p*x^3*log(c))*log(x)
```

Sympy [A]
time = 0.80, size = 49, normalized size = 0.94

$$\frac{2n^2p^2x^3}{27} - \frac{2npx^3\log\left(c(bx^n)^p\right)}{9} + \frac{x^3\log\left(c(bx^n)^p\right)^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**n)**p)**2,x)
```

```
[Out] 2*n**2*p**2*x**3/27 - 2*n*p*x**3*log(c*(b*x**n)**p)/9 + x**3*log(c*(b*x**n)
**p)**2/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(46) = 92.
time = 5.44, size = 115, normalized size = 2.21

$$\frac{1}{3}n^2p^2x^3\log(x)^2 - \frac{2}{9}n^2p^2x^3\log(x) + \frac{2}{3}np^2x^3\log(b)\log(x) + \frac{2}{27}n^2p^2x^3 - \frac{2}{9}np^2x^3\log(b) + \frac{1}{3}p^2x^3\log(b)^2 + \frac{2}{3}npx^3\log(c)\log(x) - \frac{2}{9}npx^3\log(c) + \frac{2}{3}px^3\log(b)\log(c) + \frac{1}{3}x^3\log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*n^2*p^2*x^3*log(x)^2 - 2/9*n^2*p^2*x^3*log(x) + 2/3*n*p^2*x^3*log(b)*lo
g(x) + 2/27*n^2*p^2*x^3 - 2/9*n*p^2*x^3*log(b) + 1/3*p^2*x^3*log(b)^2 + 2/3
*n*p*x^3*log(c)*log(x) - 2/9*n*p*x^3*log(c) + 2/3*p*x^3*log(b)*log(c) + 1/3
*x^3*log(c)^2
```

Mupad [B]
time = 3.80, size = 46, normalized size = 0.88

$$\frac{2\,n^2\,p^2\,x^3}{27} - \frac{2\,n\,p\,x^3\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{9} + \frac{x^3\,{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(b*x^n)^p)^2,x)
```

```
[Out] (x^3*log(c*(b*x^n)^p)^2)/3 + (2*n^2*p^2*x^3)/27 - (2*n*p*x^3*log(c*(b*x^n)^
p))/9
```

3.231 $\int x \log^2\left(c(bx^n)^p\right)\,dx$

Optimal. Leaf size=52

$$\frac{1}{4}n^2p^2x^2-\frac{1}{2}npx^2\log\left(c(bx^n)^p\right)+\frac{1}{2}x^2\log^2\left(c(bx^n)^p\right)$$

```
[Out] 1/4*n^2*p^2*x^2-1/2*n*p*x^2*ln(c*(b*x^n)^p)+1/2*x^2*ln(c*(b*x^n)^p)^2
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {2342, 2341, 2495}

$$\frac{1}{2}x^2\log^2\left(c(bx^n)^p\right)-\frac{1}{2}npx^2\log\left(c(bx^n)^p\right)+\frac{1}{4}n^2p^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(b*x^n)^p]^2,x]
```

```
[Out] (n^2*p^2*x^2)/4 - (n*p*x^2*Log[c*(b*x^n)^p])/2 + (x^2*Log[c*(b*x^n)^p]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int x\log^2\left(c(bx^n)^p\right)\,dx = \text{Subst}\left(\int x\log^2\left(b^pcx^{np}\right)\,dx, b^pcx^{np}, c(bx^n)^p\right)$$
$$= \frac{1}{2}x^2\log^2\left(c(bx^n)^p\right) - \text{Subst}\left((np)\int x\log\left(b^pcx^{np}\right)\,dx, b^pcx^{np}, c(bx^n)^p\right)$$
$$= \frac{1}{4}n^2p^2x^2 - \frac{1}{2}npx^2\log\left(c(bx^n)^p\right) + \frac{1}{2}x^2\log^2\left(c(bx^n)^p\right)$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.83

$$\frac{1}{4}x^2\left(n^2p^2 - 2np\log\left(c(bx^n)^p\right) + 2\log^2\left(c(bx^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(b*x^n)^p]^2,x]
[Out] (x^2*(n^2*p^2 - 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\ln\left(c(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^n)^p)^2,x)
[Out] int(x*ln(c*(b*x^n)^p)^2,x)
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.88

$$\frac{1}{4}\,n^2p^2x^2 - \frac{1}{2}\,npx^2\log\left((bx^n)^p\,c\right) + \frac{1}{2}\,x^2\log\left((bx^n)^p\,c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="maxima")
[Out] 1/4*n^2*p^2*x^2 - 1/2*n*p*x^2*log((b*x^n)^p*c) + 1/2*x^2*log((b*x^n)^p*c)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.
time = 0.34, size = 113, normalized size = 2.17

$$\frac{1}{2}\,n^2p^2x^2\log\left(x\right)^2 + \frac{1}{4}\,n^2p^2x^2 - \frac{1}{2}\,np^2x^2\log\left(b\right) + \frac{1}{2}\,p^2x^2\log\left(b\right)^2 + \frac{1}{2}\,x^2\log\left(c\right)^2 - \frac{1}{2}\,\left(npx^2 - 2\,px^2\log\left(b\right)\right)\log\left(c\right) - \frac{1}{2}\,\left(n^2p^2x^2 - 2\,np^2x^2\log\left(b\right) - 2\,npx^2\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*n^2*p^2*x^2*log(x)^2 + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^2
*x^2*log(b)^2 + 1/2*x^2*log(c)^2 - 1/2*(n*p*x^2 - 2*p*x^2*log(b))*log(c) -
1/2*(n^2*p^2*x^2 - 2*n*p^2*x^2*log(b) - 2*n*p*x^2*log(c))*log(x)
```

Sympy [A]
time = 0.42, size = 46, normalized size = 0.88

$$\frac{n^2p^2x^2}{4} - \frac{npx^2\log\left(c(bx^n)^p\right)}{2} + \frac{x^2\log\left(c(bx^n)^p\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**n)**p)**2,x)
```

```
[Out] n**2*p**2*x**2/4 - n*p*x**2*log(c*(b*x**n)**p)/2 + x**2*log(c*(b*x**n)**p)*
*2/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(46) = 92.
time = 7.38, size = 112, normalized size = 2.15

$$\frac{1}{2}n^2p^2x^2\log(x)^2 - \frac{1}{2}n^2p^2x^2\log(x) + np^2x^2\log(b)\log(x) + \frac{1}{4}n^2p^2x^2 - \frac{1}{2}np^2x^2\log(b) + \frac{1}{2}p^2x^2\log(b)^2 + npx^2\log(c)\log(x) - \frac{1}{2}npx^2\log(c) + px^2\log(b)\log(c) + \frac{1}{2}x^2\log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*n^2*p^2*x^2*log(x)^2 - 1/2*n^2*p^2*x^2*log(x) + n*p^2*x^2*log(b)*log(x)
 + 1/4*n^2*p^2*x^2 - 1/2*n*p^2*x^2*log(b) + 1/2*p^2*x^2*log(b)^2 + n*p*x^2*
log(c)*log(x) - 1/2*n*p*x^2*log(c) + p*x^2*log(b)*log(c) + 1/2*x^2*log(c)^2
```

Mupad [B]
time = 3.85, size = 46, normalized size = 0.88

$$\frac{n^2\,p^2\,x^2}{4} - \frac{n\,p\,x^2\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2} + \frac{x^2\,{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(b*x^n)^p)^2,x)
```

```
[Out] (x^2*log(c*(b*x^n)^p)^2)/2 + (n^2*p^2*x^2)/4 - (n*p*x^2*log(c*(b*x^n)^p))/2
```

3.232 $\int \log^2(c(bx^n)^p)\,dx$

Optimal. Leaf size=39

$$2n^2p^2x - 2npx\log(c(bx^n)^p) + x\log^2(c(bx^n)^p)$$

```
[Out] 2*n^2*p^2*x-2*n*p*x*ln(c*(b*x^n)^p)+x*ln(c*(b*x^n)^p)^2
```

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2333, 2332, 2495}

$$x\log^2(c(bx^n)^p) - 2npx\log(c(bx^n)^p) + 2n^2p^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2,x]
```

```
[Out] 2*n^2*p^2*x - 2*n*p*x*Log[c*(b*x^n)^p] + x*Log[c*(b*x^n)^p]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}
\int \log^2(c(bx^n)^p)\,dx &= \text{Subst}\left(\int \log^2(b^pcx^{np})\,dx, b^pcx^{np}, c(bx^n)^p\right)\\
&= x\log^2(c(bx^n)^p) - \text{Subst}\left((2np)\int \log(b^pcx^{np})\,dx, b^pcx^{np}, c(bx^n)^p\right)\\
&= 2n^2p^2x - 2npx\log(c(bx^n)^p) + x\log^2(c(bx^n)^p)
\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 37, normalized size = 0.95

$$x\log^2\left(c(bx^n)^p\right)-2np(-npx+x\log\left(c(bx^n)^p\right))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2,x]
[Out] x*Log[c*(b*x^n)^p]^2 - 2*n*p*(-(n*p*x) + x*Log[c*(b*x^n)^p])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \ln\left(c(b\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2,x)
[Out] int(ln(c*(b*x^n)^p)^2,x)
```

Maxima [**A**]
time = 0.28, size = 39, normalized size = 1.00

$$2\,n^2p^2x-2\,npx\log\left((bx^n)^p\,c\right)+x\log\left((bx^n)^p\,c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="maxima")
[Out] 2*n^2*p^2*x - 2*n*p*x*log((b*x^n)^p*c) + x*log((b*x^n)^p*c)^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.
time = 0.35, size = 90, normalized size = 2.31

$$n^2p^2x\log(x)^2+2\,n^2p^2x-2\,np^2x\log(b)+p^2x\log(b)^2+x\log(c)^2-2\,(npx-px\log(b))\log(c)-2\,(n^2p^2x-np^2x\log(b)-npx\log(c))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="fricas")
[Out] n^2*p^2*x*log(x)^2 + 2*n^2*p^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + x*lo
g(c)^2 - 2*(n*p*x - p*x*log(b))*log(c) - 2*(n^2*p^2*x - n*p^2*x*log(b) - n*
p*x*log(c))*log(x)
```

Sympy [**A**]
time = 0.22, size = 39, normalized size = 1.00

$$2n^2p^2x-2npx\log\left(c(bx^n)^p\right)+x\log\left(c(bx^n)^p\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2,x)
```

```
[Out] 2*n**2*p**2*x - 2*n*p*x*log(c*(b*x**n)**p) + x*log(c*(b*x**n)**p)**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(39) = 78.
time = 4.76, size = 92, normalized size = 2.36

$$n^2p^2x\log(x)^2 - 2\,n^2p^2x\log(x) + 2\,np^2x\log(b)\log(x) + 2\,n^2p^2x - 2\,np^2x\log(b) + p^2x\log(b)^2 + 2\,npx\log(c)\log(x) - 2\,npx\log(c) + 2\,px\log(b)\log(c) + x\log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] n^2*p^2*x*log(x)^2 - 2*n^2*p^2*x*log(x) + 2*n*p^2*x*log(b)*log(x) + 2*n^2*p
^2*x - 2*n*p^2*x*log(b) + p^2*x*log(b)^2 + 2*n*p*x*log(c)*log(x) - 2*n*p*x*
log(c) + 2*p*x*log(b)*log(c) + x*log(c)^2
```

Mupad [B]
time = 3.84, size = 39, normalized size = 1.00

$$2\,x\,n^2\,p^2 - 2\,x\,n\,p\,\ln\left(c\,{\left(b\,x^n\right)}^p\right) + x\,{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2,x)
```

```
[Out] x*log(c*(b*x^n)^p)^2 + 2*n^2*p^2*x - 2*n*p*x*log(c*(b*x^n)^p)
```

3.233 $\int \frac{\log^2(c(bx^n)^p)}{x} \, dx$

Optimal. Leaf size=22

$$\frac{\log^3\left(c(bx^n)^p\right)}{3np}$$

[Out] 1/3*ln(c*(b*x^n)^p)^3/n/p

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2339, 30, 2495}

$$\frac{\log^3\left(c(bx^n)^p\right)}{3np}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(b*x^n)^p]^2/x,x]

[Out] Log[c*(b*x^n)^p]^3/(3*n*p)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^2(c(bx^n)^p)}{x}\,dx = \text{Subst}\left(\int \frac{\log^2(b^p c x^{np})}{x}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= \text{Subst}\left(\frac{\text{Subst}\left(\int x^2\,dx, x, \log(b^p c x^{np})\right)}{np}, b^p c x^{np}, c(bx^n)^p\right)$$
$$= \frac{\log^3(c(bx^n)^p)}{3np}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log^3(c(bx^n)^p)}{3np}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x,x]
[Out] Log[c*(b*x^n)^p]^3/(3*n*p)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln(c(b\,x^n)^p)^3}{3pn}$	21
default	$\frac{\ln(c(b\,x^n)^p)^3}{3pn}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/3*ln(c*(b*x^n)^p)^3/p/n
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$\frac{\log\left((bx^n)^p\,c\right)^3}{3\,np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="maxima")
[Out] 1/3*log((b*x^n)^p*c)^3/(n*p)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.
time = 0.42, size = 54, normalized size = 2.45

$$\frac{1}{3} n^2 p^2 \log(x)^3 + \left(np^2 \log(b) + np \log(c)\right) \log(x)^2 + \left(p^2 \log(b)^2 + 2 p \log(b) \log(c) + \log(c)^2\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*n^2*p^2*log(x)^3 + (n*p^2*log(b) + n*p*log(c))*log(x)^2 + (p^2*log(b)^2
 + 2*p*log(b)*log(c) + log(c)^2)*log(x)
```

Sympy [A]
time = 0.78, size = 41, normalized size = 1.86

$$-\begin{cases} -\log(x) \log(b^p c)^2 & \text{for } n = 0 \\ -\log(c)^2 \log(x) & \text{for } p = 0 \\ -\frac{\log(c(bx^n)^p)^3}{3np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x,x)
```

```
[Out] -Piecewise((-log(x)*log(b**p*c)**2, Eq(n, 0)), (-log(c)**2*log(x), Eq(p, 0)
), (-log(c*(b*x**n)**p)**3/(3*n*p), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(20) = 40$.
time = 5.35, size = 59, normalized size = 2.68

$$\frac{1}{3} n^2 p^2 \log(x)^3 + np^2 \log(b) \log(x)^2 + p^2 \log(b)^2 \log(x) + np \log(c) \log(x)^2 + 2 p \log(b) \log(c) \log(x) + \log(c)^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*n^2*p^2*log(x)^3 + n*p^2*log(b)*log(x)^2 + p^2*log(b)^2*log(x) + n*p*lo
g(c)*log(x)^2 + 2*p*log(b)*log(c)*log(x) + log(c)^2*log(x)
```

Mupad [B]
time = 3.75, size = 20, normalized size = 0.91

$$\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)^3}{3\,n\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x,x)
```

```
[Out] log(c*(b*x^n)^p)^3/(3*n*p)
```

3.234 $\int \frac{\log^2(c(bx^n)^p)}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{2n^2p^2}{x}-\frac{2np\log\left(c(bx^n)^p\right)}{x}-\frac{\log^2\left(c(bx^n)^p\right)}{x}$$

```
[Out] -2*n^2*p^2/x-2*n*p*ln(c*(b*x^n)^p)/x-ln(c*(b*x^n)^p)^2/x
```

Rubi [A]

time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2342, 2341, 2495}

$$-\frac{\log^2\left(c(bx^n)^p\right)}{x}-\frac{2np\log\left(c(bx^n)^p\right)}{x}-\frac{2n^2p^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^2,x]
```

```
[Out] (-2*n^2*p^2)/x - (2*n*p*Log[c*(b*x^n)^p])/x - Log[c*(b*x^n)^p]^2/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^2(c(bx^n)^p)}{x^2}\,dx = \text{Subst}\left(\int \frac{\log^2(b^p c x^{np})}{x^2}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{\log^2(c(bx^n)^p)}{x} + \text{Subst}\left((2np)\int \frac{\log(b^p c x^{np})}{x^2}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{2n^2p^2}{x} - \frac{2np\log(c(bx^n)^p)}{x} - \frac{\log^2(c(bx^n)^p)}{x}$$

Mathematica [**A**]
time = 0.01, size = 40, normalized size = 0.87

$$-\frac{2n^2p^2 + 2np\log(c(bx^n)^p) + \log^2(c(bx^n)^p)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^2,x]
[Out] -((2*n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + Log[c*(b*x^n)^p]^2)/x)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^n)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^2,x)
[Out] int(ln(c*(b*x^n)^p)^2/x^2,x)
```

Maxima [**A**]
time = 0.27, size = 46, normalized size = 1.00

$$-\frac{2\,n^2p^2}{x} - \frac{2\,np\log\left((bx^n)^p\,c\right)}{x} - \frac{\log\left((bx^n)^p\,c\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="maxima")
[Out] -2*n^2*p^2/x - 2*n*p*log((b*x^n)^p*c)/x - log((b*x^n)^p*c)^2/x
```

Fricas [**A**]
time = 0.41, size = 81, normalized size = 1.76

$$-\frac{n^2p^2\log(x)^2 + 2\,n^2p^2 + 2\,np^2\log(b) + p^2\log(b)^2 + 2\,(np + p\log(b))\log(c) + \log(c)^2 + 2\,(n^2p^2 + np^2\log(b) + np\log(c))\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(n^2*p^2*log(x)^2 + 2*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*(n*p + p
*log(b))*log(c) + log(c)^2 + 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(x)
)/x
```

Sympy [A]
time = 0.22, size = 41, normalized size = 0.89

$$-\frac{2n^2p^2}{x}-\frac{2np\log\left(c(bx^n)^p\right)}{x}-\frac{\log\left(c(bx^n)^p\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**2,x)
```

```
[Out] -2*n**2*p**2/x - 2*n*p*log(c*(b*x**n)**p)/x - log(c*(b*x**n)**p)**2/x
```

Giac [A]
time = 2.84, size = 90, normalized size = 1.96

$$-\frac{n^2p^2\log(x)^2}{x}-\frac{2\left(n^2p^2+np^2\log(b)+np\log(c)\right)\log(x)}{x}-\frac{2\,n^2p^2+2\,np^2\log(b)+p^2\log(b)^2+2\,np\log(c)+2\,p\log(b)\log(c)+\log(c)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] -n^2*p^2*log(x)^2/x - 2*(n^2*p^2 + n*p^2*log(b) + n*p*log(c))*log(x)/x - (2
*n^2*p^2 + 2*n*p^2*log(b) + p^2*log(b)^2 + 2*n*p*log(c) + 2*p*log(b)*log(c)
 + log(c)^2)/x
```

Mupad [B]
time = 3.85, size = 40, normalized size = 0.87

$$-\frac{2\,n^2\,p^2+2\,n\,p\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)+{\ln\left(c\,{\left(b\,x^n\right)}^p\right)}^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^2,x)
```

```
[Out] -(log(c*(b*x^n)^p)^2 + 2*n^2*p^2 + 2*n*p*log(c*(b*x^n)^p))/x
```

3.235 $\int \frac{\log^2(c(bx^n)^p)}{x^3} dx$

Optimal. Leaf size=52

$$-\frac{n^2p^2}{4x^2} - \frac{np\log\left(c(bx^n)^p\right)}{2x^2} - \frac{\log^2\left(c(bx^n)^p\right)}{2x^2}$$

```
[Out] -1/4*n^2*p^2/x^2-1/2*n*p*ln(c*(b*x^n)^p)/x^2-1/2*ln(c*(b*x^n)^p)^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2342, 2341, 2495}

$$-\frac{\log^2\left(c(bx^n)^p\right)}{2x^2} - \frac{np\log\left(c(bx^n)^p\right)}{2x^2} - \frac{n^2p^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^3,x]
```

```
[Out] -1/4*(n^2*p^2)/x^2 - (n*p*Log[c*(b*x^n)^p])/(2*x^2) - Log[c*(b*x^n)^p]^2/(2
*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^2\left(c(bx^n)^p\right)}{x^3}\,dx = \text{Subst}\left(\int \frac{\log^2\left(b^p c x^{np}\right)}{x^3}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{\log^2\left(c(bx^n)^p\right)}{2x^2} + \text{Subst}\left((np)\int \frac{\log\left(b^p c x^{np}\right)}{x^3}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{n^2p^2}{4x^2} - \frac{np\log\left(c(bx^n)^p\right)}{2x^2} - \frac{\log^2\left(c(bx^n)^p\right)}{2x^2}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 0.83

$$-\frac{n^2p^2 + 2np\log\left(c(bx^n)^p\right) + 2\log^2\left(c(bx^n)^p\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^3,x]
[Out] -1/4*(n^2*p^2 + 2*n*p*Log[c*(b*x^n)^p] + 2*Log[c*(b*x^n)^p]^2)/x^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^n)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^3,x)
[Out] int(ln(c*(b*x^n)^p)^2/x^3,x)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.88

$$-\frac{n^2p^2}{4\,x^2} - \frac{np\log\left((bx^n)^p\,c\right)}{2\,x^2} - \frac{\log\left((bx^n)^p\,c\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="maxima")
[Out] -1/4*n^2*p^2/x^2 - 1/2*n*p*log((b*x^n)^p*c)/x^2 - 1/2*log((b*x^n)^p*c)^2/x^
2
```

Fricas [A]
time = 0.34, size = 87, normalized size = 1.67

$$-\frac{2\,n^2p^2\log\left(x\right)^2 + n^2p^2 + 2\,np^2\log\left(b\right) + 2\,p^2\log\left(b\right)^2 + 2\,(np + 2\,p\log\left(b\right))\log\left(c\right) + 2\,\log\left(c\right)^2 + 2\,(n^2p^2 + 2\,np^2\log\left(b\right) + 2\,np\log\left(c\right))\log\left(x\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*n^2*p^2*log(x)^2 + n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*(n
*p + 2*p*log(b))*log(c) + 2*log(c)^2 + 2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*p*
log(c))*log(x))/x^2
```

Sympy [A]
time = 0.49, size = 48, normalized size = 0.92

$$-\frac{n^2p^2}{4x^2}-\frac{np\log\left(c(bx^n)^p\right)}{2x^2}-\frac{\log\left(c(bx^n)^p\right)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**3,x)
```

```
[Out] -n**2*p**2/(4*x**2) - n*p*log(c*(b*x**n)**p)/(2*x**2) - log(c*(b*x**n)**p)*
*2/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.
time = 5.07, size = 94, normalized size = 1.81

$$-\frac{n^2p^2\log(x)^2}{2\,x^2}-\frac{\left(n^2p^2+2\,np^2\log(b)+2\,np\log(c)\right)\log(x)}{2\,x^2}-\frac{n^2p^2+2\,np^2\log(b)+2\,p^2\log(b)^2+2\,np\log(c)+4\,p\log(b)\log(c)+2\,\log(c)^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*n^2*p^2*log(x)^2/x^2 - 1/2*(n^2*p^2 + 2*n*p^2*log(b) + 2*n*p*log(c))*l
og(x)/x^2 - 1/4*(n^2*p^2 + 2*n*p^2*log(b) + 2*p^2*log(b)^2 + 2*n*p*log(c) +
 4*p*log(b)*log(c) + 2*log(c)^2)/x^2
```

Mupad [B]
time = 3.79, size = 46, normalized size = 0.88

$$-\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)^2}{2\,x^2}-\frac{n^2\,p^2}{4\,x^2}-\frac{n\,p\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^3,x)
```

```
[Out] - log(c*(b*x^n)^p)^2/(2*x^2) - (n^2*p^2)/(4*x^2) - (n*p*log(c*(b*x^n)^p))/(
2*x^2)
```

3.236 $\int \frac{\log^2(c(bx^n)^p)}{x^4} \, dx$

Optimal. Leaf size=52

$$-\frac{2n^2p^2}{27x^3}-\frac{2np\log\left(c(bx^n)^p\right)}{9x^3}-\frac{\log^2\left(c(bx^n)^p\right)}{3x^3}$$

```
[Out] -2/27*n^2*p^2/x^3-2/9*n*p*ln(c*(b*x^n)^p)/x^3-1/3*ln(c*(b*x^n)^p)^2/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2342, 2341, 2495}

$$-\frac{\log^2\left(c(bx^n)^p\right)}{3x^3}-\frac{2np\log\left(c(bx^n)^p\right)}{9x^3}-\frac{2n^2p^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(b*x^n)^p]^2/x^4,x]
```

```
[Out] (-2*n^2*p^2)/(27*x^3) - (2*n*p*Log[c*(b*x^n)^p])/(9*x^3) - Log[c*(b*x^n)^p]
^2/(3*x^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^2(c(bx^n)^p)}{x^4}\,dx = \text{Subst}\left(\int \frac{\log^2(b^p c x^{np})}{x^4}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{\log^2(c(bx^n)^p)}{3x^3} + \text{Subst}\left(\frac{1}{3}(2np)\int \frac{\log(b^p c x^{np})}{x^4}\,dx, b^p c x^{np}, c(bx^n)^p\right)$$
$$= -\frac{2n^2p^2}{27x^3} - \frac{2np\log(c(bx^n)^p)}{9x^3} - \frac{\log^2(c(bx^n)^p)}{3x^3}$$

Mathematica [A]
time = 0.00, size = 52, normalized size = 1.00

$$-\frac{2n^2p^2}{27x^3} - \frac{2np\log(c(bx^n)^p)}{9x^3} - \frac{\log^2(c(bx^n)^p)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(b*x^n)^p]^2/x^4,x]
```

```
[Out] (-2*n^2*p^2)/(27*x^3) - (2*n*p*Log[c*(b*x^n)^p])/(9*x^3) - Log[c*(b*x^n)^p]
^2/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln(c(b\,x^n)^p)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^n)^p)^2/x^4,x)
```

```
[Out] int(ln(c*(b*x^n)^p)^2/x^4,x)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.88

$$-\frac{2\,n^2p^2}{27\,x^3} - \frac{2\,np\log((bx^n)^p\,c)}{9\,x^3} - \frac{\log((bx^n)^p\,c)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] -2/27*n^2*p^2/x^3 - 2/9*n*p*log((b*x^n)^p*c)/x^3 - 1/3*log((b*x^n)^p*c)^2/x
^3
```

Fricas [A]
time = 0.33, size = 88, normalized size = 1.69

$$-\frac{9\,n^2p^2\log\left(x\right)^2+2\,n^2p^2+6\,np^2\log\left(b\right)+9\,p^2\log\left(b\right)^2+6\,\left(np+3\,p\log\left(b\right)\right)\log\left(c\right)+9\,\log\left(c\right)^2+6\,\left(n^2p^2+3\,np^2\log\left(b\right)+3\,np\log\left(c\right)\right)\log\left(x\right)}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*n^2*p^2*log(x)^2 + 2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 + 6
*(n*p + 3*p*log(b))*log(c) + 9*log(c)^2 + 6*(n^2*p^2 + 3*n*p^2*log(b) + 3*n
*p*log(c))*log(x))/x^3
```

Sympy [A]
time = 0.89, size = 51, normalized size = 0.98

$$-\frac{2n^2p^2}{27x^3}-\frac{2np\log\left(c(bx^n)^p\right)}{9x^3}-\frac{\log\left(c(bx^n)^p\right)^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**n)**p)**2/x**4,x)
```

```
[Out] -2*n**2*p**2/(27*x**3) - 2*n*p*log(c*(b*x**n)**p)/(9*x**3) - log(c*(b*x**n)
**p)**2/(3*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.
time = 4.67, size = 95, normalized size = 1.83

$$-\frac{n^2p^2\log\left(x\right)^2}{3\,x^3}-\frac{2\,\left(n^2p^2+3\,np^2\log\left(b\right)+3\,np\log\left(c\right)\right)\log\left(x\right)}{9\,x^3}-\frac{2\,n^2p^2+6\,np^2\log\left(b\right)+9\,p^2\log\left(b\right)^2+6\,np\log\left(c\right)+18\,p\log\left(b\right)\log\left(c\right)+9\,\log\left(c\right)^2}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^n)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*n^2*p^2*log(x)^2/x^3 - 2/9*(n^2*p^2 + 3*n*p^2*log(b) + 3*n*p*log(c))*l
og(x)/x^3 - 1/27*(2*n^2*p^2 + 6*n*p^2*log(b) + 9*p^2*log(b)^2 + 6*n*p*log(c
) + 18*p*log(b)*log(c) + 9*log(c)^2)/x^3
```

Mupad [B]
time = 3.88, size = 46, normalized size = 0.88

$$-\frac{\ln\left(c\,{\left(b\,x^n\right)}^p\right)^2}{3\,x^3}-\frac{2\,n^2\,p^2}{27\,x^3}-\frac{2\,n\,p\,\ln\left(c\,{\left(b\,x^n\right)}^p\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(b*x^n)^p)^2/x^4,x)
```

```
[Out] - log(c*(b*x^n)^p)^2/(3*x^3) - (2*n^2*p^2)/(27*x^3) - (2*n*p*log(c*(b*x^n)^
p))/(9*x^3)
```

3.237 $\int (ex)^q (a+b\log(c(dx^m)^n))^3 \, dx$

Optimal. Leaf size=135

$$-\frac{6b^3m^3n^3(ex)^{1+q}}{e(1+q)^4}+\frac{6b^2m^2n^2(ex)^{1+q}(a+b\log(c(dx^m)^n))}{e(1+q)^3}-\frac{3bmn(ex)^{1+q}(a+b\log(c(dx^m)^n))^2}{e(1+q)^2}+\frac{(ex)^{1+q}(a$$

```
[Out] -6*b^3*m^3*n^3*(e*x)^(1+q)/e/(1+q)^4+6*b^2*m^2*n^2*(e*x)^(1+q)*(a+b*ln(c*(d
*x^m)^n))/e/(1+q)^3-3*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)^2+(
e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^3/e/(1+q)
```

Rubi [A]

time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2342, 2341, 2495}

$$\frac{6b^2m^2n^2(ex)^{q+1}(a+b\log(c(dx^m)^n))}{e(q+1)^3}+\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))^3}{e(q+1)}-\frac{3bmn(ex)^{q+1}(a+b\log(c(dx^m)^n))^2}{e(q+1)^2}-\frac{6b^3m^3n^3(ex)^{q+1}}{e(q+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

```
[Out] (-6*b^3*m^3*n^3*(e*x)^(1 + q))/(e*(1 + q)^4) + (6*b^2*m^2*n^2*(e*x)^(1 + q)
*(a + b*Log[c*(d*x^m)^n]))/(e*(1 + q)^3) - (3*b*m*n*(e*x)^(1 + q)*(a + b*Lo
g[c*(d*x^m)^n])^2)/(e*(1 + q)^2) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])^
3)/(e*(1 + q))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^3 \, dx = \text{Subst}\left(\int (ex)^q \left(a+b\log\left(cd^n x^{mn}\right)\right)^3 \, dx, cd^n x^{mn}, c(dx^m)^n\right)$$

$$= \frac{(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^3}{e(1+q)} - \text{Subst}\left(\frac{(3bmn)\int (ex)^q \left(a+b\log\left(cd^n x^{m}\right.\right.}{1+q}\right.$$

$$= -\frac{3bmn(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{e(1+q)^2} + \frac{(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^3}{e(1+q)} +$$

$$= -\frac{6b^3m^3n^3(ex)^{1+q}}{e(1+q)^4} + \frac{6b^2m^2n^2(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)}{e(1+q)^3} - \frac{3bmn(ex)^{1}}{}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.67

$$\frac{x(ex)^q\left(\left(a+b\log\left(c(dx^m)^n\right)\right)^3 - \frac{3bmn\left((1+q)^2\left(a+b\log\left(c(dx^m)^n\right)\right)^2+2bmn\left(bmn-(1+q)\left(a+b\log\left(c(dx^m)^n\right)\right)\right)\right)}{(1+q)^3}\right)}{1+q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^3,x]
```

```
[Out] (x*(e*x)^q*((a + b*Log[c*(d*x^m)^n])^3 - (3*b*m*n*((1 + q)^2*(a + b*Log[c*(
d*x^m)^n])^2 + 2*b*m*n*(b*m*n - (1 + q)*(a + b*Log[c*(d*x^m)^n]))))/(1 + q)
^3))/(1 + q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^q \left(a+b\ln\left(c(d\,x^m)^n\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^3,x)
```

```
[Out] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.
time = 0.29, size = 285, normalized size = 2.11

$$\frac{(xe)^{q+1}b^3e^{(-1)}\log\left(\left(dx^m\right)^n c\right)^3}{q+1} + \frac{3(xe)^{q+1}ab^2e^{(-1)}\log\left(\left(dx^m\right)^n c\right)^2}{q+1} - \frac{3a^2bmnxe^{(q\log(x)+q)}}{(q+1)^2} + \frac{3(xe)^{q+1}a^2be^{(-1)}\log\left(\left(dx^m\right)^n c\right)}{q+1} + 6\left(\frac{m^2n^2xe^{(q\log(x)+q)}}{(q+1)^3} - \frac{mnxe^{(q\log(x)+q)}\log\left(\left(dx^m\right)^n c\right)}{(q+1)^2}\right)ab^2 - 3\left(\frac{mnxe^{(q\log(x)+q)}\log\left(\left(dx^m\right)^n c\right)^2}{(q+1)^2} + \frac{2\left(\frac{m^2n^2xe^{(q\log(x)+q+1)}}{(q+1)^3} - \frac{mnxe^{(q\log(x)+q+1)}\log\left(\left(dx^m\right)^n c\right)}{(q+1)^2}\right)mne^{(-1)}}{q+1}\right)b^3 + \frac{(xe)^{q+1}a^3e^{(-1)}}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="maxima")
```

```
[Out] (x*e)^(q + 1)*b^3*e^(-1)*log((d*x^m)^n*c)^3/(q + 1) + 3*(x*e)^(q + 1)*a*b^2
*e^(-1)*log((d*x^m)^n*c)^2/(q + 1) - 3*a^2*b*m*n*x*e^(q*log(x) + q)/(q + 1)
^2 + 3*(x*e)^(q + 1)*a^2*b*e^(-1)*log((d*x^m)^n*c)/(q + 1) + 6*(m^2*n^2*x*e
^(q*log(x) + q)/(q + 1)^3 - m*n*x*e^(q*log(x) + q)*log((d*x^m)^n*c)/(q + 1)
^2)*a*b^2 - 3*(m*n*x*e^(q*log(x) + q)*log((d*x^m)^n*c)^2/(q + 1)^2 + 2*(m^2
*n^2*x*e^(q*log(x) + q + 1)/(q + 1)^3 - m*n*x*e^(q*log(x) + q + 1)*log((d*x
^m)^n*c)/(q + 1)^2)*m*n*e^(-1)/(q + 1))*b^3 + (x*e)^(q + 1)*a^3*e^(-1)/(q +
 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(135) = 270$.
time = 0.35, size = 1355, normalized size = 10.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*q^3 + 3*b^3*q^2 + 3*b^3*q + b^3)*x*log(c)^3 + (b^3*n^3*q^3 + 3*b^3*n^
3*q^2 + 3*b^3*n^3*q + b^3*n^3)*x*log(d)^3 + (b^3*m^3*n^3*q^3 + 3*b^3*m^3*n^
3*q^2 + 3*b^3*m^3*n^3*q + b^3*m^3*n^3)*x*log(x)^3 + 3*(a*b^2*q^3 - b^3*m*n
+ a*b^2 - (b^3*m*n - 3*a*b^2)*q^2 - (2*b^3*m*n - 3*a*b^2)*q)*x*log(c)^2 + 3
*(2*b^3*m^2*n^2 + a^2*b*q^3 - 2*a*b^2*m*n + a^2*b - (2*a*b^2*m*n - 3*a^2*b)
*q^2 + (2*b^3*m^2*n^2 - 4*a*b^2*m*n + 3*a^2*b)*q)*x*log(c) + 3*((b^3*n^2*q^
3 + 3*b^3*n^2*q^2 + 3*b^3*n^2*q + b^3*n^2)*x*log(c) + (a*b^2*n^2*q^3 - b^3*
m*n^3 + a*b^2*n^2 - (b^3*m*n^3 - 3*a*b^2*n^2)*q^2 - (2*b^3*m*n^3 - 3*a*b^2*
n^2)*q)*x)*log(d)^2 + 3*((b^3*m^2*n^2*q^3 + 3*b^3*m^2*n^2*q^2 + 3*b^3*m^2*n
^2*q + b^3*m^2*n^2)*x*log(c) + (b^3*m^2*n^3*q^3 + 3*b^3*m^2*n^3*q^2 + 3*b^3
*m^2*n^3*q + b^3*m^2*n^3)*x*log(d) + (a*b^2*m^2*n^2*q^3 - b^3*m^3*n^3 + a*b
^2*m^2*n^2 - (b^3*m^3*n^3 - 3*a*b^2*m^2*n^2)*q^2 - (2*b^3*m^3*n^3 - 3*a*b^2
*m^2*n^2)*q)*x)*log(x)^2 - (6*b^3*m^3*n^3 - 6*a*b^2*m^2*n^2 - a^3*q^3 + 3*a
^2*b*m*n - a^3 + 3*(a^2*b*m*n - a^3)*q^2 - 3*(2*a*b^2*m^2*n^2 - 2*a^2*b*m*n
 + a^3)*q)*x + 3*((b^3*n*q^3 + 3*b^3*n*q^2 + 3*b^3*n*q + b^3*n)*x*log(c)^2
+ 2*(a*b^2*n*q^3 - b^3*m*n^2 + a*b^2*n - (b^3*m*n^2 - 3*a*b^2*n)*q^2 - (2*b
^3*m*n^2 - 3*a*b^2*n)*q)*x*log(c) + (2*b^3*m^2*n^3 + a^2*b*n*q^3 - 2*a*b^2*
m*n^2 + a^2*b*n - (2*a*b^2*m*n^2 - 3*a^2*b*n)*q^2 + (2*b^3*m^2*n^3 - 4*a*b^
2*m*n^2 + 3*a^2*b*n)*q)*x)*log(d) + 3*((b^3*m*n*q^3 + 3*b^3*m*n*q^2 + 3*b^3
*m*n*q + b^3*m*n)*x*log(c)^2 + (b^3*m*n^3*q^3 + 3*b^3*m*n^3*q^2 + 3*b^3*m*n
^3*q + b^3*m*n^3)*x*log(d)^2 + 2*(a*b^2*m*n*q^3 - b^3*m^2*n^2 + a*b^2*m*n -
 (b^3*m^2*n^2 - 3*a*b^2*m*n)*q^2 - (2*b^3*m^2*n^2 - 3*a*b^2*m*n)*q)*x*log(c
) + (2*b^3*m^3*n^3 + a^2*b*m*n*q^3 - 2*a*b^2*m^2*n^2 + a^2*b*m*n - (2*a*b^2
*m^2*n^2 - 3*a^2*b*m*n)*q^2 + (2*b^3*m^3*n^3 - 4*a*b^2*m^2*n^2 + 3*a^2*b*m*
```

```
n)*q)*x + 2*((b^3*m*n^2*q^3 + 3*b^3*m*n^2*q^2 + 3*b^3*m*n^2*q + b^3*m*n^2)*
x*log(c) + (a*b^2*m*n^2*q^3 - b^3*m^2*n^3 + a*b^2*m*n^2 - (b^3*m^2*n^3 - 3*
a*b^2*m*n^2)*q^2 - (2*b^3*m^2*n^3 - 3*a*b^2*m*n^2)*q)*x)*log(d))*log(x))*e^
(q*log(x) + q)/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^q \left(a + b\log\left(c(dx^m)^n\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**3,x)
```

```
[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. 2(135) = 270.
time = 4.39, size = 1811, normalized size = 13.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^3,x, algorithm="giac")
```

```
[Out] b^3*m^3*n^3*q^3*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*
m^3*n^3*q^2*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 3*b^3*m^3*
n^3*q^2*x*x^q*e^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*m^2*n^3*
q^2*x*x^q*e^q*log(d)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^3*n^3*q*x*x
^q*e^q*log(x)^3/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) - 6*b^3*m^3*n^3*q*x*x^q*e^q
*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*b^3*m^2*n^2*q^2*x*x^q*e^q*log
(c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^3*q*x*x^q*e^q*log(d)*log
(x)^2/(q^3 + 3*q^2 + 3*q + 1) + b^3*m^3*n^3*x*x^q*e^q*log(x)^3/(q^4 + 4*q^3
 + 6*q^2 + 4*q + 1) + 6*b^3*m^3*n^3*q*x*x^q*e^q*log(x)/(q^4 + 4*q^3 + 6*q^2
 + 4*q + 1) - 6*b^3*m^2*n^3*q*x*x^q*e^q*log(d)*log(x)/(q^3 + 3*q^2 + 3*q +
1) + 3*b^3*m*n^3*q*x*x^q*e^q*log(d)^2*log(x)/(q^2 + 2*q + 1) - 3*b^3*m^3*n^
3*x*x^q*e^q*log(x)^2/(q^4 + 4*q^3 + 6*q^2 + 4*q + 1) + 3*a*b^2*m^2*n^2*q^2*
x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^2*q*x*x^q*e^q*log(
c)*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^2*n^3*x*x^q*e^q*log(d)*log(x)
^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^3*n^3*x*x^q*e^q*log(x)/(q^4 + 4*q^3 +
6*q^2 + 4*q + 1) - 6*b^3*m^2*n^2*q*x*x^q*e^q*log(c)*log(x)/(q^3 + 3*q^2 + 3
*q + 1) - 6*b^3*m^2*n^3*x*x^q*e^q*log(d)*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 6
*b^3*m*n^2*q*x*x^q*e^q*log(c)*log(d)*log(x)/(q^2 + 2*q + 1) + 3*b^3*m*n^3*x
*x^q*e^q*log(d)^2*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m^2*n^2*q*x*x^q*e^q*log(
x)^2/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m^2*n^2*x*x^q*e^q*log(c)*log(x)^2/(q^3
 + 3*q^2 + 3*q + 1) - 6*b^3*m^3*n^3*x*x^q*e^q/(q^4 + 4*q^3 + 6*q^2 + 4*q +
```

```
1) + 6*b^3*m^2*n^3*x*x^q*e^q*log(d)/(q^3 + 3*q^2 + 3*q + 1) - 3*b^3*m*n^3*x
*x^q*e^q*log(d)^2/(q^2 + 2*q + 1) + b^3*n^3*x*x^q*e^q*log(d)^3/(q + 1) - 6*
a*b^2*m^2*n^2*q*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) - 6*b^3*m^2*n^2*x*
x^q*e^q*log(c)*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 3*b^3*m*n*q*x*x^q*e^q*log(c
)^2*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m*n^2*q*x*x^q*e^q*log(d)*log(x)/(q^2 +
 2*q + 1) + 6*b^3*m*n^2*x*x^q*e^q*log(c)*log(d)*log(x)/(q^2 + 2*q + 1) + 3*
a*b^2*m^2*n^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 6*b^3*m^2*n^2*x*
x^q*e^q*log(c)/(q^3 + 3*q^2 + 3*q + 1) - 6*b^3*m*n^2*x*x^q*e^q*log(c)*log(d
)/(q^2 + 2*q + 1) + 3*b^3*n^2*x*x^q*e^q*log(c)*log(d)^2/(q + 1) - 6*a*b^2*m
^2*n^2*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 6*a*b^2*m*n*q*x*x^q*e^q*l
og(c)*log(x)/(q^2 + 2*q + 1) + 3*b^3*m*n*x*x^q*e^q*log(c)^2*log(x)/(q^2 + 2
*q + 1) + 6*a*b^2*m*n^2*x*x^q*e^q*log(d)*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m
^2*n^2*x*x^q*e^q/(q^3 + 3*q^2 + 3*q + 1) - 3*b^3*m*n*x*x^q*e^q*log(c)^2/(q^
2 + 2*q + 1) - 6*a*b^2*m*n^2*x*x^q*e^q*log(d)/(q^2 + 2*q + 1) + 3*b^3*n*x*x
^q*e^q*log(c)^2*log(d)/(q + 1) + 3*a*b^2*n^2*x*x^q*e^q*log(d)^2/(q + 1) + 3
*a^2*b*m*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + 6*a*b^2*m*n*x*x^q*e^q*log(c
)*log(x)/(q^2 + 2*q + 1) - 6*a*b^2*m*n*x*x^q*e^q*log(c)/(q^2 + 2*q + 1) + b
^3*x*x^q*e^q*log(c)^3/(q + 1) + 6*a*b^2*n*x*x^q*e^q*log(c)*log(d)/(q + 1) +
 3*a^2*b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) - 3*a^2*b*m*n*x*x^q*e^q/(q^2
+ 2*q + 1) + 3*a*b^2*x*x^q*e^q*log(c)^2/(q + 1) + 3*a^2*b*n*x*x^q*e^q*log(d
)/(q + 1) + 3*a^2*b*x*x^q*e^q*log(c)/(q + 1) + a^3*x*x^q*e^q/(q + 1)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^q\,\left(a+b\,\ln\left(c\,\left(d\,x^m\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3,x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^3, x)
```

3.238 $\int (ex)^q \left(a+b\log\left(c(dx^m)^n\right)\right)^2 \, dx$

Optimal. Leaf size=93

$$\frac{2b^2m^2n^2(ex)^{1+q}}{e(1+q)^3}-\frac{2bmn(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)}{e(1+q)^2}+\frac{(ex)^{1+q}\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{e(1+q)}$$

```
[Out] 2*b^2*m^2*n^2*(e*x)^(1+q)/e/(1+q)^3-2*b*m*n*(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n
))/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))^2/e/(1+q)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2342, 2341, 2495}

$$\frac{(ex)^{q+1}\left(a+b\log\left(c(dx^m)^n\right)\right)^2}{e(q+1)}-\frac{2bmn(ex)^{q+1}\left(a+b\log\left(c(dx^m)^n\right)\right)}{e(q+1)^2}+\frac{2b^2m^2n^2(ex)^{q+1}}{e(q+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] (2*b^2*m^2*n^2*(e*x)^(1 + q))/(e*(1 + q)^3) - (2*b*m*n*(e*x)^(1 + q)*(a + b
*Log[c*(d*x^m)^n]))/(e*(1 + q)^2) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m)^n])
^2)/(e*(1 + q))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int (ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^2\,dx &= \text{Subst}\left(\int (ex)^q \left(a+b\log \left(cd^n x^{mn}\right)\right)^2\,dx, cd^n x^{mn}, c(dx^m)^n\right)\\ &= \frac{(ex)^{1+q}\left(a+b\log \left(c(dx^m)^n\right)\right)^2}{e(1+q)} - \text{Subst}\left(\frac{(2bmn)\int (ex)^q \left(a+b\log \left(cd^n x\right.\right.}{1+q}\right.\\ &= \frac{2b^2m^2n^2(ex)^{1+q}}{e(1+q)^3} - \frac{2bmn(ex)^{1+q}\left(a+b\log \left(c(dx^m)^n\right)\right)}{e(1+q)^2} + \frac{(ex)^{1+q}\left(a+b\right.}{e(1\ldots}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 90, normalized size = 0.97

$$\frac{x(ex)^q \left(a+b\log \left(c(dx^m)^n\right)\right)^2}{1+q} - \frac{2bmnx^{-q}(ex)^q \left(-\frac{bmnx^{1+q}}{(1+q)^2}+\frac{x^{1+q}\left(a+b\log \left(c(dx^m)^n\right)\right)}{1+q}\right)}{1+q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] (x*(e*x)^q*(a + b*Log[c*(d*x^m)^n])^2)/(1 + q) - (2*b*m*n*(e*x)^q*(-((b*m*n
*x^(1 + q))/(1 + q)^2) + (x^(1 + q)*(a + b*Log[c*(d*x^m)^n]))/(1 + q)))/((1
 + q)*x^q)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^q \left(a+b\ln \left(c(d\,x^m)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^2,x)
```

```
[Out] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^2,x)
```

Maxima [**A**]
time = 0.28, size = 152, normalized size = 1.63

$$\frac{(xe)^{q+1}b^2e^{(-1)}\log \left((dx^m)^n c\right)^2}{q+1} - \frac{2\,abmnxe^{(q\log(x)+q)}}{(q+1)^2} + \frac{2\,(xe)^{q+1}abe^{(-1)}\log \left((dx^m)^n c\right)}{q+1} + 2\left(\frac{m^2n^2xe^{(q\log(x)+q)}}{(q+1)^3} - \frac{mnxe^{(q\log(x)+q)}\log \left((dx^m)^n c\right)}{(q+1)^2}\right)b^2 + \frac{(xe)^{q+1}a^2e^{(-1)}}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

```
[Out] (x*e)^(q + 1)*b^2*e^(-1)*log((d*x^m)^n*c)^2/(q + 1) - 2*a*b*m*n*x*e^(q*log(
x) + q)/(q + 1)^2 + 2*(x*e)^(q + 1)*a*b*e^(-1)*log((d*x^m)^n*c)/(q + 1) + 2
```

*(m^2*n^2*x*e^(q*log(x) + q)/(q + 1)^3 - m*n*x*e^(q*log(x) + q)*log((d*x^m)
^n*c)/(q + 1)^2)*b^2 + (x*e)^(q + 1)*a^2*e^(-1)/(q + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(93) = 186.
time = 0.39, size = 388, normalized size = 4.17

$$\frac{\left((b^2q^2+2b^2q+b^2)x\log(c)^2+(b^2n^2q^2+2b^2n^2q+b^2n^2)x\log(d)^2+(b^2m^2n^2q^2+2b^2m^2n^2q+b^2m^2n^2)x\log(x)^2-2(b^2mn-abq^2-ab+(b^2mn-2ab)q)x\log(c)+(2b^2m^2n^2-2abmn+a^2q^2+a^2-2(abmn-a^2)q)x+2((b^2nq^2+2b^2nq+b^2n)x\log(c)-(b^2mn^2-abnq^2-abn+(b^2mn^2-2abn)q)x)\log(d)+2((b^2mnq^2+2b^2mnq+b^2mn)x\log(c)+(b^2mn^2q^2+2b^2mn^2q+b^2mn^2)x\log(d)-(b^2m^2n^2-abmnq^2-abmn+(b^2m^2n^2-2abmn)q)x)\log(x)\right)e^{(q\log(x)+q)}}{q^3+3q^2+3q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")

[Out] ((b^2*q^2 + 2*b^2*q + b^2)*x*log(c)^2 + (b^2*n^2*q^2 + 2*b^2*n^2*q + b^2*n^
2)*x*log(d)^2 + (b^2*m^2*n^2*q^2 + 2*b^2*m^2*n^2*q + b^2*m^2*n^2)*x*log(x)^
2 - 2*(b^2*m*n - a*b*q^2 - a*b + (b^2*m*n - 2*a*b)*q)*x*log(c) + (2*b^2*m^2
*n^2 - 2*a*b*m*n + a^2*q^2 + a^2 - 2*(a*b*m*n - a^2)*q)*x + 2*((b^2*n*q^2 +
 2*b^2*n*q + b^2*n)*x*log(c) - (b^2*m*n^2 - a*b*n*q^2 - a*b*n + (b^2*m*n^2
- 2*a*b*n)*q)*x)*log(d) + 2*((b^2*m*n*q^2 + 2*b^2*m*n*q + b^2*m*n)*x*log(c)
 + (b^2*m*n^2*q^2 + 2*b^2*m*n^2*q + b^2*m*n^2)*x*log(d) - (b^2*m^2*n^2 - a*
b*m*n*q^2 - a*b*m*n + (b^2*m^2*n^2 - 2*a*b*m*n)*q)*x)*log(x))*e^(q*log(x) +
 q)/(q^3 + 3*q^2 + 3*q + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^q \left(a + b\log\left(c(dx^m)^n\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**2,x)

[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(93) = 186.
time = 3.12, size = 561, normalized size = 6.03

$$\frac{b^2m^2n^2q^2xx^qe^q\log(x)^2}{q^3+3q^2+3q+1}+\frac{2b^2m^2n^2qxx^qe^q\log(x)^2}{q^3+3q^2+3q+1}-\frac{2b^2m^2n^2qxx^qe^q\log(x)}{q^3+3q^2+3q+1}+\frac{2b^2mn^2qxx^qe^q\log(d)\log(x)}{q^2+2q+1}+\frac{b^2m^2n^2xx^qe^q\log(x)^2}{q^3+3q^2+3q+1}-\frac{2b^2m^2n^2xx^qe^q\log(x)}{q^3+3q^2+3q+1}+\frac{2b^2mnqxx^qe^q\log(c)\log(x)}{q^2+2q+1}+\frac{2b^2mn^2xx^qe^q\log(d)\log(x)}{q^2+2q+1}+\frac{2b^2m^2n^2xx^qe^q}{q^3+3q^2+3q+1}-\frac{2b^2mn^2xx^qe^q\log(d)}{q^2+2q+1}+\frac{b^2n^2xx^qe^q\log(d)^2}{q+1}+\frac{2abmnqxx^qe^q\log(x)}{q^2+2q+1}+\frac{2b^2mnxx^qe^q\log(c)\log(x)}{q^2+2q+1}-\frac{2b^2mnxx^qe^q\log(c)}{q^2+2q+1}+\frac{2b^2nxx^qe^q\log(c)\log(d)}{q+1}+\frac{2abmnxx^qe^q\log(x)}{q^2+2q+1}-\frac{2abmnxx^qe^q}{q^2+2q+1}+\frac{b^2xx^qe^q\log(c)^2}{q+1}+\frac{2abnxx^qe^q\log(d)}{q+1}+\frac{2abxx^qe^q\log(c)}{q+1}+\frac{a^2xx^qe^q}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")

[Out] b^2*m^2*n^2*q^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m^2*n^2*
q*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*m^2*n^2*q*x*x^q*e^q*lo
g(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m*n^2*q*x*x^q*e^q*log(d)*log(x)/(q^2 +
 2*q + 1) + b^2*m^2*n^2*x*x^q*e^q*log(x)^2/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*

```
m^2*n^2*x*x^q*e^q*log(x)/(q^3 + 3*q^2 + 3*q + 1) + 2*b^2*m*n*q*x*x^q*e^q*lo
g(c)*log(x)/(q^2 + 2*q + 1) + 2*b^2*m*n^2*x*x^q*e^q*log(d)*log(x)/(q^2 + 2*
q + 1) + 2*b^2*m^2*n^2*x*x^q*e^q/(q^3 + 3*q^2 + 3*q + 1) - 2*b^2*m*n^2*x*x^
q*e^q*log(d)/(q^2 + 2*q + 1) + b^2*n^2*x*x^q*e^q*log(d)^2/(q + 1) + 2*a*b*m
*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + 2*b^2*m*n*x*x^q*e^q*log(c)*log(x)/(
q^2 + 2*q + 1) - 2*b^2*m*n*x*x^q*e^q*log(c)/(q^2 + 2*q + 1) + 2*b^2*n*x*x^q
*e^q*log(c)*log(d)/(q + 1) + 2*a*b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) - 2
*a*b*m*n*x*x^q*e^q/(q^2 + 2*q + 1) + b^2*x*x^q*e^q*log(c)^2/(q + 1) + 2*a*b
*n*x*x^q*e^q*log(d)/(q + 1) + 2*a*b*x*x^q*e^q*log(c)/(q + 1) + a^2*x*x^q*e^
q/(q + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (e\,x\right)}^q\,{\left (a+b\,\ln \left (c\,{\left (d\,x^m\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2,x)
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^2, x)
```

3.239 $\int (ex)^q (a+b\log(c(dx^m)^n))\,dx$

Optimal. Leaf size=51

$$-\frac{bmn(ex)^{1+q}}{e(1+q)^2}+\frac{(ex)^{1+q}(a+b\log(c(dx^m)^n))}{e(1+q)}$$

```
[Out] -b*m*n*(e*x)^(1+q)/e/(1+q)^2+(e*x)^(1+q)*(a+b*ln(c*(d*x^m)^n))/e/(1+q)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2341, 2495}

$$\frac{(ex)^{q+1}(a+b\log(c(dx^m)^n))}{e(q+1)}-\frac{bmn(ex)^{q+1}}{e(q+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] -((b*m*n*(e*x)^(1 + q))/(e*(1 + q)^2)) + ((e*x)^(1 + q)*(a + b*Log[c*(d*x^m
)^n]))/(e*(1 + q))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\begin{aligned}\int (ex)^q (a+b\log(c(dx^m)^n))\,dx &= \text{Subst}\left(\int (ex)^q (a+b\log(cd^nx^{mn}))\,dx, cd^nx^{mn}, c(dx^m)^n\right)\\ &= -\frac{bmn(ex)^{1+q}}{e(1+q)^2}+\frac{(ex)^{1+q}(a+b\log(c(dx^m)^n))}{e(1+q)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.73

$$\frac{x(ex)^q (a - bmn + aq + b(1+q)\log(c(dx^m)^n))}{(1+q)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n]),x]
[Out] (x*(e*x)^q*(a - b*m*n + a*q + b*(1 + q)*Log[c*(d*x^m)^n]))/(1 + q)^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^q \left(a + b\ln\left(c\left(d\,x^m\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a+b*ln(c*(d*x^m)^n)),x)
[Out] int((e*x)^q*(a+b*ln(c*(d*x^m)^n)),x)
```

Maxima [A]
time = 0.28, size = 63, normalized size = 1.24

$$-\frac{bmnxe^{(q\log(x)+q)}}{(q+1)^2} + \frac{(xe)^{q+1}\,be^{(-1)}\log\left((dx^m)^n\,c\right)}{q+1} + \frac{(xe)^{q+1}\,ae^{(-1)}}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
[Out] -b*m*n*x*e^(q*log(x) + q)/(q + 1)^2 + (x*e)^(q + 1)*b*e^(-1)*log((d*x^m)^n*
c)/(q + 1) + (x*e)^(q + 1)*a*e^(-1)/(q + 1)
```

Fricas [A]
time = 0.37, size = 69, normalized size = 1.35

$$\frac{((bq+b)x\log(c) + (bnq+bn)x\log(d) + (bmnq+bmn)x\log(x) - (bmn-aq-a)x)e^{(q\log(x)+q)}}{q^2+2\,q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
[Out] ((b*q + b)*x*log(c) + (b*n*q + b*n)*x*log(d) + (b*m*n*q + b*m*n)*x*log(x) -
 (b*m*n - a*q - a)*x)*e^(q*log(x) + q)/(q^2 + 2*q + 1)
```

Sympy [A]
time = 4.27, size = 110, normalized size = 2.16

$$a\left(\begin{cases} 0^q x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(ex)^{q+1}}{q+1} & \text{for } q\neq -1 \\ \log(ex) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right) - bmn\left(\begin{cases} 0^q x & \text{for } (e=0 \wedge q\neq -1) \vee e=0 \\ \frac{\begin{cases} \frac{ex(ex)^q}{q+1} & \text{for } q\neq -1 \\ \log(x) & \text{otherwise} \end{cases}}{eq+e} & \text{for } q>-\infty \wedge q<\infty \wedge q\neq -1 \\ \frac{\log(ex)^2}{2e} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} 0^q x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(ex)^{q+1}}{q+1} & \text{for } q\neq -1 \\ \log(ex) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right)\log\left(c(dx^m)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n)),x)
```

```
[Out] a*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(q + 1), Ne(q, -
1)), (log(e*x), True))/e, True)) - b*m*n*Piecewise((0**q*x, Eq(e, 0) | (Eq(
e, 0) & Ne(q, -1))), (Piecewise((e*x*(e*x)**q/(q + 1), Ne(q, -1)), (log(x),
 True))/(e*q + e), (q > -oo) & (q < oo) & Ne(q, -1)), (log(e*x)**2/(2*e), T
rue)) + b*Piecewise((0**q*x, Eq(e, 0)), (Piecewise(((e*x)**(q + 1)/(q + 1),
 Ne(q, -1)), (log(e*x), True))/e, True))*log(c*(d*x**m)**n)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(51) = 102.
time = 4.96, size = 111, normalized size = 2.18

$$\frac{bmnqxx^q e^q \log(x)}{q^2+2q+1} + \frac{bmnxx^q e^q \log(x)}{q^2+2q+1} - \frac{bmnxx^q e^q}{q^2+2q+1} + \frac{bnxx^q e^q \log(d)}{q+1} + \frac{bxx^q e^q \log(c)}{q+1} + \frac{axx^q e^q}{q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

```
[Out] b*m*n*q*x*x^q*e^q*log(x)/(q^2 + 2*q + 1) + b*m*n*x*x^q*e^q*log(x)/(q^2 + 2*
q + 1) - b*m*n*x*x^q*e^q/(q^2 + 2*q + 1) + b*n*x*x^q*e^q*log(d)/(q + 1) + b
*x*x^q*e^q*log(c)/(q + 1) + a*x*x^q*e^q/(q + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (e\,x)^q\,\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n)),x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n)), x)
```

3.240 $\int \frac{(ex)^q}{a+b\log(c(dx^m)^n)}\,dx$

Optimal. Leaf size=86

$$\frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\operatorname{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bemn}$$

```
[Out] (e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b/e/exp(a*(1+q)/b/m/n)/m/
n/((c*(d*x^m)^n)^((1+q)/m/n))
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2347, 2209, 2495}

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c(dx^m)^n\right)^{-\frac{q+1}{mn}}\operatorname{Ei}\left(\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bemn}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
```

```
[Out] ((e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/(
b*e*E^((a*(1 + q))/(b*m*n))*m*n*(c*(d*x^m)^n)^((1 + q)/(m*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx = \text{Subst}\left(\int \frac{(ex)^q}{a+b\log\left(cd^nx^{mn}\right)}\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left((ex)^{1+q}\left(cd^nx^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+q)x}{mn}}}{a+bx}\,dx, x, \log\left(cd^nx^{mn}\right)\right)}{emn}, cd^nx^{mn}, c$$

$$= \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\,\text{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bemn}$$

Mathematica [A]
time = 0.10, size = 85, normalized size = 0.99

$$\frac{e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}x^{-q}(ex)^q\text{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{bmn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n]),x]
[Out] ((e*x)^q*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)])/(b*E^((
(1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*m*n*x^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{a+b\ln\left(c\left(d\,x^m\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a+b*ln(c*(d*x^m)^n)),x)
[Out] int((e*x)^q/(a+b*ln(c*(d*x^m)^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="maxima")
[Out] integrate((x*e)^q/(b*log((d*x^m)^n*c) + a), x)
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.21

$$\frac{\mathrm{Ei}\left(\frac{aq+(bq+b)\log(c)+(bnq+bn)\log(d)+(bmnq+bmn)\log(x)+a}{bmn}\right)e^{\left(\frac{(bmn-a)q-(bq+b)\log(c)-(bnq+bn)\log(d)-a}{bmn}\right)}}{bmn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="fricas")
```

```
[Out] Ei((a*q + (b*q + b)*log(c) + (b*n*q + b*n)*log(d) + (b*m*n*q + b*m*n)*log(x
) + a)/(b*m*n))*e^(((b*m*n - a)*q - (b*q + b)*log(c) - (b*n*q + b*n)*log(d)
 - a)/(b*m*n))/(b*m*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{a+b\log\left(c\left(dx^m\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n)),x)
```

```
[Out] Integral((e*x)**q/(a + b*log(c*(d*x**m)**n)), x)
```

Giac [A]
time = 3.63, size = 140, normalized size = 1.63

$$\frac{\mathrm{Ei}\left(q\log\left(x\right)+\frac{q\log(d)}{m}+\frac{q\log(c)}{mn}+\frac{\log(d)}{m}+\frac{aq}{bmn}+\frac{\log(c)}{mn}+\frac{a}{bmn}+\log\left(x\right)\right)e^{\left(q-\frac{aq}{bmn}-\frac{a}{bmn}\right)}}{bc^{\frac{q}{mn}}c^{\frac{1}{mn}}d^{\frac{q}{m}}d^{\left(\frac{1}{m}\right)}mn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n)),x, algorithm="giac")
```

```
[Out] Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)
/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))/(b*c^(q/(m*n))
*c^(1/(m*n))*d^(q/m)*d^(1/m)*m*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x\right)}^q}{a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a + b*log(c*(d*x^m)^n)),x)
```

```
[Out] int((e*x)^q/(a + b*log(c*(d*x^m)^n)), x)
```

3.241 $\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx$

Optimal. Leaf size=127

$$\frac{e^{-\frac{a(1+q)}{bmn}}(1+q)(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\operatorname{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{b^2em^2n^2}-\frac{(ex)^{1+q}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}$$

```
[Out] (1+q)*(e*x)^(1+q)*Ei((1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)/b^2/e/exp(a*(1+q)/b
/m/n)/m^2/n^2/((c*(d*x^m)^n)^((1+q)/m/n))-(e*x)^(1+q)/b/e/m/n/(a+b*ln(c*(d*
x^m)^n))
```

Rubi [A]
time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2343, 2347, 2209, 2495}

$$\frac{(q+1)(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c(dx^m)^n\right)^{-\frac{q+1}{mn}}\operatorname{Ei}\left(\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)}{b^2em^2n^2}-\frac{(ex)^{q+1}}{bemn\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] ((1 + q)*(e*x)^(1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*
m*n)])/(b^2*e*E^((a*(1 + q))/(b*m*n))*m^2*n^2*(c*(d*x^m)^n)^((1 + q)/(m*n))
) - (e*x)^(1 + q)/(b*e*m*n*(a + b*Log[c*(d*x^m)^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(ex)^q}{(a+b\log(c(dx^m)^n))^2}\,dx = \text{Subst}\left(\int \frac{(ex)^q}{(a+b\log(cd^nx^{mn}))^2}\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$
$$= -\frac{(ex)^{1+q}}{bemn\,(a+b\log(c(dx^m)^n))} + \text{Subst}\left(\frac{(1+q)\int \frac{(ex)^q}{a+b\log(cd^nx^{mn})}\,dx}{bmn}, cd^nx^{mn},\right.$$
$$= -\frac{(ex)^{1+q}}{bemn\,(a+b\log(c(dx^m)^n))} + \text{Subst}\left(\frac{\left((1+q)(ex)^{1+q}\left(cd^nx^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Sub}}{bem^2}\right.$$
$$= \frac{e^{-\frac{a(1+q)}{bmn}}(1+q)(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\text{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{b^2em^2n^2} - \frac{(}{bemn\,(a+}$$

Mathematica [A]
time = 0.19, size = 112, normalized size = 0.88

$$\frac{(ex)^q\left(e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}(1+q)x^{-q}\text{Ei}\left(\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)-\frac{bmnx}{a+b\log(c(dx^m)^n)}\right)}{b^2m^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q/(a + b*Log[c*(d*x^m)^n])^2,x]
```

```
[Out] ((e*x)^q*(((1 + q)*ExpIntegralEi[((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)
])/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))/(b*m*n))*x^q) - (b
*m*n*x)/(a + b*Log[c*(d*x^m)^n])))/(b^2*m^2*n^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{(a+b\ln(c(d\,x^m)^n))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a+b*ln(c*(d*x^m)^n))^2,x)
```

```
[Out] int((e*x)^q/(a+b*ln(c*(d*x^m)^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="maxima")
```

```
[Out] (q*e^q + e^q)*integrate(x^q/(b^2*m*n*log((x^m)^n) + a*b*m*n + (m*n^2*log(d)
 + m*n*log(c))*b^2), x) - x*e^(q*log(x) + q)/(b^2*m*n*log((x^m)^n) + a*b*m*
n + (m*n^2*log(d) + m*n*log(c))*b^2)
```

Fricas [A]
time = 0.38, size = 198, normalized size = 1.56

$$-\frac{bmnxe^{(q\log(x)+q)} - (aq + (bq+b)\log(c) + (bnq+bn)\log(d) + (bmnq+bmn)\log(x) + a)\mathrm{Ei}\left(\frac{aq+(bq+b)\log(c)+(bnq+bn)\log(d)+(bmnq+bmn)\log(x)+a}{bmn}\right)e^{\left(\frac{(bmn-a)q-(bq+b)\log(c)-(bnq+bn)\log(d)-a}{bmn}\right)}}{b^3m^3n^3\log(x) + b^3m^2n^3\log(d) + b^3m^2n^2\log(c) + ab^2m^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="fricas")
```

```
[Out] -(b*m*n*x*e^(q*log(x) + q) - (a*q + (b*q + b)*log(c) + (b*n*q + b*n)*log(d)
 + (b*m*n*q + b*m*n)*log(x) + a)*Ei((a*q + (b*q + b)*log(c) + (b*n*q + b*n)
*log(d) + (b*m*n*q + b*m*n)*log(x) + a)/(b*m*n))*e^(((b*m*n - a)*q - (b*q +
 b)*log(c) - (b*n*q + b*n)*log(d) - a)/(b*m*n)))/(b^3*m^3*n^3*log(x) + b^3*
m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^q}{\left(a + b\log\left(c\left(dx^m\right)^n\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q/(a+b*ln(c*(d*x**m)**n))**2,x)
```

```
[Out] Integral((e*x)**q/(a + b*log(c*(d*x**m)**n))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. 2(127) = 254.
time = 4.49, size = 1540, normalized size = 12.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q/(a+b*log(c*(d*x^m)^n))^2,x, algorithm="giac")
```

```
[Out] -b*m*n*x*x^q*e^q/(b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log
(c) + a*b^2*m^2*n^2) + b*m*n*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) +
log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m
*n) - a/(b*m*n))*log(x)/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2
*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*n
*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log
(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(d)/((b^
3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)
*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*m*n*Ei(q*log(x) + q*log(d)/m
+ q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(
x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(x)/((b^3*m^3*n^3*log(x) + b^3*m^2*n
^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(
q/m)*d^(1/m)) + b*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m +
a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*
m*n))*log(c)/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c)
 + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*n*Ei(q*log(x
) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a
/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(d)/((b^3*m^3*n^3*log
(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*
c^(1/(m*n))*d^(q/m)*d^(1/m)) + a*q*Ei(q*log(x) + q*log(d)/m + q*log(c)/(m*n
) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q - a*q/
(b*m*n) - a/(b*m*n))/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^
2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + b*Ei(q
*log(x) + q*log(d)/m + q*log(c)/(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*
n) + a/(b*m*n) + log(x))*e^(q - a*q/(b*m*n) - a/(b*m*n))*log(c)/((b^3*m^3*n
^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(
m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m)) + a*Ei(q*log(x) + q*log(d)/m + q*log(c)/
(m*n) + log(d)/m + a*q/(b*m*n) + log(c)/(m*n) + a/(b*m*n) + log(x))*e^(q -
a*q/(b*m*n) - a/(b*m*n))/((b^3*m^3*n^3*log(x) + b^3*m^2*n^3*log(d) + b^3*m^
2*n^2*log(c) + a*b^2*m^2*n^2)*c^(q/(m*n))*c^(1/(m*n))*d^(q/m)*d^(1/m))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^q}{{\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2,x)
```

```
[Out] int((e*x)^q/(a + b*log(c*(d*x^m)^n))^2, x)
```

3.242 $\int (ex)^q (a+b\log (c(dx^m)^n))^p \, dx$

Optimal. Leaf size=134

$$\frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\Gamma\left(1+p,-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)}{e(1+q)}$$

```
[Out] (e*x)^(1+q)*GAMMA(1+p,-(1+q)*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)
^n))^p/e/exp(a*(1+q)/b/m/n)/(1+q)/((c*(d*x^m)^n)^((1+q)/m/n))/((-(1+q)*(a+b
*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2347, 2212, 2495}

$$\frac{(ex)^{q+1}e^{-\frac{a(q+1)}{bmn}}\left(c(dx^m)^n\right)^{-\frac{q+1}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{(q+1)(a+b\log(c(dx^m)^n))}{bmn}\right)}{e(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] ((e*x)^(1 + q)*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]*
(a + b*Log[c*(d*x^m)^n])^p)/(e*E^((a*(1 + q))/(b*m*n))*(1 + q)*(c*(d*x^m)^n
)^((1 + q)/(m*n))*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (ex)^q\left(a+b\log\left(c(dx^m)^n\right)\right)^p\,dx = \text{Subst}\left(\int (ex)^q\left(a+b\log\left(cd^nx^{mn}\right)\right)^p\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left((ex)^{1+q}\left(cd^nx^{mn}\right)^{-\frac{1+q}{mn}}\right)\text{Subst}\left(\int e^{\frac{(1+q)x}{mn}}(a+bx)^p\,dx, x, \log\left(cd\right.\right.}{emn}\right.$$

$$= \frac{e^{-\frac{a(1+q)}{bmn}}(ex)^{1+q}\left(c(dx^m)^n\right)^{-\frac{1+q}{mn}}\Gamma\left(1+p, -\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log}{e(1+q)}$$

Mathematica [A]
time = 0.17, size = 133, normalized size = 0.99

$$\frac{e^{-\frac{(1+q)(a-bmn\log(x)+b\log(c(dx^m)^n))}{bmn}}x^{-q}(ex)^q\Gamma\left(1+p,-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)^p\left(-\frac{(1+q)(a+b\log(c(dx^m)^n))}{bmn}\right)^{-p}}{1+q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^q*(a + b*Log[c*(d*x^m)^n])^p,x]
[Out] ((e*x)^q*Gamma[1 + p, -(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n))]*(a + b
*Log[c*(d*x^m)^n])^p)/(E^(((1 + q)*(a - b*m*n*Log[x] + b*Log[c*(d*x^m)^n]))
/(b*m*n))*(1 + q)*x^q*(-(((1 + q)*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)))^p)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (ex)^q\left(a+b\ln\left(c(d\,x^m)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^p,x)
[Out] int((e*x)^q*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^q (a + b \log (c(dx^m)^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**q*(a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral((e*x)**q*(a + b*log(c*(d*x**m)**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^q*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((x*e)^q*(b*log((d*x^m)^n*c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (e\,x\right)}^q\,{\left (a+b\,\ln \left (c\,{\left (d\,x^m\right)}^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int((e*x)^q*(a + b*log(c*(d*x^m)^n))^p, x)
```

3.243 $\int x^2(a+b\log(c(dx^m)^n))^p\,dx$

Optimal. Leaf size=117

$$3^{-1-p}e^{-\frac{3a}{bmn}}x^3(c(dx^m)^n)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)}{bmn}\right.$$

```
[Out] 3^(-1-p)*x^3*GAMMA(1+p,-3*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n)
)^p/exp(3*a/b/m/n)/((c*(d*x^m)^n)^(3/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^
p)
```

Rubi [A]

time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2347, 2212, 2495}

$$3^{-p-1}x^3e^{-\frac{3a}{bmn}}(c(dx^m)^n)^{-\frac{3}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int x^2(a+b\log(c(dx^m)^n))^p\,dx = \text{Subst}\left(\int x^2(a+b\log(cd^nx^{mn}))^p\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left(x^3(cd^nx^{mn})^{-\frac{3}{mn}}\right)\text{Subst}\left(\int e^{\frac{3x}{mn}}(a+bx)^p\,dx, x, \log(cd^nx^{mn})\right)}{mn}, cd^nx$$

$$= 3^{-1-p}e^{-\frac{3a}{bmn}}x^3(c(dx^m)^n)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log($$

Mathematica [A]
time = 0.13, size = 117, normalized size = 1.00

$$3^{-1-p}e^{-\frac{3a}{bmn}}x^3(c(dx^m)^n)^{-\frac{3}{mn}}\Gamma\left(1+p,-\frac{3(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((3*a)/(b*m*n))*(c*(d*x^m)^n)^(3/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2(a+b\ln(c(d\,x^m)^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d*x^m)^n))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\log\left(c(dx^m)^n\right))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d*x**m)**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d*x^m)^n))^p, x)
```

3.244 $\int x(a+b\log(c(dx^m)^n))^p\,dx$

Optimal. Leaf size=117

$$2^{-1-p}e^{-\frac{2a}{bmn}}x^2(c(dx^m)^n)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n}{bmn}\right.$$

```
[Out] 2^(-1-p)*x^2*GAMMA(1+p,-2*(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n)
)^p/exp(2*a/b/m/n)/((c*(d*x^m)^n)^(2/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^
p)
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2347, 2212, 2495}

$$2^{-p-1}x^2e^{-\frac{2a}{bmn}}(c(dx^m)^n)^{-\frac{2}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int x(a+b\log(c(dx^m)^n))^p\,dx = \text{Subst}\left(\int x(a+b\log(cd^nx^{mn}))^p\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$
$$= \text{Subst}\left(\frac{\left(x^2(cd^nx^{mn})^{-\frac{2}{mn}}\right)\text{Subst}\left(\int e^{\frac{2x}{mn}}(a+bx)^p\,dx, x, \log(cd^nx^{mn})\right)}{mn}, cd^n$$
$$= 2^{-1-p}e^{-\frac{2a}{bmn}}x^2(c(dx^m)^n)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log($$

Mathematica [A]
time = 0.12, size = 117, normalized size = 1.00

$$2^{-1-p}e^{-\frac{2a}{bmn}}x^2(c(dx^m)^n)^{-\frac{2}{mn}}\Gamma\left(1+p,-\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d*x^m)^n])^p,x]
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*
Log[c*(d*x^m)^n])^p)/(E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*(-((a + b*L
og[c*(d*x^m)^n])/(b*m*n)))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c(d\,x^m)^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d*x^m)^n))^p,x)
[Out] int(x*(a+b*ln(c*(d*x^m)^n))^p,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \log (c(dx^m)^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral(x*(a + b*log(c*(d*x**m)**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \ln \left(c \left(d x^m\right)^n\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d*x^m)^n))^p, x)
```

3.245 $\int (a+b\log (c(dx^m)^n))^p \, dx$

Optimal. Leaf size=108

$$e^{-\frac{a}{bmn}}x(c(dx^m)^n)^{-\frac{1}{mn}}\Gamma\left(1+p,-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}$$

```
[Out] x*GAMMA(1+p,(-a-b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/exp(a/b/m
/n)/((c*(d*x^m)^n)^(1/m/n))/(((-a-b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2337, 2212, 2495}

$$xe^{-\frac{a}{bmn}}(c(dx^m)^n)^{-\frac{1}{mn}}(a+b\log(c(dx^m)^n))^p\left(-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log(c(dx^m)^n)}{bmn}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^n
])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/(b
*m*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int (a+b\log (c(dx^m)^n))^p \, dx = \text{Subst}\left(\int (a+b\log (cd^n x^{mn}))^p \, dx, cd^n x^{mn}, c(dx^m)^n\right)$$

$$= \text{Subst}\left(\frac{\left(x(cd^n x^{mn})^{-\frac{1}{mn}}\right) \text{Subst}\left(\int e^{\frac{x}{mn}}(a+bx)^p \, dx, x, \log (cd^n x^{mn})\right)}{mn}, cd^n x^{mn},\right.$$

$$= e^{-\frac{a}{bmn}} x(c(dx^m)^n)^{-\frac{1}{mn}} \Gamma\left(1+p, -\frac{a+b\log (c(dx^m)^n)}{bmn}\right) (a+b\log (c(dx^m)^n))^p \, ($$

Mathematica [**A**]
time = 0.11, size = 108, normalized size = 1.00

$$e^{-\frac{a}{bmn}} x(c(dx^m)^n)^{-\frac{1}{mn}} \Gamma\left(1+p, -\frac{a+b\log (c(dx^m)^n)}{bmn}\right) (a+b\log (c(dx^m)^n))^p \left(-\frac{a+b\log (c(dx^m)^n)}{bmn}\right)^{-p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p,x]
```

```
[Out] (x*Gamma[1 + p, -((a + b*Log[c*(d*x^m)^n])/(b*m*n))]*(a + b*Log[c*(d*x^m)^n
])^p)/(E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*(-((a + b*Log[c*(d*x^m)^n])/(b
*m*n)))^p)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (a+b\ln (c(d\,x^m)^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d*x^m)^n))^p,x)
```

```
[Out] int((a+b*ln(c*(d*x^m)^n))^p,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.09, size = 73, normalized size = 0.68

$$e^{\left(-\frac{bmnp\log\left(-\frac{1}{bmn}\right)+bn\log(d)+b\log(c)+a}{bmn}\right)}\Gamma\left(p+1,-\frac{bmn\log\left(x\right)+bn\log\left(d\right)+b\log\left(c\right)+a}{bmn}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="fricas")
```

```
[Out] e^(-(b*m*n*p*log(-1/(b*m*n)) + b*n*log(d) + b*log(c) + a)/(b*m*n))*gamma(p
+ 1, -(b*m*n*log(x) + b*n*log(d) + b*log(c) + a)/(b*m*n))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c(dx^m)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p, x)
```

3.246 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx$

Optimal. Leaf size=33

$$\frac{(a+b\log(c(dx^m)^n))^{1+p}}{bmn(1+p)}$$

```
[Out] (a+b*ln(c*(d*x^m)^n))^(1+p)/b/m/n/(1+p)
```

Rubi [A]

time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2339, 30, 2495}

$$\frac{(a+b\log(c(dx^m)^n))^{p+1}}{bmn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x,x]
```

```
[Out] (a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x}\,dx = \text{Subst}\left(\int \frac{(a+b\log(cd^nx^{mn}))^p}{x}\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$
$$= \text{Subst}\left(\frac{\text{Subst}\left(\int x^p\,dx, x, a+b\log(cd^nx^{mn})\right)}{bmn}, cd^nx^{mn}, c(dx^m)^n\right)$$
$$= \frac{(a+b\log(c(dx^m)^n))^{1+p}}{bmn(1+p)}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{(a+b\log(c(dx^m)^n))^{1+p}}{bmn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x,x]
[Out] (a + b*Log[c*(d*x^m)^n])^(1 + p)/(b*m*n*(1 + p))
```

Maple [A]
time = 0.04, size = 34, normalized size = 1.03

method	result	size
derivativedivides	$\frac{(a+b\ln(c(dx^m)^n))^{1+p}}{bmn(1+p)}$	34
default	$\frac{(a+b\ln(c(dx^m)^n))^{1+p}}{bmn(1+p)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d*x^m)^n))^p/x,x,method=_RETURNVERBOSE)
[Out] (a+b*ln(c*(d*x^m)^n))^(1+p)/b/m/n/(1+p)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.00

$$\frac{(b\log((dx^m)^n c)+a)^{p+1}}{bmn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="maxima")
[Out] (b*log((d*x^m)^n*c) + a)^(p + 1)/(b*m*n*(p + 1))
```

Fricas [A]
time = 0.38, size = 49, normalized size = 1.48

$$\frac{(bmn\log(x)+bn\log(d)+b\log(c)+a)(bmn\log(x)+bn\log(d)+b\log(c)+a)^p}{bmnp+bmn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="fricas")
```

```
[Out] (b*m*n*log(x) + b*n*log(d) + b*log(c) + a)*(b*m*n*log(x) + b*n*log(d) + b*l
og(c) + a)^p/(b*m*n*p + b*m*n)
```

Sympy [A]
time = 1.46, size = 80, normalized size = 2.42

$$-\begin{cases} -a^p\log(x) & \text{for } b=0 \\ -(a+b\log(cd^n))^p\log(x) & \text{for } m=0 \\ -(a+b\log(c))^p\log(x) & \text{for } n=0 \\ -\frac{\begin{cases} \frac{(a+b\log(c(dx^m)^n))^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(a+b\log(c(dx^m)^n)) & \text{otherwise} \end{cases}}{bmn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x,x)
```

```
[Out] -Piecewise((-a**p*log(x), Eq(b, 0)), (-(a + b*log(c*d**n))**p*log(x), Eq(m,
 0)), (-(a + b*log(c))**p*log(x), Eq(n, 0)), (-Piecewise(((a + b*log(c*(d*x
**m)**n))**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*log(c*(d*x**m)**n)), Tru
e))/(b*m*n), True))
```

Giac [A]
time = 5.62, size = 36, normalized size = 1.09

$$\frac{(bmn\log(x)+bn\log(d)+b\log(c)+a)^{p+1}}{bmn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x,x, algorithm="giac")
```

```
[Out] (b*m*n*log(x) + b*n*log(d) + b*log(c) + a)^(p + 1)/(b*m*n*(p + 1))
```

Mupad [B]
time = 4.07, size = 33, normalized size = 1.00

$$\frac{(a+b\,\ln(c\,(d\,x^m)^n))^{p+1}}{b\,m\,n\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x,x)
[Out] (a + b*log(c*(d*x^m)^n))^(p + 1)/(b*m*n*(p + 1))
```

3.247 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx$

Optimal. Leaf size=107

$$-\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\Gamma\left(1+p,\frac{a+b\log(c(dx^m)^n)}{bmn}\right)\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x}$$

[Out] -exp(a/b/m/n)*(c*(d*x^m)^n)^(1/m/n)*GAMMA(1+p,(a+b*ln(c*(d*x^m)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/x/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p)

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2347, 2212, 2495}

$$-\frac{e^{\frac{a}{bmn}}\left(c\left(dx^m\right)^n\right)^{\frac{1}{mn}}\left(a+b\log\left(c\left(dx^m\right)^n\right)\right)^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,\frac{a+b\log(c(dx^m)^n)}{bmn}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]

[Out] -((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)^n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*m*n))^p))

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^2}\,dx = \text{Subst}\left(\int \frac{(a+b\log(cd^nx^{mn}))^p}{x^2}\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$
$$= \text{Subst}\left(\frac{(cd^nx^{mn})^{\frac{1}{mn}}\text{Subst}\left(\int e^{-\frac{x}{mn}}(a+bx)^p\,dx, x, \log(cd^nx^{mn})\right)}{mnx}, cd^nx^{mn}, c(\right.$$
$$= -\frac{e^{\frac{a}{bmn}}(c(dx^m)^n)^{\frac{1}{mn}}\Gamma\left(1+p, \frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(}{bmn}\right.}{x}$$

Mathematica [A]
time = 0.12, size = 107, normalized size = 1.00

$$-\frac{e^{\frac{a}{bmn}}(c(dx^m)^n)^{\frac{1}{mn}}\Gamma\left(1+p, \frac{a+b\log(c(dx^m)^n)}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^2,x]
```

```
[Out] -((E^(a/(b*m*n))*(c*(d*x^m)^n)^(1/(m*n))*Gamma[1 + p, (a + b*Log[c*(d*x^m)^
n])/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x*((a + b*Log[c*(d*x^m)^n])/(b*m*
n))^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c(d\,x^m)^n))^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d*x^m)^n))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d*x^m)^n))^p/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log{(c(dx^m)^n)})^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right))^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p/x^2, x)
```

3.248 $\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx$

Optimal. Leaf size=117

$$-\frac{2^{-1-p}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}\,\Gamma\left(1+p,\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log\left(c(dx^m)^n\right))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x^2}$$

```
[Out] -2^(-1-p)*exp(2*a/b/m/n)*(c*(d*x^m)^n)^(2/m/n)*GAMMA(1+p,2*(a+b*ln(c*(d*x^m
)^n))/b/m/n)*(a+b*ln(c*(d*x^m)^n))^p/x^2/(((a+b*ln(c*(d*x^m)^n))/b/m/n)^p)
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2347, 2212, 2495}

$$-\frac{2^{-p-1}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}(a+b\log\left(c(dx^m)^n\right))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}\text{Gamma}\left(p+1,\frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
+ b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*Lo
g[c*(d*x^m)^n])/(b*m*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
```

```
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{(a+b\log(c(dx^m)^n))^p}{x^3}\,dx = \text{Subst}\left(\int \frac{(a+b\log(cd^nx^{mn}))^p}{x^3}\,dx, cd^nx^{mn}, c(dx^m)^n\right)$$

$$= \text{Subst}\left(\frac{(cd^nx^{mn})^{\frac{2}{mn}}\,\text{Subst}\left(\int e^{-\frac{2x}{mn}}(a+bx)^p\,dx, x, \log(cd^nx^{mn})\right)}{mnx^2}, cd^nx^{mn}, c(d$$

$$= -\frac{2^{-1-p}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}\,\Gamma\left(1+p, \frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a\cdot}{}\right.}{x^2}$$

Mathematica [A]
time = 0.12, size = 117, normalized size = 1.00

$$-\frac{2^{-1-p}e^{\frac{2a}{bmn}}(c(dx^m)^n)^{\frac{2}{mn}}\,\Gamma\left(1+p, \frac{2(a+b\log(c(dx^m)^n))}{bmn}\right)(a+b\log(c(dx^m)^n))^p\left(\frac{a+b\log(c(dx^m)^n)}{bmn}\right)^{-p}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])^p/x^3,x]
[Out] -((2^(-1 - p)*E^((2*a)/(b*m*n))*(c*(d*x^m)^n)^(2/(m*n))*Gamma[1 + p, (2*(a
+ b*Log[c*(d*x^m)^n]))/(b*m*n)]*(a + b*Log[c*(d*x^m)^n])^p)/(x^2*((a + b*Lo
g[c*(d*x^m)^n])/(b*m*n))^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\ln(c(d\,x^m)^n))^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d*x^m)^n))^p/x^3,x)
[Out] int((a+b*ln(c*(d*x^m)^n))^p/x^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log (c(dx^m)^n))^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))**p/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d*x**m)**n))**p/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((d*x^m)^n*c) + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, \ln (c\, (d\, x^m)^n))^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d*x^m)^n))^p/x^3, x)
```

3.249 $\int \frac{a+b\log(c(dx^m)^n)}{e+fx^2}\,dx$

Optimal. Leaf size=111

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log\left(c(dx^m)^n\right))}{\sqrt{e}\,\sqrt{f}}-\frac{ibmn\text{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{2\sqrt{e}\,\sqrt{f}}+\frac{ibmn\text{Li}_2\left(\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{2\sqrt{e}\,\sqrt{f}}$$

```
[Out] arctan(x*f^(1/2)/e^(1/2))*(a+b*ln(c*(d*x^m)^n))/e^(1/2)/f^(1/2)-1/2*I*b*m*n
*polylog(2,-I*x*f^(1/2)/e^(1/2))/e^(1/2)/f^(1/2)+1/2*I*b*m*n*polylog(2,I*x*
f^(1/2)/e^(1/2))/e^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {211, 2361, 12, 4940, 2438, 2495}

$$-\frac{ibmn\text{PolyLog}\left(2,-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{2\sqrt{e}\,\sqrt{f}}+\frac{ibmn\text{PolyLog}\left(2,\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{2\sqrt{e}\,\sqrt{f}}+\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)(a+b\log\left(c(dx^m)^n\right))}{\sqrt{e}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[f]*x)/Sqrt[e]]*(a + b*Log[c*(d*x^m)^n]))/(Sqrt[e]*Sqrt[f]) -
((I/2)*b*m*n*PolyLog[2, ((-I)*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]) + ((I/
2)*b*m*n*PolyLog[2, (I*Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2361

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> With[{u = IntHide[1/(d + e*x^2), x]}, Simp[u*(a + b*Log[c*x^n]), x] - Di
st[b*n, Int[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\log\left(c(dx^m)^n\right)}{e+fx^2}\,dx = \text{Subst}\left(\int \frac{a+b\log\left(cd^n x^{mn}\right)}{e+fx^2}\,dx, cd^n x^{mn}, c(dx^m)^n\right)$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\,\sqrt{f}} - \text{Subst}\left((bmn)\int \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{\sqrt{e}\,\sqrt{f}\,x}\,dx, c\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\,\sqrt{f}} - \text{Subst}\left(\frac{(bmn)\int \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)}{x}\,dx}{\sqrt{e}\,\sqrt{f}}, cd^n\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\,\sqrt{f}} - \text{Subst}\left(\frac{(ibmn)\int \frac{\log\left(1-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{x}\,dx}{2\sqrt{e}\,\sqrt{f}}, cd\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(a+b\log\left(c(dx^m)^n\right)\right)}{\sqrt{e}\,\sqrt{f}} - \frac{ibmn\text{Li}_2\left(-\frac{i\sqrt{f}\,x}{\sqrt{e}}\right)}{2\sqrt{e}\,\sqrt{f}} + \frac{ibmn\text{Li}_2\left(\frac{i\sqrt{f}}{\sqrt{e}}\right.}{2\sqrt{e}\,\sqrt{f}}$$

Mathematica [A]

time = 0.06, size = 113, normalized size = 1.02

$$\frac{-\left((a+b\log\left(c(dx^m)^n\right))\left(\log\left(1+\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)-\log\left(1+\frac{e\sqrt{f}\,x}{(-e)^{3/2}}\right)\right)\right)+bmn\text{Li}_2\left(\frac{\sqrt{f}\,x}{\sqrt{-e}}\right)-bmn\text{Li}_2\left(\frac{e\sqrt{f}\,x}{(-e)^{3/2}}\right)}{2\sqrt{-e}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d*x^m)^n])/(e + f*x^2),x]
```

```
[Out] (-((a + b*Log[c*(d*x^m)^n])*(Log[1 + (Sqrt[f]*x)/Sqrt[-e]] - Log[1 + (e*Sqr
t[f]*x)/(-e)^(3/2)])) + b*m*n*PolyLog[2, (Sqrt[f]*x)/Sqrt[-e]] - b*m*n*Poly
Log[2, (e*Sqrt[f]*x)/(-e)^(3/2)])/(2*Sqrt[-e]*Sqrt[f])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c(d\,x^m)^n\right)}{f\,x^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d*x^m)^n))/(f*x^2+e),x)
```

```
[Out] int((a+b*ln(c*(d*x^m)^n))/(f*x^2+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] a*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/sqrt(f) + b*integrate((n*log(d) + log
(c) + log((x^m)^n))/(f*x^2 + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] integral((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c(dx^m)^n\right)}{e + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d*x**m)**n))/(f*x**2+e),x)
[Out] Integral((a + b*log(c*(d*x**m)**n))/(e + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d*x^m)^n))/(f*x^2+e),x, algorithm="giac")
[Out] integrate((b*log((d*x^m)^n*c) + a)/(f*x^2 + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\ln\left(c\,{\left(d\,x^m\right)}^n\right)}{f\,x^2 + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d*x^m)^n))/(e + f*x^2),x)
[Out] int((a + b*log(c*(d*x^m)^n))/(e + f*x^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 1276
4.2 Listing of Grading functions 1276

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```